

SIMPLIFIED SOLUTION OF SURGING AT THE SURGE CHAMBER AT THE INITIALLY FULL-FLOWING TAIL-RACE TUNNEL DUE TO LOAD DECREASE

(放水路サージチャンバー サージングの簡易計算法)

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Synopsis : Simplification is shown of solution of the surging at the surge chamber at the initially full-flowing tail-race tunnel developed in the previous paper by the same author. Approximation is made to the governing equations to make the graphical solution of Schoklitsch applicable. Comparison of the solutions by the graphical method is made with the results obtained from the laboratory investigation described in the previous paper. Good agreement between theory and experiment is affirmed. The simplified method of solution is shown to be sufficiently adequate for practical applications.

INTRODUCTION

In the previous publications^{1), 2), 3), 4)} the author proposed the fundamental equations for surging at the initially full-flowing tail-race tunnel due to load decrease. These equations have been adequately affirmed experimentally. However, as these equations may be integrated only arithmetically, the solution of them has sometimes been considered more roundabout and troublesome than all what might have been desired for practical design purposes.

Messrs. Senshû and Akimoto made a contribution on modification of these equations and solution thereof⁵⁾.

In this paper, a slight modification is made to the original equations, which modification makes the solution of the present problem much more speedy than before by applying the graphical method of Schoklitsch.

RESUME OF THE PREVIOUS THEORY

When load is decreased from a hydro-electric power station with a full-flowing tail-race tunnel at the top of which a surge chamber is provided,

the inertia of the water flowing in the tunnel lowers the water level in the surge chamber until, in general, the free surface penetrates into the tunnel. The mechanism immediately responsible for the actual change in water surface elevation is a procession of translatory negative waves traversing the length of the tunnel against a negative hydraulic gradient. In order to make an approximate representation of the wave phenomenon possible with the oscillation equation similar to the conventional surging equations, assumptions were made by referring to Fig. 1 as follows :

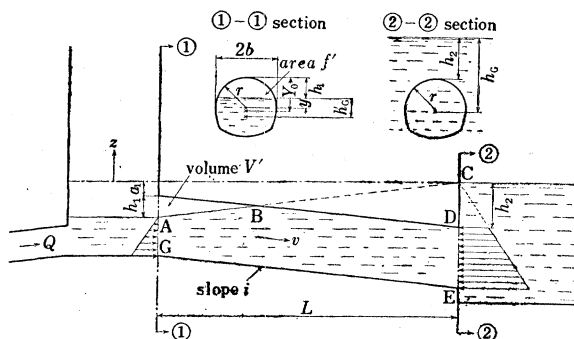


Fig. 1 Assumed shape of water surface and hydraulic gradient during the surging at load decrease.

1. Hydraulic gradient is represented by the straight line connecting point C and point B, the latter being the front of the negative wave.
2. Point B is on the straight line connecting point C and point A, the latter being at the water surface at the entrance of the tunnel.
3. The inertia of flowing water which contributes to the equation of motion is that which is due to the mass of the portion ABDEGA.

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The dynamic equation which results from these three assumptions is

$$\frac{W}{g} \frac{dv}{dt} = P_1 - P_2 + Wi \mp K v^2 \dots \dots (1)$$

where

W : mass of water of the portion ABD-EGA,

P_1 : resultant of hydrostatic pressure acting at the entrance of the tunnel upon the flowing water in the tunnel,

P_2 : resultant of hydrostatic pressure acting at the exit of the tunnel upon the flowing water in the tunnel,

i : slope of the tunnel,

v : mean velocity of water at full-flowing part of the tunnel,

t : time,

g : acceleration of gravity,

K : coefficient such that $K v^2$ = total frictional resistance in the tunnel.

Equation of hydraulic continuity which results from the three assumptions is written

$$-F \Delta z + \Delta V' = (fv - Q_t) \Delta t \dots \dots (2)$$

or

$$-F \frac{dz}{dt} + \frac{dV'}{dt} = fv - Q_t \dots \dots (3)$$

where

z : water level in the surge chamber measured vertically upwards from the tail-race elevation,

F : horizontal section of the surge chamber,

V' : volume of the upper space above the free surface in the tunnel (Fig.

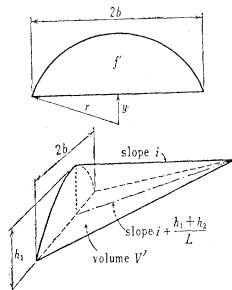


Fig. 2 Volume V' .

2),

Q_t : turbine discharge,

f : cross section of the tail-race tunnel,

According to the three assumptions V' is determined by geometrical relation as follows:

$$V' = \frac{L}{h_1 + h_2} r^3 \left[\frac{2}{3} \left(\frac{b}{r} \right)^3 - \frac{r - h_1}{r} \left\{ \sin^{-1} \frac{b}{r} - \left(1 - \frac{h_1}{r} \right) \frac{b}{r} \right\} \right] \dots \dots (4)$$

where

r : radius of the upper semi-circle of the section of the tunnel,

b : semi-width of water at the section of the entrance of the tunnel,

h_1, h_2 : vertical distances as shown in Fig.

1.

Expressing $P_1 - P_2$ and W in functions of z , and substituting those expressions in eqs. (1) and (3), we obtain

$$\frac{dv}{dt} = \frac{z \mp [c v^2 \{1 - (V'/L f)\}]}{L/g} \dots \dots (5)$$

$$\frac{dz}{dt} = \frac{Q_t - f v}{F + \frac{L}{h_1 + h_2} \left(f' - \frac{V'}{L} \right)} \dots \dots (6)$$

where

c : coefficient such that $c v^2$ = total losses of head at full-flowing flow in the

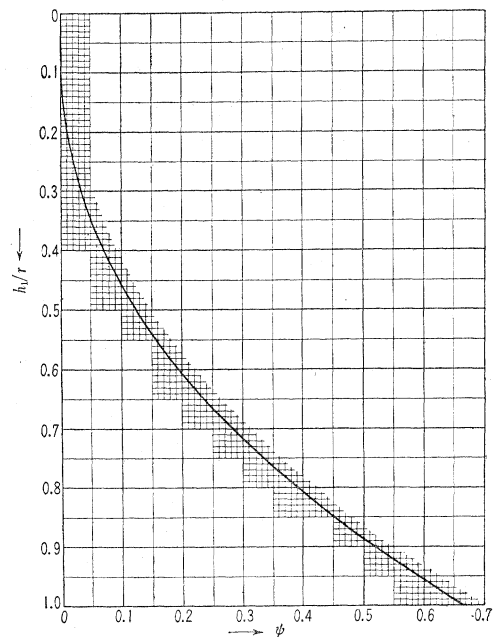


Fig. 3 Values of dimensionless quantity ψ .

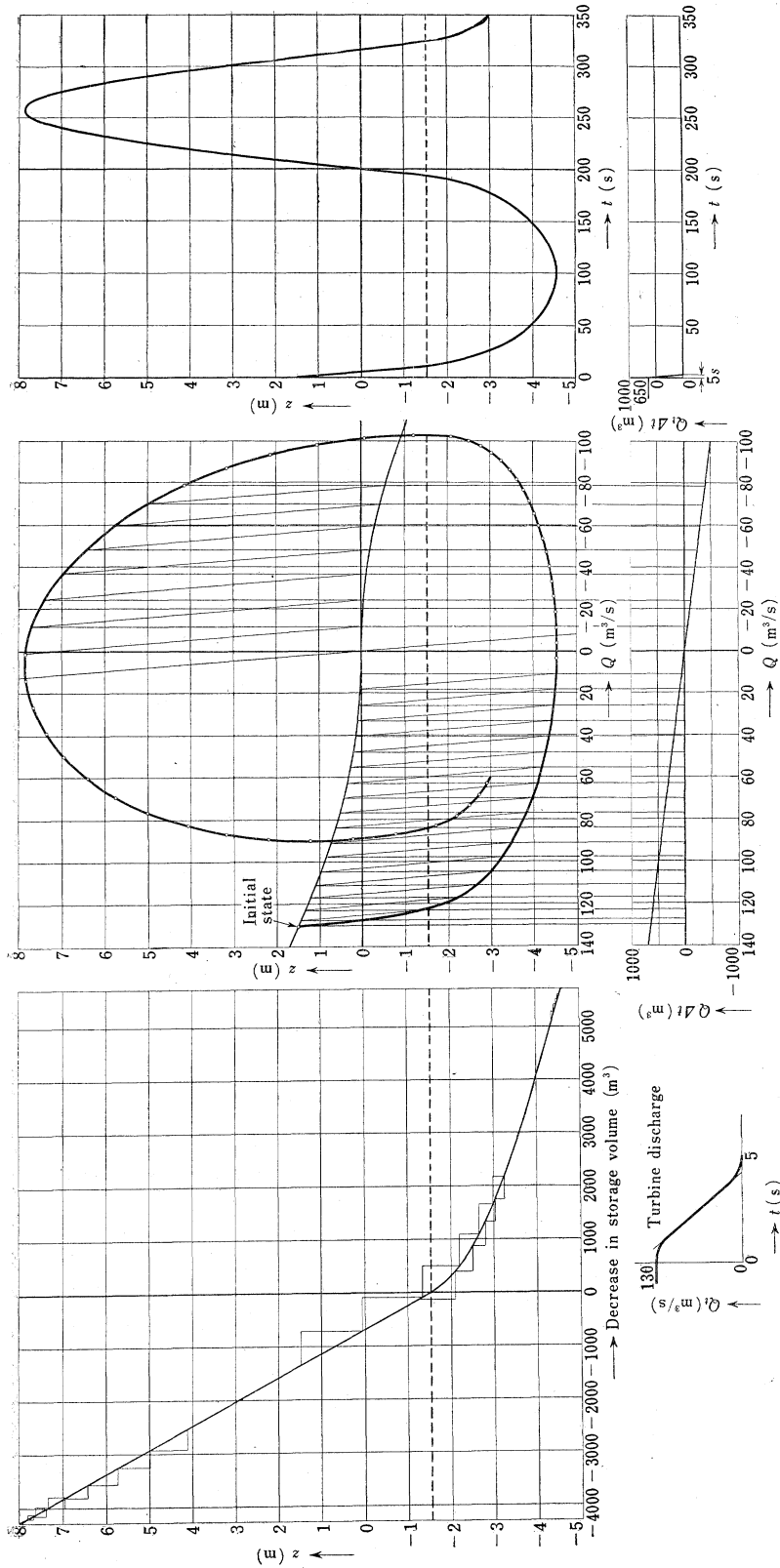


Fig. 4 Example of practical application. ($Q=fv$)

tunnel,

f' : area of the upper space above the water surface at the section of the entrance of the tunnel.

Eqs. (5) and (6) thus derived were the fundamental equations in the previous papers.

Dimensionless form for V' which is defined by

$$\psi = V' / \left(\frac{L}{h_1 + h_2} r^3 \right) \dots \dots \dots (7)$$

is drawn to scale in Fig. 3.

By arithmetic integration of eqs. (5) and (6) and by the use of Fig. 3 for V' , we can calculate the surging at the surge chamber at the tail-race tunnel.

In a special case when the location of the tunnel is deep enough to prevent the penetration of air into the tunnel,

$$f' = 0 \quad \text{and} \quad V' = 0 \quad \dots \dots \dots (8)$$

hence eqs. (5) and (6) reduce respectively to

$$\frac{dv}{dt} = \frac{z \mp cv^2}{L/g} \quad \dots \dots \dots (9)$$

$$\frac{dz}{dt} = \frac{Q_t - fv}{F} \quad \dots \dots \dots (10)$$

which are nothing but the conventional surging equations.

SIMPLIFIED SOLUTION BY THE METHOD OF GRAPHICAL SOLUTION

Assuming approximatley

$$V' \ll Lf \quad \dots \dots \dots (11)$$

dynamic equation (5) becomes

$$\frac{dv}{dt} = \frac{z \mp cv^2}{L/g} \quad \dots \dots \dots (12)$$

This equation may be written in the form

$$f dv = - \frac{gf}{L} dt \cdot (-z \pm cv^2) \quad \dots (13)$$

which, together with the equation of continuity (2) in finite differences, becomes the funda-

mental equation; these two equations make the graphical method of Schoklitsch applicable for solution.

The only alteration for the purpose of applying the graphical method for the surge tank in the head-race tunnel to the present case consists of replacement of cubic content $\int Fdz$ of the surge chamber by cubic content $\int Fdy$ of the surge chamber plus "storage volume of the tunnel" V' , where $y = -z$.

As is easily seen, it is convenient to choose the origin of the curve ($\int Fdy + V'$) at $y = a_1$ (see Fig. 1); above elevation a_1 this curve consists of only $\int Fdy$, while below the elevation it consists of the summation $\int Fdy$ and V' . For plotting the curve for V' we can make use of the graph for ψ (Fig. 3).

An example of practical application is shown in Fig. 4, the characteristics used in which are as follows:

Total closure (Time of closure = 5 sec)

$$Q_t = 130 \text{ m}^3/\text{sec}$$

$$L = 1,230.20 \text{ m}$$

$$f = 40.269 \text{ m}^2$$

$$r = 3.500 \text{ m}$$

$$F = 450 \text{ m}^2$$

$$v_0 = 3.228 \text{ m/sec}$$

$$h_f = cv_0^2 = 1.488 \text{ m}$$

$$c = 0.1428 \text{ m}/(\text{m/s})^2$$

$$a_1 = 1.515 \text{ m}$$

$$h_2 = 2.745 \text{ m}$$

COMPARISON BETWEEN THEORY AND EXPERIMENT

Although the fundamental equations (5) and (6) were already affirmed experimentally in the previous papers, it may still be necessary to check by experiment the theory furnished by eqs. (2)

Table 1

	L (m)	d (m)	f (m ²)	F (m ²)	Q (l/s)	v (m/s)	h_f (m)	c (m/(m/s) ²)	Time of closure
Case 1	21.62	0.1307	0.01344	0.1143	4.09	0.304	0.0272	0.2935	Instantaneous
Case 2	14.88	0.1307	0.01344	0.0364	4.07	0.303	0.0235	0.2561	Instantaneous

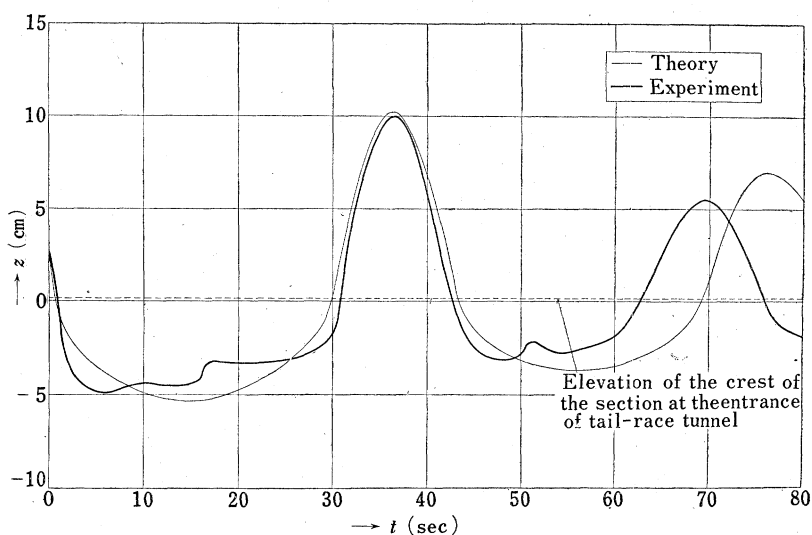


Fig. 5 Comparison between theory and experiment for Case 1.

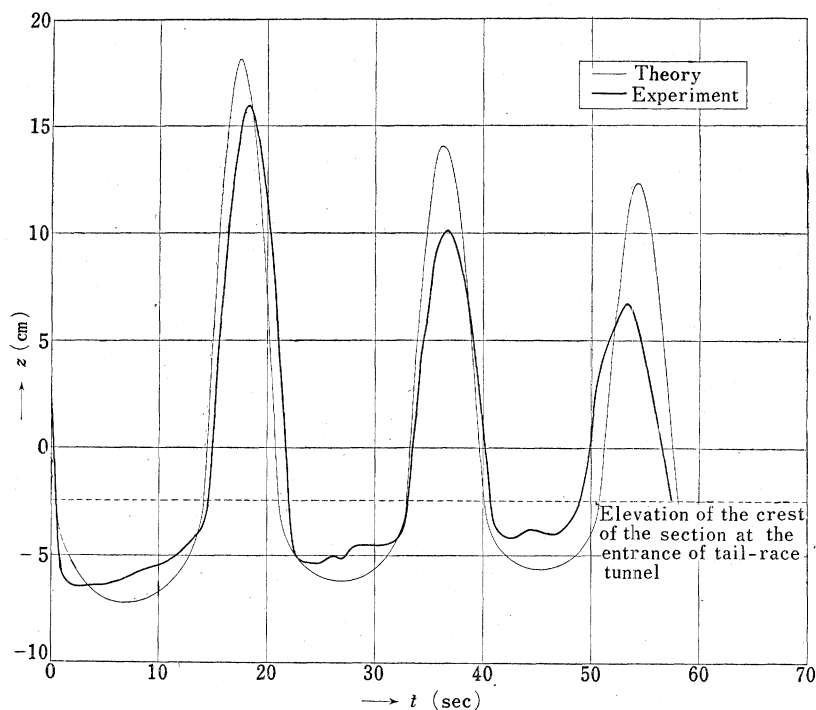


Fig. 6 Comparison between theory and experiment for Case 2.

and (13). For this purpose surging waves are solved by the graphical method developed in this paper and they are compared with those given by experiment. A few typical examples of comparison are shown in Fig. 5 and Fig. 6, where the characteristics are as shown in Table 1.

The experimental curves in these figures are those reproduced from the previous papers.^{3), 4)}

Agreement between theory and experiment is at every time very close in spite of the simplification of the fundamental equations.

SUMMARY AND CONCLUSION

Simplification has been shown of solution of the surging at the surge chamber at the initially full-flowing tail-race tunnel developed in the

previous papers by the same author. Basic equations become eqs. (2) and (13), which make the graphical solution of Schoklitsch applicable.

A few typical examples of comparison of the theoretical results by the simplified solution with experimental results are shown in Fig. 5 and Fig. 6. The agreement between theory and experiment is always very close in spite of a few basic assumptions made at the derivation of the basic equations.

The simplified solution may thus be applied with confidence for all practical design purposes.

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