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LOGIT MODEL AND ITS EXTENSIVE STUDIES IN THE CONTEXT OF CONSUMER THEORY

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Abstract: The formulation and development of the logit model is known to be one of the most widely used probability choice model that is generated based on the framework of random utility theory. Since the standard logit model treats the total demand as given, there remains the problem of dealing with the generated transport demand. Also, for the newly introduced service/facility, the reliability of these services to the user may not be justified by the random theory-based demand model. In the context of consumer behavior theory, this study proposes a different derivation of the logit model which then further extends to the study of the nested logit and the generalized extreme value (GEV) models. In this framework, without loss of generality, the model enables us to analyze the generated demand, and due to the consumer maximization background, the proposed model is more reliable in a certain applications.
Keyword: logit model, quasi-nested logit, multinomial logit, conditional nested logit, generalized extreme value, generated demand, absolute quantities.

1. INTRODUCTION

Application of the logit model has been given increasing attention in the transport behavior analysis due to its simple formulation. However, the application of the logit model in the demand behavior often assumes that the total consumption of goods is fixed exogenously at N. While this assumption is explainable in some context, it is not always completely satisfied. For example, the number of generated demand due to the introduction of new policy/service is quite a significant element in many economic analyses. Also, since the conventional logit demand model is constructed based on the probability model where all the alternatives of the choice set are always assigned a portion of the total demand without any constraints, as far as the reliability of the alternative to the consumer is concerned, the application of the conventional model may not always be credible. For the fulfillment of these problems, this paper will present a different derivation of the logit demand model that has been formulated and discussed in the context of consumer behavior theory. In this formulation, the linkages between the consumer indirect utility function and the demand function are fully utilized to directly derive the general logit demand model from any given indirect utility function through the application of Roy's identity. Further, the proposed model is developed to derive the *quasi-nested logit model* and the conditional generalized extreme value (CGEV) model as well. Finally, the conclusion is added to sum up all the results which we have obtained in the study.

The main model developed, shows that the consumption level of the commodity/service i, i=1,..., n, (for both one dimensional and multidimensional choice situations), is formulated as the product form of two components, the total consumption of all or one group of commodities/ services and the probability (or a nested probability in the multidimensional case) of purchasing commodity/ service i among all or the group of commodities/ services. The model also shows that since the total consumption of a group of commodities/ services can be determined endogenously, it thus enables us to extensively analyze the generated demand. Moreover, the functional form of the proposed model shows that if one specifies the functional form of the indirect utility function the explicit form of the total demand function and the probability models are obtained. By using these functions, the absolute quantities demanded for other commodities can also be acquired through only the data of the transport service.
(for detail, see Morisugi and Le dam hanh (1993)[13]).

Evidently, this approach tracked out a new orientation in deriving the logit model, the quasi-nested logit model and the conditional nested logit and GEV models in the context of continuous behavior theory, beside the random utility theory. In this respect, we found that our paper has an analogy to the approach of Anderson, Palma, and Thisse (1992)[6]. Their study puts emphasis, however, on the functional form of the indirect utility function of the consumer representing a discrete choice model. Based on the assumption that an individual chooses only one of the variants offered, they showed that the logit model can be described to be consistent with a representative consumer model through the specification of the linear random utility model, LRUM. The difference with our model is that the logit model developed by them are independent of income as long as all consumers can afford each variant (see Anderson, Palma, Thisse (1992), pp. 73). Also, the restriction on the fixed value of the total demand still existed in their model. While in this approach we place more emphasis on the functional form of the travel demand as a logit type. Though the proposed framework have reached a certain considerable results, as far as the completion of

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the this model is concerned, the significant of this model is still in question. However, its contribution to the completion of the conventional model in practice is remarkable.

2. SOME IMPLICATIONS OF CONSUMER BEHAVIOR THEORY

Consumer Behavior

In this paper, we study the multinomial logit model by utilizing the microeconomic theory of consumer behavior in depicting the household's decision problem which can be transformed into a demand function. The representative consumer demand function expresses the action of a consumer. It is assumed that the consumer maximizes his/her direct utility under a given budget constraint

$$Max.\, U(X_1, X_2, \ldots, X_n), \quad s.t. \sum_{j=1}^{n} p_j X_j = y \qquad (1)$$

where U(.): direct utility function; X_j: the demand of commodity/service j that is generally assumed to be non-negative continuous variable; p_j: price of commodity/service j; y: income of an individual; j=1,...,n are commodities/services. Then the solution which gives optimal amounts of X_i to the above maximization problem (1) is as follows

$$X^*_i = X_i(p, y),\; i=1,2,\ldots,n. \qquad (2)$$

Equation (2) known as the demand function for the commodity i, describes the choice of consumer with respect to the consumption level of the commodity i for a given price vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and income y. The demand functions which provide an expression for the optimal consumption of commodities /services can now be substituted into the given direct utility function to obtain the maximum utility level that is achievable under the given price vector and income. This is known as the indirect utility function V(**p**,y) which is defined by the following

$$V(p, y) \equiv \qquad (3)$$
$$U(X^*_1, \ldots, X^*_n) = Max.\, [U(X), \sum_{j=1}^{n} p_j X_j = y],$$

where **X** is the consumption vector, $\mathbf{X}=(X_1,\ldots,X_n)$. It is well known that this function has the following four properties for the reasonable consumer theory:

(1) V(**p**,y) is continuous at all **p**>>0,y>0,
(2) V(**p**,y) is monotonically increasing in y, and decreasing in price p,
(3) V(**p**,y) is homogeneous of degree 0 in (**p**,y),
(4) V(**p**,y) is convex in **p**.

Roy's identity

One of the important identities of the indirect utility function which will be used as main access to this research work is Roy's identity. Roy's identity which shows the observed market demand function is expressed by a system of partial derivatives of V(p,y). It is stated that if **X**(**p**,y) is the Marshallian demand function, then

$$X_i(p, y) = -\frac{\partial V(p, y)/\partial p_i}{\partial V(p, y)/\partial y},\; i=1,2,\ldots,n, \qquad (4)$$

provided that the right hand side is well defined and that **p**>>0, y>0, (see Varian (1992), p.106).

In many applications it is convenient to deal with the normalized indirect utility function. The normalized indirect utility function is an indirect utility function where prices are divided by income so that the expenditure is identical to one. From this definition and Roy's law, the demand function can be expressed as the function of the normalized indirect utility function (Varian (1992), p.155)

$$X_i(q) = \frac{\partial V(q)/\partial q_i}{\sum_{j=1}^{n} q_j * \partial V(q)/\partial q_j} \qquad (5)$$

where $q_j = p_j/y$, and j=1,2,...,n. Obviously, Eq.(5) can directly be derived from Eq.(4).

3. A DERIVATION OF LOGIT MODEL

General logit model

In this section, we shall provide a new derivation of the logit model within the framework of the consumer behavior theory which is quite different from the conventional approach of deriving the logit model in the context of discrete choice theory. Following Roy's identity, suppose that given any indirect utility function V(**q**), the resulting demand function consistent with the given indirect utility function is formulated as equation (5). Multiplying the numerator and denominator of (5) by $\sum_{j\in J} \partial V(\mathbf{q})/\partial q_j$, where J is a set of commodities/ services, and rewriting the total demand for commodity i, equation (5) now becomes

$$X_i(q) = N_J(q) * x_{iJ}(q),\; i=1,2,\ldots,n, \qquad (6)$$

where:

- $N_J(\mathbf{q})$ is the total consumption of a given group of commodities/services J, that is explicitly obtained as

$$N_J(q) = \frac{\sum_{j\in J} \partial V(q)/\partial q_j}{\sum_{j=1}^{n} q_j * \partial V(q)/\partial q_j}, \qquad (7)$$

- $x_{iJ}(\mathbf{q})$ denotes the frequency (probability) of purchasing commodity i within the group J. It is formulated in the form of a share model

$$x_{iJ}(q) = \frac{\partial V(q)/\partial q_i}{\sum_{j\in J} \partial V(q)/\partial q_j}, i\in J. \qquad (8)$$

It is clearly recognized that the total demand for commodity i, $X_i(\mathbf{q})$, as shown in Eq. (6) is formulated as the product of the total consumption of a group of commodities/services J, $N_J(\mathbf{q})$ and the share of the commodity i in the total consumption of the focused group of commodities/services J, $x_{ij}(\mathbf{q})$. This share model takes the form of a logit model and satisfies the requirement that $\Sigma_i x_i=1$, $i\in J$. As a result, it shows that with any given indirect utility function, we can derive the logit type model as in equations (6), (7) & (8). Having these properties, it is worth pointing out that the total consumption of a focused group of commodities/ services J, $N_J(\mathbf{q})$, is not an exogenously given variable any more. Rather it is an endogenous variable that changes according to the variation in the related factors that sum all commodities/services.(e.g. the prices of commodities/services, and all relative parameters). Moreover, the function of $N_J(.)$ as formulated in (7) is defined by $N_J(\mathbf{q})= \Sigma_{j\in J} X_j(\mathbf{q})$, Thus, the model can be used to easily and explicitly analyze the generated demand. By that, it has overcome the restriction of the conventional approach.

A specific Indirect Utility Function

In practice, it is necessary to specify the indirect utility functions in all applications. There are a number of different functional forms of V(.) from which to choose, depending on the purposes/requirements of the analysis. The following specific indirect utility function is one of the most significant function which is chosen to verify the proposed model and to further discuss the interpretation of the share model under the framework of this study. Suppose, the indirect utility function is given as

$$V(q) = F[\sum_{j=1}^{n} \int_{q_j}^{\infty} \exp k_j(q_j)\, dq_j]. \qquad (9)$$

Where, F is the increasing monotonous function of $\Sigma_j \int \exp k_j(q_j)dq_j$, and $k_j(.)$ are arbitrary functions such that the equation (9) satisfies the characteristic properties of an indirect utility function, with $q_j=(p_j/y)$. Equation (9) is a separable and additive utility function in relation to a single variable, q_j. Obviously, from (7) & (8) the explicit equations for the total consumption $N_J(\mathbf{q})$, and the choice probability of commodity i, x_{ij}, are easily obtained. Here, we place more emphasis on the study of travel forecasting through the share type of logit model. For more convenience, the discussion is restricted on three goods case, n=1,2,3, where the subscript n=1 denotes the composite goods and n=2,3 are transport modes. Following the proposed framework, the total demand of the transport mode 2 is computed by using formulation (5). The result is given as

$$X_2(q) = \frac{\exp k_2(q_2)}{\sum_{j=1}^{3} q_j \exp k_j(q_j)}. \qquad (10)$$

Once again, the multiplication technique that has been used so far, proves to be significant in this discussion. Here, the only difference is that the multiplied component relates only with the transport modes. By multiplying the nominator and denominator of (10) by $\Sigma^3_{j=2} \exp k_j(q_j)$, the demand for transport mode 2 now becomes

$$X_2(q) = \frac{\sum_{j=2}^{3} \exp k_j(q_j)}{\sum_{j=1}^{3} q_j * \exp k_j(q_j)} * \frac{\exp k_2(q_2)}{\sum_{j=2}^{3} \exp k_j(q_j)}. \qquad (11)$$

Equation (11) is the product of the two components. The first component of RHS of (11) is

$$N(q_1, q_2, q_3) = \frac{\sum_{j=2}^{3} \exp k_j(q_j)}{\sum_{j=1}^{3} q_j * \exp k_j(q_j)}, \qquad (12)$$

which presents the total demand of transport modes that is the function of the prices of the transport goods q_2, q_3, as well as the other commodities q_1. The second component of RHS of (11) is the share of the transport mode 2 in the total transport demand. It is the function of q_2 and q_3 and can be formulated as follows

$$x_2(q_2, q_3) = \frac{\exp k_2(q_2)}{\exp k_2(q_2) + \exp k_3(q_3)}. \qquad (13)$$

By equations (12) & (13), the demand for transport mode 2 is obtained as extension

$$X_2(q_1, q_2, q_3) = N(q_1, q_2, q_3) * x_2(q_2, q_3). \qquad (14)$$

The formulation (14) implies that the demand function can be expressed by the total demand of transport modes N times the share x_2. Though this formula is identified with (8), the difference is with the exponential component of (13), $\exp k_i(q_i)$ which is a separable function in terms of q_i. Moreover, due to the summation component of (12) that takes all commodities/services, i=1,...,n, equation (14) has the possibility to include the related influences of any factors of other commodities into the travel forecasting. This merit provides a comprehensive technique to the transport demand forecasting, and through this, the demand for the composite goods can be also easily computed.

4.MODELS OF MULTIDIMENSIONAL CHOICE AND THE QUASI NESTED LOGIT MODEL

General nested probability model

In the previous section, we have shown a different derivation of the logit model in the context of the consumer behavior theory. In this section, our interest is to show how the proposed model can be extended to the multidimensional choice situation in deriving the general nested probability model, a quasi nested logit model, and to show how the conditional nested logit model can be derived as a special case.

Suppose we have a multidimensional choice set $\mathbf{C}_n$ whose elements are defined as mode and destination combinations. We will denote D_n and M_n as the destination and mode choice sets, respectively which present the set of all destinations and modes of at least one element in the multidimensional choice set $\mathbf{C}_n$. We also denote the conditional destination choice set as D_m, defined as the subset of destinations in D_n feasible for person n if he/she used mode m. Thus M_d will be the conditional mode choice set, defined as subset of modes in M_n which are feasible for person n going to destination d.

Let $V_{dm}(\mathbf{q}_{dm})$ be defined as the indirect utility function of the element in the mode and destination choice set example, where q_{dm} is vector price of all feasible combination of modes and destinations. For convenience, the following presentation of V_{dm} and q_{dm} are simplified by V(.) and **q**, respectively, except for the case where the subscripts are implicit. Using the above notations, the solution to the Roy's identity for the combination of the destination (i) and mode (m) is straightforwardly obtained as

$$X_{im}(q) = \frac{V(q)_i}{\sum_{m \in M_n} (\sum_{j \in D_m} V(q)_j * q_j)},$$
$$\text{where: } V(q)_i = \frac{\partial V_{dm}(q_{dm})}{\partial q_{im}}. \qquad (15)$$

Based on the general demand formulation that was first derived in Eq.(6), equation (15) can logically be rewritten as follows

$$X_{im}(q) = N_{DM}(q) * x(m) * x(i|m), \qquad (16)$$

where:
- $N_{DM}(\mathbf{q})$ denotes the total demand of all feasible combinations of mode and destination in choice set C_n where i=1,...,D and m=1,...,M. Its formulation is given as

$$N_{DM}(q) = \frac{\sum_{m \in M_n} \sum_{j \in D_m} V(q)_j}{\sum_{m \in M_n} [\sum_{j \in D_m} V(q)_j * q_j]}, \qquad (17)$$

- x(i,m) defines the formulation of a nested probability model where its logistic components are defined by the so-called *marginal choice probability* (x(m)) in the context of random theory which chooses mode m (the first term of the RHS of (18) and the *conditional choice probability* (x(i:m)) wherein the destination i is chosen conditional to the use of mode m (the second term of (18)). Its formulation is

$$x(i,m) = \frac{\sum_{j \in D_{mi}} V(q)_j}{\sum_{m \in M_n} \sum_{j \in D_m} V(q)_j} * \frac{V(q)_i}{\sum_{j \in D_{mi}} V(q)_j}. \qquad (18)$$

Equation (18) is the formulation of the general nested probability model that can be obtained by any given function of the indirect utility function. More significant, the conditional choice probability given in Eq.(18), that is the probability of the lower level in the mode–destination choice situation, is given exactly the same as the share model, the so–called general logit type model that is derived in equation (8).

Quasi nested logit model

We have mentioned that the nested probability model can be derived within the framework of consumer behavior by any given *indirect utility function.* Suppose that given the nested indirect utility function as

$$V(q) = \sum_{m=1}^{M} (\sum_{j \in D_m} y_j)^{\mu} \qquad (19)$$

where:
- The choice set (1,...,n) is partitioned into M nonoverlapping subsets D_m, m=1,...,M,
- μ is the scale parameter, $0<\mu<1$,
- $$y_i = \int_{q_i}^{\infty} \exp k_i(q_{im})\, dq_{im}, \qquad (20)$$
- $k_{im}(q_{im})$ is an arbitrary function such that the equation (19) satisfies the characteristic properties of a indirect utility function, with $q_{im} = p_{im}/y$. Note that p_{im} now is the cost of transport mode m that goes to destination i.

Provided that the first derivative of $V_{dm}(\mathbf{q})$ w.r.t. q_{im} is well defined and non–negative, and considering Eq.(19) and Eq.(20), we obtain

$$\frac{\partial V(q)}{\partial q_i} = \mu (\sum_{j \in D_{mi}} y_j)^{\mu - 1} * e^{k_i(q_i)}, \text{ for } i \in D_m. \qquad (21)$$

Recalling that D_{mi} denotes the subset of modes that includes choice i.

Following the Roy's identity (5), the demand of transport mode m that goes to destination i, $X_{im}(\mathbf{q})$ can straightforward be obtained as

$$X_{im}(q) = \frac{e^{k_i(q_i)}\left(\sum_{j\in D_{mi}} y_j\right)^{\mu-1}}{\sum_{m\in M_n}\left[\sum_{j\in D_m} q_j e^{k_i(q_i)}\left(\sum_{j\in D_m} y_j\right)^{\mu-1}\right]}, \quad (22)$$

for m=1,...,M and i=1,...,n.

Consciously, equation (22) is rewritten and given in the following equation.

$$X_{im} = \left[\frac{\sum_{m\in M_n}\left[\sum_{j\in D_m} e^{k_j(q_j)}\left(\sum_{j\in D_m} y_j\right)^{\mu-1}\right]}{\sum_{m\in M_n}\left[\sum_{j\in D_m} q_j e^{k_j(q_j)}\left(\sum_{j\in D_m} y_j\right)^{\mu-1}\right]}\right] * \left[\frac{e^{k_i(q_i)}\left(\sum_{j\in D_{mi}} y_j\right)^{\mu-1}\sum_{j\in D_{mi}} e^{k_j(q_j)}}{\sum_{j\in D_{mi}} e^{k_j(q_j)}\sum_{m\in M_n}\left[\left(\sum_{j\in D_m} y\right)^{\mu-1}\sum_{j\in D_m} e^{k_j(q_j)}\right]}\right]. \quad (23)$$

It is recognized that the first term of the RHS of Eq.(23) presents the total demand of all feasible combinations of modes and destinations in the choice set C_n, $N_{DM}(\mathbf{q})$. Obviously, $N_{DM}(\mathbf{q})$ is not an exogenous variable, rather it can be endogenously computed. This result concurs with the one we first obtained in Eq.(7). Now our interest turns on what the second term of the RHS of (23) means. To be more visible, the second term can be rewritten and defined as

$$x(i,m) = x(i|m) * x(m) = \frac{e^{k_{im}(q_{im})}}{\sum_{j\in D_{mi}} e^{k_{jm}(q_{jm})}} * \frac{\sum_{j\in D_{mi}} e^{k_{jm}(q_{jm})}\left(\sum_{j\in D_{mi}} y_j\right)^{\mu-1}}{\sum_{m\in M_n}\left[\sum_{j\in D_m} e^{k_{jm}(q_{jm})}\left(\sum_{j\in D_m} y_j\right)^{\mu-1}\right]} \quad (24)$$

Equation (24), x(i,m) is called a joint probability in the context of random theory, wherein the combination of mode (m) and destination (i) is chosen. It results in the product of two probabilities, the conditional choice and the marginal choice probabilities as already derived in Eq.(19) and are defined as:

$$x(i|m) = \frac{\exp k_i(q_i)}{\sum_{j\in D_{mi}} \exp k_j(q_j)}, \; for\, i\in D_{mi}, \quad (25)$$

As shown in Eq.(25) the conditional choice probability is given by a logit model with the exponential component, exp $k_i(q_i)$ that is a separable function of q_i and was first derived in equation (13), and also by

$$x(m) = \frac{\sum_{j\in D_{mi}} e^{k_j(q_j)}\left(\sum_{j\in D_{mi}} y_j\right)^{\mu-1}}{\sum_{m\in M_n}\left[\sum_{j\in D_m} e^{k_j(q_j)}\left(\sum_{j\in D_m} y_j\right)^{\mu-1}\right]}, \; m=1,\ldots M \quad (26)$$

By a simple mathematical technique, Eq.(26) can be rewritten as

$$x(m) = \frac{e^{\ln\sum_{j\in D_{mi}} e^{k_i(q_i)} + (\mu-1)\ln\sum_{j\in D_{mi}} y_j}}{\sum_{m\in M_n} e^{\ln\sum_{j\in D_m} e^{k_j(q_j)} + (\mu-1)\ln\sum_{j\in D_m} y_j}}. \quad (27)$$

It shows that the marginal choice probability, x(m) is given in the form of the logit model but with an extra twist that is induced by the attribute of function $y_j(.)$, j=1,...,n.

Due to this extra twist problem, the nested probability as derived in equation (24) is not a perfect nested logit model. Rather, in the context of this paper, we prefer to name it as *the Quasi–Nested Logit Model*. Moreover, if the scale parameter μ normalized to 1, the marginal choice probability given in Eq.(24) becomes the logit mode. Thus, the nested probabilities (24) have become the multinomial logit model with the normalized scale parameter μ. As the result shows, though we may not able to derive the general nested logit model directly from the theory of consumer behavior, we have succeeded in generating the general probability model, as well as the multinomial logit model within the framework of continuous behavior.

As the result shows, the demand of the mode–destination combination, X_{im} is again generally formulated as the product of the total demand over all the feasible mode–destination combinations in the mode and destination choice set C_n and the nested probability wherein the combination (i,m) is chosen. The only difference is that in this model, the probability model is given by the quasi–nested logit model where its logistic components are depicted in Eq.(25) and (27). Noticeably, all the merits of this general demand formula with regards to the generated demand that have been discussed in section 3, are still reliable for this model.

Furthermore, since the so-called quasi–nested logit model is partitioned into two separate probabilities, the conditional and the marginal probabilities, its parameter estimation procedure can be conducted by applying the conventional estimation technique, for example, the log likelihoods method that estimate the parameters of different levels by using the separated conditional and marginal log likelihood functions. In this model, first the functional form of $k_j(q_j)$, j=1,...,n, are specified, thus the conditional choice probability (25) is used to estimate the parameters of the lower level, for j=1,..,n. Then those values can be used as inputs to estimate the parameter μ of the upper level through the marginal choice probability equation (27). Also, the Newton–lapson or the Homotopy methods can be used to estimate the parameters through the equation system that is directly obtained from equation (23), when given the related information and the observable data of the travel demands.

Conditional Nested Logit Model

Though we have yet to derive the nested logit model, we have already derived the so-called quasi-nested logit model due to the extra twist component as shown in equation (27). This problem itself have suggested us that we can derive the nested logit model by specifying the function of $y_j(q_j)$, given (19). Suppose that a very simple function of y_j is given as

$$y_j = e^{q_j},\ j=1,\ldots,J. \tag{28}$$

Having this specification and recalling equation (19) we obtain

$$\frac{\partial V(q)}{\partial q_i} = \mu\Big(\sum_{j\in D_{mi}} e^{q_j}\Big)^{\mu-1} * e^{q_j}. \tag{29}$$

Thus the demand of the destination–mode choice X_{im} is defined as

$$X_{im}(q) = \frac{\Big(\sum_{j\in D_{mi}} e^{q_i}\Big)^{\mu-1} e^{q_i}}{\sum_{m\in M_n}\Big[\Big(\sum_{j\in D_m} e^{q_j}\Big)^{\mu-1}\sum_{j\in D_m} q_j e^{q_j}\Big]}, \tag{30}$$

for m=1,...,M and i=1,...,n.

By a simple multiplication technique that we have been using so far, Eq.(30) can identically be formulated as the model shown in Eqs.(17) & (18), whose components are defined as

$$N_{MD}(q) = \frac{\sum_{m\in M_n}\Big[\sum_{j\in D_m} e^{q_j}\Big(\sum_{j\in D_m} e^{q_j}\Big)^{\mu-1}\Big]}{\sum_{m\in M_n}\Big[\sum_{j\in D_m} q_j e^{q_j}\Big(\sum_{j\in D_m} e^{q_j}\Big)^{\mu-1}\Big]}, \tag{31}$$

and

$$x(i,m) = x(i|m) * x(m) = \frac{e^{q_i}}{\sum_{j\in D_{mi}} e^{q_j}} * \frac{\Big(\sum_{j\in D_{mi}} e^{q_j}\Big)^{\mu}}{\sum_{m\in M_n}\Big[\Big(\sum_{j\in D_m} e^{q_j}\Big)^{\mu}\Big]}. \tag{32}$$

This is a formulation of the nested logit model with the scale parameter μ. The exponential component in (32) appears as the one variable function or more accurately, it is the variable q_i itself.

Coming back with the twist problem that brought about the so-called quasi-nested logit model in the context of this study, the twist of the two under sum components in equation (27) (e.g. $\Sigma \exp k_j(q_j)$ and $(\mu-1)\ln(\Sigma y_j)$ has been solved to derive the nested logit model (Eq.32) by specifying the function of $y_j = \exp q_j$. This specification serves as a sufficient condition for $\partial y_i/\partial q_i = y_i$. This statement leads to the conclusion that the sufficient and necessary condition for deriving the nested logit model in the context of the proposed model is the exponential component $(k_i(q_{im}))$ should be a function where its first derivation w.r.t. q_{im} is unity. (e.g., $k_{im}(q_{im}) = q_{im}$, in the above example). Obviously, the nested logit model would be well derived even when $y_i = \exp(q_{im}+c_i)$ or $y_i = A \exp q_{im}$ and etc., where A is a parameter which is assumed to be common over all choices.

Having the results of the logit model, the multinomial logit model, the quasi-nested logit model and the conditional nested logit model which are all derived in the framework of the consumer behavior theory, the study is extensively carried out in the following section to discuss whether this proposed mode can be further developed to derive the generalized extreme value (GEV) mode.

5. A DERIVATION OF CONDITIONAL GEV MODEL

By definition, the generalized extreme value is actually a large class of models which includes MNL and nested logit model. The GEV model was first derived by McFadden (1978) directly from the concept of random utility theory. This theory is well known as the basic foundation for generating a series of probability models. However, challenged by the significant generalization of the proposed model, the possibility of deriving the GEV model in the framework of consumer behavior is answered as follows. Suppose that there exits a function V(.) such that

$$V_i = \frac{\partial V(.)}{\partial q_i} = G_i e^{k_i(q_i)},\ i=1,\ldots,J, \tag{33}$$

where:

$$-G = G(e^{k_1(q_1)},\ldots,e^{k_J(q_J)}),$$

$$-G_i = \frac{\partial G}{\partial(e^{k_i(q_i)})},\ i=1,\ldots,n.$$

Then the potential associated with the indirect utility function V(.) is suggested as

$$V(.) = \oint_{\infty}^{q}\sum_{i\in J} G_i e^{k_i(q_i)}\, dq_i\,, \tag{34}$$

where: $-\mathbf{q} = (q_1,\ldots,q_J)$, and

- $\oint$ is a line integral with respect to vector q along an arbitrary path.

Given V(.), from Roy's identity, the solution to this particular function is

$$X_i(.)=\frac{\sum_{j\epsilon J} G_j e^{k_j}}{\sum_{j\epsilon J} G_j e^{k_j} q_j} * \frac{G_i e^{k_i}}{\sum_{j\epsilon J} G_j e^{q_j}}, \qquad (35)$$

for i=1,2,...,J. Assuming that G is a μ homogeneity function, thus by Euler's theorem it is true that

$$\mu G(\ldots)=\sum_{j=1}^{J} G_j(\ldots)\, e^{k_j}. \qquad (36)$$

Hence equation (35) now becomes

$$X_i(q)=N(q)*x_i(q_{j\epsilon J}) = \frac{\sum_{j\epsilon J} G_j e^{k_j}}{\sum_{j\epsilon J} q_j G_j e^{k_j}} * \frac{G_i e^{k_i}}{\mu G(e^{k_1},\ldots,e^{k_J})}. \qquad (37)$$

The first component of the RHS of (37) is the so-called total consumption of a group of good/service in section 3 (e.g., in Eq.7 and Eq.12 for the total transport demand). As the result shows, the second component is given by the generalized extreme value (GEV) model.

Undoubtedly, we have derived the GEV model in the context of consumer behavior as shown in Eq.37. This derivation, however, is totally based on the assumption given in Eq.(34). Accordingly, it raises the question of what the function $k_i(q_i)$ has to be to satisfy Eq.(33) or in other words, is there any possibility for the function given in Eq.(34) to exist. The only feasible function of k(.) is to be the function that is formulated in such a way that $\partial k_i/\partial q_j = \partial k_j/\partial q_i$, for i,j = 1,...,n. Thus, to meet with this condition, the only possible function is $k_i(q_i) = a_i + bq_i$, where b is a parameter assumed to be common over all choices. It is good to note that this condition includes the function that is sufficient and necessary condition to derive the nested logit model in the context of consumer behavior, recalling $k_i(q_i)= q_i$, for i=1,...,n. Evidently, the conditional GEV model can also directly be derived in the framework of the proposed mode from the specified function (34), given the following nested indirect utility function (38). The derivation is briefly shown as follows. Suppose the nested indirect utility is given as

$$V(q)=G[y_1,y_2,\ldots,y_n], \quad where: y_i=e^{k_i(q_i)}=e^{q_i}. \qquad (38)$$

Where function G is a μ homogeneity function. Thus by Roy's identity the demand function is defined as

$$X_i(q)=N_J(q)*x_i(q) = \frac{\sum_{j\epsilon J} G_j e^{q_j}}{\sum_{j\epsilon n} G_j e^{q_j} * q_j} * \frac{G_i e^{q_i}}{\mu G[e^{q_1},e^{q_2},\ldots,e^{q_J}]}. \qquad (39)$$

The second term of the RHS of (39) is the so-called GEV model in the context of Ben-Akiva and Francois (1983) where function F is homogeneous of degree μ.

6. CONCLUSION

The study shows a different approach to derive the general as well as the specific functional forms of the logit type of share model, the modified nested logit model or the so-called quasi nested logit and even the conditional GEV model within the framework of the consumer behavior theory. Also, the discussion on the possibility of deriving the nested logit as well as the GEV modes in the context of consumer behavior have been brought out. The four significant results that have been obtained from this study are as follows:

Firstly, the study has shown that from any functional form of an indirect utility function with the normal characteristics, the share type of the logit model and its development in the multidimensional choice situation for the nested probability model can be derived within the framework of consumer behavior theory. In the context of the proposed framework, the demand function can be generally expressed as the product of a total demand model, and the probability model which are given by the logit type of share model (Eq. 8), the nested probability model (Eq.24 and Eq.32), and the GEV model (Eq.37 & 39). Although this demand formulation appears more generalized, it has a restriction that the functional form of each component of the multiplicative terms are the partial derivative with respect to prices. This restriction is relaxed later by specifying the functions of the indirect utility to derive the conditional logit, nested logit and GEV models in some special cases.

Secondly, in the framework of proposed model, since the total demand functions are explicitly formulated,(as shown in Eq. (12), (23) and (31)), its value is no longer fixed at an exogenously given value. Rather it can be determined endogenously by the given observable data. This merit leads to a better general model since we can explicitly deal with the additional or in other words, the generated demand. In this aspect the model has overcome the limitation of the conventional logit model with fixed total demand.

Thirdly, in the conventional travel fôrecasting, the effects of factors such as prices of other goods/services to the transport demand behavior are ignored. However, it is clearly shown in this study that, since the total demand model as obtained in (12) takes into account all prices and other factors, the demand functions of the transport modes (11) are also a function of the price of other commodities. This formulation leads to the possibility of this model to fully consider and obtain the absolute figure of the transport demand and influences of the other commodities in the travel forecasting process. Based on the product of the transport demand function as (14), this study proposes a new comprehensive parameter estimation technique that uses the results of estimation on the share type of logit model (13). It can also estimate the total transport demand function (12) only by using transport related data. It remains for future study, however, to compare our proposed estimation technique with the conventional method in term of efficiency, accuracy, and practically.

Fourthly, the application of the proposed model to the study of the newly introduced transport facility is proved to be more significant compare with the conventional model which was generated in the probability background. Since the proposed model is generated based on the consumer behavior theory where the consumer maximizes his utility under her/his individual characteristic constraints. The model enables us accurately analyze the reliability of the new service to the user, while in the conventional model, the portion of the new mode will always be assigned without any constraint. Even the new mode may not be reliable to the user, such that it induces a larger error in the application of the conventional model to the newly introduced facility situation. Thus the derivation of the demand model in the context of the consumer behavior theory does not only theoretically give a new insight to the study of the demand model but also provides a significant model for practical usage.

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