METHODS OF EVALUATING TENSILE CREEP AND STRESS-RELAXATION OF CONCRETES SUBJECTED TO CONTINUOUSLY INCREASING LOADS

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INTRODUCTION

Reported herein are the results of a study on methods evaluating creep and strees-relaxation of concrete under continuously increasing tensile stress. The laboratory tests of this study were performed in the Engineering Materials Laboratory, University of California at Berkeley, California, U.S.A., as two individual graduate student research projects. These research projects were carried out as the partial fulfillment of the requirements for the Master of Science degree in Civil Engineering during the period from June 1961 to January 1962, and were reported to the University[1,2]. Since these two projects included the common problem of tensile creep and stress-relaxation of concrete, the authors discussed the problem and extended analysis upon the completion of their study at the University. The final completion of this report was carried out by Yuzo Akatsuka with consent of Shao-Chien Chang and Milos Polivka who was the faculty adviser for the projects.

1. Scope

When a reinforced concrete member is subjected to drying, shrinkage will take place in the concrete and it will be restrained to some degree by the reinforcement, consequently producing compressive stress in reinforcement and tensile stress in concrete. If this concrete will crack or not under this tension will depend upon the shrinkage and creep characteristics and the ultimate tensile strength of the concrete as well as the degree of restraint of shrinkage. Carlson proposed a method of test for cracking tendency of concrete utilizing this phenomenon[3].

Extending this principle, methods of test can be developed for tensile creep and stress-relaxation of concrete. The authors present herein the results of a study on methods of test for creep and stress-re laxation of concrete subjected to the continuously increasing tensile stress. Two method were employed to produce the tensile stress in concrete through the introduction of restraint of shrinkage; one provided external restraint by placing three steel restraining rods on the outside of a concrete specimen which prevented the concrete from free shrinkage through two loading plates glued to the specimen; the other provided internal restraint by embedding a steel rod on the inside of a concrete specimen. The former method is described in the test program Part I, and the latter in Part II. To evaluate the applicability of the restraining methods for tensile creep studies, creep tests in tension as well as in compression were carried out and compared with results obtained either by the external or the internal restraining method.

2. Basic Concepts

Shrinkage and Restrained-Shrinkage

In the course of drying, concrete shrinks due to loss of moisture and carbonation of hydrated gel. When shrinkage is not uniformly distributed or is restrained, tensile stresses will develop and the concrete will crack if it reaches its ultimate tensile strength. Being given two concrete specimens of the same dimensions, made of the identical concrete mixture, cured likewise, and then exposed to the same drying atmosphere, there will be no difference in the magnitude of shrinkage between the two specimens unless one of them is restrained to some extent from shrinking. After a certain duration of exposure to the drying atmosphere,

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the unrestrained specimen will shrink by the amount δ_F and the restrained specimen will shrink by the amount δ_R. Since δ_F will be larger than δ_R, a tensile stress will develop in the restrained-shrinkage specimen.

Stress-Relaxation and Creep

If the concrete is entirely elastic, the stress developed in the restrained-shrinkage specimen will be $E(\delta_F-\delta_R)$, in which E is the modulus of elasticity of concrete. It has been conventionally conceived, however, that the actual stress in the restrained-shrinkage specimen is much lower than the value $E(\delta_F-\delta_R)$. The difference between the actual stress σ and the value $E\,(\delta_F-\delta_R)$, i.e., $E\,(\delta_F-\delta_R)-\sigma$, is defined here as stress-relaxation in concrete. Although there are several factors which may cause this stress-relaxation, creep of concrete is considered the most predominant factor. A typical behavior of concrete under sustained load, initially loaded at time K_1, is illustrated in Fig. 1. After removal of the sustained load at time K_2, the concrete shows elastic recoveries of σ/E_2, which is in general smaller than σ/E_1, the elastic deformation upon application of laod σ at time K_1. The value of E_2 is larger than E_1 due to the aging of the concrete. A substantial part of the creep is irrecoverable (permanent deformation), and only a small part shows time-dependent recovery. Conventionally the creep at time K_2 is defined as the deformation $C=\varepsilon_2-\varepsilon_1$ in Fig. 1[4]. In this study, however, creep at time K_2 is defined as the difference between the total deformation and the elastic deformation, i.e., $\varepsilon=\varepsilon_2-\sigma/E_2$, in which the aging effect of concrete upon the modulus of elasticity E and the time dependent recovery of creep is assumed as negligible and therefore not considered.

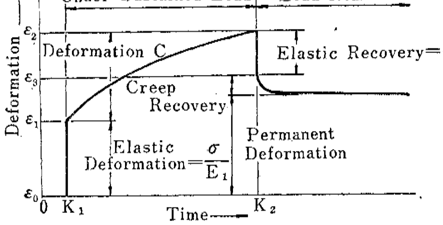

Fig. 1 Concrete Develops Both Elastic and Creep Deformations under Sustained Load, and Shows Both Elastic and Creep Recoveries after Removal of the Load.

Creep and Stress-Relaxation by Restraining Methods

Both the internal and external restraining methods and their unrestrained control specimens provide the necessary data on shrinkage and restrained-shrinkage of concrete, fromwhich stress-relaxation and creep under the tensile stress developed in the restrained-shrinkage specimen can be found. Shrinkage and restrained-shrinkage data can be plotted as illustrated in Fig. 2(A). The tensile stress developed in the restrained-shrinkage specimen can be obtained as follows:

$$\sigma=\frac{A_s}{A_c}\cdot E_s\cdot\delta_R\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

in which σ = tensile stress of concrete in restrained-shrinkage specimen,

A_s = cross sectional area of restraining steel rod,

A_c = net cross sectional area of concrete,

E_s = modulus of elasticity of steel,

δ_R = restrained-shrinkage per unit length.

Once tensile stress σ of concrete is known, stress-relaxation is given by :

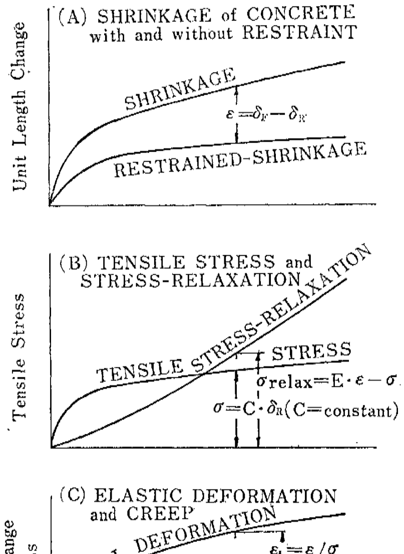

Fig. 2 Illustrative Presentation for Computation of Stress-Relaxation and Creep under Continuously Increasing Tensile Stress.

$$\sigma_{\text{relax}} = E(\delta_F - \delta_R) - \sigma \quad \cdots\cdots (2)$$

where σ_{relax} = stress-relaxation in concrete,
E = modulus of elasticity of concrete,
δ_F = unrestrained shrinkage per unit length.

The tensile stress and stress-relaxation will be somewhat like illustrated in Fig. 2(B). Since the total deformation per unit length due to the tensile stress is $(\delta_F - \delta_R)$, the total deformation per unit stress is obtained by :

$$\varepsilon_{\text{total}} = \frac{(\delta_F - \delta_R)}{\sigma} \quad \cdots\cdots (3)$$

The elastic deformation per unit stress is given by :

$$\varepsilon_{\text{elastic}} = \frac{1}{E} \quad \cdots\cdots (4)$$

Then creep due to the tensile stress is obtained by subtracting the elastic deformation from the total deformation, which is given by :

$$\varepsilon_{\text{creep}} = \frac{(\delta_F - \delta_R)}{\sigma} - \frac{1}{E} \quad \cdots\cdots (5)$$

The stress-relaxation can be exactly determined by Eq. (2), provided tensile stress in the restrained-shrinkage specimen is properly determined, as shown above, and the tensile modulus of elasticity of concrete is known exactly. However, Eq. (5) will yield only an approximation for creep, even if the stress-relaxation is obtained exactly. The deformation δ_F and δ_R are not correlated linearly to each other since the effect of loading history and load intensity upon creep as well as on modulus of elasticity was neglected in this method.

Creep Curves Corresponding to Loading at Various Ages

As pointed out above, the restraining methods are considered to yield only an approximation for creep, although they give fair results for stress-relaxation. To investigate the validity of the restraining methods for creep studies, creep tests in tension and compression were performed for this purpose. The tensile creep tests were made by directly applying a tensile stress on concrete specimens. This tensile stress was intermittently increased and sustained to simulate the development of tensile stress in the restrained-shrinkage specimens. Creep curves due to a compressive unit stress initially applied at various ages were first obtained.

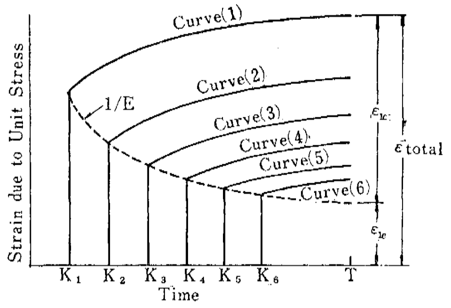

Fig. 3 Creep Curves Obtained from Identical Specimens under Sustained unit Stress, Loaded at Different Ages.

Then using these curves, the elastic deformation and the creep due to stress increments at various ages were computed. Fig. 3 illustrates a set of typical creep curves obtained from identical concrete specimens initially loaded at various ages. The dotted curve indicates the elastic deformation due to a unit stress. The difference between two deformations represented by curves (1) and $1/E$ at a certain time is considered as the creep of concrete initially loaded at time K_1. At time T, for instance, the elastic deformation and creep in the concrete are ε_{1e}, and ε_{1c}, respectively. When load was applied at ages K_2, K_3 and so on, the creep can be found using curves (2), (3), and so on, respectively.

If an increment of load $\Delta\sigma_1$ is applied to the concrete specimen at time K_1, the resulting deformation will be obtained through multiplying the value of strain represented by curve (1) by $\Delta\sigma_1$. If an increment of load $\Delta\sigma_2$ is added to the same specimen at time K_2, the deformation due to $\Delta\sigma_2$ will be obtained by the same manner using curve (2), and the deformation thereafter is obviously caused both by $\Delta\sigma_1$, and $\Delta\sigma_2$, hence it is the superposition of these two. By use of this principle of superposition, the deformation due to various loads applied intermittently can be computed.

TEST PROGRAM PART I : EXTERNAL RESTRAINING METHOD

3. Concrete Materials and Mix

a) Portland Cement The portland cement used in this part of the test program was an

Table 1 Chemical Analyses of Cements.

Composition	Percent	
	Test Program I	Test Program II
	ASTM Type II Brand s	ASTM Type II Brand C
SiO_2	23.4	23.3
Al_2O_3	3.9	4.2
Fe_2O_3	3.7	2.6
CaO	64.2	64.4
MgO	1.6	2.3
SO_3	2.0	1.8
Alkalies as Na_2O	0.5	0.6
Ignition Lass	0.8	0.9
C 3 S	46	48
C 2 S	33	31
C 3 A	4	7
C 4 AF	11	8

Table 2 Physical Properties of Coast Range Aggregates.

Sieve Size (mm)	Cumul ative Percent Passing				
	Test Program Part I			Test Program Part II	
	Sand		Gravel I	Sand C*	Gravel II
	Fine	Coarse			
25	—	—	100	—	99
20	—	—	96	—	58
13	—	—	69	—	27
10	—	—	30	—	10
5	100	100	0	100	0
2.5	99	85	—	74	—
1.2	98	56	—	52	—
0.6	93	32	—	32	—
0.3	33	14	—	16	—
0.15	7	5	—	6	—
Fineness Modulus	1.72	3.08	6.74	3.20	7.32
Specific Gravity	2.60	2.64	2.68	2.64	2.67
Absorption (%)	0.9	1.9	1.3	2.1	1.7

* 6 Percent of Passing 0.15 mm was Introduced; Fine Part of Sand D or Pulverized Gravel II

Table 3 Concrete mix Proportioning.

Material	Quantities of Materials, kg/m³		
	Part I	Test Program Part II	
	Mix No. 1	Mix No. 2	Mix No. 3
Cement	331 *	331**	332**
Water	194	152	153
Sand			
Fine-A	106	—	—
Coarse-B	668		—
Sand-D	—	44	—
Pulv. Gravel-II	—	—	44
Sand-C	—	689	689
Gravel			
I	1 124	—	—
II	—	1 090	1 090

* ASTM Type II Cement, Brand-S
** ASTM Type II Cement, Brand-C

ASTM type II portland cement, brand S. Its chemical analysis is given in Table 1.

Table 4 Properties of Concretes*

Items	Test Progran Part I	Test Program Part II	
	Mix No. 1	Mix No. 2	Mix No. 3
Cement Cont., kg/m³	331	331	332
S/A by WT	0.41	0.40	0.40
W/C by WT	0.59	0.46	0.46
Slump, cm	10.8	3.8	2.5
Air Cont., %	1.5	1.4	1.5

Age in Days	Strength (kg/cm²) : Average of three 7.5 by 15 cm Cylinders					
	Tensile	Comp.	Tensile	Comp.	Tensile	Comp.
3	11.3	86	17.9	143	17.9	148
7	18.6	140	24.6	215	25.8	232
14	25.2	193	39.0	290	35.0	291
28	29.4	249	45.2	374	43.8	379

*20 mm Max. Size Aggregates

b) Aggregates The aggregates used were from the Coast Range of Central California. They included a gravel of 20 mm maximum size and a blend of a fine (F.M. 1.72) and a coarse (F.M. 3.08) sands. The physical properties of the aggregates are given in Table 2.

c) Concrete Mix The concrete mix proportion used is Mix No. 1 given in Table 3. The nominal cement content was 330 kg per cubic-meter and the slump was 10 cm. The properties of fresh concrete are given in Table 4.

4. Manufacture of Specimens

a) Mixing of Concrete The concrete was mixed in a 70 liters, pan-type, Lancaster mixer. The aggregates and cement were placed in the mixer first, mixed together for one minute, then water was added during the next full minute of mixing. the mixing was then continued for another two minutes. The concrete so mixed was used for casting specimens. Mixing and casting were done at 21°C.

Shrinkage and Creep Specimens

All specimens were cast in 7.5 by 30 cm cardboard molds. The molds were filled in three successive 10 cm. layers and a table vibrator was used for consolidation of each layer of concrete. Immediately after the casting, specimens were stored in the curing room (21°C and 100 percent relative humidity). They were stripped after 24 hours and remained in the curing room until age of test. The specimens are classified into four groups according to their functions in the test program, namely : (1) shrinkage, (2) restrained-shrinkage, (3) creep, and (4) control specimen.

a) Shrinkage Specimen The shrinkage

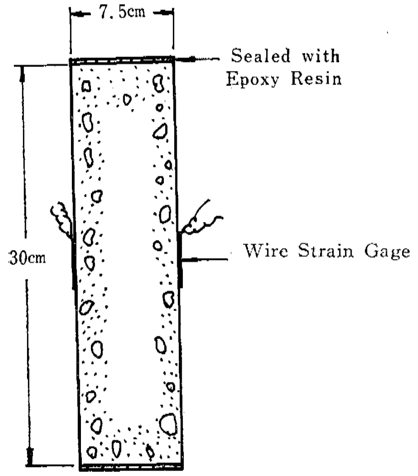

Fig. 4 Free Shrinkage Specimen, 7.5 by 30 cm Cylinder with two Strain Gages at mid-height, 180° Apart.

specimen is illustrated in Fig. 4. Both ends of the specimen were sealed with epoxy-resin in order to obtain the same condition of exposure to the surrounding atmosphere as the restrained-shrinkage specimen. Two strain gages were fixed on the surface of each specimen, placed 180° apart.

b) Restrained-Shrinkage Specimen The scheme of external restraint upon shrinkage is illustrated in Fig. 5. A 16 mm thick slice of concrete was cut away from the top end of each specimen to provide stronger bond with the top plate, then both ends of the cylinder were sand-blasted and glued to the aluminum triangular plates with epoxy-resin. Specimens were restrained by three steel rods fixed to the end plates by nuts as shown in Fig. 5. After the epoxy-resin glue had hardened, the nuts were tightened against the inner sides of the plates. As the specimens contracted the steel rods were subjected to compression and the concrete to tension. Two strain gages were attached to each rod, placed 180° apart and connected in series to cancel the bending effect on the rods. The elastic shortening of the rods, as measured by the strain gages, provided data for computing the tensile stress in concrete. The specimens cracked after certain duration of storage in the low humidity room (21°C and 50 percent relative humidity).

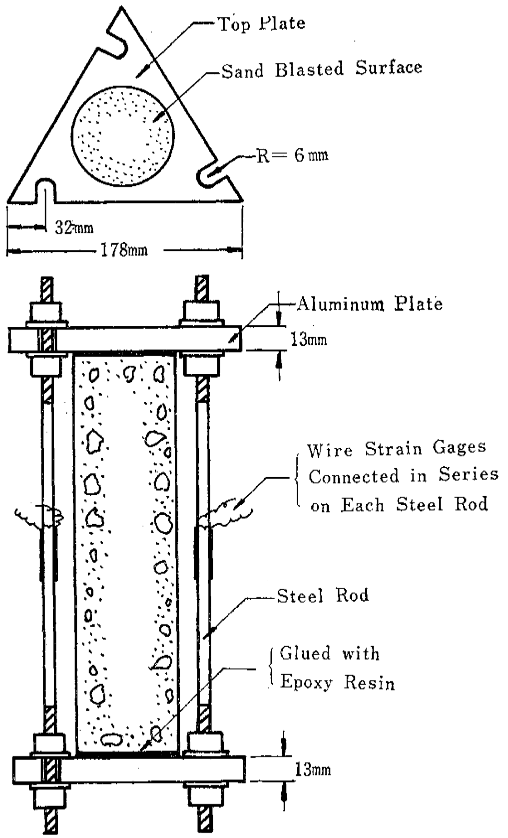

Fig. 5 Restrained Specimen, Originally 7.5 by 30 cm Cylinder, 6 mm was Spliced off from the Top End to Provide Better Bond.

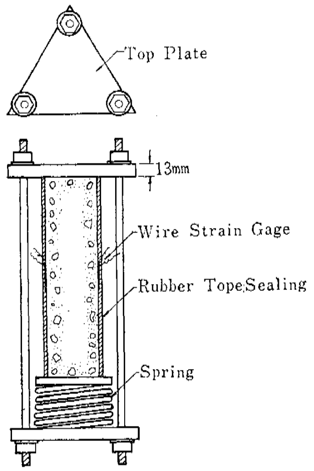

Fig. 6 Creep Specimen in Compression, 7.5 by 30 cm Cylinder, Sealed with Rubber Tape.

c) Creep Specimen Tensile creep specimens were prepared to obtain a comparison with the creep computed from shrinkage and restrained-shrinkage. Compressive creep specimens were also prepared to furnish the data for creep curves as shown in Fig. 3. The loading and

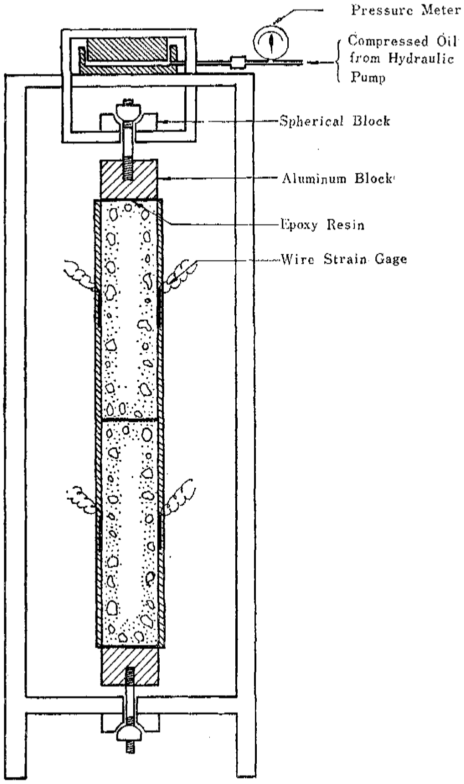

Fig. 7 Installation for Tensile Creep of Concrete, two 7.5 by 30 cm Cylinders Installed in Series, Glued to each other with Epoxy Resin.

instrumentation of creep specimens are presented in Figs. 6 and 7.

d) Control Specimen Control specimens were cast and tested to determine both tensile and compressive strengths as well as modulus of elasticity of concrete. The specimens were 7.5 by 15 cm cylinders, cut from the original 7.5 by 30 cm cylinders. The modulus of elasticity obtained from the compression test was substituted for the tensile modulus. One of the control specimens was employed to investigate non-uniform distribution of shrinkage over the cross section of a concrete cylinder, embedding wire strain gages at several locations. Results of this test were used to eliminate the effect of non-uniform distribution of shrinkage upon measurements of deformation in the shinkage specimen.

5. Measurements

a) Shrinkage, Restrained-Shrinkage, and Creep After completion of the moist-curing period of 3 days, initial measurements were

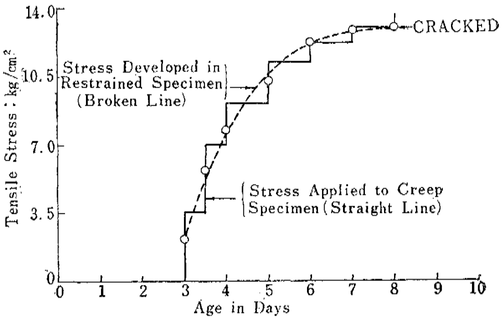

Fig. 8 Stress Increments Applied to Creep Specimen to Obtain the Similar Effect of Tensile Creep as obtained in Restrained Specimen. At the Age of 8 days, Restrained Specimen Cracked.

taken on shrinkage, restrained-shrinkage, and creep specimens and then these specimens were transfered to the low humidity room (50 percent relative humidity) where measurements of length change were taken every 24 hours for two weeks. The tensile stress developed in restrained-shrinkage specimens was computed by Eq. (1), in which $A_s=2.14$ sq. cm, $E_s=2.1\times10^6$ kg/cm², $A_c=44.7$ sq. cm, and δ_R is given in cm/cm,

The tensile creep specimens were first loaded at age 3 days and subsequently subjected to loads corresponding to the tensile stress developed in the restrained-shrinkage specimens : stress increments for loading these specimens were selected so that the stress in the creep specimen at any time was close to the actual stress in the restrained-shrinkage specimen at the corresponding time. The stress increments applied are shown in Fig. 8. Compressive creep specimens were loaded to 14 kg/cm² at ages of 3, 4, 5, 7 and 8 days. Based on the creep curves obtained, the compressive creep values were computed for the loads equivalent to the ones applied on the tensile creep specimens.

b) Strengths and Modulus of Elasticity Compressive strength tests on 7.5 by 15 cm cylinders at ages of 3, 7, 14 and 28 days were made in accordance with ASTM Method C 31-61. Hydrostone was used for capping the ends of cylinders. Tensile strength was obtained by the splitting test method, in accordance with Japan Industrial Standard A 1113-1951. Again 7.5 by 15 cm cylinders were employed and tested at the same ages as used in the compressive

Table 5 Modulus of Elasticity, Shrinkage, Restrained Shrinkage, and Tensile Stress due to Restraint.

Age in Days	Modulus of Elasticity (10^6 kg/cm^2)	Shrinkage (10^{-6}cm/cm)	Restrained Shrinkage (10^{-6} cm/cm)	Tensile Stress (kg/cm^2)
3	0.103	—	24	2.1
3 1/2	0.113	124	57	5.7
4	0.123	172	78	7.7
5	0.143	227	103	10.2
6	0.159	266	124	12.3
7	0.175	284	130	12.8
8	0.189	295	131	13.0

Table 6 Elastic Deformation Plus Creep Under Sustained Unit Stress, as Obtained by Compressive Creep Test on 7.5 by 30 cm Cylinders.

Age in Days	Unit Length Change Per Unit Stress, 10^{-6} cm/cm/(kg/cm^2)						
	Age of Initial Loading in Days						
	3	3 1/2	4	5	6	7	8
3	9.72	—	—	—	—	—	—
3 1/2	11.14	8.86	—	—	—	—	—
4	12.00	9.72	8.14	—	—	—	—
5	13.00	10.86	9.57	7.00	—	—	—
6	13.58	11.72	10.43	8.29	6.29	—	—
7	14.00	12.43	11.00	9.14	7.57	5.72	—
8	14.29	12.86	11.43	9.57	8.14	7.00	5.29
9	14.57	13.00	11.71	9.72	8.58	7.57	6.43

strength tests. The modulus of elasticity in compression was obtained on 7.5 by 15 cm cylinders by the compressometer method specified in Test Method CRD-C 19-55, Corps of Engineers, U.S. Army.

6. Test Results

a) Strengths and Modulus of Elasticity Compressive and tensile strengths at ages of 3, 7, 14

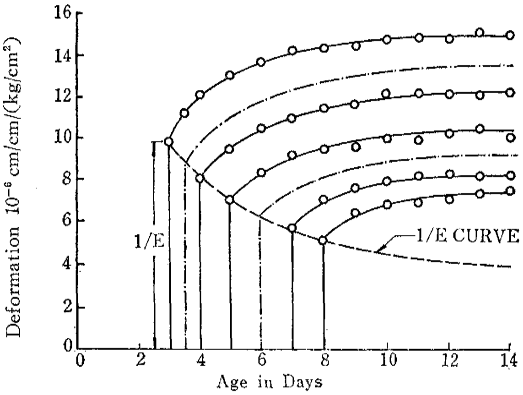

Fig. 9 Total Deformation due to Sustained Unit Stress, 7.5 by 30 cm Cylinder Initially Loaded at Various Ages, and Stored at 21°C. Dotted Lines are Interpolated.

Table 7 Computation for Creep of Concrete Subjected to an Intermittently Increased and Sustained Loading on 7.5 by 30 cm Cylinders.

Row \ Column	(1) Age of Loading Days	(2) Stress Increment $\Delta\sigma$ kg/cm^2	(3) Accum. Stress σ kg/cm^2	(4) Elastic Deform. $1/E$ 10^{-6}cm^2/kg	(5) Deformation due to $\Delta\sigma$ at Each Age, $\varepsilon=\Delta\sigma\times\varepsilon_U$. ε_U=(Elastic Deformation +Creep) due to Unit Stress. ε_U Are Found in Table 6 and Quoted Here in the Upper Right Corner of Each Block. $\varepsilon=10^{-6}$ cm/cm, $\varepsilon_U=10^{-6}$ cm/cm/(kg/cm^2).							
					3 DA.	3 1/2 DA.	4 DA.	5 DA.	6 DA.	7 DA.	8 DA.	9 DA.
(1)	3	3.5	3.5	9.72	9.72 34	11.14 39	12.00 42	13.00 46	13.58 48	14.00 49	14.29 50	14.57 51
(2)	3 1/2	3.5	7.0	8.86		8.86 31	9.72 34	10.86 38	11.72 41	12.43 44	12.86 45	13.00 46
(3)	4	2.1	9.1	8.14			8.18 17	9.57 20	10.43 22	11.00 23	11.43 24	11.71 25
(4)	5	2.1	11.2	7.00				7.00 15	8.29 17	9.14 19	9.57 20	9.72 20
(5)	6	1.1	12.3	6.29					6.29 7	7.57 8	8.14 9	8.58 9
(6)	7	0.7	13.0	5.72						5.72 4	7.00 5	7.57 5
(7)	8	0.7	13.7	5.29							5.29 5	6.43 5
(A)	Elastic Deformation Plus Creep=Σ col. (5) 10^{-6} cm/cm				34	70	93	119	135	147	157	161
(B)	Elastic Deformation=$\sigma\times1/E$ 10^{-6} cm/cm				34	62	74	78	77	74	72	70
(C)	Compressive Creep=(A)−(B) 10^{-6} cm/cm				0	8	19	41	58	73	85	91
(D)	Elastic Deformation Plus Creep by Tensile Creep Test. 10^{-6} cm/cm				32	65	98	122	145	159	167	--
(E)	Tensile Creep by Tensile Creep Test=(D)−(B) 10^{-6} cm/cm				—	3	24	44	68	85	95	-

and 28 days are given in Table 4. Modulus of elasticity of concrete are listed in Table 5 up to the age of 8 days, when the restrained-shrinkage specimens cracked.

b) Shrinkage and Restrained-Shrinkage Also listed in Table 5 are the values of shrinkage, restrained-shrinkage, and the tensile stress due to the restraint of shrinkage.

c) Creep The total deformations, i.e., elastic deformation plus creep, due to a sustained unit stress observed on compressive creep specimens, are given in Table 6 and they are also illustrated in Fig. 9. Results of the tensile creep tests are given in row (D) of Table 7.

7. Computation of Creep and Stress-Relaxation

a) Tensile Creep by Tensile Creep Test Tensile creep by the direct tensile creep test was obtained simply subtracting the elastic deformation from the total observed deformation as shown in row (D) of Table 7.

b) Compressive Creep The reader is refered to Table 7 and Fig. 3. The ages of initial loadings, stress increments, and accumulated stress are given in columns (1), (2) and (3), in Table 7, respectively. Column (4) presents the elastic deformation due to a unit stress at various ages. Column (5) is in itself a table containing of many blocks. Each block contains a smaller block at the uper right corner, where a number is given to repersent the elastic deformation plus creep due to a unit stress at age K_2 when the load is applied at age K_1 ($K_2 \geq K_1$) (obtained from Table 6). The total deformation due to stress increment $\Delta\sigma$ applied at age K_1 is obtained by multiplying the number given in the smaller block by $\Delta\sigma$, the result of which is given in the larger block. For example, the total deformation at age 6 days due to a stress increment $\Delta\sigma = 3.5\ \mathrm{kg/cm^2}$ applied at age 3 1/2 days was computed as follows :

total deformation per unit stress $= 11.72 \times 10^{-6}\ \mathrm{cm/cm/(kg/cm^2)}$ $\Delta\sigma = 3.5\ \mathrm{kg/cm^2}$

$\varepsilon_{\mathrm{total}} = 11.72 \times 10^{-6} \times 3.5 = 41 \times 10^{-6}\ \mathrm{cm/cm}$

All the deformation values (given in column 5 of Table 7) at any age due to stress increments applied at various ages were found in this manner. Making the summation of the deformations for each age, the total deformations due to an intermittently increased load were computed and are given in Table 7, row (A). Compressive creep was obtained as shwon in row (C) by subtracting the elastic deformation given in row (B) from the total deformation of row (A).

c) Tensile Creep by External Restraining Methods Tensile creep by the external restraining method was computed from shrinkage and the restrained-shrinkage by Eq. (5), and is given in Table 8. To permit a comparison with the values of creep obtained by tensile and compressive creep tests, the values of creep in Table 8 are given in terms of a unit length

Table 8 Creep as Determined by Three Test Methods and Stress-Relaxation on 7.5 by 30 cm Cylinders Moist-Cured for 3 Days.

Age in Days	Creep 10^{-6} cm/cm			Stress Relaxation, (kg/cm^2)
	Tensile*	Compressive**	Restraint***	
3	--	—	—	—
3 1/2	3	8	5	0.6
4	24	19	20	2.5
5	44	41	46	6.6
6	68	58	65	10.4
7	85	73	80	14.0
8	95	85	92	17.4

* by Direct Tensile Creep Test
** by Compressive Creep Test
*** by External Restraining Method

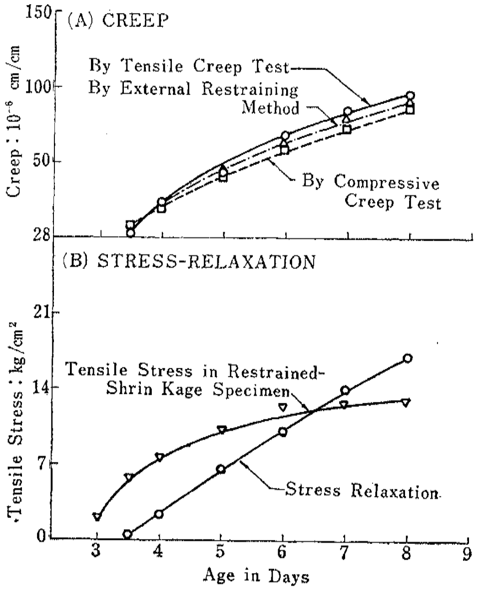

Fig. 10 Creep and Stress-Relaxation by External Method, and Tensile and Compressive Creep Tests. 7.5 by 30 cm Cylinders, Moist-Cured for 3 days.

change, cm/cm.

d) Stress-Relaxation Stress-relaxation by the external restraining method was computed by Eq. (2) and shown in Table 8, and in Fig. 10.

e) Summary on Creep Test Results The final results obtained from the three types of creep tests are summarized in Table 8 and illustrated in Fig. 10. From these results it may be observed that the tensile creep obtained by the external restraining method is fairly close to those obtained in tensile and compressive creep tests. Since none of these creep values represent what actually takes place in the restrained-shrinkage specimens in the strict sense of the word, it may be erroneous to make distinct conclusions from the above presented results. At least, however, these tests and analysis are considered to suggest the applicability of the restraining methods of test for the study of tensile creep.

TEST PROGRAM PART II : INTERNAL RESTRAINING METHOD

In the following sections are reported the results of tests in which the internal restraining method was employed. In this method both the shrinkage and the restrained-shrinkage were obtained by measuring the total length change of the specimens using a horizontal extensometer. This procedure of measuring the total length of a specimen is considered to give the average value of shrinkage more or less eliminating any error due to the nonuniform distribution of shrinkage in the specimen.

8. Concrete Materials and Mixes

a) Portand Cement The portland cement employed in this part of the test program was a type II portland cement, brand C. Its chemical analysis is given in Table 1.

b) Aggregates Gravel of 20 mm maxium size (gravel II) and sand (sand C) from the Coast Range of Central California were employed. Sand C contained either one of two types of fine materials passing 0.15 mm sieve in the amount of 6 percent of the total sand. One of these was the fine part of sand D and the other was pulverized gravel II. The sand D was also from a Coast Range in California. The only reason for adding these fines was to obtain a sand that would have sufficient material (6 percent) passing the 0.15 mm sieve. The physical properties of the aggregates are given in Table 2.

c) Concrete Mixes Two mixes were prepared, namely, Mix No. 2 and Mix No. 3, the former cantains sand D as extra fines and the latter pulverized gravel II as extra fines. Both mixes were almost identical as shown in Table 3. The nominal cement content was 330 kg per cubic-meter for both mixes.

9. Manufacture of Specimens

a) Mixing of Concrete The same method of mixing was employed as described in the test program of Part I.

b) Test Specimens Shrinkage and restrained-shrinkage specimens were cast in 7.5 by 7.5 by 100 cm molds. In order to produce the restraining effect for the restrained-shrinkage bars, a steel rod of 20 mm diameter was secured in the mold longitudinally along the centerline of the mold. As shown in Fig. 11, this steel rod was threaded at each end for a distance of 7.5 cm to provide anchorage. To prevent bond along the remaining 85 cm length, the bar was covered with a 3 mm-thick soft rubber tube. All molds used for casting of shrinkage specimens had 13 mm diameter brass plugs secured in the ends of the molds for embedment in the concrete. These plugs projected 13 mm out of the concrete specimens to permit length measurements. A similar 13 mm projection was provided on the

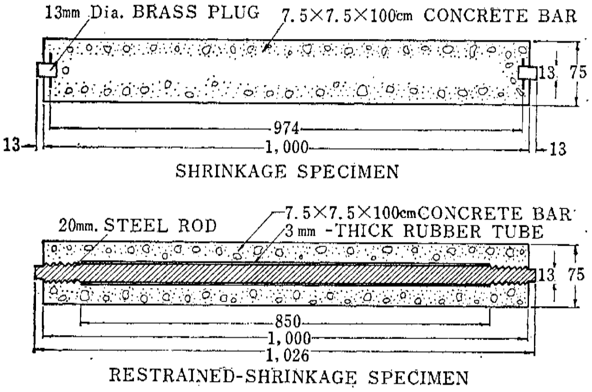

Fig. 11 Shrinkage and Restrained-Shrinkage Specimens for Internal Restraining Method, unit=mm.

steel bars of the restrained-shrinkage specimens. Specimens for compressive strength, tensile strength, and modulus of elasticity were cast in 7.5 by 15 cm cylinder molds.

c) Casting of Specimens The molds for shrinkage and restrained-shrinkage bars were filled in two successive 3.8 cm layers. A table vibrator was used for consolidation of the concrete. Immediately after casting, specimens were stored in the moist curing room. They were stripped after 24 hours and remained there until age of test. Standard procedure was employed for casting of 7.5 by 15 cm concrete cylinders.

10. Measurements

After completion of a 14 days moist-curing period, initial measurements were taken on both the shrinkage and restrained-shrinkage bars. The bars were then transfered to the low humidity room (50 percent relative humidity), where length measurement were taken every 24 hours until a crack occured in the restrained-shrinkage bars. All measurements were made using a 100 cm horizontal extensometer.

a) Shrinkage and Restrained-Shrinkage The shrinkage was computed by dividing the total change in length by the effective length (97.4 cm) of the concrete bars. The length change of the steel bar was observed for the restrained-shrinkage and then tensile stress in the concrete was calculated using Eq. (1), in which $A_s=2.85$ sq. cm, $A_c=52.9$ sq. cm ,and $\delta_R=\varDelta L/L$. $\varDelta L$ is the measured deformation of steel rod and L is the effective length (92.5 cm) of the restrained bars. This 92.5 cm effective length is based on the assumption that the stress distribution at the end of the bar varies linearly from 0 kg/cm² to maximum over the 7.5 cm threaded part at each end.

b) Strengths and Modulus of Elasticity Tests for compressive strength, tensile strength, and modulus of elasticity of the concrete were performed on 7.5 by 15 cm cylniders employing the same methods as described in the test program Part I.

11. Test Results

a) Strengths and Modulus of Elasticity Compressive and tensile strengths at ages of 3, 7, 14 and 28 days are shown in Table 4. Modulus of elasticity tests were made on concrete cylinders moist-cured for 14 days and then subjected to the identical drying atmosphere as shrinkage specimens. The results are given in Table 9.

Table 9 Shrinkage, Restrained Shrinkage, Tensile Stress, and Modulus of Elasticity of Concretes Moist-Cured for 14 Days.

Days at 21°C and 50% R.H.	Shrinkage 10^{-6} cm/cm	Restrained Shrinkage 10^{-6} cm/cm	Tensile Stress kg/cm²	Modulus of Elasticity 10^6 kg/cm²
		Mix No. 2		
0	—	—	—	0.319
1	110	88	10.5	0.325
2	173	116	13.8	0.332 *
3	210	135	15.4	0.338
4	244	150	17.9	0.344 *
5	274	162	19.3	0.350 *
6	301	Cracked	Cracked	0.354
		Mix No. 3		
0	—	—	—	0.332
1	118	81	9.7	0.335
2	165	99	11.8	0.341 *
3	204	118	14.1	0.349
4	236	134	16.0	0.352 *
5	272	148	17.6	0.357 *
6	300	160	19.1	0.362
7	330	175	20.9	0.370
8	357	188	22.4	0.376 *
9	386	198	23.7	0.382 *
10	409	Cracked	Cracked	0.389

* Graphycally Interpolated

b) Shrinkage and Restrained-Shrinkage Shrinkage and restrained-shrinkage tests were made on concrete specimens moist-cured for 14 days prior to drying at 21°C and 50 percent relative humidity. Values of shrinkage and restrained-shrinkage for the two concrete mixes are given in Table 9. The tensile stress due to the restraint is also listed in Table 9.

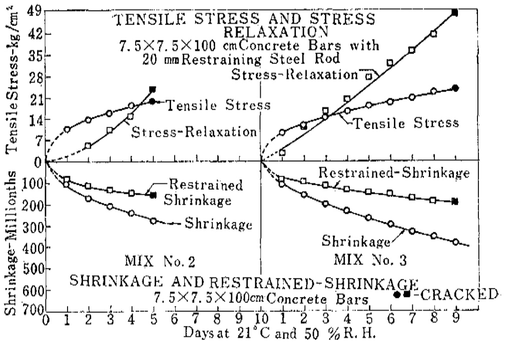

Fig. 12 Tensile Stress, Stress Relaxation, Shrinkage and Restrained-Shrinkage of Concretes Moist-Cured for 14 days.

Table 10 Tensile Creep and Stress Relaxation by Internal Restraining Method.

Mix	Days at 21°C & 50 % R.H.	Stress Relax. $E(\delta_F-\delta_R)-\sigma$ kg/cm²	Total Deform. $(\delta_F-\delta_R)/\sigma$ 10^{-6} cm/cm/(kg/cm²)	Elastic Deform. $1/E$ 10^{-6} cm/cm/(kg/cm²)	Creep $(\sigma_F-\delta_R)/\sigma-1/E$ 10^{-6} cm/cm/(kg/cm²)
No. 2	1	—	2.10	3.09	—
	2	5.1	4.13	3.01	1.12
	3	9.9	4.73	2.96	1.77
	4	14.4	5.27	2.92	2.35
	5	23.8	5.80	2.86	2.94
No. 3	1	2.7	3.83	2.99	0.84
	2	10.6	5.59	2.93	2.66
	3	15.9	6.11	2.87	3.24
	4	19.8	6.36	2.84	3.52
	5	26.6	7.03	2.80	4.23
	6	31.6	7.33	2.76	4.57
	7	36.5	7.43	2.70	4.73
	8	41.1	7.54	2.66	4.88
	9	48.1	7.94	2.62	5.32

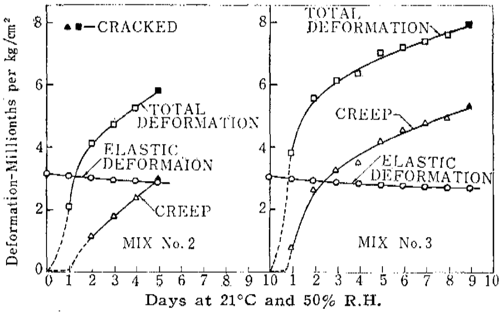

Fig. 13 Total and Flastic Deformations, and Creep per unit Stress. 7.5 by 7.5 by 100 cm Concrete Bars with 20 mm Restraining Steel Rod, Moist-Cured for 14 days.

12. Computation of Creep and Stress-Relaxation

Based on the data shown in Table 9, stress-relaxation and tensile creep of the two concrete mixes were computed using Eqs. (2) and (5), respectively, and the results are presented in Table 10 and Figs. 12 and 13.

DISCUSSION AND RECOMMENDATIONS

1. To complete the study under the limitted conditions, it was inevitable to substitute the compressive modulus of elasticity of concrete for the tensile in computations for creep and stress relaxation assuming the identity of the two. As far as the comparison of creep data obtained by several methods is concerned, no serious discrepancy has resulted from this assumption, since the modulus was introduced as a common factor. This assumption may possibly be justified at low stress levels, but not necessarily, for the high tensile stress level obtained prior to cracking of the concrete; this was not investigated. It is recommended, therefore, to employ the actual tensile modulus of elasticity obtained through tests in future studies.

In this investigation, restraining steel rods were of one size for each part of the program, consequently the effect of only a simple percentage of restraint upon shrinkage was studied. By employing various sizes of restraining rods, it would be possible to select a particular restraining effect upon shrinkage which would enable observations of tensile creep or stress-relaxation of concrete over a longer period of time.

2. In the external restraining method, it is necessary to use a glue such as an epoxy-resin to fix the loading plates or wire strain gages to the concrete. It was quite difficult to obtain sufficient bond between them without losing some moisture from the concrete. This moisture loss might seriously affect the initial measurements of deformation. An aluminum plate was found to provide for a better bond than a steel plate. In this test program the creep or elastic deformation of the hardened epoxy resin was neglected, although a minimum amount of the resin was used and sufficient time of curing provided. The deformation of the resin should be considered in future studies to improve accuracy of results. The same will apply for the direct tensile creep test of concrete.

When the deformation of concrete is measured on the surface of the specimens, attention should be paid to the effect of nonuniform distribution of shrinkage upon the measurements. In general, shrinkage is larger on the surface than inside the body of the concrete. In this test program, the maximum difference between the two amounted to about 70×10^{-6} cm/cm in a case of the shrinkage specimen of 7.5 by 30 cm cylinder. The measurement of elastic deformation on restraining steel rods is considered to yield more or less the average value of deformation of the specimen.

3. In the internal restraining method, the

effective lengths for shrinkage and restrained-shrinkage bars were set as 97.4 and 92.5 cm, respectively. They were determined considering the embedded length of end plugs and a uniform distribution of shrinkage along the center line of the bar for shrinkage specimens, and a uniform distibutrion of tensile stress over the rubber tubed length of 85 cm and a linear stress distribution over the threaded part of 7.5 cm at both ends for restrained shrinkage specimens. Alternate effective lengths can be employed on proper patterns of shrinkage and stress distribution assumed.

4. In both restraining methods, the tensile stress developed in the restrained-shrinkage specimen increases continuously until the specimen cracks, and naturally the creep under such a loading will be different from the one under a sustained load or under an intermittently increased load. Both the plastic and elastic characteristics of concrete in tension or in compression are considered to be affected by the loading history and magnitude of load intensity.

The restraining methods presented are considered to be applicable for such practical purposes as an investigation of the tensile stress-relaxation or the tensile cracking in reinforced concrete members. However, further studies should be made if these methods of test are to evaluate tensile creep of concrete.

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REFERENCES

1) Akatsuka, Yuzo : EFFECTS OF CLAY-LIKE MATERIALS ON STRENGTH, SHRINKAGE, AND CRACK-RESISTANCE OF CONCRETE, Graduate Student Research Report, Univ. of Calif. at Berkeley, Sept. (1961).
2) Chang, Shao-Chien : TENSILE CREEP OF CONCRETE DUE TO VARIOUS STRESSES CAUSED BY RESTRAINED-SHRINKAGE, Graduate Student Research Report, Univ. of Calif. at Berkeley, Jan. (1962).
3) Carlson, Roy W. : ATTEMPTS TO MEASURE THE CRACKING TENDENCY OF CONCRETE, Proc. of ACI, Vol. 36, pp. 533-540, June. (1940).
4) Troxell, George E. and Harmer E. Davis : COMPOSITION AND PROPERTIES OF CONCRETE, McGraw-Hill Book Co., New York, p. 246, (1956).

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