ON AN APPROXIMATE ANALYSIS OF SIMPLY SUPPORTED BOX GIRDER BRIDGE

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SYNOPSIS

As a means of the beam bridge design, hitherto, has been adopted two-dimensional calculation method; hence, main beams, floor systems and floor slabs were calculated separately. With the recent progress and development of steel highway bridge constructions and the generalization of stress measurings, it is insufficient to treat in customary means the beam bridge design, and the necessity of three-dimensional consideration has been recognized. Inasmuch as main beams of box girder bridges have especially considerable width, it is evidently inadequate to treat them as an assemblage of rods. Accordingly, in this paper, the deformation and stress of a simply supported box girder bridge with steel plate floor were three dimensionally analyzed and its approximate solution was proposed, the reliability of which was confirmed by model experiment. Further the cooperation of the floor slab in the load distribution was considered.

Analytical consideration

A system of two parallel box girders is considered which are connected with a steel plate floor and m cross beams as shown in Fig. 1, each box girder being supported simply but torsionlessly. In order to derive its solution, an elemental system is considered which consists of two parallel box girders connected by a steel plate floor without cross beam, as shown in Fig. 2. Firstly the analysis of this elemental system is considered, and the original system which has cross beams to be analyzed secondly. If, for this purpose, a section at midspan of each cross beam is assumed and the moments and shearing forces as statical redundancy are put on (Fig. 3), it is reduced to a solution of an elemental system with cross beams which are cut across their midspan.

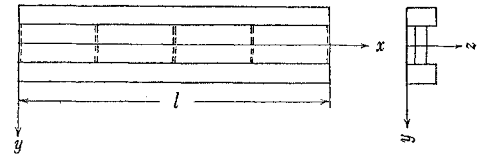

Fig. 1

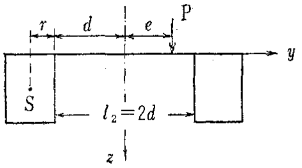

Fig. 2

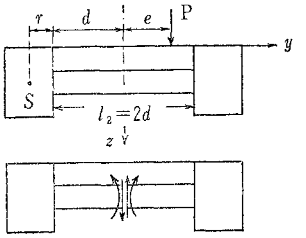

Fig. 3

Assumptions are made here for analysis as follows:—

1. The cross sectional form of a box girder is invariable for any loads.
2. Each box girder is supported simply but torsionlessly.
3. The steel slab part between the two box girders is orthotropic plate.
4. The steel slab part is simply supported at both ends along x-direction.
5. Cross beams exists indifferently to the floor slab and their torsional rigidity is neglected.
6. The neutral axis is determined by the whole cross section in the case of stress calculation.

Since the elemental system is symmetrical with respect to x-axis, it is sufficient to deal with the positive part only with respect to y-axis. Then, depending on the position and condition of loads, the following six cases are considered.

(a) Actual load——load position $o\leqq e\leqq d$
  i) symmetrical loading
  ii) skew-symmetrical loading
(b) Actual load——load position $d\leqq e$
  i) symmetrical loading
  ii) skew-symmetrical loading
(c) Statical redundancy
  i) symmetrical loading
  ii) skew-symmetrical loading

In the case (a) equations of deflection of the plate and their boundary and continuity conditions are described, as follows:—

$o\leqq y\leqq e$

$$w=\sum_n L_n(A_n \cosh \alpha_n y+B_n \sinh \alpha_n y$$
$$+C_n \alpha_n y \sinh \alpha_n y+D_n \alpha_n y \cosh \alpha_n y)$$
$$\times \sin \alpha_n x$$

$e \leqq y \leqq d$

$$w'=\sum_n L_n(A_n' \cosh \alpha_n y+B_n' \sinh \alpha_n y$$
$$+C_n' d_n y \sinh \alpha_n y+D_n' \alpha_n y \cosh \alpha_n y)$$
$$\times \sin \alpha_n x$$

	y	symmetrical loading	skew-symmetrical loading
1	0	$\frac{\partial w}{\partial y}=0$	$w=0$
2	0	$\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2 \partial y}=0$	$\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}=0$
3	e	$w-w'=0$	the same as the left
4	e	$\frac{\partial w}{\partial y}-\frac{\partial w'}{\partial y}=0$	〃
5	e	$D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)-D\left(\frac{\partial^2 w'}{\partial y^2}+\nu\frac{\partial^2 w'}{\partial x^2}=0\right)$	〃
6	e	$D\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2 \partial y}\right)-D\left(\frac{\partial^3 w'}{\partial y^3}+\frac{\partial^3 w'}{\partial x^2 \partial y}\right)=-p$	〃
7	d	$w'=\phi r+\delta$	〃
8	d	$\frac{\partial w'}{\partial y}=\phi$	〃

ϕ and δ in the above table are given by the following two equations respectively :—

$$-EC_w\frac{d^4 \phi}{dx^4}+GJ\frac{d^2 \phi}{dx^2}=D\left(\frac{\partial^2 w'}{\partial y^2}+\nu\frac{\partial^2 w'}{\partial x^2}\right)_{y=d}$$
$$+D\left(\frac{\partial^3 w'}{\partial y^3}+[2-\nu]\frac{\partial^3 w'}{\partial x^2 \partial y}\right)_{y=d}\cdot r$$
$$EI_1\frac{d^4 \delta}{dx^4}=D\left(\frac{\partial^3 w'}{\partial y^3}+[2-\nu]\frac{\partial^3 w'}{\partial x^2 \partial y}\right)_{y=d}$$

in which ϕ and δ are extended by Fourier series as follows :—

$$\phi=\sum_n L_n R_n \sin \alpha_n x, \quad \delta=\sum_n L_n S_n \sin \alpha_n x$$

In the above equations

EI_1 = flexural rigidity of main beam,
GJ = simple torsional rigidity of main beam,
EC_w = bending torsional rigidity of main beam,
D = flexural rigidity of floor slab plate.

By the above treatment the unknown coefficients in the plate deflection equations can be determined. The cases (b) and (c) can be treated similarly.

When the elemental system is solved as mentioned above the use of this result enables one to obtain equilibrium equations from continuity conditions at the mid-section of cross beam and then obtain statical redundancies.

The load distribution effect of the floor slab

Essential points ars as follows :—

1. When there are no cross beams, the load distribution effects by flexural rigidity of the floor slab vary considerably depending on its magnitude.
2. When there exist cross beams which have even slight cross section, the load distribution effects become much better, compared with the system without cross beams.
3. When the system has cross beams, the load distribution differs very little irrespective of the magnitude of the flexural rigidity of the floor slab. Then the load distribution effects become nearly equal to those of the system without the floor slab.
4. From the above facts, in the case of the system with cross beams, the load distribution effect of the floor slab can be neglected.

Experimental consideration

In order to verify the above analysis, an organic glass model box girder of the span 1.20 m. was made, of which there are three different types as follows——the original system, the elemental system and the system without floor slab between the two box girders but with cross beams; and a concentrated load is applied to them in order to measure their deformation and stresses, and experimental and theoretical values were compared and the result was that as to the deflection the experimental values were very similar to the theoretical ones, and as to the stress both were similar in tendency, as shown in Fig. 4. Accordinglly the present analysis method is considered almost appropriate.

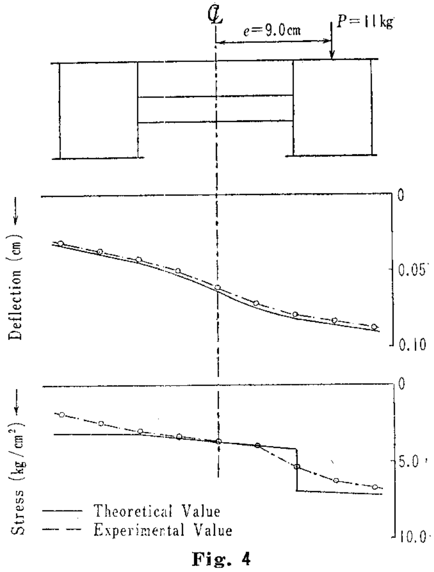

Fig. 4