ON THE ESTIMATION OF PROBABLE RAINFALL-INTENSITY-FORMULA BY SPECIFIC COEFFICIENT METHODS

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SYNOPSIS

Succeeding to the author's previous report on the estimation method of the probable rainfall-intensity-formula based on the Logarithmic Normal distributions, another estimation method to which the Extreme Limiting Theory is applied is proposed in the first part of this paper. Moreover, an eatimation method by the "Specific Coefficient" is further proposed, which can be determined by rainfall charactristics of a short duration time in Japan, with considerations of the type of rainfall-intensity curves. For any types of the curves, and various formulas in the past can be reduced to merely one generalized form by this method, which gives the simpler estimation of the curve with higher accuracy than the former methods. It is also an advantage of this method that it can be applicable, even if the rainfall records cannot be obtained for each duration of rainfall over a long period as in the case of local cities.

1) Rigorous estimation method of probable rainfall intensities formula

The author proposes a new rigorous estimation methods, the procedure of which may be expressed as shown in Fig. 1 from ① to ⑤, inclusive, using the data of rainfall intensity of each duration. The first step of the procedure is to convert the rainfall of each duration to the rainfall intensity in terms of mm per hour, the frequency curve of each duration is as shown in Fig. 1, ①. From Fig. 1, ① the cumulative frequency curve or probability curve of the rainfall intensities data is obtained as shown in Fig. 1, ②. The cumulative frequency curves may be plotted in a straight line which is convenient for practical purposes as shown in Fig. 1, ③. In this case the Logarithmic-Normal distributions theory and Extreme Limiting theory can be applied to the calculation method of return period. However, the latter theory has been used in the present paper.

The probability of occurrence of a value in the annual series which is equal to or less than y is given by

$P(y)=\exp(-e^{-\eta})\cdots(1)$, $\eta=(a+y)/c\ \cdots(2)$

where a and c may be obtained from the following equations; $a=\gamma c-\bar{y}$, $c=(\sqrt{6}/\pi)\sigma\cdots\cdots(3)$

where $\bar{y}$ is the sample mean and σ is the sample standard deviation and $\gamma=0{,}577\ 216$ which is so called Euller's constant.

There are two analytical methods of curve fitting; the method of moments and the other, the method of least squares. The value of return period for rainfall intensities of each duration could be computed by these two methods. These relations are shown in Fig. 1 ④ and the probable values thus computed are named "2nd data". The probable rainfall intensity formula can be calculated from the 2nd data. And then, a best-fitting curve can be computed by the method of least squares for every probability of the (return period of same value) in each duration, and as a result the probable rainfall intensity curves can be set up as shown in Fig. 1, ⑤.

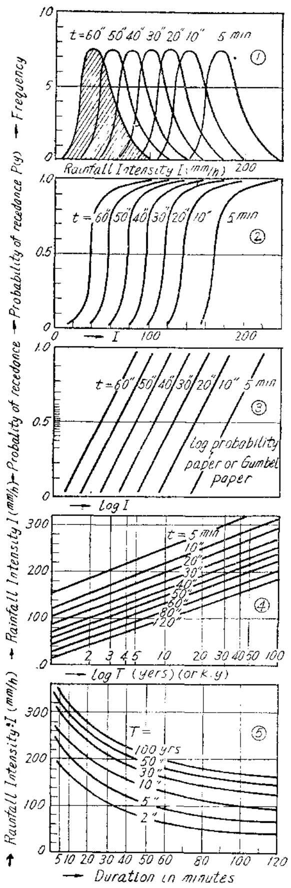

Fig. 1. Rigorous estimation method of probable rainfall intensity.

2) Estimation method of probable rainfall intensity formula by the Specific Coefficient Method and substance of rainfall intensity formula.

A number of existing rainfall intensity formula can be inclusively represented by mere one equation as follows :

$I_N = \beta_N \cdot R_N$ (this is called "Specific Coefficient formula") ··············(4)

where I_N is rainfall intensity (mm/hr), β_N is Specific Coefficient and it becomes 1.0 when t is 60 minutes, R_N is one-hour rainfall or one-hour rainfall intensity (mm/hr), and the suffix N to these three symbols designates the number of return period in terms of years. Therefore, various formulas in the past can be represented concisely as Eq. (4), which shows that the rainfall intensity curve is equal to the product of the Specific Coefficient, designating the slope of rainfall intensity curve and one-hour rainfall. This relation is the true nature of the rainfall intensity formula. Since it is not nesesary to discuss R_N in Eq. (4), because it is the value of generalized return period, here β_N only is discussed. Inasmuch as β_N can be of any type of formula showing the rainfall intensity in the past, it can be also so chosen as to take certain types of formula which is simple and of high accuracy, as standard types as follows :

$I = a/(\sqrt{t} \pm b)$, $I = a/(t+b)$, and $I = a/(t^n)$

where I is rainfall intensity, a, b and n are regional constants. And then, according to Eq. (4). β_N is represented for three cases as follows:

Case Ⅰ : $\beta_N = a'/(\sqrt{t} \pm b)$ ···········(5)

Case Ⅱ : $\beta_N = a'/(t+b)$ ··················(6)

Case Ⅲ : $\beta_N = a'/(t^n)$ ····················(7)

where β_N is Specific Coefficient of N-year probability; and also is an expression showing the ratio of the rate of rainfall in any time duration to the one-hour rainfall : t is the rainfall duration in minutes; a, b and n are regional constants. The three constants in Eqs. (5), (6) and (7) can be computed as follows;

$$\text{Case I}: \begin{cases} a' = b + \sqrt{60} \\ b = (\sqrt{60} - \beta_N{}^t \cdot \sqrt{t})/(\beta_N{}^t - 1) \end{cases} \quad \cdots(8)$$

$$\text{Case II}: \begin{cases} a' = b + 60 \\ b = (60 - \beta_N{}^t \cdot t)/(\beta_N{}^t - 1) \end{cases} \quad \cdots\cdots(9)$$

$$\text{Case III}: \begin{cases} a' = \{\text{Log}\, \beta_N{}^t (\text{Log}\, 60 - \text{Log}\, 1)\} \\ \quad /(\text{Log}\, 60 - \text{Log}\, t) \\ n = \text{Log}\, a/\text{Log}\, 60 \end{cases} \quad \cdots\cdots\cdots(10)$$

where $\beta_N{}^t$ is the value of the ratio of the rate of rainfall for a duration of t minutes to the one-hour rainfall of N-year probability, this ratio is named as "Specific Coefficient Values", and the same time duration must be taken as the value of t for each case. Fig. 2, shows the values of β_N and $\beta_N{}^{10}$ for case I, and the values of $\beta_N{}^t$ in arbitrary durations can be innummerably derived from Eq. (8). And we can express other two cases in the same manner. Therefore, the rainfall-intensity-duration formula makes the complete calculation possible for the specific coefficient method, basing on the only two data–60-minute and random durations, even if the rainfall data covering the entire time duration such as 5, 10, 20, ······120 minutes are not available.

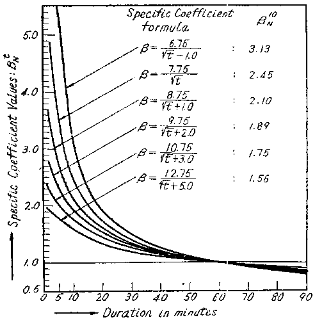

Fig. 2 Specific Coefficient Curves of $\beta = \dfrac{a'}{\sqrt{t} \pm b}$

There are many methods to determine $\beta_N{}^t$, but a most reasonable method of all is to calculate $\beta_N{}^t$ on the basis of the ratio between 60-minute rainfall intensity and t-minute rainfall intensity in a same return period, which can be analytically as well as statistically calculated from the two kinds of data on the rate of rainfall.

As the result of application of this calculation method to 147 places, the fact that the value of $\beta_N{}^t$ varies, if the return period is not same, has been ascertained. This fact has been noticed at the places 90% of the total.

Accuracy of the specific coefficient method depends on the choice of any the types of rainfall intensity formula. Therefore an optimum type of formula must be chosen for the places in question, then the author studied what type of formula is most appropriate for the rainfall data at the 147 places in Japan. The study has revealed the following : Case I is the most appropriate types for the cases in Japan. This type fits to all of the places with a mean deviation of ±5% from the data, if the time duration is set up to 60 minutes, and Cases Ⅱ and Ⅲ also fit to all of the places, excepting certain special places for which the time duration is set up to 120 minutes, with a mean deviation of ±5% from the data. In conclusion it might be mentioned that as a result of applying the specific coefficient method to the cases in Japan, two kinds of chart have been presented—the isohyetal chart of R_N is one and the specific coefficient distribution of $\beta_N{}^{10}$ in Japan is the other.

As a result of this study the probable rainfall intensity formula can be now easely obtained with a high accuracy for any places throughout Japan.