THE SOLUTION OF THE SKEWED PLATE

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Synopsis : The author searched the solution of the skewed plate with two opposite edges simply supported and the other edges free by the method of Fourier's transformation and this solution can apply to the other skewed plate with two opposite edges simply supported and the other edges various conditions.

1. The general solution of rectangular plate

The deflection $\zeta(x,y)$ of the middle surface of a thin homogeneous isotropic plate must satisfy the following differential equation over the region bounded by the plate in **Fig.1**.

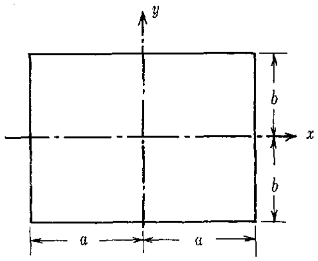

Fig. 1 Rectangular plate and system of coordinates.

$$\frac{\partial^4\zeta}{\partial x^4}+2\frac{\partial^4\zeta}{\partial x^2 y^2}+\frac{\partial^4\zeta}{\partial y^4}=\frac{p(xy)}{D} \quad \cdots\cdots\cdots\cdots(1)$$

where $p(x,y)$ = load per unit area normal to surface of plate.

$D=\frac{Eh^3}{12(1-\nu^2)}$, h=thickness of plate

E=young's modulus

ν=poisson's ratio of material

We assume that a solution of Equation (1) may be written is the form.

$$\zeta=\zeta_0+\zeta_1 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where

ζ_0= a particular solution decided by load functions.

ζ_1= a general solution decided by boundary conditions of rectangular plate.

A general solution ζ_1 consist in potential function of the form.

$\varphi_0(xy)=f(x+iy)+g(x-iy)$

And bi-potential function in the form $x\varphi_1\cdot(x,y)$ or $y\varphi_2(x,y)$. In the case of full uniform load the equation (2) is expressed in the form.

$$\zeta=\frac{pb^4}{24D}\left(\frac{y^4}{b^4}-6\frac{y^2}{b^2}+5\right)+f_1(x+iy)+g_1(x-iy)$$
$$+x\{f_2(x+iy)+g_2(x-iy)\}\cdots\cdots\cdots\cdots\cdots(3)$$

2. The solution of the skewed plate.

In **Fig. 2**, the solution of the skewed plate is expressed from Equation (3) in the form.

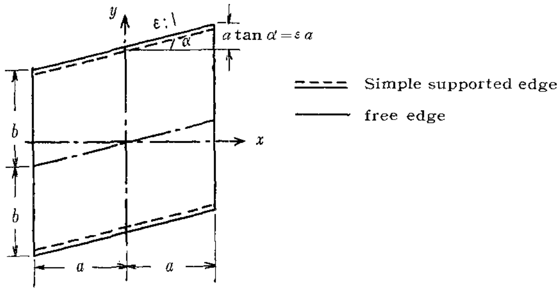

Fig. 2 The Skewed plate with two opposite edges simply supported and the other edges-free.

$$\zeta=\frac{p}{24D(1+\varepsilon^2)^2}\left\{(y-\varepsilon x)^4\frac{1}{b^4}-6\frac{1}{b^2}(y-\varepsilon x)^2\right.$$
$$\left.+5\right\}+g_1(x-iy)+f_1(x+iy)+(x+\varepsilon y)$$
$$\cdot\left\{\frac{1}{1+i\varepsilon}f_2(x+iy)+\frac{1}{1-i\varepsilon}g_2(x-iy)\right\} \quad (4)$$

The equation must satisfy the following boundary conditions at the sides $y=\varepsilon x+b$.

$$\zeta=0, \qquad \frac{\delta^2\zeta}{\partial y^2}+\frac{\delta^2\zeta}{\partial x^2}=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

Substituting the condition (5) in the equation (4), we obtain the following equation as result of calculation,

$$\zeta=\frac{p}{24D(1+\varepsilon^2)^2}\{(y-\varepsilon x)^4-6b^2(y-\varepsilon x)^2$$
$$+5b^4\}\frac{1}{b^4}+\left[2\frac{(x+\varepsilon y)}{b}\varepsilon\sum_{n=2,4,6\cdots}C_n\cdot\sin\frac{n\pi}{2b}(\varepsilon x\right.$$
$$-y)\cosh\frac{n\pi}{2b}(x+\varepsilon y)+2\frac{(x+\varepsilon y)}{b}\sum_{n=1,3,5\cdots}D_n$$
$$\cdot\cos\frac{n\pi}{2b}(\varepsilon x-y)\cdot\sinh\frac{n\pi}{2b}(x+\varepsilon y)-\varepsilon\sum_{n=2,4,6\cdots}A_n$$
$$\cdot\sin\frac{n\pi}{2b}(\varepsilon x-y)\sinh\frac{n\pi}{2b}(\varepsilon y+x)-\sum_{n=1,3,5\cdots}B_n$$
$$\left.\cdot\cos\frac{n\pi}{2b}(\varepsilon x-y)\cdot\cosh\frac{n\pi}{2b}(x+\varepsilon y)\right]$$
$$\frac{1}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

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The constants of integration A_n, B_n, C_n, D_n, can be determined from the boundary conditions on the sides $x=\pm a$.

3. The analysis of the skewed plate

The boundary conditions of the skewed plate on the sides $x=\pm a$ are

$$\left.\begin{aligned}&\left|\frac{\partial^2\zeta}{\partial x^2}+\nu\frac{\partial^2\zeta}{\partial y^2}\right|_{x=\pm a}=0\\&\left|\frac{\partial^3\zeta}{\partial x^3}+(2-\nu)\frac{\partial^3\zeta}{\partial x\partial y^2}\right|_{x=\pm a}=0\end{aligned}\right\}\quad\cdots\cdots(7)$$

Appling the conditions (7) to the equation (6), we obtain the following equations.

$$\sum_{n=2,4,6\cdots} g_nC_n+\sum_{n=1,3,5\cdots} h_nD_n+\sum_{n=2,4,6\cdots} i_nA_n+\sum_{n=1,3,5\cdots} j_nB_n=R(y)\quad\cdots\cdots(8)$$

$$\sum_{n=2,4,6,\cdots} g_n'C_n+\sum_{n=1,3,5,\cdots} h_n'D_n+\sum_{n=2,4,6,\cdots} i_n'A_n+\sum_{n=1,3,5,\cdots} j_n'B_n=R'(y)\quad\cdots\cdots(9)$$

where

$$g_n=2\frac{n}{\pi}\{\varepsilon^2(1-\nu)f_n^{(a)}+\varepsilon(1+\nu\varepsilon^2)\phi_n^{(a)}\}+\frac{1}{2}(1-\nu)\varepsilon^2(a'-y')n^2\{\varphi_n^{(a)}(1-\varepsilon^2)+2\,\varepsilon\tau_n^{(a)}\}$$

$$h_n=2\frac{n}{\pi}\{(1+\nu\varepsilon^2)f_n^{(a)}-\varepsilon(1-\nu)\phi_n^{(a)}\}+\frac{1}{2}(1-\nu)\varepsilon(a'-y')n^2\{\tau_n^{(a)}(1-\varepsilon^2)-2\,\varepsilon\varphi_n^{(a)}\}$$

$$j_n=-\frac{n^2}{4}(1-\nu)\{f_n^{(a)}(1-\varepsilon^2)-2\,\varepsilon\phi_n^{(a)}\},$$

$$i_n=-\frac{n^2}{4}\varepsilon(1-\nu)\{\phi_n^{(a)}(1-\varepsilon^2)+2\,\varepsilon\, f_n^{(a)}\},$$

$$R(y')=-\frac{(y'^2-1)}{2\,\pi^2(1+\varepsilon^2)^2}(\nu+\varepsilon^2)\quad\cdots\cdots(8)'$$

and

$$g_n'=\frac{n^2}{\pi}[\{\varepsilon(1-\nu)(\varepsilon^3-2\,\varepsilon)+\varepsilon^2(1+\varepsilon^2)\}\tau_n^{(a)}+\{\varepsilon(1+\varepsilon^2)+\frac{1}{2}(1-\nu)(5\,\varepsilon^2-1)\varepsilon\}\varphi_n^{(a)}]+\frac{n^3}{4}(a'-y')(1-\nu)\varepsilon^2\{f_n^{(a)}(\varepsilon^3-3\,\varepsilon)+\phi_n^{(a)}(3\,\varepsilon^2-1)\}$$

$$h_n'=\frac{n^2}{n}[\{(1-\nu)(2\,\varepsilon-\varepsilon^3)-\varepsilon(1+\varepsilon^2)\}\varphi_n^{(a)}+\{(1+\varepsilon^2)+\frac{1}{2}(1-\nu)(5\,\varepsilon^2-1)\}\tau_n^{(a)}]+\frac{n^3}{4}(a'-y')(1-\nu)\varepsilon\{(3\,\varepsilon^2-1)f_n^{(a)}-(\varepsilon^3-3\,\varepsilon)\phi_n^{(a)}\}$$

$$j_n'=-(1-\nu)\frac{n^3}{8}\{\tau_n^{(a)}(3\,\varepsilon^2-1)-\varphi_n^{(a)}(\varepsilon^3-3\,\varepsilon)\}$$

$$i_n'=-(1-\nu)\frac{n^3}{8}\varepsilon\{\tau_n^{(a)}(\varepsilon^3-3\,\varepsilon)+\varphi_n^{(a)}(3\,\varepsilon^2-1)\}$$

$$R_n'=-\frac{\varepsilon}{\pi^3(1+\varepsilon^2)}\left(1+\frac{1-\nu}{1+\varepsilon^2}\right)y'\quad\cdots\cdots(9)'$$

Writing $y'=\varepsilon a-y$, $a'=\left(\varepsilon+\frac{1}{\varepsilon}\right)\frac{a}{b}$,

In the expression (7)′ (8)′, the value of $\varphi_n^{(a)}$, $f_n^{(a)}$, $\tau_n^{(a)}$, $\phi_n^{(a)}$, are these of φ_n, τ_n, f_n, ϕ_n in $x=a$.

where

$$\left.\begin{aligned}\varphi_n&=\sin\frac{n\pi}{2b}(\varepsilon x-y)\ \cosh\frac{n\pi}{2b}(x+\varepsilon y),\ \frac{1}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\\\tau_n&=\cos\frac{n\pi}{2b}(\varepsilon x-y)\ \sinh\frac{n\pi}{2b}(x+\varepsilon y),\ \frac{1}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\\\phi_n&=\sin\frac{n\pi}{2b}(\varepsilon x-y)\ \sinh\frac{n\pi}{2b}(x+\varepsilon y),\ \frac{1}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\\f_n&=\cos\frac{n\pi}{2b}(\varepsilon x-y)\ \cosh\frac{n\pi}{2b}(x+\varepsilon y),\ \frac{1}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\end{aligned}\right\}\ (10)$$

From the expression (10), these values are represented by Fourier Sine series for $0\leqq y\leqq 1$ as follows.

i.e.

$$\left.\begin{aligned}\varphi_n^{(a)}&=\sum_{s=1}^{\infty}\varphi_{ns}^{(1)}\sin s\pi y=\sum_{s=1}^{\infty}(\vartheta'_{ns}{}^{(1)}-\vartheta_{ns}^{(1)})\sin s\pi y\\\tau_n^{(a)}&=\sum_{s=1}^{\infty}\tau_{ns}^{(1)}\sin s\pi y=\sum_{s=1}^{\infty}(w_{ns}^{(1)}-w'_{ns}{}^{(1)})\sin s\pi y\\\phi_n^{(a)}&=\sum_{s=1}^{\infty}\phi_{ns}^{(1)}\sin s\pi y=\sum_{s=1}^{\infty}(z'_{ns}{}^{(1)}-z_{ns}^{(1)})\sin s\pi y\\f_n^{(a)}&=\sum_{s=1}^{\infty}f_{ns}^{(1)}\sin s\pi y=\sum_{s=1}^{\infty}(u_{ns}^{(1)}-u'_{ns}{}^{(1)})\sin s\pi y\end{aligned}\right\}\ (11)$$

$$\left.\begin{aligned}yf_n^{(a)}&=\sum_{n=1}^{\infty}f'_{ns}{}^{(1)}\sin s\pi y-\sum_{s=1}^{\infty}f_{ns}^{(2)}\sin s\pi y\\y\,\varphi_n^{(a)}&=\sum_{n=1}^{\infty}\varphi'_{ns}{}^{(1)}\sin s\pi y-\sum_{s=1}^{\infty}\varphi_{ns}^{(2)}\sin s\pi y\\y\,\tau_n^{(a)}&=\sum_{n-1}^{\infty}\tau'_{ns}{}^{(1)}\sin s\pi y-\sum_{s=1}^{\infty}\tau_{ns}^{(2)}\sin s\pi y\\y\,\phi_n^{(a)}&=\sum_{n=1}^{\infty}\phi'_{ns}{}^{(1)}\sin s\pi y\end{aligned}\right\}\ (12)$$

$$-\sum_{s=1}^{\infty}\phi_{nt}{}^{(2)}\sin s\pi y$$

where

$$f_{ns}{}'^{(1)}=(\bar{u}_{ns}{}^{(1)}-\bar{u}_{ns}{}'^{(1)}),$$
$$f_{ns}{}^{(2)}=(u_{ns}{}^{(2)}-u_{ns}{}'^{(2)}),\cdots\cdots\cdots\text{etc.}$$

the value of $\bar{u}_{ns}{}^{(1)}$, $\bar{u}_{ns}{}'^{(1)}$,$\cdots\cdots$etc. is the value of the expression (12) substituting a_{ns}', b_{ns}', c_{ns}', d_{ns}', instead of a_{ns}, b_{ns}, c_{ns}, d_{ns},

writing $y=\frac{1}{2}(1+y')$. $(0\leqq y\leqq 1)$

The new symbols of $u_{ns}{}^{(1)}$, $u_{ns}{}^{(2)}$, $\vartheta_{ns}{}^{(1)}$, $\vartheta_{ns}{}^{(2)}$, $\cdots\cdots$etc are noted as follows.

$$\left.\begin{aligned}
u_{ns}{}^{(1)}&=\frac{1}{\pi\lambda_{ns}}\{n\varepsilon a_{ns}-(n+s)b_{ns}\}\\
u'_{ns}{}^{(1)}&=\frac{1}{\pi\lambda'_{ns}}\{n\varepsilon a_{ns}-(n-s)b_{ns}\}\\
\vartheta_{ns}{}^{(1)}&=\frac{1}{\pi\lambda_{ns}}\{n\varepsilon c_{ns}+(n+s)d_{ns}\}\\
\vartheta_{ns}{}'^{(1)}&=\frac{1}{\pi\lambda'_{ns}}\{n\varepsilon c_{ns}+(n-s)d_{ns}\}\\
w_{ns}{}^{(1)}&=\frac{1}{\pi\lambda_{ns}}\{-n\varepsilon d_{ns}+(n+s)c_{ns}\}\\
w'_{ns}{}^{(1)}&=\frac{1}{\pi\lambda'_{ns}}\{-n\varepsilon d_{ns}+(n-s)c_{ns}\}\\
z_{ns}{}^{(1)}&=\frac{-1}{\pi\lambda_{ns}}\{n\varepsilon b_{ns}+(n+s)a_{ns}\}\\
z'_{ns}{}^{(1)}&=\frac{-1}{\pi\lambda'_{ns}}\{n\varepsilon b_{ns}+(n-s)a_{ns}\}
\end{aligned}\right\}\quad(13)$$

and

$$u_{ns}{}^{(2)}=\frac{-1}{\pi\lambda_{ns}}\{n\varepsilon w_{ns}{}^{(1)}+(n+s)\vartheta_{ns}{}^{(1)}\},$$
$$u_{ns}{}'^{(2)}=\frac{-1}{\pi\lambda'_{ns}}\{n\varepsilon w'^{(1)}+(n-s)\vartheta'_{ns}{}^{(1)}\},$$
$$\vartheta_{ns}{}^{(2)}=\frac{1}{\pi\lambda_{ns}}\{-n\varepsilon z_{ns}{}^{(1)}+(n+s)u_{ns}{}^{(1)}\},$$
$$\vartheta_{ns}{}'^{(2)}=\frac{1}{\pi\lambda'_{ns}}\{-u\varepsilon z'_{ns}{}^{(1)}+(n-s)u'_{ns}{}^{(1)}\},$$
$$w_{ns}{}^{(2)}=\frac{-1}{\pi\lambda_{ns}}\{n\varepsilon u_{ns}{}^{(1)}+(n+s)z_{ns}{}^{(1)}\},$$
$$w_{ns}{}'^{(2)}=\frac{-1}{\pi\lambda'_{ns}}\{n\varepsilon u'_{ns}{}^{(1)}+(n-s)z'_{ns}{}^{(1)}\},$$
$$z_{ns}{}^{(2)}=\frac{1}{\pi\lambda_{ns}}\{-n\varepsilon\vartheta_{ns}{}^{(1)}+(n+s)w_{ns}{}^{(1)}\},$$
$$z_{ns}{}'^{(2)}=\frac{1}{\pi\lambda'_{ns}}\{-n\varepsilon\vartheta'_{ns}{}^{(1)}+(n-s)w'_{ns}{}^{(1)}\},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(14)$$

writing

$$\lambda_{ns}=n^2\varepsilon^2+(n+s)^2,\ \lambda'_{ns}=n^2\varepsilon^2+(n-s)^2,$$

$$a_{ns}=\frac{-(-1)^{\frac{n-1}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\left\{(-1)^s\sinh\frac{n\pi}{2}\varepsilon(a'-1)+\sinh\frac{n\pi}{2}\varepsilon(a'+1)\right\}$$

$$b_{ns}=\frac{(-1)^{\frac{n}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\left\{(-1)^s\cosh\frac{n\pi}{2}\varepsilon(a'-1)-\cosh\frac{n\pi}{2}\varepsilon(a'+1)\right\}$$

$$c_{ns}=\frac{-(-1)^{\frac{n}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\left\{(-1)^s\sinh\frac{n\pi}{2}\varepsilon(a'-1)-\sinh\frac{n\pi}{2}\varepsilon(a'+1)\right\}$$

$$d_{ns}=\frac{(-1)^{\frac{n-1}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\left\{(-1)^s\cosh\frac{n\pi}{2}\varepsilon(a'-1)+\cosh\frac{n\pi}{2}\varepsilon(a'+1)\right\}$$

$$a'_{ns}=\frac{-(-1)^{\frac{n-1}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}\left\{(-1)^s\sinh\frac{n\pi}{2}\varepsilon(a'-1)\right\},$$

$$b'_{ns}=\frac{(-1)^{\frac{n}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}(-1)^s\cosh\frac{n\pi}{2}\varepsilon(a'-1),$$

$$c'_{ns}=\frac{-(-1)^{\frac{n}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}(-1)^s\sinh\frac{n\pi}{2}\varepsilon(a'-1),$$

$$d'_{ns}=\frac{(-1)^{\frac{n-1}{2}}}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)}(-1)^s\cosh\frac{n\pi}{2}\varepsilon(a'-1),$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

and then, expanding Equation (8), (9) to Fourier Sine series, each value of every terms in expression (8)$'$ are expressed as follows.

$$\begin{aligned}
g_{ns}=&2\frac{n}{\pi}\{\varepsilon^2(1-\nu)f_{ns}{}^{(1)}+\varepsilon(1+\nu\varepsilon^2)\phi_{ns}{}^{(1)}\}\\
&+\frac{1}{2}(1-\nu)\varepsilon^2n^2[(a'+1)\{(1-\varepsilon^2)\varphi_{ns}{}^{(1)}\\
&+2\varepsilon\tau_{ns}{}^{(1)}\}-2\{(1-\varepsilon^2)(\varphi_{ns}{}'^{(1)}-\varphi_{ns}{}^{(2)})\\
&+2\varepsilon(\tau'_{ns}{}^{(1)}-\tau_{ns}{}^{(2)})\}]
\end{aligned}$$

$$\begin{aligned}
h_{ns}=&2\frac{n}{\pi}\{(1+\nu\varepsilon^2)f_{ns}{}^{(1)}-\varepsilon(1-\nu)\phi_{ns}{}^{(1)}\}\\
&+\frac{1}{2}(1-\nu)\varepsilon n^2[(a'+1)\{(1-\varepsilon^2)\tau_{ns}{}^{(1)}\\
&-2\varepsilon\varphi_{ns}{}^{(1)}\}-2\{(1-\varepsilon^2)(\tau_{ns}{}'^{(1)}-\tau_{ns}{}^{(2)})\\
&-2\varepsilon(\varphi'_{ns}{}^{(1)}-\varphi_{ns}{}^{(2)})\}]
\end{aligned}$$

$$j_{ns}=-\frac{n^2}{4}(1-\nu)\{(1-\varepsilon^2)f_{ns}{}^{(1)}-2\varepsilon\phi_{ns}{}^{(1)}\}$$

$$i_{ns}=-\frac{n^2}{4}(1-\nu)\varepsilon\{(1-\varepsilon^2)\phi_{ns}{}^{(1)}+2\varepsilon f_{ns}{}^{(1)}\}$$

$$R_s=\frac{8(\nu+\varepsilon^2)}{\pi^5s^3(1+\varepsilon^2)^2}\{1-(-1)^s\}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(16)$$

and

$$\begin{aligned}
g'_{ns}=&\frac{n^2}{\pi}\varepsilon(c_1\tau_{ns}{}^{(1)}+c_2\varphi_{ns}{}^{(1)})+\frac{n^3}{4}\varepsilon^2[(a'+1)\\
&\cdot(c_4f_{ns}{}^{(1)}+c_3\phi_{ns}{}^{(1)})-2\{c_4(f_{ns}{}'^{(1)}
\end{aligned}$$

$$-f_{ns}{}^{(2)})+c_3(\phi_{ns}{}'^{(1)}-\phi_{ns}{}^{(2)})\}]$$

$$h'_{ns}=\frac{n^2}{\pi}(-c_1\varphi_{ns}{}^{(1)}+c_2\tau_{ns}{}^{(1)})+\frac{n^3}{4}\varepsilon[(a'+1)\cdot(c_3f_{ns}{}^{(1)}-c_4\phi_{ns}{}^{(1)})-2\{c_3(f_{ns}{}'^{(1)}-f_{ns}{}^{(2)})-c_4(\phi_{ns}{}'^{(1)}-\phi_{ns}{}^{(2)})\}]$$

$$j'_{ns}=-\frac{n^3}{8}(c_3\tau_{ns}{}^{(1)}-c_4\varphi_{ns}{}^{(1)})$$

$$i'_{ns}=-\frac{n^3}{8}\varepsilon(c_4\tau_{ns}{}^{(1)}+c_3\varphi_{ns}{}^{(1)})$$

$$R_s'=\frac{2}{\pi^3}\cdot\frac{\varepsilon}{1+\varepsilon^3}\left(1+\frac{1-\nu}{1+\varepsilon^2}\right)\frac{1+(-1)^s}{s\pi}\quad\cdots\cdots(17)$$

where

$$c_1=\varepsilon(1+\varepsilon^2)-(1-\nu)(2\varepsilon-\varepsilon^3),$$

$$c_2=1+\varepsilon^2+\frac{1}{2}(1-\nu)(5\varepsilon^2-1),$$

$$c_3=(1-\nu)(3\varepsilon^2-1),\qquad c_4=(1-\nu)(\varepsilon^3-3\nu)\quad\cdots\cdots(17)'$$

Thus, we have two groups of equations providing relation for determining the unknown constants A_n, C_n, B_n, and D_n. Furthermore, the value of $\sinh\frac{n\pi}{2}\varepsilon(a'-1)$ or $\cosh\frac{n\pi}{2}\varepsilon(a'-1)$ becomes negligible in comparison with $\sinh\frac{n\pi}{2}\cdot\varepsilon(a'+1)$ when $n\rightarrow\infty$. In that case, the value of f_{ns}, τ_{ns}, ϕ_{ns}, φ_{ns}, is expressed as follows.

$$\underset{n=1,3,5,\cdots}{f_{ns}{}^{(1)}}=\underset{n=1,3,5,\cdots}{\tau_{ns}{}^{(1)}}=W_{ns}{}^{(1)}\frac{a_{ns}}{\pi},$$

$$\underset{n=1,3,5,\cdots}{\phi_{ns}{}^{(1)}}=\underset{n=1,3,5,\cdots}{\varphi_{ns}{}^{(1)}}=V_{ns}{}^{(1)}\frac{a_{ns}}{\pi}$$

$$\underset{n=1,3,5,\cdots}{f_{ns}{}^{(2)}}=\underset{n=1,3,5,\cdots}{\tau_{ns}{}^{(2)}}=W_{ns}{}^{(2)}\frac{a_{ns}}{\pi^2},$$

$$\underset{n=1,3,5,\cdots}{\phi_{ns}{}^{(2)}}=\underset{n=1,3,5,}{\varphi_{ns}{}^{(2)}}=V_{ns}{}^{(2)}\frac{a_{ns}}{\pi^2},\quad\cdots\cdots(18)_a$$

and

$$\underset{n=2,4,6,\cdots}{f_{ns}{}^{(1)}}=\underset{n=2,4,6,\cdots}{\tau_{ns}{}^{(1)}}=-V_{ns}{}^{(1)}\frac{b_{ns}}{\pi},$$

$$\underset{n=2,4,6,\cdots}{\phi_{ns}{}^{(1)}}=\underset{n=2,4,6,\cdots}{\varphi_{ns}{}^{(1)}}=W_{ns}{}^{(1)}\frac{b_{ns}}{\pi},$$

$$\underset{n=2,4,6,\cdots}{f_{ns}{}^{(2)}}=\underset{n=2,4,6,\cdots}{\tau_{ns}{}^{(2)}}=-V_{ns}{}^{(2)}\frac{b_{ns}}{\pi^2},$$

$$\underset{n=2,4,6,\cdots}{\phi_{ns}{}^{(2)}}=\underset{n=2,4,6,\cdots}{\varphi_{ns}{}^{(2)}}=W_{ns}{}^{(2)}\frac{b_{ns}}{\pi^2},\quad\cdots\cdots(18)_b$$

where when $n\gg s$, the value of W_{ns}, V_{ns} is expressed as follows.

$$W_{ns}{}^{(1)}=-\frac{4\varepsilon s}{n^2(1+\varepsilon^2)^2},\quad V_{ns}{}^{(1)}=-\frac{2(1-\varepsilon^2)s}{n^2(1+\varepsilon^2)^2},$$

$$W_{ns}{}^{(2)}=\frac{4s(3\varepsilon^2-1)}{n^3(1+\varepsilon^2)^3},\quad V_{ns}{}^{(2)}=\frac{4s\varepsilon(3-\varepsilon^2)}{n^3(1+\varepsilon^2)^3}\quad\cdots\cdots(18)_c$$

Applying the expression (18) to the expression (16),

$$j_n=0,\quad i_n=0,\quad g_n=-\frac{8\varepsilon^2 s}{n\pi^2(1+\varepsilon^2)}(-1)^{n/2},$$

$$h_n=-\frac{8\varepsilon s}{n\pi^2(1+\varepsilon^2)}(-1)^{n/2},\quad\cdots\cdots(19)_a$$

Similary applying to the expression (17),

$$j_n'=\frac{1}{4}\varepsilon(1-\nu)n\frac{s}{\pi}(-1)^{\frac{n-1}{2}},$$

$$i_n'=\frac{1}{4}\varepsilon^2(1-\nu)n\frac{s}{\pi}(-1)^{n/2},$$

$$h_n'=\left\{\frac{2\varepsilon}{\pi}+\frac{1}{2}\varepsilon^2 n(1-\nu)(a'+1)\right\}\frac{s}{\pi}(-1)^{\frac{n-1}{2}},$$

$$g_n'=\left\{\frac{2\varepsilon}{\pi}+\frac{1}{2}\varepsilon^2 n(1-\nu)(a'+1)\right\}\frac{s}{\pi}\varepsilon(-1)^{\frac{n-1}{2}},\quad\cdots\cdots(19)_b$$

From the expression $(19)_b$, we obtain the following results.

$$\text{i.e.}\quad h'_{ns}=\frac{s}{s+1}h'_{ns+1},\quad g'_{ns}=\frac{s}{s+1}g'_{ns+1},\quad j'_{ns}=\frac{s}{s+1}j'_{ns+1},\quad i'_{ns}=\frac{s}{s+1}i'_{ns+1}\quad (19)_c$$

From the expression $(19)_c$, the expression (17) can be transformed as follows.

$$\text{i.e.}\quad \bar{h}'_{ns}=\frac{s}{s-1}h'_{ns-1}-h'_{ns},\quad \bar{g}'_{ns}=\frac{s}{s-1}g'_{ns-1}-g'_{ns},\quad \bar{j}'_{ns}=\frac{s}{s-1}j'_{ns-1}-j'_{ns},\quad \bar{i}'_{ns}=\frac{s}{s-1}i'_{ns-1}-i'_{ns},\quad\text{for } s\geqq 2\quad\cdots\cdots(20)$$

In the case of $s=1$

$$\bar{h}'_{n1}=\int_0^1 h_n dy-h'_{n1},\quad \bar{g}'_{n1}=\int_0^1 g_n dy-g'_{n1},\quad \bar{j}'_{n1}=\int_0^1 j'_n dy-j'_{n1},\quad \bar{i}'_{n1}=\int_0^1 i_n dy-i'_{n1},\quad\cdots\cdots(20)'$$

where

$$\int_0^1 h_n dy=-\frac{\varepsilon}{\pi}\left[\frac{2}{\pi}(1+\gamma_2)+\frac{1}{2}(1-\nu)\varepsilon n\cdot\left\{(a'+1)-\frac{2\gamma_1}{\alpha_1+\gamma_1}\right\}(\alpha_1+\gamma_1)\right](-1)^{\frac{n-1}{2}},$$

$$\int_0^1 g_n dy=-\frac{\varepsilon}{\pi}\left[\frac{2\varepsilon}{\pi}(\alpha_1-\gamma_1)+\frac{1}{2}(1-\nu)\varepsilon^2 n\cdot\left\{(a'+1)+\frac{2\gamma_2}{1-\gamma_2}\right\}(1-\gamma_2)\right](-1)^{n/2},$$

$$\int_0^1 j_n dy=\frac{n}{4}(1-\nu)\frac{\varepsilon}{\pi}(1+\gamma_2)(-1)^{\frac{n-1}{2}},$$

$$\int_0^1 i_n dy=\frac{n}{4}(1-\nu)\frac{\varepsilon^2}{\pi}(\alpha_1-\gamma_1)(-1)^{n/2},$$

$$\text{and}\quad \gamma_1=\frac{\sinh\frac{n\pi}{2}\varepsilon(a'-1)}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)},\quad \gamma_2=\frac{\cosh\frac{n\pi}{2}\varepsilon(a'-1)}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)},$$

$$\alpha_1=\frac{\sinh\frac{n\pi}{2}\varepsilon(a'+1)}{\cosh\frac{n\pi}{2}\varepsilon(a'+1)},$$

Finally we obtain following two groups of equations from the expression (16), (20).

i.e.

$$\left.\begin{array}{l}\sum_{n=1,3,5,\cdots} h_{1n}D_n+\sum_{n=2,4,6,\cdots} g_{1n}C_n+\sum_{n=1,3,5,\cdots} j_{1n}B_n+\sum_{n=2,4,6,\cdots} i_{1n}A_n=R_1\\ \sum_{n=1,3,5,\cdots} h_{2n}D_n+\sum_{n=2,4,6,\cdots} g_{2n}C_n+\sum_{n=1,3,5,\cdots} j_{2n}B_n+\sum_{n=2,4,6,\cdots} i_{2n}A_n=R_2\\ \vdots\\ \sum_{n=1,3,5,\cdots} h_{kn}D_n+\sum_{n=2,4,6,\cdots} g_{kn}C_n+\sum_{n=1,3,5,\cdots} j_{kn}B_n+\sum_{n=2,4,6,\cdots} i_{kn}A_n=R_k\\ \vdots\end{array}\right\}\quad(21)$$

and

$$\left.\begin{array}{l}\sum_{n=1,3,5,\cdots} \overline{h'}_{1n}D_n+\sum_{n=2,4,6,\cdots} \overline{g'}_{1n}C_n+\sum_{n=1,3,5,\cdots} \overline{j'}_{1n}B_n+\sum_{n=2,4,6,\cdots} \overline{i'}_{1n}A_n=R_1'\\ \sum_{n=1,3,5,\cdots} \overline{h'}_{2n}D_n+\sum_{n=2,4,6,\cdots} \overline{g'}_{2n}C_n+\sum_{n=1,3,5,\cdots} \overline{j'}_{2n}B_n+\sum_{n=2,4,6,\cdots} \overline{i'}_{2n}A_n=R_2'\\ \vdots\\ \sum_{n=1,3,5,\cdots} \overline{h'}_{kn}D_n+\sum_{n=2,4,6,\cdots} \overline{g'}_{kn}C_n+\sum_{n=1,3,5,\cdots} \overline{j'}_{kn}B_n+\sum_{n=2,4,6,\cdots} \overline{i'}_{kn}A_n=R_3'\\ \vdots\end{array}\right\}\quad(22)$$

where two growps of these equations have special relations each other from the expression $(13)_a$, $(18)_b$ and $(18)_c$. i.e. these equations are written as follows.

$$\sum_{n=1,3,5,\cdots k-1} h_{1n}D_n+\sum_{n=2,4,6,\cdots k-2} g_{1n}C_n+\sum_{n=1,3,5,\cdots k-1} j_{1n}B_n+\sum_{n=2,4,6,\cdots k-2} i_{1n}A_n$$
$$+h_{1k}(C_k+\alpha_{1\cdot k+1}D_{k+1}+\alpha_{1\cdot k+2}C_{k+2}+\cdots\cdots)$$
$$+j_{1k}(A_k+\beta_{1\cdot k+1}B_{k+1}+\beta_{1\cdot k+2}A_{k+2}\cdots\cdots)=R_1$$

in k th equation

$$\sum_{n=1,3,5,\cdots k-1} h_{kn}D_n+\sum_{n=2,4,6,\cdots k-2} g_{kn}C_n+\sum_{n=1,3,5,\cdots k-1} j_{kn}B_n+\sum_{n=2,4,6,\cdots k-2} i_{kn}A_n$$
$$+h_{kk}(C_k+\alpha_{kk+1}D_{k+1}+\alpha_{k\cdot k+2}C_{k+2}+\cdots\cdots)$$
$$+j_{kk}(A_k+\beta_{k\cdot k+1}B_{k+1}+\beta_{k\cdot k+2}A_{k+2}+\cdots\cdots)=R_k$$

Similary

$$\sum_{n=1,3,5,\cdots k-1} \overline{h}'_{1n}D_n+\sum_{n=2,4,6,\cdots k-2} \overline{g}'_{1n}C_n+\sum_{n=1,3,5,\cdots k-1} \overline{j}'_{1n}B_n+\sum_{n=2,4,6,\cdots k-2} \overline{i}'_{1n}A_n$$
$$+\overline{h}'_{1k}(C_k+\alpha'_{1\cdot k+1}D_{k+1}+\alpha'_{1\cdot k+2}C_{k+2}+\cdots\cdots)$$
$$\overline{j}'_{1k}(A_k+\beta'_{1\cdot k+1}B_{k+1}+\beta'_{1\cdot k+2}A_{k+2}+\cdots\cdots)=R_1'$$

in k th equation

$$\sum_{n=1,3,5,\cdots k-1} \overline{h}'_{kn}D_n+\sum_{n=2,4,6,\cdots k-2} \overline{g}'_{kn}C_n+\sum_{n=1,3,5,\cdots k-1} \overline{j}'_{kn}B_n+\sum_{n=2,4,6,\cdots k-2} \overline{i}'_{kn}A_n$$
$$+\overline{h}'_{kk}(C_k+\alpha'_{k\cdot k+1}D_{k+1}+\alpha'_{k\cdot k+2}C_{k+2}+\cdots\cdots)$$
$$+\overline{j}'_{kk}(A_k+\beta'_{k\cdot k+1}B_{k+1}+\beta'_{k\cdot k+2}A_{k+2}+\cdots\cdots)=R'_k$$

where each coefficient of unknown constant have approximately the following relations.

i.e. in the $k+t$ the coefficients.

$$\alpha_{1\cdot k+t}\fallingdotseq\alpha_{2\cdot k+t}\fallingdotseq\alpha_{3\cdot k+t}\cdots\cdots\fallingdotseq\alpha_{k\cdot k+t}\cdots\cdots$$
$$\fallingdotseq\alpha_1'{}_{k+t}\fallingdotseq\alpha_2'{}_{k+t}\cdots\cdots\fallingdotseq\alpha'_{k\cdot k+t}\cdots\cdots(t=1,2,3,\cdots)$$

and

$$\beta_{1\cdot k+t}\fallingdotseq\beta_{2\cdot k+t}\fallingdotseq\beta_{3\cdot k+t}\cdots\cdots\fallingdotseq\beta_{k\cdot k+t}\cdots\cdots$$
$$\fallingdotseq\beta'_{1\cdot k\cdot t}\fallingdotseq\beta'_{2\cdot k+t}\cdots\cdots\fallingdotseq\beta'_{k\cdot k+t}$$

By these relations, we can select the most properble and balanced value of each coefficient, where the value γ_t and γ_t' equals nearly each coefficients, $\alpha_{k\cdot k+t}$, $\alpha'_{k\cdot k+t}$, and $\beta_{k\cdot k+t}$, $\beta'_{k\cdot k+t}$.

Therefore, unkown constants group containing these coefficient γ_t, γ_t', are expressed as follows.

$$\overline{C}_2=C_2+\gamma_3D_3+\gamma_4C_4+\gamma_5D_5+\cdots\cdots\gamma_kC_k+\cdots\cdots$$
$$\overline{A}_2=A_2+\gamma_3'D_3+\gamma_4'A_4+\gamma_5'B_5+\cdots\cdots\gamma_k'A_k+\cdots\cdots$$
$$\overline{D}_3=D_3+\frac{1}{\gamma_3}(\gamma_4C_4+\gamma_5D_5+\gamma_6C_6+\cdots\cdots\gamma_kC_k+\cdots\cdots)$$
$$\overline{B}_3=B_3+\frac{1}{\gamma_3'}(\gamma_4'A_4+\gamma_5'B_5+\gamma_6'A_6+\cdots\gamma_k'A_k+\cdots)$$

in the k th unknown constant group

$$\overline{C}_k=C_k+\frac{1}{\gamma_k}(\gamma_{k+1}D_{k+1}+\gamma_{k+2}C_{k+2}+\gamma_{k+3}D_{k+3}\cdots\cdots)$$
$$\overline{A}_k=A_k+\frac{1}{\gamma'_k}(\gamma'_{k+1}B_{k+1}+\gamma'_{k+2}A_{k+2}$$
$$+\gamma'_{k+3}B_{k+3}+\cdots\cdots)\qquad k\geqq 4\quad\cdots\cdots\cdots(23)$$

Furthermore, from the expression (23)

$$\left.\begin{array}{l}\overline{D}_3=\frac{1}{\gamma_3}(\overline{C}_2-C_2),\ \overline{C}_4=\frac{1}{\gamma_4}(\overline{C}_2-C_2-\gamma_3D_3),\\ \overline{B}_3=\frac{1}{\gamma_3'}(\overline{A}_2-A_2),\ \overline{A}_4=\frac{1}{\gamma_4'}(\overline{A}_2-A_2-\gamma_3'B_3),\\ \overline{C}_k=\frac{1}{\gamma_k}(\overline{C}_2-C_2-\gamma_3D_3-\gamma_4C_4-\cdots-\gamma_{k-1}D_{k-1}),\\ \overline{A}_k=\frac{1}{\gamma_k'}(\overline{A}_2-A_2-\gamma_3'B_3-\gamma_4'A_4-\cdots\gamma'_{k-1}B_{k-1},\end{array}\right\}\quad(24)$$

Substituting into the equation (21), we obtain finally the following equations.

i.e.

$$h_{11}D_1+j_{11}B_1+g_{12}\overline{C}_2+i_{12}\overline{A}_2=R_1+\Delta_1\quad\cdots\cdots\cdots(a_1)$$
$$h_{21}D_1+j_{21}B_1+g_{22}\overline{C}_2+i_{22}\overline{A}_2=R_2+\Delta_2\quad\cdots\cdots\cdots(a_2)$$
$$\left(g_{32}-\frac{1}{\gamma_3}h_{33}\right)C_2+\left(i_{32}-\frac{1}{\gamma_3'}j_{33}\right)A_2=R_3-h_{31}D_1$$
$$-j_{31}B_1-\frac{1}{\gamma_3}h_{33}\overline{C}_2-\frac{1}{\gamma_3'}j_{33}\overline{A}_2+\Delta_3\quad\cdots\cdots\cdots\cdots(a_3)$$

therefore in k th equation

$$\left(h_{k\cdot k-1}-\frac{\gamma_{k-1}}{\gamma_k}g_{kk}\right)C_{k-1}+\left(j_{k\cdot k-1}-\frac{\gamma'_{k-1}}{\gamma_k'}i_{kk}\right)A_{k-1}$$
$$=R_k+\Delta_k-h_{k1}D_1-g_{k2}C_2-h_{k3}D_3\cdots\cdots$$
$$-h_{k\cdot k-2}D_{k-2}-\frac{1}{\gamma_k}g_{kk}(\overline{C}_2-C_2-\gamma_3D_3-\gamma_4C_4\cdots\cdots$$
$$-\gamma_{k-2}D_{k-2})-j_{k1}B_1-j_{k2}A_2-j_{k3}B_3-\cdots\cdots$$
$$-j_{k\cdot k-2}B_{k-2}-\frac{1}{\gamma_k'}i_{kk}(\overline{A}_2-A_2-\gamma_3'B_3$$
$$-\gamma_4'A_4\cdots\cdots-\gamma'_{k-2}B_{k-2})\quad\cdots\cdots\cdots\cdots\cdots\cdots(a_k)$$

Similary from the equation (22)

$$\overline{h'}_{11}D_1+\overline{j'}_{11}B_1+\overline{g'}_{12}\overline{C}_2+\overline{i'}_{12}\overline{A}_2=R_1'+\Delta_1'\quad\cdots\cdots(b_1)$$
$$\overline{h'}_{21}D_1+\overline{j'}_{21}B_1+\overline{g'}_{22}\overline{C}_2+\overline{i'}_{22}\overline{A}_2=R_2'+\Delta_2'\quad\cdots\cdots(b_2)$$

in the k th equation

$$\left(\overline{h}'_{k\cdot k-1}-\frac{\gamma_{k-1}}{\gamma_k}\overline{g}'_{kk}\right)C_{k-1}+\left(\overline{j}'_{kk}-\frac{\gamma_{k-1}'}{\gamma_k'}\overline{i}'_{kk}\right)A_{k-1}$$
$$=R_k'+\Delta_k'-\overline{h}'_{k1}D_1-\overline{g}'_{k2}C_2-\overline{h}'_{k3}D_3\cdots\cdots$$
$$-\overline{h}'_{k\cdot k-2}D_{k-2}-\frac{1}{\gamma_k}\overline{g}'_{kk}(\overline{C}_2-C_2-\gamma_3D_3-\gamma_4C_4\cdots\cdots$$
$$-\gamma_{k-2}D_{k-2})-\overline{j}'_{k1}B_1-\overline{i}'_{k2}A_2-\overline{j}'_{k3}B_3\cdots\cdots$$

$$-\bar{j}'_{k\cdot k-2}B_{k-2}-\frac{1}{\gamma'_k}\bar{i}'_{kk}(\bar{A}_2-A_2-\gamma_3'B_3\cdots\cdots$$
$$-'\gamma_{k-2}B_{k-2}) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (b_k)$$

Where Δ_k and Δ_k' $(k=1,2,3,\cdots\cdots)$ are residues of difference between γ_t, γ_t' and α_{kk+t}, α'_{kk+t}, β_{kk+t}, β'_{kk+t}, and also Δ_k, $\Delta_k'(k=1,2,3)$ are negligible small.

Therefore, from the first four sest of equations, (a_1), (a_2), (b_1), (b_2), we obtain the approximate value of the unknown constant, D_1, B_1, $\bar{C}_2$, and $\bar{A}_2$, thereby substituting these values into the next two sets of equations (a_3), (b_3), we obtain the approximate value of the unknown constant, C_2, A_2.

And then, we obtain successively the each values of the unknown constant, D_k, B_k $(k=3,4,5,\cdots\cdots)$ from each two sets of equations $(a_{k+1})(b_{k+1})$. Again substituting these approximate each value into the first equations (21) and (22), we obtain the more accurate approximate value.

4. Numerical Example

We take the example of the skewed plate with two opposite edges simply supported and the other edges free in the case of $a=b$, $\varepsilon=0.70$, $\nu=0.30$, full uniform load.

Taking the unknown constant B_n, D_n, to $n=10$, we obtain the following **Table 1** from the equations (21) and (22).

Table 1(a) Array of Coefficients of Eqations (21)

	j_{k1}	i_{k2}	j_{k3}	i_{k4}	j_{k5}	i_{k6}	j_{k7}	i_{k8}	j_{k9}	i_{k10}	
$k=1$	−0.06355	0.01363	0.00817	−0.00316	−0.00286	0.00136	0.00145	−0.00077	−0.00087	0.00049	
$k=2$	−0.09702	0.09802	0.07068	−0.02692	−0.02397	0.01145	0.01188	−0.00632	−0.00709	0.00400	
$k=3$	−0.05099	0.14959	0.21239	−0.09388	−0.08497	0.04039	0.04159	−0.02197	−0.02453	0.01380	
$k=4$	−0.04537	0.12201	0.31134	−0.19847	−0.20198	0.09881	0.10218	−0.05385	−0.05989	0.03355	
$k=5$	−0.02905	0.09899	0.30457	−0.27809	−0.35446	0.19012	0.20300	−0.10801	−0.12032	0.06731	
$k=6$	−0.02936	0.07864	0.26509	−0.29547	−0.47705	0.29775	0.34198	−0.18774	−0.21174	0.11896	
$k=7$	−0.02038	0.06780	0.22563	−0.27620	−0.52674	0.38752	0.49625	−0.28891	−0.33543	0.19103	
$k=8$	−0.02177	0.05706	0.19513	−0.24704	−0.51835	0.43259	0.62832	−0.39700	−0.48365	0.28277	
$k=9$	−0.01573	0.05144	0.17040	−0.21932	−0.48317	0.44328	0.71037	−0.49137	−0.63804	0.38826	
$k=10$	−0.01732	0.04480	0.15177	−0.19545	−0.44122	0.42698	0.73967	−0.55609	−0.77488	0.49625	

	h_{k1}	g_{k2}	h_{k3}	g_{k4}	h_{k5}	g_{k6}	h_{k7}	g_{k8}	h_{k9}	g_{k10}	R_k
$k=1$	0.54295	−0.20105	−0.16694	0.08192	0.08979	−0.05094	−0.06115	0.03691	0.04633	−0.02892	0.018615
$k=2$	0.56424	−0.62243	−0.56876	0.25799	0.26399	−0.14207	−0.16378	0.09575	0.11716	−0.07162	0
$k=3$	0.31593	−0.78549	−1.20500	0.60597	0.61008	−0.31698	−0.35305	0.20026	0.23877	−0.14275	0.000691
$k=4$	0.25352	−0.63996	−1.57382	1.06137	1.16961	−0.61415	−0.67527	0.37594	0.43979	−0.25327	0
$k=5$	0.17586	−0.50851	−1.51033	1.37496	1.82685	−1.03376	−1.16032	0.64554	0.74879	−0.43490	0.000150
$k=6$	0.16167	−0.40708	−1.30673	1.42808	2.32312	−1.49616	−1.78561	1.01706	1.18650	−0.68758	0
$k=7$	0.12237	−0.34449	−1.11031	1.32723	2.50937	−1.86557	−2.44788	1.46670	1.75165	−1.02418	0.000054
$k=8$	0.11917	−0.29277	−0.95566	1.18269	2.45184	−2.05428	−2.99736	1.93088	2.40534	−1.43798	0
$k=9$	0.09410	−0.25959	−0.83345	1.04951	2.27984	−2.07536	−3.32970	2.32655	3.06891	−1.90110	0.000025
$k=10$	0.09454	−0.22872	−0.73935	0.93371	2.07898	−1.99295	−3.43859	2.59270	3.64604	−2.36560	0

Table 1(b) Array of Coefficients of Equations (22)

	j'_{k1}	$\bar{i}'_{k2}$	j'_{k3}	$\bar{i}'_{k4}$	j'_{k5}	$\bar{i}'_{k6}$	j'_{k7}	$\bar{i}'_{k8}$	j'_{k9}	$\bar{i}'_{k10}$	
$k=1$	0.04820	−0.01052	−0.01013	0.00487	0.00547	−0.00315	−0.00383	0.00233	0.00295	−0.00185	
$k=2$	−0.03926	0.10102	0.08229	−0.03710	−0.03848	0.02120	0.02502	−0.01496	−0.01867	0.01161	
$k=3$	0.04414	0.09260	0.24378	−0.12234	−0.12336	0.06416	0.07280	−0.04223	−0.05154	0.03152	
$k=4$	0.01036	−0.03260	0.23293	−0.23739	−0.27576	0.14618	0.16242	−0.09185	−0.10956	0.06574	
$k=5$	0.01699	−0.04251	0.02266	−0.23095	−0.43107	0.26455	0.30455	−0.17203	−0.20265	0.11573	
$k=6$	0.00240	−0.03890	−0.08026	−0.08665	−0.42325	0.36456	0.47870	−0.28315	−0.33731	0.19882	
$k=7$	0.00903	−0.02491	−0.09772	0.03206	−0.23323	0.35983	0.60911	−0.40423	−0.50589	0.30372	
$k=8$	0.00036	−0.02256	−0.08493	0.08051	−0.02802	0.23960	0.60318	−0.48754	−0.67426	0.42507	
$k=9$	0.00586	−0.01439	−0.07259	0.08903	0.09537	0.08646	0.44815	−0.48384	−0.78322	0.53852	
$k=10$	−0.00033	−0.01433	−0.05843	0.08363	0.14623	−0.02971	0.22896	−0.38598	−0.77845	0.60864	

	h'_{k1}	$\bar{g}'_{k2}$	h'_{k3}	$\bar{g}'_{k4}$	h'_{k5}	$\bar{g}'_{k6}$	h'_{k7}	$\bar{g}'_{k8}$	h'_{k9}	$\bar{g}'_{k10}$	$\bar{R}'_k$
$k=1$	−0.11252	0.02903	0.03282	−0.01722	−0.01960	0.01165	0.01523	−0.00894	−0.00625	0.00813	−0.008413
$k=2$	0.43614	−0.33148	−0.27830	0.13309	0.14339	−0.08106	−0.09750	0.05911	0.07461	−0.04685	−0.014177
$k=3$	0.03412	−0.56118	−0.89705	0.44933	0.46210	−0.24761	−0.28555	0.16773	0.20677	−0.12752	0.021267
$k=4$	−0.04000	−0.15258	−1.15995	0.92388	1.05930	−0.56933	−0.64125	0.36670	0.44135	−0.26678	−0.007089
$k=5$	−0.03289	0.06528	−0.54288	1.06911	1.73068	−1.04537	−1.21103	0.69013	0.81949	−0.48766	0.008862
$k=6$	−0.02083	0.09735	0.01387	0.65249	1.89899	−1.49203	−1.92857	1.14312	1.36951	−0.81189	−0.004727

$k=7$	−0.01899	0.08346	0.21773	0.15236	1.34493	−1.59046	−2.52504	1.65140	2.06600	−1.24467	0.005516
$k=8$	−0.01106	0.07231	0.25272	−0.13152	0.55431	−1.23409	−2.64538	2.03944	2.78345	−1.75167	−0.003546
$k=9$	−0.01196	0.05576	0.23876	−0.24041	−0.02990	−0.66533	−2.17905	2.11291	3.29823	−2.24104	0.003988
$k=10$	−0.00658	0.04897	0.20784	−0.26449	−0.34188	−0.16510	−1.37605	1.81351	3.39118	−2.57616	−0.002836

Assuming the most properble coefficient value r_t, r_t', to the following value, where

$r_3=0.90,\quad r_4=-0.40,\quad r_5=-0.44,\quad r_6=0.23,$
$r_7=0.27,\quad r_8=-0.18,\quad r_9=-0.22,\quad r_{10}=0.14,$
and $r_3'=0.80,\quad r_4'=-0.30,\quad r_5'=-0.30,$
$r_6'=0.15,\quad r_7'=0.18,\quad r_8'=-0.10,$
$r_9'=-0.12,\quad r_{10}'=0.077,$

we have the first approximate value from (a_1), (a_2),……(a_{10}), and (b_1), (b_2),……(b_{10}).

i.e.

$B_1^{(1)}=-0.19446,\qquad D_1^{(1)}=0.01943,$
$\bar{A}_2^{(1)}=-0.29534,\qquad \bar{C}_2^{(1)}=0.001344,$
$A_2^{(1)}=-0.57925,\qquad C_2^{(1)}=-0.005428,$
$B_3^{(1)}=0.66492,\qquad D_3^{(1)}=0.03506,$
$A_4^{(1)}=3.24861,\qquad C_4^{(1)}=0.74938,$
$B_5^{(1)}=-4.19376,\qquad D_5^{(1)}=-1.17940,$
$A_6^{(1)}=-4.81379,\qquad C_6^{(1)}=-1.35297,$
$B_7^{(1)}=1.47483,\qquad D_7^{(1)}=0.30354,$
$A_8^{(1)}=1.11620,\qquad C_8^{(1)}=0.19116,$
$B_9^{(1)}=-0.47793,\qquad D_9^{(1)}=-0.15862,$

Substituting these value into the equation (24), we have the approximate value of $\bar{B}_3$, $\bar{D}_3$,…… $\bar{A}_{10}$, $\bar{C}_{10}$.

i.e. $\bar{B}_3^{(1)}=0.35489,\qquad \bar{D}_3^{(1)}=0.00751,$
$\bar{A}_4^{(1)}=0.82666,\qquad \bar{C}_4^{(1)}=0.06207,$
$\bar{B}_5^{(1)}=-2.42200,\qquad \bar{D}_5^{(1)}=-0.62483,$
$\bar{A}_6^{(1)}=-3.54391,\qquad \bar{C}_6^{(1)}=-1.06180,$
$\bar{B}_7^{(1)}=1.05831.\qquad \bar{D}_7^{(1)}=0.24909,$
$\bar{A}_8^{(1)}=0.74968,\qquad \bar{C}_8^{(1)}=0.08178,$
$\bar{B}_9^{(1)}=-0.30594,\qquad \bar{D}_9^{(1)}=0.08943,$
$\bar{A}_{10}^{(1)}=-0.26735,\qquad \bar{C}_{10}^{(1)}=-0.10808$

Therefore, substituting these value B_1, D_1, C_2, D_3,…… and $\bar{A}_k$,, $\bar{C}_k$ into the first equations (21) and (22), we get the secondary approximate value $B_k^{(2)}$, $D_k^{(2)}$,……, i.e. we get the value of $D_1^{(2)}$ from the first equation (a_1) of (21), and $B_1^{(2)}$ from the first equation (b_1) of (22), and then successively we get the value of $C_k^{(2)}$ from the k th equation (a_k), and $A_k^{(2)}$ from the k th equation (b_k).

Thereby, these values are as follows.

i.e. $B_1^{(2)}=-0.18041,\qquad D_1^{(2)}=0.02267,$
$A_2^{(2)}=-0.54949,\qquad C_2^{(2)}=-0.00443,$
$B_3^{(2)}=0.69287,\qquad D_3^{(2)}=0.04781,$
$A_4^{(2)}=3.49605,\qquad C_4^{(2)}=0.76078,$
$B_5^{(2)}=-4.34633,\qquad D_5^{(2)}=-1.12583,$
$A_6^{(2)}=-5.17175,\qquad C_6^{(2)}=-1.36515,$
$B_7^{(2)}=1.55240,\qquad D_7^{(2)}=0.32658,$
$A_8^{(2)}=1.05188,\qquad C_8^{(2)}=0.18154,$
$B_9^{(2)}=-0.50276,,\qquad D_9^{(2)}=-0.17457,$

As these value equals nearly the first value, we can consider them as a true value.

Therefore, substituting the above-mentioned value into the expression of moment, we obtain the following table.

Table 2 Moment of Uniformly Loaded Skewed Plate.

Location	M_y	M_x	M_{xy}	Location	M_y	M_x	M_{xy}
a_1	0.12791	0.06366	0.04669	d_3	0.28662	−0.09258	0.08093
a_2	0.20806	0.11015	0.09475	d_4	0.28613	0.05804	0.14578
a_3	0.25415	0.14006	0.12792	d_5	0.29501	0.06623	0.17618
a_4	0.26886	0.14726	0.13996	d_6	0.17371	0.13176	0.18779
c_1	0.14657	−0.02468	0.01717	d_7	0.06889	0.11498	0.10529
c_2	0.27281	0.00780	0.09021	e_3	0.28276	−0.10393	0.14592
c_3	0.27468	0.09179	0.10612	e_4	0.32166	0.03247	0.15942
c_4	0.29975	0.10383	0.14668	e_5	0.28111	0.10531	0.18806
c_5	0.26609	0.11972	0.15970	e_6	0.25188	0.08735	0.26012
c_6	0.18585	0.12821	0.12100	e_7	0.07847	0.12220	0.16133
c_7	0.09732	0.08913	0.06909				

Coeff. pa^2

From the **Table 2**, the maximum moment and the direction is obtained as follows.

Location	M_n	M_t	α	Location	M_n	M_t	α
a_1	0.15224	0.03914	27°45′	d_3	0.30318	−0.10914	11°35′
a_2	0.26576	0.05246	31°20′	d_4	0.35627	−0.01209	26°0′
a_3	0.33720	0.05700	33°0	d_5	0.39068	−0.02943	28°30′
a_4	0.36071	0.05541	33°15′	d_6	0.34180	−0.03633	41°50′
c_1	0.14109	−0.01920	5°40′	d_7	0.19958	−0.01572	38°20′
c_2	0.30082	−0.02025	17°15′	e_3	0.33192	−0.15308	18°0′
c_3	0.32334	0.04313	24°40′	e_4	0.38922	−0.03508	23°30′
c_4	0.37679	0.02619	28°5′	e_5	0.40076	−0.01432	31°50′
c_5	0.36790	0.01792	32°45′	e_6	0.44312	−0.10388	36°15′
c_6	0.28123	0.03283	38°20′	e_7	0.26313	−0.06246	41°8′
c_7	0.16248	0.02398	43°20′				

Coeff. pa^2

From the **Table 3**, we obtain the following principal moment diagram.

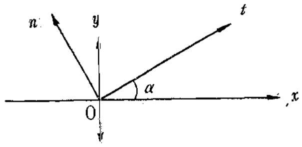

Fig. 3(a) Indicates the location of x, y in the each point of moment, shear, and reaction.

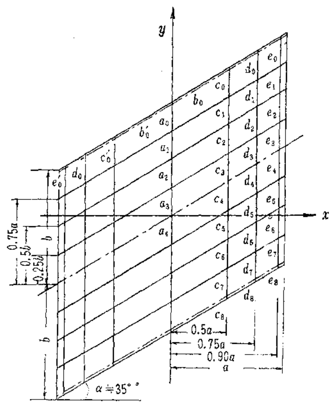

Fig. 3(b)

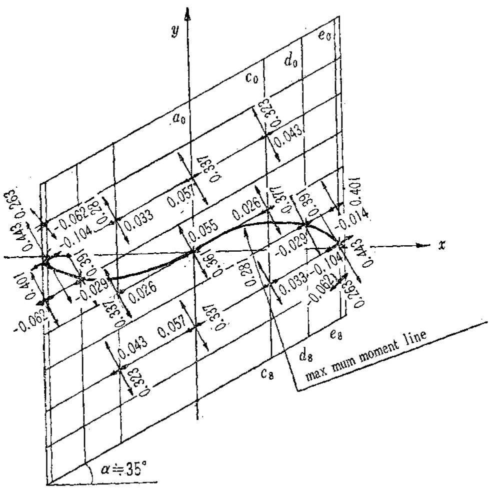

Fig. 4 Principal Moment Diagram Coeff. pa^2.

Similary, substituting the above-mentioned value into the expression of shear, we obtain the following table.

Table 4 Shear of Uniformly Loaded Skewed Plate.

Location	Q_y	Q_x	Location	Q_y	Q_x
a_0	0.63452	−0.44434	d_2	0.38184	−0.20717
a_1	0.48174	−0.32560	d_3	0.19033	−0.10954
a_2	0.32422	−0.21299	d_4	0.11580	−0.01145
a_3	0.16315	−0.10600	d_5	−0.08518	0.13523
a_4	0	0	d_6	−0.29076	0.22750
c_0	0.69382	−0.48563	d_7	−0.46803	0.33892
c_1	0.56761	−0.33240	d_8	−0.64514	0.45270
c_2	0.34406	−0.21328	e_3	0.21538	−0.13385
c_3	0.16818	−0.04806	e_4	0.17983	−0.03385
c_4	0.02764	0.03770	e_5	−0.05725	0.10425
c_5	−0.13384	0.12732	e_6	−0.21427	0.21259
c_6	−0.31178	0.23105	e_7	−0.47197	0.32313
c_7	−0.48046	0.34061	e_8	−0.61383	0.45183
c_8	−0.64972	0.45499			

Coeff. pa

Furthermore, substituting into the expression of reaction, we obtain the following table.

Location	e_0'	d_0'	c_0'	b_0'	a_0	b_0	c^0
$R_{y=sx+b}$	0.94637	0.85867	0.82949	0.81282	0.78783	0.70931	0.50500

Coeff. pa

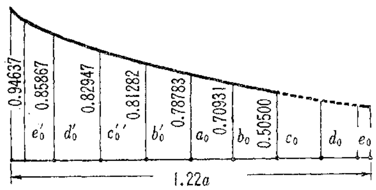

Fig. 5 Distribution of Reaction.

Table 5 indicates the numerical results of the square plate under the uniform load, comparing these values with the numerical ones for the skewed plate.

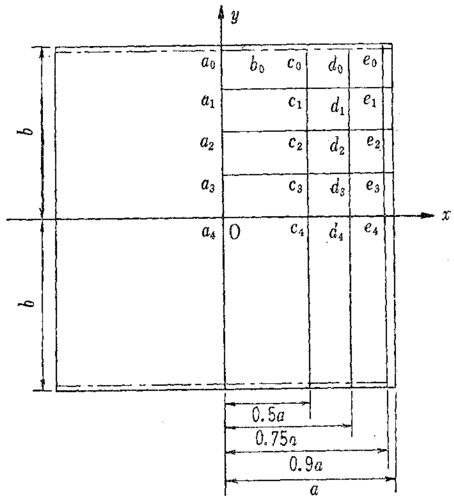

Fig. 6 Indicates the location of x, y in each point of moment, shear, and reaction.

Table 5(a) Moment of Uniformly Loded Square Plate.

Location	M_y	M_x	M_{xy}	Location	M_y	M_x	M_{xy}
a_1	0.21341	0.05121	0	d_0	0	0	0.05251
a_2	0.36515	0.08591	0	d_1	0.21850	0.03079	0.04496
a_3	0.45497	0.10558	0	d_2	0.37448	0.04819	0.03713
a_4	0.48606	0.11237	0	d_3	0.46716	0.05631	0.02009
c_0	0	0	0.02828	d_4	0.49926	0.05902	0
c_1	0.21531	0.04292	0.02613	e_0	0	0	0.07355
c_2	0.36864	0.07077	0.02000	e_1	0.22166	0.01975	0.06794
c_3	0.45984	0.08554	0.01082	e_2	0.38038	0.02772	0.05194
c_4	0.49101	0.09066	0	e_3	0.47489	0.02958	0.02815
				e_4	0.50761	0.02884	0

Coeff. pa^2

Table 5(b) Shear of Uniformly Loaded Square Plate.

Location	Q_y	Q_x	Location	Q_y	Q_x
a_0	0.93678	0	d_0	0.88919	0
a_1	0.69242	0	d_1	0.64763	0.03504
a_2	0.45593	0	d_2	0.42165	0.06473
a_3	0.22615	0	d_3	0.20760	0.08458
a_4	0	0	d_4	0	0.09155
c_0	0.91745	0	e_0	0.86433	0
c_1	0.67373	0.02070	e_1	0.62464	0.04609
c_2	0.44163	0.03825	e_2	0.40406	0.08517
c_3	0.21841	0.04998	e_3	0.19807	0.11128
c_4	0	0.05410	e_4	0	0.12044

Coeff. pa

and then, the reaction is expressed as follows.

Table 5(c) Distribution of Reaction

Location	a_5	b_5	c_0	d_0	e_0
$R)_{y=b}$	0.88949	0.87741	0.83874	0.76575	0.69977

Coeff. pa

5. Conclusion

The calculation of the skewed plate is in the case of full uniform load, but this can apply in the case of any load and any skew angle of the plate.

And this solution can not only to the other skewed plate with two opposite edges simply supported and the other edges various conditions, but can apply to a rectangular plate with any boundary condition.

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References

1) W. Fuchsteiner, : Entwicklungs functionen für polygonal begrenzte dünne platten (Der Bauingenieur, 1953, Heft 7)

2) Homberg, Marx, Schiefe Stäbe und Platten.

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