

ON THE MOTION OF THE FLOOD-FLOW RUNNING DOWN THROUGH THE RIVER

—Some Deduced Characters—

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Synopsis : Combining the equations of motion of fluid with the equation of continuity, the author solved numerically differential equations of flood-flow which is one kind of unsteady flows. The system of the flood-flow under consideration is nonlinear with a finite amplitude. In this paper, some characters are deduced from this method by numerical computation.

1. Effect of Friction (Chezy's Coefficient)

In the previous report¹⁾ the author showed how to solve equations of motion and continuity numerically with nonlinearity and to apply this method to the actual floods. This paper is its continuation. Some characters of floods, for example crest and front velocities, decaying and so on, are deduced from the numerical method in this paper. The equations and notations are the same as in the previous report. The author investigates floods in the various model channels as follows.

Chezy's coefficient C is the reciprocal of the friction coefficient. Supposing other conditions are equal, the water velocity of mean flow is proportional to this coefficient C .

River improvement often means to make the river regular, for example, to smooth up irregularities of bottom inclination and cross sectional area in order to pour the flood into the sea as fast as possible. This work produces results to decrease the friction and hence to increase C .

When we attempt the river improvement, we must be able to predict quantitatively its effects, in other words, how high the flood rises up at maximum stage or when its crest arrives at certain point by means of the changing of C after the improvement.

If we substitute $\frac{1}{n} h^{\frac{2}{3}}$ for C , n is called the Manning's friction coefficient, where h is the water depth. Roughly speaking, C corresponds to $\frac{1}{n}$. The author will use Chezy's coefficient C in this paper for convenience of computation process. There is no essential difference between C and $\frac{1}{n}$.

In this section, model channels are assumed to be uniformly rectangular and extending to infinity. Of course, we can discuss characters of floods in the arbitrary channel, but, first it is important to study them for simple cases.

Constants are chosen as follows.

$$\begin{array}{lll} i=1/1000 & 2 \Delta x=1000 \text{ m} & \Delta t=100 \text{ sec} \\ \text{initial depth (initial condition)} & & =1 \text{ m} \\ \text{maximum depth at the upstream boundary} & & =2 \text{ m} \end{array}$$

Flood hydrograph at the upstream boundary is given by symmetrical function.

$$h=1+\frac{t^2(t-T)^2}{10^{12}} \quad 0 \leq t \leq T \quad \dots\dots\dots(1)$$

$$=1 \quad T < t$$

where T is a duration. $T=20 \Delta t \text{ sec}=2000 \text{ sec}$ in all cases. The hydrograph h is a continuous function of time t which is illustrated in **Fig. 10 (a)**. The author computed five floods for different C 's.

$$5 \sqrt{10}=15.8, \quad 7.5 \sqrt{10}=23.7, \quad 10 \sqrt{10}=31.6, \quad 12.5 \sqrt{10}=39.5, \quad 15 \sqrt{10}=47.4 (\text{m}^{\frac{1}{2}} \text{ sec}^{-1})$$

These values are chosen for the sake of simplicity of calculation.

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The foregoing $\Delta t-\Delta x$ relation is satisfied with restrictions (16)~(19) in Chapter 4 in the previous report in the range of these C 's. Characters of floods are investigated as follows.

(a) Movement of crest The most important information concerning flood protection is the time when the crest of the flood arrives at a certain point. The author intends to find the difference of velocities of crests due to various Chezy's coefficients.

Velocities of crests vary gradually as crests become flattened, so we cannot express clearly their characters corresponding to Chezy's coefficients. A general features can be seen in their time-distance curves, which are drawn in **Fig. 1** by connecting the time when the water-level rises up to maximum at every point, that is to say, according to mathematical expression $\left(\frac{\partial h}{\partial t}\right)_x = 0$.

The reciprocals of their gradients equal to crest velocities (phase velocities) because the abscissa is measured by distance and the ordinate is measured by time. These curves are bending a little as crest velocities become gradually slow due to crest decaying. According to these curves we can see that the larger C is, the faster the flood crest runs down. The crest velocity is roughly proportional to C .

For further study, we draw a graph whose abscissa is measured by distance and ordinate is measured by U/u_0 where U is a crest velocity and $u_0 = C\sqrt{h_0 i}$ is the velocity of steady flow. If U is proportional to C , U is also proportional to u_0 , consequently U/u_0 of various floods overlap together perfectly. **Fig. 2 (a)** is drawn in such an intention. It shows a bit of discrepancy of U/u_0 curves, that is to say, the crest velocity U is not perfectly proportional to Chezy's coefficient C . The flood in the channel of large C is inclined to travel faster than expected by proportion. **Fig. 2 (b)** is a diagram of $\frac{2}{3}u^*/u_0$ on the same co-ordinate where u^* is the water velocity at the maximum water-level. Seddon's formula says **Fig. 2 (a)** lies upon **Fig. 2 (b)**.

Fig. 1 Time-distance curves for crests in channels of various Chezy's coefficients.

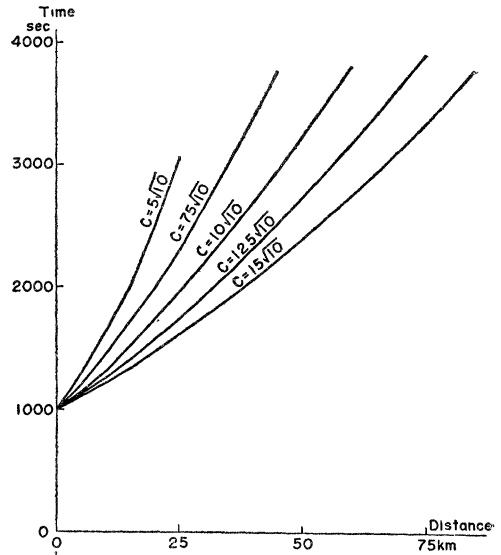


Fig. 2 (a) U/u_0 -distance curves for various Chezy's coefficients.

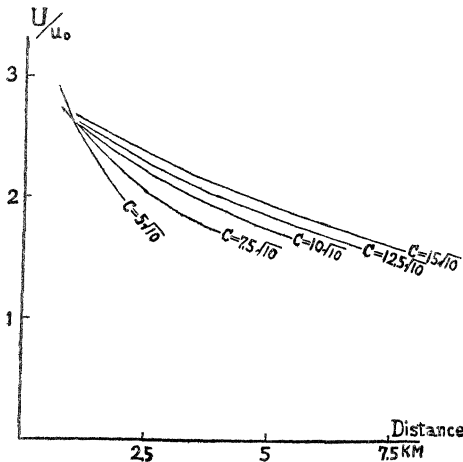
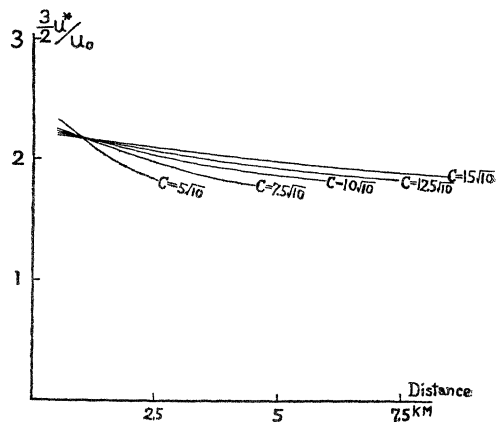


Fig. 2 (b) $\frac{2}{3}u^*/u_0$ -distance curves for various Chezy's coefficients.



because $U = \frac{2}{3} u^*$. Two figures roughly coincide with each other but decreasing of velocities have different tendencies.

(b) Movement of front As the equation is of diffusion type, the flood front runs away very rapidly. In this paper the front is defined by a part whose water-level is 1.001 m. In other words the front velocity is the velocity of the phase 1.001 m depth. The time-distance curves of flood fronts are drawn in **Fig. 3** in the same co-ordinate as in **Fig. 1**. Comparing them with each other, we see fronts are flowing down faster than the crests. The fundamental equation includes the diffusion and the transportation, so the crest is transported and the front velocity is the sum of transportation and diffusion velocities. The front velocity is roughly proportional to Chezy's coefficient C . The front velocity in the channel of large C is not so fast as expected by proportion.

(c) Decaying of crest **Fig. 4** indicates decaying of crests. The ordinate is elevation and the abscissa is measured by distance. This representation is somewhat deficient, because conception about time is excluded in this diagram. But we can see intuitively how crests are decaying. Though hydrographs in the upstream boundary are same, the flood form in the channel of small C is flattened quickly.

(d) Water volume One of the most important purposes of river improvements is to hurry the flood away into the ocean. The hydrograph is easily deformed by the conditions of the river, so it may not give a very good measure regarding how fast the flood travels down. On the other hand it may be more desirable that the water volume passed by at a certain point during some time interval is chosen for the measure.

If the discharge of steady flow is written by Q_0 , $Q-Q_0$ is the pure flood discharge. The time integration of $Q-Q_0$ means the water volume of the pure flood discharge during t sec.

$$V(t) = \int_0^t (Q - Q_0) dt$$

$V(\infty) = \int_0^\infty (Q - Q_0) dt$ is the total water volume of the flood discharge.

Fig. 5 shows the ratios $V(t)/V(\infty)$ at $x=2.5$ km.

The larger C is, the greater water volume flows down till a certain time at a certain point. This fact is not inconsistent with **Figs. (1)~(4)**. The increment of Chezy's coefficient ΔC due to river improvement at the channel of smaller C has a larger effect.

Fig. 3 Time-distance curves of fronts for various Chezy's coefficients.

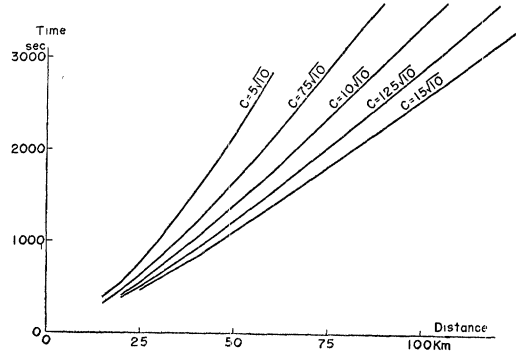


Fig. 4 Decaying of crests on distance co-ordinate,

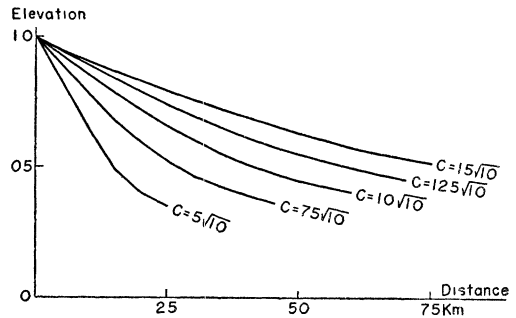
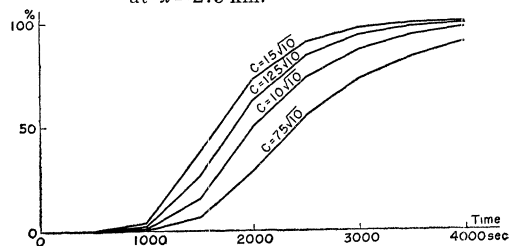


Fig. 5 Ratios $V(t)/V(\infty)$, where V is the water volume of the flood, defined by the integration of the discharge at $x=2.5$ km.



2. Effect of Bottom Inclination

Isikari River and other rivers on alluvial plains had been meandering like snakes. But recently we often have cut off the meandering parts of the river connecting with the artificial channel. Consequently the river is shortened and the bottom inclination becomes steeper than ever. This improvement answers the requirement to pour the flood water into the ocean as fast as possible.

If the bottom inclination i becomes steep, what kind of change takes place in the flood form? The author sets up model channels which are uniformly rectangular. Constants are chosen as follows.

$$C=10\sqrt{10} \text{ m}^{1/2}\text{sec}^{-1} \quad \Delta x=2000 \text{ m} \quad \Delta t=1000 \text{ sec}$$

initial depth (initial condition)=1 m

maximum depth at the upstream boundary=2 m

Flood hydrograph at the upstream boundary is given by a symmetrical function.

$$h=1.5-0.5 \cos\left(\frac{2\pi}{T}t\right) \quad 0 \leq t \leq T \quad \dots\dots(2)$$

$$=1 \quad T < t$$

where T is a duration, $T=20 \Delta t=20000 \text{ sec}$ in all cases. Hydrograph h is a continuous function of time t . This hydrograph (2) seems to differ from the hydrograph (1) in previous section, but (2) is almost similar to (1). The author computed four floods for different i 's.

$$i = \frac{1}{10000}, \frac{2}{10000}, \frac{3}{10000}, \frac{4}{10000}$$

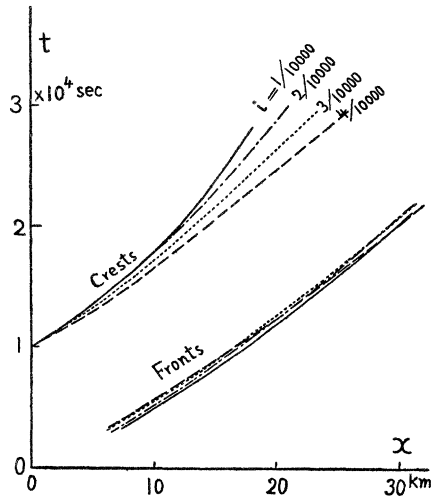
The foregoing $\Delta t \sim \Delta x$ relation is satisfied with restriction (16)~(18) in Chapter 4 in the previous report in the range of these i 's. Characters of floods are investigated as follows.

(a) Movements of crest and front Fig. 6 shows the time-distance curves for crests and fronts in four cases. In advance, the front was defined as the point where the water-level is 1.001 m. It is remarkable that all front velocities in this range do not differ appreciably from each other. Though crest velocities differ slightly from each other, front velocities are not so separated as in Fig. 3 which shows the relation between the front velocity and Chezy's coefficient. Thus we notice the bottom inclination has little influences upon the front velocity. Crest velocities are apt to increase little by little with the bottom inclination, however we cannot find such a fine relation as in Fig. 1.

As the bottom inclination i increases by means of short-cuts of meandering parts, the mean velocity of water becomes greater. Therefore phase velocities *i. e.* crest velocity and front velocity are presumed to increase with the bottom inclination, because the flood form is transmitted by the mean velocity of water, but phase velocities are almost independent upon the bottom inclination as we see in Fig. 6. In these cases the bottom inclinations are gentle, so the depth inclination $\frac{\partial h}{\partial x}$ plays an important role. Consequently the bottom inclination does not contribute to phase velocities remarkably.

It is said that at the downstream of the cut-off river the flood rises up more quickly than before cut off. This phenomenon seems to be caused by the effect rather of shortening of the channel than of increasing of the bottom inclination. Though the time-distance curves for $i = \frac{1}{10000}, \frac{2}{10000}$ nearly overlap together, the curves of $i = \frac{3}{10000}, \frac{4}{10000}$ separate a little. If we compute floods in steeper channels, we may say a more general trend of flood movement.

Fig. 6 Time-distance curves for crests and fronts in channels of various bottom inclinations.



In the case of $i = \frac{1}{10\,000}$ the time-distance curves for the crest and the front are slowly diverging, while in the case of $i = \frac{4}{10\,000}$ the curves of the crest and the front are almost parallel. In other words, as the crest travels slowly in the gentle channel, the crest separates from the front gradually. On the other hand, as the crest runs a little quickly in the steep channel, the crest runs after the front with a constant interval.

(b) Hydrograph The relation between crests and fronts is found in **Fig. 7**, in which hydrographs at the same distance in four channels are drawn on the same co-ordinate; the abscissa is measured by time and the ordinate is elevation. We can say from this figure that the flood in the steeper channel is flattened more gradually, the crest reaches somewhat earlier and the tail of the hydrograph is cut off shorter comparing with the flood in the more gentle channel. Therefore it does not seem that in the steep channel a diffusion property is remarkable.

The diffusion coefficient of the fundamental equation (3) (of course, not in a rigorous meaning but in a rough correspondence) decreases with increasing of the bottom inclination. If the diffusion coefficient is small, the wave form will not be flattened easily. Consequently, in the steep channel the flood hydrograph does not deform as we see in **Fig. 7**. The qualitative discussion is not sufficient, but the quantitative result is illustrated in **Fig. 7**.

In steeper channels $i = \frac{5}{10\,000}$, $\frac{6}{10\,000}$, the water velocity is greater than in foregoing channels. Therefore new relation between Δt and Δx must be considered according to (16)~(19). So we cannot compare them in detail under the same computational condition. In the more gentle channels $i = \frac{0.8}{10\,000}$, $\frac{0.6}{10\,000}$, water runs down so slowly that the flood remains high for some time at the last stage of the flood. The water-level at $x=0$ is compelled to fall down as a given function of time (that is to say, the boundary condition). The surface gradient of water becomes negative near the upstream boundary at the last stage, which seems perplexing. Moreover, if the bottom inclination of the channel is very gentle, the surface gradient, the friction and the acceleration are balanced together, in other words, we must consider the acceleration in the gentle channel. But it is difficult to solve the fundamental equation (3) with acceleration terms. One trial to solve them will be shown in section 5 in this report. Therefore the author does not investigate floods in more gentle channels.

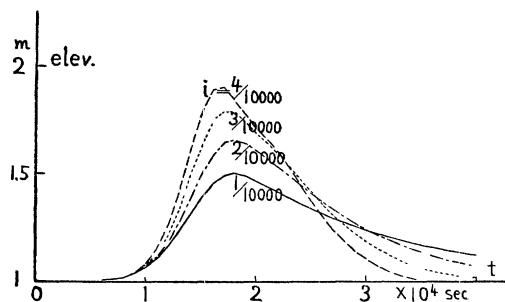
3. Broadened channel

In preceding consideration the breadth was assumed to be constant all through the channel. In this section the author is going to investigate how the flood flow is deformed in the channel whose breadth in the downstream is wider than in the upstream (three times in this case) with gradual transition. The channel is extended to infinity, Chezy's coefficient and the bottom inclination are assumed to be constant everywhere. The channel is linearly broadened from $x=1500$ m to $x=2500$ m where the breadth is three times the upstream part. At the further downstream the breadth is constant.

The initial water-level in the upstream is about 1 m, in the downstream is 0.4807 m and in the transition part it varies gradually so as to conserve the discharge constantly all through the channel at the steady stage.

Constants are chosen as follows.

Fig. 12 Hydrographs for various bottom inclinations.



$$C=10\sqrt{10}=31.6 \text{ (m}^{1/2}\text{sec}^{-1}) \quad i=1/1000$$

$$2 \Delta x=500 \text{ m} \quad \Delta t=50 \text{ sec}$$

The boundary condition at the upstream boundary is the same form as (1)

$$h=1+\frac{t^2(t-T)^2}{2 \times 10^{11}} \quad 0 \leq t \leq T$$

$$=1 \quad T < t$$

where T is a duration, $T=20 \Delta t=1000$ sec. The maximum water-level is 1.3125 m. The Δt - Δx relation is satisfied with restrictions (16)~(18) in the previous report.

For comparison the author computed which flowed under the same condition as above except for the channel form: its width is constant.

(a) Crest velocity In **Table 1**, we find differences of crest velocities in three parts of the channels.

Table 1 Crest Velocity

	Beginning part	Next part	Transition part
Broadened channel	2.67 m/sec	2.38 m/sec	1.18 m/sec
Uniform channel	2.66	2.31	1.74

The crest velocity in the broadened channel falls down faster than in the uniform one. This trend is also represented in deformation of hydrographs.

(b) Hydrograph Hydrographs at every 500 m are drawn in **Fig. 8**, where the ordinate is elevation and the abscissa is time. Suppose the widths and depths of two channels are denoted by b, b', h and h' respectively and discharges in steady state are constant everywhere,

$$Q=Cb h^{3/2} I^{1/2}=C b' h'^{3/2} I^{1/2}$$

then $(b/b')^{2/3}=(h'/h)$.

As $b'=3b$ in this case, we determine the depth in the broadened channel $h'=0.4807h$ in steady state.

In this section, we want to see whether the flood elevation in the broadened channel is 0.4807 times one in the uniform channel, that is to say, whether we can discuss the flood according to the law of steady flow. The answer is "no", as we see in **Fig. 8**. The elevation of flood in the broadened channel is higher than 0.4807 times, which deserves attention for design of the river improvement.

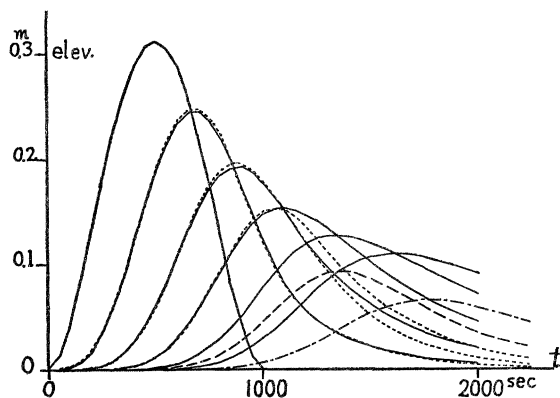
The author computed only one case, that is, the breadth is three times. Of course, the flood in the channel whose width changes in other way can easily be computed by the same method as occasion demands.

4. Two-crested Flood

When one river pours a flood first and another does somewhat later and two rivers

Fig. 8 Hydrographs in uniform and broadened channel.

full lines : in the uniform channel.
dotted lines : at $x=0.5 \text{ km}, 1.0 \text{ km}$ and 1.5 km , where the channel is not yet broadened in the broadened channel.
broken line : at $x=2.0 \text{ km}$ in the broadened channel.
chain line : at $x=2.5 \text{ km}$, where the channel width is three times that of the upstream in the broadened channel.



joins, or when two cyclones pass by a basin successively, the flood hydrograph has two crests, which run down interacting each other. This phenomenon had already been investigated under an assumption of a linear interaction. However the interaction is nonlinear, of course. The author's method is powerful for such a phenomenon as two-crested flood with non-linearity.

In this example constants are chosen as follows.

$$\begin{aligned}
 C &= 10 \sqrt{10} = 31.6 \text{ (m}^{1/2}\text{sec}^{-1}) & i &= 1/10\,000 \\
 \Delta x &= 2\,000 \text{ m} & \Delta t &= 1\,000 \text{ sec} \\
 \text{initial depth (initial condition)} & & &= 1 \text{ m} \\
 \text{maximum depth at the upstream boundary} & & &= 2 \text{ m}
 \end{aligned}$$

Flood hydrograph at the upstream boundary is of the same form as the function (2), but in this case the duration is $2T$, that is, as T is the duration of one wave, two sine waves are given at the boundary.

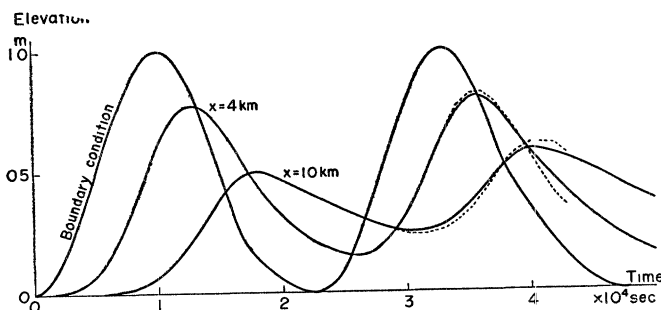
Hydrographs are drawn in Fig. 9, for comparing with the linear superposition. The second crest is travelling a little faster and its height is lower than expected by linear superposition.

This trend seems to be caused by following reasons. The second flood comes into the channel in which the first flood has not perfectly gone away and the water-level is yet higher than the initial water-level. Consequently the front of the second flood is running fast and its crest decays easily. As the second flood travels faster than the first, the second catches up with the first soon, then they are combined to one. Strictly speaking, the trough between two crests becomes shallow and they become undistinguished. The second has no influence upon the first obviously.

According to this computation, within the range of this example, it may be roughly allowed that the first approximation is equal to the linear superposition. But it deserves attention that the second crest arrives earlier than expected as linear.

Fig. 9 Two-crested flood.

full lines : hydrographs at the upstream boundary, at $x=4 \text{ km}$ and at 10 km .
 dotted lines : hydrographs of linear superposition of a one-crested flood.



5. Acceleration

The author neglected acceleration terms in the fundamental equation, in the preceding discussion, because they are negligibly small comparing with other terms and they make the boundary condition inconvenient. However at the river-mouth where the bottom inclination i is very small, or at the part where the cross sectional area increases or decreases abruptly, the friction does not balance against the surface gradient, and the acceleration cannot be neglected.

The author first has discussed characters of floods in channels whose conditions are good, that is to say, meet requirements in Chapter 5 in the previous report. The second approach is how to investigate characters of floods in more general rivers. For this purpose we must make efforts to solve equation of motion with acceleration terms. There are many mathematical difficulties in non-linear

differential equations in physics, for example the existence of solution, boundary conditions and so on. They are briefly described in Chapter 2 in the previous report.

It is possible to get the numerical solution in difference form on (t, x) domain as follows. Equation of motion is transformed to the difference equation.

$$\frac{\eta}{g} \frac{u - u'}{\Delta t} + \frac{\alpha}{g} u \frac{u - u^*}{\Delta x} - i + \frac{\partial h}{\partial x} + \frac{u^2}{C^2 R} = 0 \dots\dots\dots(3)$$

where u', u and u^* are the mean velocities of water at $(t_1 - \Delta t, x_1)$, (t_1, x_1) and $(t_1, x_1 - \Delta x)$ respectively, and R and $\frac{\partial h}{\partial x}$ are functions of the water-level h . Other terms are related to the water velocity. Suppose that the water-level is known everywhere at $t = t_1$, the water velocity is known everywhere at $t = t_1 - \Delta t$ and the water velocity at Δx^m upstream neighbor is known at $t = t_1$, then the water velocity and the discharge will be derived from (3) by solving a second order algebraic equation about u . As the water velocity at $x = x_1$ becomes known by means of the foregoing process, we can compute the water velocity at $x = x_1 + \Delta x$, and then at $x = x_1 + 2\Delta x$. Repeating this process, water velocities at all points at $t = t_1$ become known successively. Next, we use equation of continuity in order to calculate the increment of the cross sectional area $\frac{\partial A}{\partial t} \Delta t$ for $t = t_1 + \Delta t$. As we know new water-levels in this process, we again use equations of motion and continuity alternately. Consequently h and u become known in (t, x) domain, like the solution without acceleration terms.

There are three simple ways to change $\frac{\partial u}{\partial t}$ into the difference form : backward, forward and central differences. In the way to change $u \frac{\partial u}{\partial x}$ into the difference form simply, there are three backward, three forward and four central differences. They are two-point or three-point approximations. We can make many varieties of four-point, five or more point approximations for this transformation. But many point approximation requires complicated procedure in calculation. So the author uses simple approximations : backward difference for time derivative

$$\frac{\partial u}{\partial t} = \frac{u(t, x) - u(t - \Delta t, x)}{\Delta t}$$

and also backward difference for x derivative

$$u \frac{\partial u}{\partial x} = u(t, x) \frac{u(t, x) - u(t, x - \Delta x)}{\Delta x}$$

They are easy and advantageous because solutions h and u are computed successively from the upstream boundary where the boundary conditions are given. Therefore the difference equation for u makes (3) and can be solved easily by the formula of root.

$$u = \frac{-\left(\frac{\eta}{g} \frac{1}{\Delta t} - \frac{\alpha}{g} \frac{u^*}{\Delta x}\right) + \sqrt{\left(\frac{\eta}{g} \frac{1}{\Delta t} - \frac{\alpha}{g} \frac{u^*}{\Delta x}\right)^2 + 4\left(\frac{1}{c^2 R} + \frac{\alpha}{g} \frac{1}{\Delta x}\right)\left(i - \frac{\partial h}{\partial x} + \frac{\eta}{g} \frac{u'}{\Delta t}\right)}}{2\left(\frac{1}{c^2 R} + \frac{\alpha}{g} \frac{1}{\Delta x}\right)}$$

The solution with acceleration terms needs the initial and boundary conditions which are different from ones for the solution without acceleration terms. In other words, the solution without acceleration terms needs only the water-level for the initial and boundary conditions and the solution with acceleration terms needs the water-level and the water velocity for both the initial and boundary conditions as discussed in Chapter 2 in the previous report.

First, the author wants to explain how to define the initial conditions. In general cases the initial instant $t = 0$ is taken just before the flood begins to intrude into the channel (computational region). At this time, the river flow is still steady, that is $\frac{\partial}{\partial t} = 0$, then the water-level and the water velocity are defined as functions of only x so that the discharge Q may be constant everywhere.

This method is applicable to actual rivers whose bottom inclinations may be very gentle or cross sectional areas may change largely. But in this paper the author will show a simple example, com-

paring it with another example in which acceleration terms are neglected. The channel is assumed to be uniformly rectangular and infinitely long.

Constants are chosen as follows, same as one of examples shown in Section 1 in this paper.

$$C = 15 \sqrt{10} = 47.4 \text{ (m}^{1/2}\text{/sec}^{-1}) \quad i = 1/1000$$

$$2 \Delta x = 1000 \text{ m} \quad \Delta t = 100 \text{ sec}$$

initial depth (initial condition) = 1 m
 maximum depth at the upstream boundary = 2 m

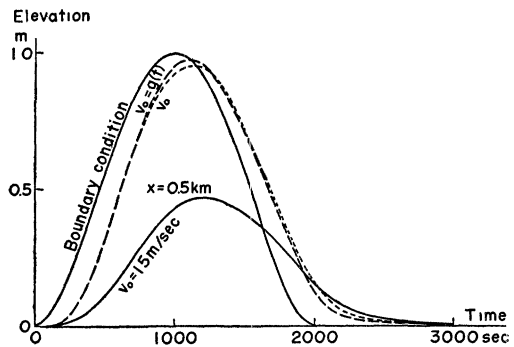
Flood hydrograph at the upstream boundary (boundary condition of the water-level) is the same function as (1). It is assumed that $\alpha=1$, $\eta=1$ and $g=9.8 \text{ m sec}^{-2}$. Such values as above are chosen because of convenience in comparing the result with one of examples in Section 1: solution without acceleration terms.

It is very difficult to introduce the boundary condition for the water velocity. For the boundary condition we can define the water velocity $u=v_0$ at $x=0$ independently from the water-level $h=h(t)$ at $x=0$.

In this paper the author tries to compute two cases to define it. (1) first, he puts $v_0=1.5 \text{ m/sec}$ always at $x=0$. The arbitrary choice of this condition may be justified by the consideration that the water velocity as the boundary condition did not contribute to the result very much and it might not be necessary to define strictly the water velocity of the boundary condition. (2) second, he applies the water velocity v_0 , which had been derived from one of the solutions without acceleration terms in Section 1 named case (3), to the boundary condition. Shortly speaking, he compares three cases: (1) v_0 is defined to be always constant, (2) $v_0=g(t)$ is applied the solution without acceleration terms and (3) v_0 is not given because of neglecting acceleration terms.

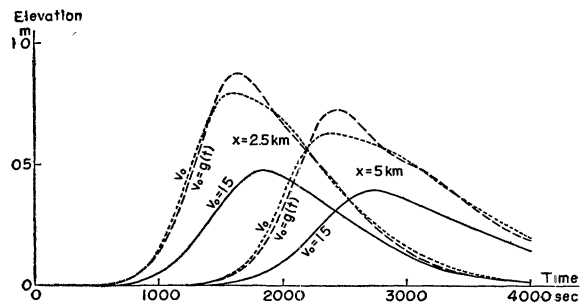
(a) Hydrograph Hydrographs at $x=0.5 \text{ km}$, 2.5 km and 5 km are drawn in Fig. 10. in which the abscissa is measured by time t .

Fig. 10 (a) Hydrographs at the upstream boundary (boundary condition which is common for all cases) and at $x=0.5 \text{ km}$.



Hydrographs of case (1) are abnormally low. This trend appears more conspicuously at $x=\Delta x=0.5 \text{ km}$, that is, the next mesh point of the upstream boundary. The unnatural figure at this point is caused by compensating the given water velocity of the boundary condition with the abnormal change in the water-level. At the downstream the water-level is also low and the crest arrives a little later than in other cases (2) and (3). Though its form is flattened, its general trend is similar to others.

Fig. 10 (b) Hydrographs at $x=2.5 \text{ km}$ and 5 km .
 full line : case (1)
 broken line : case (2)
 dotted line : case (3)



Hydrographs in case (2) give a little higher water than case (3). The solution with acceleration terms consists of three components: transportation, diffusion and wave motion, while the solution without acceleration terms consists of two components: transportation and diffusion ex-

cepting wave motion. Deviations between two hydrographs (2) and (3) only appear at crests. Fronts and tails of two are overlapping respectively, so they do not seem to be affected by acceleration terms. Consequently the wave property manifests itself at the crest and tends to raise the crest.

The difference in size of hydrographs seems to be caused by the difference in discharges. Three cases are given for the same variation of the water-level at the boundary. Cases (2) and (3) contain the same variation in the water velocity, while we give the constant water velocity to case (1). As the discharge is a product of the water-level and the water velocity, the discharge in case (1) is much smaller than in other cases. Consequently hydrographs for (1) are different from (2) and (3) in their height.

(b) Decaying of crest The decay is decreasing of the crest height, indicating how important the diffusion property of equation is to the solution.

The crest in case (1) is deformed at $x = \Delta x$ as we see at (a) previously. Though such a boundary condition for the water velocity as case (1) is obviously disagreeable, the crest is decaying gradually, almost parallel to case (2) at the downstream. The crest in case (2) is decaying slowly. The diffusion property is not so distinguished in (2) and the wave property which conserves the flood form comes to the fore. Case (3) which does not include the wave equation decays a little faster than case (2).

These trends have been found in deformations of hydrographs in Fig. 10.

(c) Movement of crest Time-distance curves for crests are shown in Fig. 12, in which the abscissa is measured by distance and the ordinate is time. However the crest for case (1) starts with a little delay according to an unnatural behaviour of the flood at the beginning of the channel, crests for three cases are running with roughly the same speeds, that is, their curves are roughly parallel to each other. As the gradient of the time-distance curves means the reciprocal of the crest velocity, crests velocities for three cases are not so separated. Strictly speaking, the crest for case (3) travels more quickly than case (2) in the upstream and gradually becomes slow. The crest for case (2) catches up with (3) and outruns it in the downstream.

Curves (1) and (2) which represent solutions with acceleration terms are almost straight lines, while curves for solutions without acceleration terms are generally bent a little upward concave in Figs. 1, 6 and 12.

The author refrains from more extensive discussions due to various uncertainties concerning following questions: (1) Mathematical difficulty in non-linear differential equation. (2) How to define the boundary condition practically. (3) Which is the best, a backward, a forward, a central difference or others? (4) To establish the new relation $\Delta t - \Delta x$ in order to suppress errors. (5) We cannot

Fig. 11 Decaying of crests on distance co-ordinate.

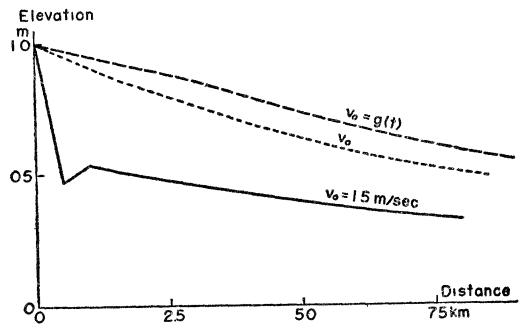
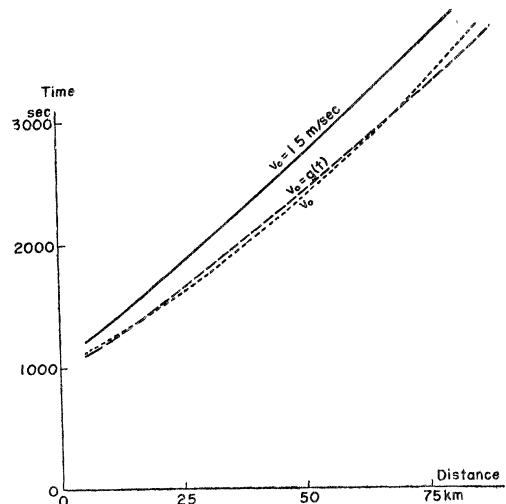


Fig. 12 Time-distance curves for crests.



compare the calculated result with the actual observation because of difficulty of measurement. These questions will be solved by further investigations.

Solutions illustrated in this section are therefore tentative ones.

Summary

Equations (1) and (2), which are called equations of motion and continuity expressed in terms of the mean flow of water, can be solved numerically. For this computation we do not need assumptions such that the channel is rectangular, the friction coefficient is constant, the flood has an infinitesimal amplitude and so on.

The author intended to deduce many informations from this method. Many characters of floods are investigated in model channels. Crest and front velocities increase roughly proportionally with Chezy's coefficient. The bottom inclination has influence upon the diffusion property of the flood form. The steeper a channel is, the less diffusive the flood form is. The broadened channel is investigated. The flood which is running through it is higher than expected from estimation for a steady flow. The two-crested flood is computed. The second crest is lower and runs more quickly than expected from linear superposition.

The acceleration terms are neglected in the most parts. Because they are smaller than others, and if they are under consideration, the boundary condition of the water velocity is needed and utilities in practical application would be reduced. But the author attempted to solve equations with acceleration terms. We cannot find serious differences between solutions with and without them.

The numerical analysis has recently been developed by high-speed computers. It gives us quantitative results as illustrated in diagrams and tables.

This numerical method is available for the preparation of the river improvement and the flood prediction.

Acknowledgement

The author thanks heartily Prof. K. Hidaka, Prof. K. Aki, Prof. K. Yoshida, Prof. S. Inokuti and Dr. Keiiti Aki who gave him kind advices and significant suggestions. He expresses his gratitude to River Bureau, Ministry of Construction which offered him useful data, and to Science and Technics Agency which supported him to compute various examples. He was helped high-speed relay computers E.T.L. Mark II and FACOM 128 which were made in Japan. He thanks deeply Yurin Electrical Co. which allowed him to operate FACOM 128 at any time on occasion.

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昭和34年9月10日 印刷
 昭和34年9月15日 発行

土木学会論文集第64号

定価 120 円 (〒20 円)

編集兼発行者 東京都新宿区四谷1丁目
 印刷所 東京都港区赤坂溜池5番地

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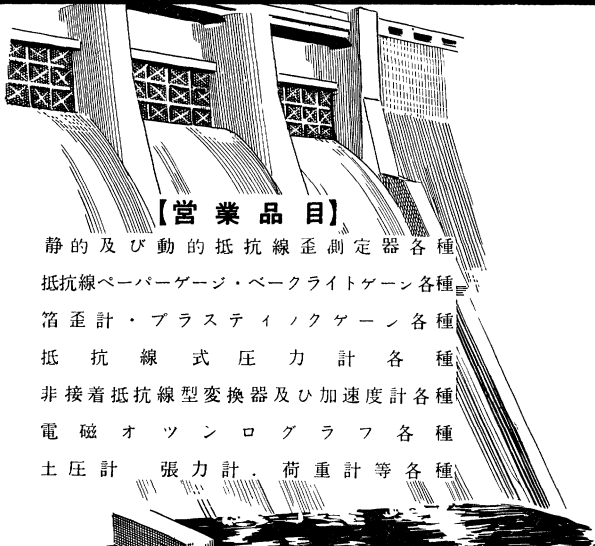
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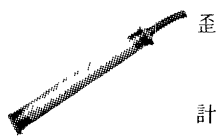
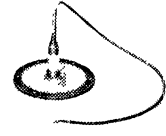

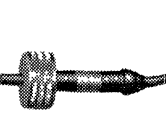
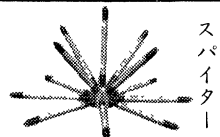
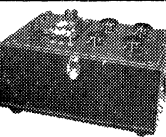
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