THE LATERAL MOTION OF SUSPENSION BRIDGES*

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Synopsis : The fundamental equations of small lateral motion of a suspension bridge structure are presented under the proper assumptions. Then, the author derived the formulas to compute the natural frequencies of lateral vibrations, which contain some dimensionless parameters. The coupling oscillations between cables and stiffening frame that is missing in other papers should be taken into account. The effect of the vertical distortion of the structure incidental to its lateral movement is estimated by means of the energy method. Finally, the results are applied to the analysis of suspension bridges subjected to the static lateral forces.

1. Introduction

Recently, long-span suspension bridges tend to be proposed in Japan, and engineers become to feel interest in the projects concerned. As well known, suspension bridges are unique, to span long distance beyond several hundred meters, as compared with other types of bridges, but its substantial defects lie in the large deflection due to live load and the unfavorable dynamic behaviors under external forces. Therefore, various problems, such as the aerodynamic stability and quake-resistant design, have been tendered so far. It goes without saying that the understanding of the modes and natural frequencies of a structure is necessary for discussing its dynamic behaviors, especially when the modal-analysis method is employed.

In this paper, the author would like to place emphasis on a revised solution for small lateral vibrations of suspension bridge structure, and also to discuss the general characteristics of suspension bridges subjected to lateral forces.

Although many investigators have studied about its behavior under the lateral forces, any convincible solution relating to the lateral vibrations of a suspension bridge seems not to be found hitherto. In dealing with its stability under wind action, A. Hirai[2] established the differential equations of the coupling oscillations for vertical and torsional displacements, and H. Chikuma[3] made some approximate corrections to these equations considering the effect of lateral displacement of the suspended structure. Nevertheless, their theoretical treatment is only valid to the wind action which contains uplift and torque components, and can not explain lateral vibrations. A formula to compute natural frequencies of vibratory lateral motion of a suspension bridge was proposed by I.K. Silverman[4], but the author has some doubts in point of the interaction between cables and suspended structure. Because the cables and stiffening frame are connected with deformable suspenders, the coupling oscillations are believed to yield. In the old papers written by N. Mononobe[5] or T. Mogami[6], even the action of cables was not considered.

In reality, when a suspension bridge deflects laterally, vertical distortion will be accompanied, and its effect will be taken into account, as the second order approximation, in the later part of this paper. If the acting point of lateral force applied to the stiffening frame or the gravity center of its cross section deviates from the twisting center of the section[7], the angular rotation of the structure is also to be considered. However, in order to avoid unnecessary confusion, this is neglected in the present analysis and the following treatment may be proper in estimating the natural frequencies of the structure.

2. Oscillations in the Horizontal Plane

In analyzing the lateral vibration of a suspension bridge, the conventional assumptions are made : that is, the elongation of suspenders is neglected, the spacing of suspenders is considered very small as compared with span length, and the rigidity and weight of structure are

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constant through the span. Furthermore, the additional cable tension caused by the inertial forces is neglected as a small quantity, and the effects of the deflection of towers and the presence of side spans may be also negligible in this case.

Refering to Fig. 1, the small lateral vibrations of a suspension bridge are discussed. The notations are seen in the figure : $p_c(x,t)$ and $p_f(x,t)$ represent the horizontal external forces applied to cables and suspended structure, respectively, and $u(x,t)$ and $v(x,t)$ are the horizontal deflection of cable and suspended structure at distance x from end of span, respectively. As seen in Fig. 1 (c), the force of restitution due to inclination of suspenders per unit length of bridge is

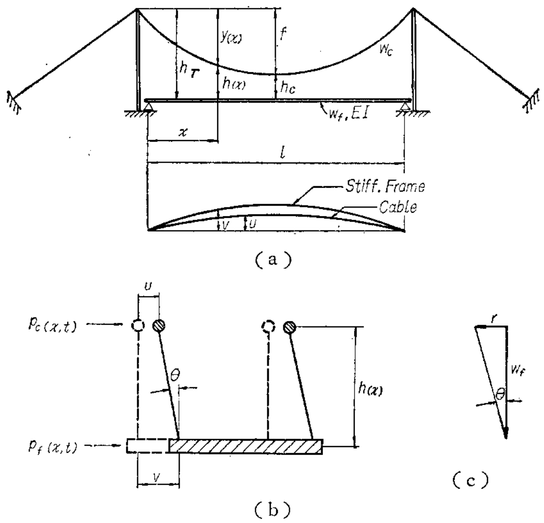

Fig. 1 Smal Lateral Displacement of a Suspension Bridge.

$$r(x)=\frac{w_f}{h(x)}(v-u) \quad \cdots\cdots(1)$$

where w_f is the total vertical load carried by suspenders per unit length. Accordingly, small vibrations in the horizontal plane about the position of equilibrium are described by means of the following differential equations :

$$\left.\begin{aligned}\frac{w_f}{g}\frac{\partial^2 v}{\partial t^2}+EI_h\frac{\partial^4 v}{\partial x^4}+\frac{w_f}{h(x)}(v-u)=p_f(x,t)\\ \frac{w_c}{g}\frac{\partial^2 u}{\partial t^2}-H_w\frac{\partial^2 u}{\partial x^2}-\frac{w_f}{h(x)}(v-u)=p_c(x,t)\end{aligned}\right\} \quad (2)$$

where EI_h represents the lateral bending rigidity of the stiffening frame, H_w is the horizontal component of cable tension due to dead load, w_c/g the mass of cables per unit length of bridge, g the acceleration due to gravity, and $h(x)$ denotes the length of suspender at distance x from end of span, which is expressed by

$$h(x)=h_T-\frac{4f}{l^2}x(l-x) \quad \cdots\cdots(3)$$

h_T representing the cable sag plus the hanger length at mid-span. The fundamental equations (2) indicate obviously the coupled vibratory motion.

In order to obtain the natural frequencies, putting the right side of Eqs. (2) to be zero and assuming the displacement at any instant t as

$$\left.\begin{aligned}v=a_n\sin\frac{n\pi x}{l}\cdot\sin\omega_n t\\ u=b_n\sin\frac{n\pi x}{l}\cdot\sin\omega_n t\end{aligned}\right\} \quad \cdots\cdots(4)$$

the principle of virtual work was applied to Eqs. (2). Then, the requirement of this principle leads to the following frequency equation

$$\begin{vmatrix} EI_h\left(\frac{n\pi}{l}\right)^4+\frac{w_f}{h_{mn}}-\omega_n^2\frac{w_f}{g} & -\frac{w_f}{h_{mn}} \\ -\frac{w_f}{h_{mn}} & H_w\left(\frac{n\pi}{l}\right)^2+\frac{w_f}{h_{mn}}-\omega_n^2\frac{w_c}{g}\end{vmatrix}=0 \quad (5)$$

$$\left.\begin{aligned}h_{mn}&=h_T-2f\left(\frac{1}{3}+\frac{1}{n^2\pi^2}\right)\\ &=h_c+f\left(\frac{1}{3}-\frac{2}{n^2\pi^2}\right)\end{aligned}\right\} \quad \cdots\cdots(6)$$

where h_c is the length of suspender at mid-span. Since it requires the intricate calculations in treating the forced lateral vibrations of a suspension bridge that the length of suspenders along a span is a function of x, the use of the constant length of hangers is desirable in practical analyses. Equating the result of Eq. (5) and that by assuming h constant, it is found that the reduced or mean length of hangers coincides with h_{mn} in Eq. (6). This h_{mn} is the value which corresponds to the n-th mode of deflection form : namely, the reduced length of suspender for the first mode ($n=1$) is $h_{m1}=h_c+0.131f$ and for higher modes $h_{mn}\doteqdot h_c+\frac{f}{3}$

Introducing three dimensionless parameters

$$\nu_h=l\sqrt{\frac{H_w}{EI_h}} \quad \cdots\cdots(7)$$

$$\beta=\frac{w_c}{w_f} \quad \cdots\cdots(8)$$

and $$\lambda_n=\frac{h_{mn}}{f} \quad \cdots\cdots(9)$$

the roots of *Eq.* (5) becomes

$$\omega_n^2=n^2\frac{g\,\pi^2}{16f}\frac{1+\beta}{\beta}\Bigg[1+\beta\frac{n^2\pi^2}{\nu_h^2}+\frac{8}{n^2\pi^2\lambda_n}\pm$$
$$\pm\sqrt{(1-\beta\frac{n^2\pi^2}{\nu_h^2})(1-\beta\frac{n^2\pi^2}{\nu_h^2}+\frac{16}{n^2\pi^2\lambda_n}\frac{1-\beta}{1+\beta}+\frac{64}{n^4\pi^4\lambda_n^2}}\quad\cdots\cdots(10)$$

Two solutions of the natural frequency ω_n obtained for each value of n in the above equation are corresponding to the same and opposite phase of vibratory motions of cables and stiffening frame, respectively.

For a given integer value of n the parameter λ_n is generally confined within narrow limits of variation, while the values of ν_h and β have a definite tendency to increase with span length. The results of investigations for existing suspension bridges including those under contemplation are plotted in Fig. 2 and 3. On the other hand, the most important parameter to characterize the behavior of a suspension bridge under vertical loads is

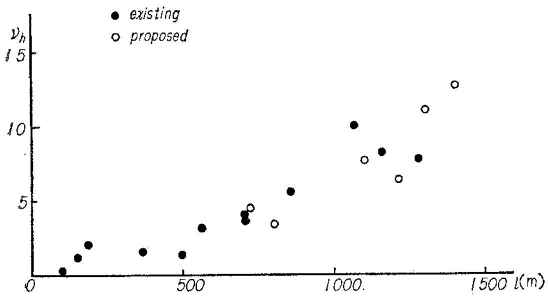

Fig. 2 ν_n to Span Length.

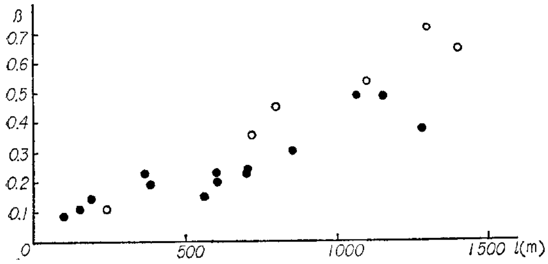

Fig. 3 β to Span Length.

$$\nu=l\sqrt{\frac{H_w}{EI}}\quad\cdots\cdots(11)$$

EI being the flexual rigidity of stiffening frame. This value of ν is the reciprocal of the stiffness factor defined by D.B. Steinman though, the above expression is more convenient to the analyses of long-span suspension bridges for which the deflection theory is applied. With increasing span length the ν-value increases almost linearly. Between ν_h and ν there is an approximate relationship

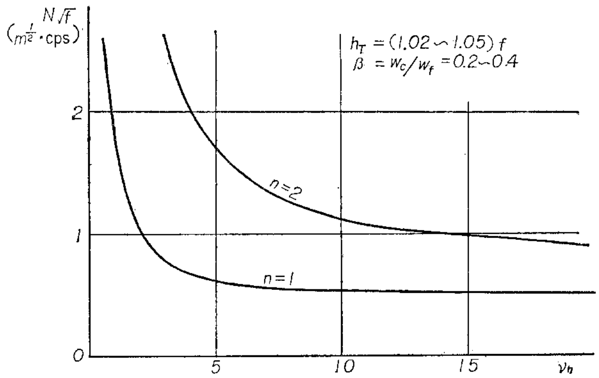

Fig. 4 Natural Frequencies (In Phase)

$$\nu_h \fallingdotseq \nu\ (d/b)\quad\cdots\cdots(12)$$

where d and b are the depth and the spacing of stiffening trusses, respectively. Fig. 2 will be understood from these facts.

The natural frequencies of the first (symmetric) and second (asymmetric) mode of lateral vibration are shown in Fig. 4 as a function of ν_h. They are calculated from Eq. (10) where n is taken as 1 and 2, and are the case that cables and stiffening frame vibrate in phase. A little variations in the values of β and λ_n do not affects much on the frequencies. From this figure it is found that in suspension bridges having very long span the fundamental natural frequency is almost inversely proportional to the square root of cable sag and hardly influenced by any other factor. It is very similar to the results in their vertical vibrations.

When the cables are fastened to the stiffening frame at mid-span, as seen in the Mackinac Bridge and the Tancarville Bridge, the assumption in Eq. (4) will be erroneous, because suspenders can not be considered as always vertical. The better solution for this kind of structure will be discussed later on. The forced lateral vibration of a suspension bridge can be solved using Eq. (2). As an example of the problems, the author dealt previously[1]) with the lateral stability of a suspension bridge subject to foundation-motion.

3. Numerical Examples

The circular natural frequencies of the first and second modes of lateral vibrations in a few existing suspension bridges were calculated and

Table 1 Circular Frequencies of Lateral Vibration in Existing Bridges
(Effect of center ties is neglected)

Name of Bridge	Golden Gate (USA)	Mackinac (USA)	Wakato (Japan)	Ohdomari (Japan)
l (m)	1 280	1 158	367	150
f (m)	143.3	105	35.0	15.0
h_T(m)	150	107	36.5	15.8
w_f(t/m)	22.8	10.2	12.4	3.6
w_c(t/m)	8.5	4.9	2.8	0.4
H_w(t)	44 300	28 864	7 312	757.5
EI_h(t·m²)	15.56×10^8	5.38×10^8	3.58×10^8	1.08×10^7
ν (−)	18	14	7.2	2.2
ν_h(−)	7	8.2	1.6	1.2
β (−)	0.37	0.48	0.23	0.11
ω_1 (rad/sec)	0.33 or 1.19	0.37 or 1.31	1.255 or 3.250	2.433 or 6.650
ω_2 (rad/sec)	0.72 or 1.36	0.91 or 1.53	3.327 or 5.030	7.090 or 9.675

Table 2 Circular Natural Frequencies of the Wakato Bridge (rad/sec)

Motion \ Mode	ω_1 (symmetric)	ω_2 (Asymmetric)
Vertical	2.08	1.56
Torsional	3.42	4.13
Lateral	1.26	3.33

Table 3 Amplitude-ratio of Cables to Suspended Frame (Wakato Bridge)

Phase lag \ Mode	ω_1	ω_2
0°	0.960	16.275
180°	−0.077	−0.272

Table 4 Results of Model Test (Circular frequencies in rad/sec)

	ω_1	ω_2	ω_3
Measured	38.8	150	369
From Eq. (5)	41.1	151	340
According to Ref. 4)	68.7	181	356

are shown in Table 1.

In Table 2, the theoretical natural frequencies of vertical, torsional, and lateral vibrations in the Wakato Bridge are compared. In this bridge which has a center span of 367 m, the largest in the Orient, cables are actually fastened to stiffening frame at the center of span, but the computed frequencies shown in Table 2 do not take into account this effect. The amplitude ratios a_n/b_n—see Eq. (4)—of cables and stiffening frame in this case of lateral vibrations are seen in Table 3, in which the amplitude of stiffening frame corresponding to the second mode of vibration in phase is very small compared with that of cables. Accordingly, in this case only the cables apparently vibrates and such a phenomenon was observed in the model tests.

Next the result of a small-scaled model test conducted to verify the theoretical treatment is refered. The suspension bridge model used in the experiment has a single span, straight backstays, and the following dimensions :

$l=300$ cm, $f=30$ cm, $h_T=33$ cm

$EI=0.14\times10^8$gr-cm², $EI_h=18.3\times10^8$gr-cm²

$w_f=15.14$ gr/cm, $w_c=3.17$ gr/cm, $w=18.31$ gr/cm

$H_w=6.9$ kg, $\beta=0.21$, $\nu_h=0.6$

Stiffening frame and towers designed as a rigid frame were made of brass and steel, and each main cable is a ϕ 2 mm stranded wire attached by distributed dead weights. Making use of the shaking-table especially designed for this purpose, the natural periods of lateral vibration were read from the recorded oscillopapers. The test results are shown and compared with theoretical values in Table 4. The theoretical values calculated from Eq. (5) are in fairly good agreement with the test results, but in order to get the better results for the fundamental period of lateral vibration the upward distortion of suspended span should be taken into consideration and it will be necessary to assume the following several terms expression instead of Eq. (4).

$$\left.\begin{aligned} v&=\sum_k a_k \sin\frac{k\pi x}{l}\cdot\sin\omega_n t\\ u&=\sum_k b_k \sin\frac{k\pi x}{l}\cdot\sin\omega_n t \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(13)$$

4. Energy Method applied to Lateral Vibrations

The energy method gives a more satisfactory approximation to the true natural frequencies

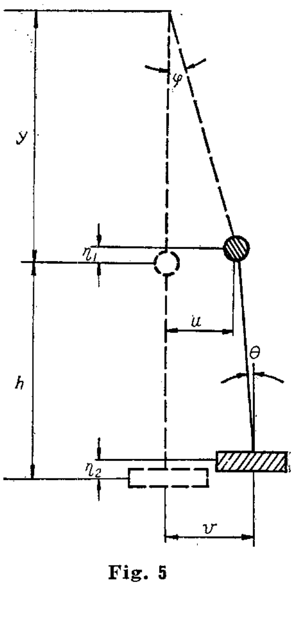

Fig. 5

by taking into account the upward deflection of cables and suspended structures, η_1 and η_2 respectively, incidental to their lateral movements u and v (Fig. 5). Since u and v are very small quantities as compared with h (length of hangers) or $y=h_T-h$ etc., the upward distortion η_1 and η_2 under the lateral motion are given by

$$\left.\begin{array}{ll}\eta_1 \doteqdot \dfrac{u^2}{2y} & \text{(cables)} \\ \eta_2 \doteqdot \dfrac{u^2}{2y}+\dfrac{(v-u)^2}{2h} & \text{(frame)}\end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots(14)$$

The maximum value of the potential energy stored in the whole structure vibrating laterally is assorted as follows, provided u and v are the amplitudes of vibration :

1. The strain energy of lateral bending of the suspended structure

$$V_1=\frac{EI_h}{2}\int_0^l\left(\frac{d^2v}{dx^2}\right)^2dx$$

2. Due to the restoring force of suspenders

$$V_2=\frac{w_f}{2}\int_0^l\frac{(v-u)^2}{h}dx$$

3. Due to the deflection of cables

$$V_3=\frac{H_w}{2}\int_0^l\left(\frac{du}{dx}\right)^2dx$$

4. Due to the upward distortion of the cables

$$V_4=w_c\int_0^l\eta_1dx=\frac{w_c}{2}\int_0^l\frac{u^2}{y}dx$$

5. Due to the upward distortion of the suspended structure

$$V_5=w_f\int_0^l\eta_2dx=\frac{w_f}{2}\int_0^l\left[\frac{u^2}{y}+\frac{(v-u)^2}{h}\right]dx$$

Consequently, the maximum value of the total potential energy is given as a function of u, v and their derivatives

$$V=\frac{EI_h}{2}\int_0^l\left(\frac{d^2v}{dx^2}\right)^2dx+\frac{H_w}{2}\int_0^l\left(\frac{du}{dx}\right)^2dx+\frac{w_f}{h_{mn}}\int_0^l(v-u)^2dx+\frac{w_c+w_f}{2(h_T-h_{mn})}\int_0^l u^2dx\cdots\cdots(15)$$

where the length of suspenders are assumed as the constant value of h_{mn} (the reduced length) given by Eq. (6) for the simplicity of calculation.

The maximum value of the total kinetic energy of the entire structure, denoted by T, is approximately given by the integral

$$T=\frac{\omega^2}{2g}\left(w_f\int_0^l v^2dx+w_c\int_0^l u^2dx\right)\cdots\cdots\cdots\cdots\cdots(16)$$

when the kinetic energy caused from the incidental vertical motion is neglected, because η_1 and η_2 are small quantities of the higher order than u and v as seen in Eq. (14).

The Ritz Method is employed to obtain the natural frequencies. Assuming the deflection as in Eq. (4), the requirements are

$$\left.\begin{array}{l}\dfrac{\partial(T-V)}{\partial a_n}=0 \\ \dfrac{\partial(T-V)}{\partial b_n}=0\end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(17)$$

As $T-V$ obtained from Eqs. (15) and (16) is a quadratic form of a_n and b_n, Eq. (17) represents a system of two linear equations determing the paramaters a and b. Since the equations (17) are homogeneous ones, the following frequency equation determing ω_1 for the first mode of lateral vibration is derived.

$$\begin{vmatrix}\dfrac{2}{\lambda_1}+\dfrac{1+\beta}{8}\dfrac{\pi^4}{\nu_h^2}-\dfrac{f}{g}\omega_1^2 & -\dfrac{2}{\lambda_1} \\ -\dfrac{2}{\lambda_1} & \dfrac{2}{\lambda_1}+\dfrac{1+\beta}{8}\pi^2+\dfrac{1+\beta}{\dfrac{h_T}{f}-\lambda_1}-\dfrac{f}{g}\omega_1^2\beta\end{vmatrix}=0 \quad (18)$$

The effect of the upward distortion of structures was taken into account in this equation. Upon putting the numerical data of the Wakato Bridge, solution of Eq. (18) yields the two roots $\omega_1=1.365$ and 4.561. As compared with the results in Table 1, the lowest frequency is augmented by 9%. If the effect of the upward distortion of structures is not considered, namely V_4 and V_5 are neglected, the total potential energy is written as

$$V=\frac{EI_h}{2}\int_0^l\left(\frac{d^2v}{dx^2}\right)^2dx+\frac{H_w}{2}\int_0^l\left(\frac{du}{dx}\right)^2dx+\frac{w_f}{2h_{mn}}\int_0^l(v-u)^2dx\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(19)$$

The same process as just mentioned leads to the frequency equation (5), as a matter of course.

Here the suspension bridge with cables fastened to the stiffening frame at the center of span is dealt with. As the first symmetric mode of lateral vibration the following assumption is employed

$$\left.\begin{array}{l}v=a\sin\dfrac{\pi x}{l} \\ u=b\sin\dfrac{\pi x}{l}+(b-a)\sin\dfrac{3\pi x}{l}\end{array}\right\} \quad (20)$$

Using the expression of the total potential energy V in Eq. (19), the Ritz Method leads to the frequency equation

$$\begin{vmatrix} \frac{1+\beta}{8}\pi^2\left(9+\frac{\pi^2}{\nu_h{}^2}\right)+\frac{2}{\lambda_1}-\frac{f}{g}\omega_1{}^2 & -\frac{9}{8}(1+\beta)\pi^2-\frac{2}{\lambda_1}-\frac{f}{g}\omega_1{}^2\beta \\ -\frac{9}{8}(1+\beta)\pi^2-\frac{2}{\lambda_1}-\frac{f}{g}\omega_1{}^2\beta & \frac{5}{4}(1+\beta)\pi^2+\frac{2}{\lambda_1}-2\frac{f}{g}\omega_1{}^2\beta \end{vmatrix}=0 \qquad (21)$$

The solution of this equation for the Wakato Bridge is $\omega_1=1.256$ and 4.240, that is, the fundamental natural frequency in this case is a little augmented. Putting $a=1$, the vibration mode of the cable corresponding to $\omega_1=1.256$ becomes

$$u=0.990\ \sin\frac{\pi x}{l}-0.010\sin\frac{3\pi x}{l}$$

Thus a far better approximation is possible for the analysis of the small lateral oscillations of suspension bridges. However, the author believes that the result of Eq. (10) is of practice with satisfactory accuracy.

5. Behaviors under Static Lateral Forces

When the cables and the stiffening frame are subjected to uniformly distributed lateral forces p_c and p_f, respectively, the following equations may be obtained in connection with Eq. (2)

$$\left.\begin{aligned} EI_h v''''(x)+\frac{w_f}{h(x)}[v(x)-u(x)]&=p_f \\ -H_w u''(x)-\frac{w_f}{h(x)}[v(x)-u(x)]&=p_c \end{aligned}\right\} \cdots\cdots(22)^*$$

*where a prime denotes the differentiation with respect to x. Assuming the displacements as

$$\left.\begin{aligned} v(x)&=\sum_n a_n \sin\frac{n\pi x}{l} \\ u(x)&=\sum_n b_n \sin\frac{n\pi x}{l} \end{aligned}\right\} (n=1,3,5,) \cdots\cdots\cdots\cdots(23)$$

and making use of the variation method, the equation to determine the unknown constants a_n and b_n is obtained. A. Selberg[8] already derived the same expression as Eq. (22) for this problem, but he solved them by expanding the second term of the left side of the equations into Fourier series. Since the convergence of Eq. (23) is very rapid, first two or three terms approximation of Eq. (23) will give easier and satisfactory results in this case. Similar problem was discussed by L.S. Moisseiff[9] whose solution is known as the uniform distribution method and the elastic distribution method. However, to obtain accurate results his method also leads to very intricate calculations. If the two term approximation is taken in Eq. (23), the unknowns a_n and b_n $(n=1,3)$ are determined by the following equation :

$$\begin{pmatrix} 1+\frac{\pi^4}{8}\frac{\lambda_1}{\nu_h{}^2}(1+\beta) & \frac{243\pi^2}{16\nu_h{}^2}(1+\beta) & -1 & 0 \\ \frac{3\pi^2}{16\nu_h{}^2}(1+\beta) & 1+\frac{81\pi^4}{8}\frac{\lambda_3}{\nu_h{}^2}(1+\beta) & 0 & -1 \\ -1 & 0 & 1+\frac{\pi^2}{8}\lambda_1(1+\beta) & \frac{27}{16}(1+\beta) \\ 0 & -1 & \frac{3}{16}(1+\beta) & 1+\frac{9\pi^2}{8}\lambda_3(1+\beta) \end{pmatrix} \begin{pmatrix} a_1 \\ a_3 \\ b_1 \\ b_3 \end{pmatrix} = \frac{4}{\pi}\frac{h_T}{w_f} \begin{pmatrix} \left(1-\frac{8}{\pi^2}\frac{f}{h_T}\right)p_f \\ \left(1-\frac{8}{9\pi^2}\frac{f}{h_T}\right)p_f/3 \\ \left(1-\frac{8}{\pi^2}\frac{f}{h_T}\right)p_c \\ \left(1-\frac{8}{9\pi^2}\frac{f}{h_T}\right)p_c/3 \end{pmatrix} \cdots\cdots(24)$$

The term $r(x)=\frac{w_f}{h(x)}(v-u)$ in Eq. (22) indicates the force transmitted into cables from suspended structure in consequence of the lateral deflection of the bridge. Actually, the suspended structure sustains the force p_f-r and the cables do p_c+r, and this was pointed out by Moisseiff et al. When the cables are fastened to stiffening frame at the center of the bridge, similar treatment is made under the condition that the deflections of cables and suspended structure should be equal at mid-span. Fig. 6 indicates the variation of $r(x)$ along span in the Wakato Bridge, when $p_c=0$ and the two-term solution is employed. It is found from these results that the presence of center ties diminishes the force transmitted from stiffening truss to cables.

6. Conclusions and Acknowledgements

The motion of a suspension bridge in the horizontal plane was discussed. In the lateral vibrations, vertical distortion is accompanied, but their contribution to the kinetic energy of entire structure is so small to be neglected. The effects of the presence of side spans and the center ties may be also negligible in the

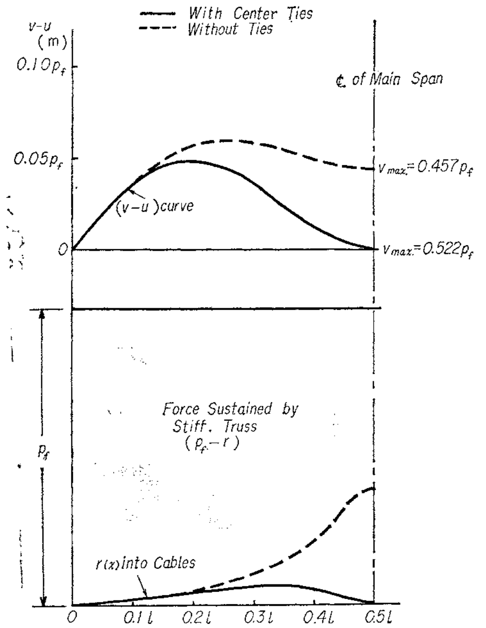

Fig. 6 Static Load to the Wakato Bridge ($p_c=0$; $h_c=1.5$ m)

lateral motion of a suspension bridge. The present theoretical treatments for computing the natural frequencies and the displacements under the action of lateral forces are believed to yield practical solutions with sufficient accuracy.

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