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USE OF A PROBIT MODEL IN TRAFFIC ASSIGNMENT*

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The existing methods of traffic assignment to a road network may be roughly classified into two groups; one is the "All or Nothing" method based upon "minimum path" and the other is the "Assignment Rate" method.With the assignment rate method,the problem is how to determine the assignment rates. The Multinomial Logit(MNL) model is widely used but appears to be less appropriate for path choice because of its property of Independence of Irrelevant Alternatives.

This paper considers the use of the Probit model in determining the assignment rate.

1. INTRODUCTION

Various procedures and techniques for determining the assignment rate when the assignment rate method of traffic assignment is used have been developed in the past several years. Because of the importance of traffic assignment in transportation planning and design, a considerable amount of research has been directed towards developing better solution methods which can increase the accuracy and reduce the cost of many transportation analyses.

This paper discusses the assignment rate method by computation using simultaneous equations. The Multinomial Probit(MNP) model is used in determining the assignment rate. The MNP model provides a more attractive alternative to the widely used MNL model in route choice modelling where correlation between alternatives is significant. It has been shown by Daganzo and Sheffi[2] that the probit model does not exhibit the inherent weaknesses of the logit model when applied to sets of routes which overlap heavily.

As preliminary steps to our presentation,section 2 gives a general outline of the Simultaneous Equation model. A detailed description of the model is given in reference[1]. The terminology and notation used here are identical to that in reference[1].

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*Key word

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2 SIMULTANEOUS EQUATION MODEL

Let $\bar{Q}_{ij}$ be the O-D traffic volume between origin i and destination j to be assigned. If $\bar{Q}_{ij}$ is assigned to m routes, then

$$\bar{Q}_{ij} = \Sigma_r {}_rQ_{ij} \quad \cdots\cdots (1)$$

Where ${}_rQ_{ij}$ is the assigned volume in the r^{th} route. This is named the CONTINUATION EQUATION.

When a typical point of each link is chosen, the total traffic volume ,q_l, is the sum of the total assigned volume, ${}_rQ_{ij}$, and the local traffic ${}_kq_l$.

$$q_l = {}_r\partial_{ij} \cdot {}_rQ_{ij} + {}_kq_l \quad \cdots\cdots (2)$$

Where ${}_r\partial_{ij}$ =0 or 1. This is named the CUMULATION EQUATION.

The method of assigning $\bar{Q}_{ij}$ to each route is defined by the assignment rate, ${}_r\beta_{ij}$, defined as :

$${}_r\beta_{ij} = {}_rQ_{ij} / \bar{Q}_{ij} \quad \cdots\cdots (3)$$

This is named the ASSIGNMENT RATE DEFINITION FUNCTION.

When a motorist selects a route among many routes, he makes some evaluation of the route and judge it to be the most attractive one. The grounds of the evaluation are thought to be minimum travel time,cheap toll rate,comfort,minimum driving stress etc. These factors are referred to as the EVALUATION VALUE, ${}_rE_{ij}$, of the road links or route.

The assignment rate can be expressed as a function of the route and other possible routes.

$${}_r\beta_{ij} = {}_rf_{ij}({}_1E_{ij}, \cdots\cdots, {}_mE_{ij}) \quad \cdots\cdots (4)$$

This is named the ASSIGNMENT EQUATION.

The evaluation value , ${}_rE_{ij}$, is considered to be a function of various factors such as travel time, ${}_rT_{ij}$, toll rate, ${}_rF_{ij}$, comfort and a constant , ${}_rK_{ij}$, which is specific to each route.

$${}_rE_{ij} = {}_rg_{ij}({}_rT_{ij}, {}_r\partial_{ij} \cdot {}_rF_{ij}, {}_rK_{ij}) \quad \cdots\cdots (5)$$

This is named the EVALUATION EQUATION.

The total travel time on each route , ${}_rT_{ij}$, is the sum of the travel time of all the links that constitute the route; and the travel time is expressed as a function of the link distance, L_l, and the total traffic volume ,q_l.

$${}_rT_{ij} = F_l (L_l, q_l) \quad \cdots\cdots (6)$$

Where the suffix l is taken for all sections of the road which constitute the route,r, between i and j. This is named the Q-T EQUATION.

By solving the above simultaneous equations, given $\bar{Q}_{ij}$ and ${}_kq_l$, as known quantities and deciding the coefficients ${}_rf_{ij}$, ${}_rg_{ij}$, F_l, one can determine all the unknown quantities ${}_rQ_{ij}$, q_l, ${}_r\beta_{ij}$ and ${}_rE_{ij}$.

3. THE ASSIGNMENT RATE

(1) Multinomial Probit Model

The route evaluation equation (eqn.5) corresponds to the utility which motorists consider as an index for choosing their routes in stochastic modelling.i.e. each route is characterised by a utility (evaluation value) that includes a random component, ξ, that captures the additive effects of unobserved and unquantifiable characteristics of both the choice maker and the alternative. Thus, eqn. 5 can be re-written as follows:

$$ {}_rE_{ij} = {}_rU_{ij} = \theta_T \cdot {}_rT_{ij} + \theta_F \cdot {}_r\partial_{ij} \cdot {}_rF_{ij} + {}_r\xi_{ij} \quad \text{.......(7a)} $$

$$ = {}_rV_{ij} + {}_r\xi_{ij} \quad \text{.......(7b)} $$

Where,

${}_rV_{ij} = \theta_T \cdot {}_rT_{ij} + \theta_F \cdot {}_r\partial_{ij} \cdot {}_rF_{ij}$ is the measured attractiveness of the link or route. θ_T and θ_F are parameters or weights in a users preference function; ${}_r\xi_{ij}$ is a random component that captures additive effects of unobserved and unquantifiable characteristics of both the user and the alternative.

Every motorist evaluates the utilities on all the routes k= 1,2,..........,m and selects the route k_{min}, with the minimum or most attractive utility.

$$ {}_{k_{min}}U_{ij} < {}_kU_{ij} \; ; \quad k \neq k_{min} \quad \text{.......(8)} $$

In this case, ${}_kU_{ij}$ is the utility of the k^{th} route between origin i and destination j as believed by the user of the system at the time he makes route decision . This is a form of the Stochastic User Equilibrium(S.U.E.); a modified Wardrops User Equilibrium which states that " In a stochastic user equilibrium network, no user believes he can improve his travel time by unilaterally changing routes".(Daganzo and Sheffi,1977).

Based on the following two postulates of motorist behaviour;

(1) Nonoverlapping sections of the road are perceived in independently,

(2) Sections of the road of equal length are perceived in identical fashion,

the following expressions of route choice can be obtained.

Defining Θ as the variance of the error term, ξ, on a section of road of unit length, the correspondening term on a section of road of length V, has a zero mean and variance ΘV. The mean of ${}_kU$, $E({}_kU)$, is by definition ${}_kV$ and the variance and covariance are:

$$ Var({}_kU) = \Theta_k V \quad \text{.......(9a)} $$

$$ Cov({}_kU, {}_sU) = \Theta_{ks} V \quad \text{.......(9b)} $$

Where ${}_{ks}U$ and ${}_{ks}V$ denote respectively the perceived utility and the measured attractiveness of the road sections shared by route k and s.

The probability of selecting route k is given by:

$$P_k = Pr[\,_kU < \,_iU \;;\quad k \neq i\,] \qquad \cdots\cdots (10a)$$

$$= Pr[\,_kU - \,_iU < 0 \;;\quad k \neq i] \qquad \cdots\cdots (10b)$$

$$= Pr[\,_kV - \,_iV < \xi_i - \xi_k \;;\quad k \neq i] \qquad \cdots\cdots (10c)$$

Where the error terms , ξ , are multivariate normal(MVN) distributed with zero mean and covariance-variance matrix whose components are given by eqn. 9.

(2) Parameter Values

The values of the parameters given in table 1 were determined by fitting the model to data provided by the Japan Highway Corporation(Nihon Doro Kodan) on the use of expressways and toll roads.See reference [7].

4. EXAMPLES

The model was applied to two examples;one was a model network designed to bring out the effects of tolls on route choice and the weakness of the logit model when applied to routes that overlap, and the other was an application of the model to a practical case of route choice.

PROBIT MODEL				LOGIT MODEL		
Mean $V=\theta_T T+\theta_F F$ Var(U) $=\Theta V$				$\gamma_i = \dfrac{\exp[\beta_1(T_i+\beta_F F_i+\beta_P P_i)]}{\sum_{\forall k}\exp[\beta_1(T_k+\beta_F F+\beta_P P_k)]}$		
VEHICLE TYPE	θ_T	θ_F	Θ	β_1	β_F	β_P
Passenger Car(PC)	-1.0	-1/2000	1/30	-15.2	-1/2000	-1/20.0
Small Truck (ST)	-1.0	-1/2000	1/30	-11.6	-1/1600	-1/13.3
Large Truck (LT)	-1.0	-1/2000	1/30	-9.2	-1/2000	-1/5.5
F=Toll Rate(Yen), T=Travel Time(Min), P=Toll Road Ratio						

TABLE 1: PARAMETER VALUES.

(1) Example one

Figure 1 shows the network geometry for which the model was applied.The route table and the O-D table are given in tables 2 and 3 respectively. The results of the computation are shown in table 5.

A comparison of the results with those of the logit model are given in table 4.

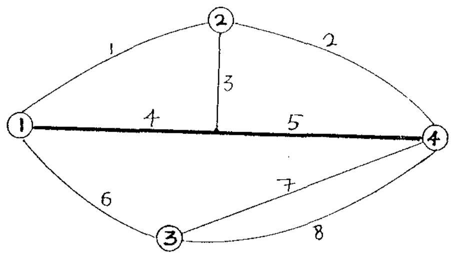

FIG.1: A NETWORK EXAMPLE

Figures on the Links represent Link numbers.
Figures in represent origin and destination numbers.
Bold lines represent Toll Roads.

(2) Example two

The model was applied to a road network between Yokohama and Tokyo. The routes considered are as follows:

(1) A route via the third Keihin Highway.

(2) National Highway No.1

(3) National Highway Nos.1 and 15

(4) Metropolitan (Shuto) Expressway(Yokohane Road).

Table 6 gives the route length,toll rate and travel time. The results of the computation are shown in table 7. Table 7 also shows a comparison of the results with those obtained when the logit model was used.

(3) Computation

The computation was done using a computer programme which incorporates a slightly streamlined version of the code developed by Daganzo, Bouthelier and Sheffi[3].
Travel times and measured attractivess were computed using the simultaneous equation models coded programme.

Table 2: Route Table

O	D	ROUTE	
1	4	1	1-2
		2	4-5
		3	6-7
		4	6-8
1	2	1	1
		2	4-3
1	3	1	6
2	4	1	2
		2	3-5
3	4	1	7
		2	8

Table 3. O-D Table.

D \ O	1	2	3	4
1	0	60	40	200
	0	50	30	160
	0	40	20	120
2	60	0	0	150
	50	0	0	100
	40	0	0	50
3	40	0	0	50
	30	0	0	40
	20	0	0	30
4	200	150	0	0
	160	100	0	0
	120	50	0	0

Figures at the top are for Passenger Car, those in the middle for Small Trucks and bottom figures are for Large Trucks.

TABLE 4: TRAVEL TIME AND ASSIGNMEMT RATE

			ASSIGNMENT RATE					
		ROUTE	PROBIT MODEL			LOGIT MODEL		
O	D	NUMBER	PC	ST	LT	PC	ST	LT
1	4	1	0.75	0.76	0.85	0.88	0.86	0.84
		2	0.09	0.09	0.00	0.04	0.02	0.01
		3	0.14	0.13	0.13	0.07	0.11	0.13
		4	0.02	0.02	0.02	0.01	0.01	0.02
1	2	1	0.93	0.94	0.98	0.94	0.94	0.94
		2	0.07	0.06	0.02	0.06	0.06	0.06
1	3	1	1.00	1.00	1.00	1.00	1.00	1.00
2	4	1	0.92	0.90	0.99	0.95	0.96	0.97
		2	0.08	0.10	0.01	0.05	0.04	0.03
3	4	1	0.85	0.83	0.84	0.94	0.88	0.85
		2	0.15	0.17	0.16	0.06	0.12	0.15

TABLE 5:LINK DISTANCE ,TRAVEL TIME AND ASSIGNED VOLUME

LINK	LINK DISTANCE	TRAVEL TIME (MIN)			LOCAL TRAFFIC	ASSIGNED VOLUME (VEH./HR.)				TOTAL VOLUME
NUMBER	(KM)	PC	ST	LT	(VEH/HR)	PC	ST	LT	TOTAL	(VEH/HR)
1	10.0	8.86	9.62	10.69	2400	206	169	141	516	2916
2	15.0	13.41	14.35	15.89	3600	288	212	151	651	4251
3	3.5	5.26	5.50	6.07	4800	16	13	2	31	4831
4	15.0	8.58	10.88	10.89	0	22	17	1	40	40
5	20.0	11.58	14.54	14.86	0	30	24	1	55	55
6	8.0	9.17	9.89	10.96	4500	72	54	38	164	4664
7	20.0	23.41	25.32	28.08	3900	70	54	41	165	4065
8	25.0	34.64	35.98	39.68	5700	12	10	7	29	5729

5. COMMENTS

TABLE 6:LENGTH AND TRAVEL TIME

ROUTE	LENGTH (KM)	TOLL(YEN)			TRAVEL TIME(MIN)		
		PC	ST	LT	PC	ST	LT
Via the third Keihin Highway	38.3	150	150	250	44.40	55.10	55.61
National Highway No. 1	35.8	-	-	-	56.34	58.81	60.65
National Highway Nos. 1 and 15	31.2	-	-	-	62.39	64.36	65.78
Metropolitan(Shuto) Expressway. Yokohane road	39.6	500	500	800	35.29	42.56	43.69

TABLE 7:ASSIGNED VOLUME

ROUTE	ASSIGNED VOLUME(VEH. /DAY)							
	PROBIT MODEL				LOGIT MODEL			
	PC	ST	LT	TOTAL	PC	ST	LT	TOTAL
Via the third Keihin Highway	2331	1090	1573	4994	411	124	136	671
National Highway No.1	888	1293	1953	4134	637	1699	2240	4546
National Highway Nos. 1 and 15	388	686	1085	2159	152	642	1275	2069
Metropolitan(Shuto) Expressway. Yokohane road	1942	969	813	3724	4349	1603	1773	7725
SUM	5549	4038	5424	15011	5549	4038	5424	15011

The results of the two examples given here show clearly that toll rates have a profound effect on route choice decisions.

In the first example, it can be seen that for O-D 1-4, even though route no.4 has the best travel time ,less traffic was assigned to it than routes 1,2 and 3 which are relatively slower but are all freeways. Similar results were obtained in the second example(table 7) where more trffic is assigned to the route via the third Keihin highway than the Metropolitan expressway, even though travel time on the metropolitan expressway is faster by approximately 10 minutes.

The superiority of the probit model over the logit model is clearly seen from the results shown in table 7; where about 50% of the traffic is assigned to the metropolitan expressway by the logit model apparently because of its superior travel time,whilst paradoxically, the Keihin highway which has the second best travel time is assigned the least volume.

The probit model on the other gave results which are very consistent and reasonable.

6. GENERAL CONCLUSION

This paper has attempted to demonstrate the feasibility and desirability of using the multinomial probit models in determining assignment rates.

Section 2 gave a brief description of the simultaneous equation model which is used to compute the travel times and measured attractiveness of the links and routes and forms the basis of our choice model.

Section 3 introduced the MNP model which is used in calculating the assignment rates.

The purpose of section 4 is two fold; first to demonstrate the effect of tolls on route choice; and second, to show the superiority of the MNP model over the MNL model in route choice modelling.

The results of the computation show that determination of assignment rates with the MNP model in route choice produce what is generally considered reasonable behaviour.

The parameter values and data presented herein are not for predicting route choice in general, but are presented to describe the empirical behaviour of the model.

We cannot say that our model has a universal application until we conclude a large scale test we are now working on.

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