

PRACTICAL FORMULAS OF THE CURVED BRIDGE  
WITH MULTIPLE PLATE GIRDER

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1. Introduction

The practical formulas for design work of the curved bridge with multiple plate girders spaced along the concentric circles is derived. In previous papers, authors developed the fundamental theory of most general curved girder bridge and found its explicit solutions by the three dimensional analysis. Applying the results, we can rationally calculate about this kind of bridge structure through the following process;

(a) taking in account of the radius of curvature, we firstly calculate the important fundamental quantities concerned with the transverse section of bridge,

(b) and then calculate both stress resultants and distortions in the whole section of bridge by the analysis in three dimensions based on the assumption that the transverse section of bridge does not deform at all,

(c) finally determine the maximum stress as well as the reactions from the stress resultants by using described formulas that contain influence of the curvature.

The process (a) shall be given at section 3 ~ 6.

2. Loading condition in the case of torsion bending

In the case where the resultants of external load pass through the center of curvature  $O$ , the torsion bending will be able to occur without symmetrical bending.

3. Centroid of transverse section of curved bridge. (Fig. 2)

The rectangular coordinates  $(\bar{y}_0, \bar{z}_0)$  to determine the situation of centroid are given by formula (10), where  $F_{sj}$  is the cross sectional area,  $R_j$  is the radius of curvature, and  $(y_j^*, z_j^*)$  are rectangular coordinates of a centroid of the  $j$  th girder.

Furthermore  $\sum_j$  denotes the summation over the transverse section of bridge.

4. Flexural properties of the transverse section of bridge. (Fig. 3)

The moment of inertia about  $O_n y$ ,  $O_n z$  and the product of inertia with reference to these axes are given by formulas (12), where  $I_{\bar{y}j}$ ,  $I_{\bar{z}j}$  and  $I_{\bar{y}zj}$  are the moments of inertia and the product of inertia about  $O_j \bar{y}_j$ ,  $O_j \bar{z}_j$  at the section of the  $j$  th girder alone,  $O_n$ ,  $O_j$  are the centroids of the transverse section of bridge and the  $j$  th girder respectively,  $(y_{oj}, z_{oj})$  are  $(y, z)$  coordinates of centroid  $O_j$ , and finally  $(y, z)$  are rectangular coordinates taken in the horizontal and vertical directions through an origin  $O_n$ .

## 5. Shear center in transverse section of bridge

Denoting the  $z$  coordinate of shear center  $S$  in the transverse section of bridge by  $z_s$ , and that of the  $j$  th girder alone by  $z_j$ , we can determine the situation of the shear center  $S$  by formulas (22) as well as (24)', where  $R_o$  is distance between shear center  $S$  and the center of curvature  $O$  and  $R_{oj}$  is that of shear center  $S_j$  concerned with the  $j$  th girder alone.

In most general bridge, we may take  $R_{oj}$  to be equal to  $R_j$ .

## 6. Torsional resistance and torsion bending constant

The torsional resistance  $J$  as well as the torsion bending constant  $C_w$  of the transverse section of bridge are given by formulas (31) and (32) respectively.

## 7. Distributed shear forces in each main girder

Under arbitrary vertical loading, the cross girder in cooperation with the lateral bracing have important action in distributing the load to each main girders. Due to this phenomena, the shear force  $Q_j$  induced in the cross section of the  $j$  th girder is given as a function of four stress resultants i.e. total shear force  $Q$ , total torsional moment  $T$ , St. Venant's torsional moment  $T_s$  and secondary torsional moment  $T_w$  by formula (34), where  $J_j$ ,  $C_{wj}$  represent torsional resistance and torsion bending constant of the  $j$  th girder respectively, and  $Y_j = R_{oj} - R_o$ .

## 8. Distributed torsional moments in each main girder

The torque  $T_j$  developed by each girder is obtained by formula (36).

## 9. Distributed bending moments in each main girder

The bending moment  $M_{y_j}$  induced in the  $j$  th girder is given by formula (37), where  $d^2\theta/d\varphi^2 + \theta$  is nothing but distortions and some solutions for them are shown as the function of situation of transverse section in Table-1 corresponding to various loading conditions.

## 10. Reactions at supports

### (1) I-girder bridge (Fig. 5a)

A reaction  $R_{ja}$  at the support  $\varphi = a$  under the web plate in the  $j$  th girder is readily obtained as the difference between two shear forces  $Q_{j,\varphi=a} - 0$  at just front section and  $Q_{j,\varphi=a} + 0$  at just behind section of the support. The relation among them is seen in formula (38)

### (2) Box girder bridge (Fig. 5b)

In the case of a box girder, two supports are generally placed on a pier to resist to torsion, since we need to find the reactions at each supports. So both outside reaction  $R_{ja}$  and the inside reaction  $R_{ji}$  are readily obtained in formulas (39)<sub>1</sub> and (39)<sub>2</sub> respectively.

#### 11. Torsional function

If the  $j$  th girder has one cellular section, and its breadth is considerably small in comparison with the radius of curvature, the torsional function  $q_j$  may be given by formula (40) approximately.

#### 12. Stress formulas

##### (1) Normal stress

The normal stress  $\sigma_j$  at any point in cross section of the  $j$  th girder is easily obtained by well-known formula (41).

##### (2) Shear stress

The shear stress  $\tau_j$  at any point in cross section of the  $j$  th girder is also obtained by formula (42). In this formula the 1st term is shear stress due to bending and the 2nd one is due to St. Venant's torsion.

#### 13. Secondary stress in I-girder bridge

Under arbitrary vertical loading the tension or compression along the axis of flange plate always applies. Due to resultant of them in direction of the radius through a center of curvature  $O$ , the local bending can be developed in the flange plate, therefore the secondary stress must be added to primary stress  $\sigma_j$  described in section 12 for evaluating the maximum stress of cross section.

Let me propose four approximate formulas (43)~(46) for the secondary stress  $\sigma_2$  corresponding to the situations of the panels and the cross sections in question of the  $j$  th girder. In order to decrease the secondary stress in flange plate, it is much favorable to make us of box girder for the curved bridge.

#### 14. Stress resultants in cross girder (Fig. 7)

The bending moment and the shear force at the middle section of cross girder situated between the  $j$  th main girder and the  $j + 1$  th one are gradually found by solving a pair of formulas (47) and (48) in the order of the magnitude of number  $j$ .

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ON THE BOND STRESS DISTRIBUTION ALONG ROUND BARS,  
DEFORMED BARS AND TWISTED DEFORMED BARS

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The authors measured the bond stress distribution along various reinforcing bars by improving the method of Mains. At first a half section of a bar was milled out and then the other half section was milled to provide a longitudinal channel to allow several electric strain gages to be mounted and the lead wires to be drawn out. Two halves with strain gages were combined together into a single specimen by means of the bonding compound, instead of tack welding by Mains. This method was effective for prevention of heating of the gages as well as for water-proof. A special enamel wire 0.35 mm in diameter was used as a lead wire.

As the bond stress is computed from the difference of normal forces acting on two adjacent sections, the former can be translated from the normal stress distribution measured by a series of strain gages.

The authors made the pull-out tests, the push-out tests, the beam bond tests, and at the same time the beam crack tests.

In the pull-out tests, a marked difference in the tendency of bond stress distribution was recognized between the round bars with the deformed bars and twisted deformed bars.

In the case of round bars, we can distinguish the following three stages. At the beginning of test, the maximum bond stress and the effective bond length increase as well, according as the load increases. At the next stage, the increase of the effective bond length is remarkable, but the increase of the stress itself is not so much. At last, the maximum bond stress increases rapidly and the failure takes place.

On the contrary, in the cases of deformed bars and twisted bars, the effective bond length is shorter than in the case of round bars and its enlargement is comparatively slow, in spite of the remarkable increase of the maximum bond stress due to the load increase. The peak point of the bond stress curve lies near the pulling end, staying almost immovable, and the tensile stress curve tends to zero asymptotically toward the unloaded end.

In the push-out tests, there are always two crests of the bond stress distribution curves. The heights of these crests grow almost proportionally to the load. But for round bars, after the beginning of slip, the crest near the pushing end goes down and the other crest continues to rise, their locations not moving. For deformed bars and twisted bars, the cracks appear from the loaded end and the failures take place immediately.

In the beam bond tests, the tensile stress of bars does not necessarily increase linear from the support to the loaded point, especially for deformed bars and twisted bars. In the region of constant moment between loads, the tensile stress in round bars shows quite uniform, but the tensile stress distributions for deformed bars and twisted bars are found to be wavy. Accordingly the bond stress curves in the last cases have several positive and negative peaks.

In the beam crack tests, it was shown that for the beams reinforced with deformed bars, the number of cracks is much more than in case of the beams with round bars, but the cracks in the former are quite narrower than in the latter. These facts make the former beams beneficial to increase the ultimate load and to prevent the various troubles caused by the wide cracks.

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# ON THE PROFILE OF SEDIMENTATION UPSTREAM OF DAMS

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## 1. Synopsis

When we plan to construct hydraulic structures across a river, such as dams, sluice gates, and jetties, or to make river improvement works widening the river width or changing the stream course, it is very important for engineers to know how the river bed profile will be changed after the improvement. This paper describes a theoretical approach to estimate the sedimentation profile upstream of low dams after the pools are filled up perfectly with bed load materials transported from upstream of rivers. To estimate the bed profiles in existing two rivers of which the river widths along the streams change considerably, dynamic and static equilibrium bed profile theories were applied. As a result of calculations, it was made clear that the static equilibrium bed profile theory is more preferable to such rivers than the dynamic equilibrium bed profile theory, and that the control flow discharge (dominant discharge)  $Q_n$  for the equilibrium bed profile plays an important role. Moreover, some statistical studies were made to find the correlation between  $Q_n$  and the annual peak discharge of flood flow.

## 2. Procedure of Calculations

Suppose an alluvial channel having rectangular cross section and gradually varied width in which the flow condition is considered gradually varied steady flow. Under the condition that the rate of sediment transport is constant along a stream, so that neither scouring nor deposition occurs at any section for a given flow discharge  $Q$ , the dynamic equilibrium bed profile is calculated, while the static equilibrium bed profile is estimated under the condition that the tractive force acting on a bed is equal to the critical tractive force at any section. Experimental formula proposed by Dr. Iwagaki is used to determine the critical shear velocity for given bed materials. For the resistance law and the bed-load formula, the experimental formulas proposed by the author are used. In case where the river width changes greatly as is seen upstream of most dams, it is convenient and practical to divide a stream into several reaches and proceed with the calculation from downstream toward upstream step by step, so as to satisfy the foregoing conditions.

## 3. Calculations for Model Channel

For a channel having a rectangular cross section and a plan of a spindle shape with a low dam at the downstream end, the dynamic and static equilibrium bed profiles were calculated for various discharges and bed material sizes to investigate the influence of the changes in channel width and bed materials upon the bed profile. As a result of calculations, the following may be stated:

- 1) The bed slope in the reach where the channel width increases toward downstream is rather gentle, and sometimes reverse slope appears.
- 2) In a channel of which the river width changes considerably, the influence of change in the channel width is more noticeable on the dynamic equilibrium bed profile than on the static one.
- 3) Large flow discharge tends to make the equilibrium bed slope gentle, and small flow discharge makes the equilibrium bed slope steep, if other conditions are the same.
- 4) Large size of bed materials is apt to make the slope of the equilibrium bed profile steep, if other conditions are the same.

#### 4. Examples Applied to Existing Rivers

##### (I) In the case of A Dam

Because about twenty six years have elapsed since A Dam was built, the reach of 11.5 km upstream of the dam is considered to be in the equilibrium state. The dynamic equilibrium bed profile calculated for various flow discharges are noticeably in disagreement with the existing profiles of both bed and water level, especially within the reach of narrow width. Accordingly, if there exist reaches of very narrow width and very wide width in a river, it seems to be inadequate to assume that the dynamic equilibrium state along the whole reach is established for a certain flood discharge. Moreover, some calculations of the static equilibrium bed profile were made to find the flood flow discharge which gives a relatively similar bed profile to the existing one, and its discharge was found to be  $500 \text{ m}^3/\text{s}$ . Therefore, the control flow discharge  $Q_n$  for the equilibrium bed profile was estimated at  $500 \text{ m}^3/\text{s}$  in this river.

##### (II) In the case of B Dam

B Dam is one of the most typical debris barriers in Japan, and there exists about six million cubic meters of deposited sediment upstream of the dam now. The river shape upstream of the dam is like a spindle, and the river width upstream of the dam is about 580 m at the widest section and 110 m at the most narrow section. It was built about twenty six years ago, the reach of 2 km upstream of the dam is considered to be in the equilibrium state. The calculations were performed by the static equilibrium bed profile theory, and  $Q_n$  was estimated at  $600 \text{ m}^3/\text{s}$ .

#### 5. Relation between $Q_n$ and annual peak discharge of flood flow

The relation between  $Q_n$  and the probability of annual peak flow discharge for the past thirty-six years at the B Dam site was investigated. It was found that  $Q_n$  corresponds to the discharge of 93% probability of exceedance. In the same way,  $Q_n$  at the B Dam site corresponds to the discharge of 80% probability, according to the records for the past twenty-nine years. As is seen in the foregoing two examples, there exist some difference in the values of  $Q_n$ . But, generally speaking, it may be said that  $Q_n$  is a

little larger than the least value among the annual peak discharge of flood flows. Provided the evaluation of  $Q_n$  and the size distribution of bed materials along the reach are made, it may be possible to estimate the equilibrium bed profile upstream of a dam.

## 6. Summary

This paper describes some fundamental factors to estimate the river bed profile upstream of a dam, and especially discusses the meaning of the control flow discharge (dominant discharge)  $Q_n$  for the equilibrium bed profile. The following are concluded:

- 1) A practical method to estimate the equilibrium bed profile upstream of a dam was proposed.
- 2) It is not preferable to make use of the dynamic equilibrium bed profile theory for the flood discharge, when the change of river width along the reach is considerable as is often seen upstream of a dam.
- 3) In the above case, the static equilibrium bed profile theory for a certain discharge  $Q_n$ , which is named the control flow discharge, gives a relatively similar bed profile to the existing one.
- 4) This process of calculation by the static equilibrium bed profile theory does not include any bed load formula which is generally unreliable in accuracy, so it is considered to yield more accurate results and lead to considerable simplification of calculations in comparison with the dynamic equilibrium bed profile theory.
- 5)  $Q_n$  will be possible to be estimated statistically from the past flood flow records. In the two examples of the existing rivers it was found that  $Q_n$  corresponds to the discharges of 93% and 80% probabilities of exceedance for the annual peak discharge of flood flow respectively. Generally speaking, that discharge seems to be a little larger than the least value among the annual peak discharges of flood flows.

In any existing river, therefore, if  $Q_n$  is estimated from the past records and the size distribution of bed materials can also be adequately estimated after the pool of a dam has been filled up completely with bed load materials, it will be possible to evaluate the equilibrium bed profile upstream of the dam.

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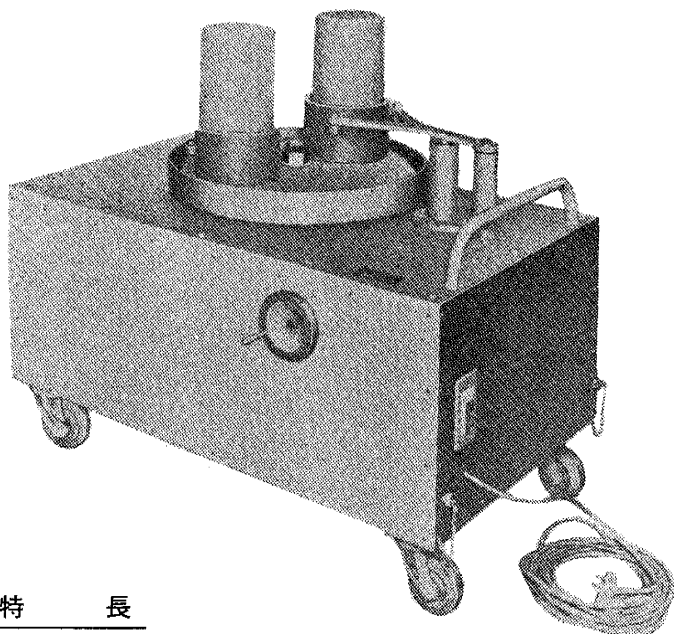
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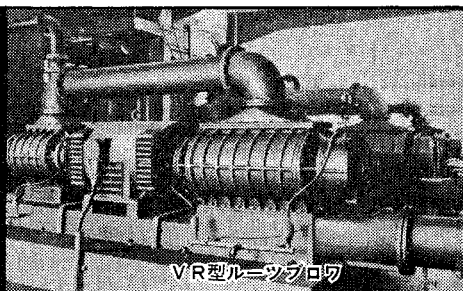
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