ON THE MOTION OF THE FLOOD-FLOW RUNNING DOWN THROUGH THE RIVER

——Mathematical Treatment and its Applicability——

By Takeo Kinosita, C,E. Member

Synopsis: Combining the equations of motion of fluid with the equation of continuity, the author solved numerically differential equations of flood-flow which is one kind of unsteady flows. The system of the flood flow under cosideration is nonlinear with a finite amplitude. In this paper, first, his method is justified by comparing calculated floods with observed floods.

Chapter 1 Introduction

We have improved many rivers since very old days. The term "river improvement" means to change physical constants and coefficients of a river, for examples to increase the bottom inclination by means of short-cutting, to adjust the friction coefficient by means of dredging or spur dikes, to smooth up the river-bed and so on. When we make a plan to improve the river, we must know quantitatively how much influence this improvement has upon the flood-flow which is not a steady flow, of course. But it is hard in general to predict these influences due to river improvements. We had closed up open levees or cut off meandering parts for land utilization, thereupon we often experienced that the flood crest ran down faster and it became higher than ever. The damages are swelling up recently with economical developments of river basins. Although we cannot conclude easily the causality of flood damages, we ought to make efforts to save the cost of improvements and avoid unexpected damages by means of investigating previously natures of floods. The river improvement is based upon social and economical requirements, but it is the problem of hydrodynamics to research and predict its effects upon flood-flows.

In this paper, the author wants to find natures of flood-flows through the mathematical computation based on hydraulics. Another way of investigation of floods is the model experiment. However the similarity law restricts unfortunately our experimental approach, that is, Froud's numbers for an actual phenomenon and an experiment must coincide with each other on one hand, and both Reynolds' numbers must do also on the other hand, so that water cannot be used in experiment for phenomenon of water. Unsteady flows are made in a small glass channel at Institute of Industrial Science, Tokyo University[1], without consideration of similarity law, and friction formula in the model channel is quite different from one in the actual river, because the flow in the channel is neither turbulent nor laminar but in complicated state. There are some objections in such an experimental research.

The flood-flow is unsteady, of which the water-level and the discharge vary with time and space. In this case, the motion of a water particle is so complicated that we can hardly formulate it strictly with respect to every water particle. For this reason some simplifications are necessary, and their results give differential equations for the mean velocity in the open channel.

From upstream to downstream, x-axis is taken along the river. At a certain time and a certain cross section, taking the average of the water velocity, we define it the mean velocity of water u. The differential equations consist of u and the water depth h on x and t co-ordinates. There are velocity components which are perpendicular to the x-axis. We call them secondary flows and take them for a kind of turbulence, their effect being included in a friction term. Through these procedures, the equation of motion is simplified as given by

* Geophysical Institute, Tokyo University

$$\frac{\eta}{g}\frac{\partial u}{\partial t}+\frac{\alpha}{g}u\frac{\partial u}{\partial x}-i+\frac{\partial h}{\partial x}+f'\frac{u^2}{2gR}=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

The first and second terms are related with the acceleration of mean velocity, the third and fourth terms express the bottom inclination and the depth gradient respectively, so that the sum of both expresses the slope of water surface, and the last term means the friction. Two coefficients η and α which are determined by the cross sectional form and the velocity distribution equal to 1 approximately. $R=\frac{A}{P}$ is named the hydraulic mean depth and almost equals to h if the channel is rectangular. A is the cross sectional area, and P is the wetted perimeter. $\frac{f'}{2g}$ which is a friction coefficient is rewritten by $\frac{1}{C^2}$. Then Chezy's formula which has been well known as an empirical formula for steady flows in open channels is introduced into our fundamental equation for unsteady flows. We call C Chezy's coefficient, which represents how easily water flows down.

$$\frac{\partial A}{\partial t}+\frac{\partial (Au)}{\partial x}=0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

(2) shows the equation of continuity, that is, the increment of A is defined by the difference between inflow and outflow at a certain section. Of course (2) is derived from div $u=0$.

As these equations are nonlinear with respect to u, we are compelled to simplify them largely for analytical developments. A few examples will be written down below.

The cross section A is assumed to be a primitive function of the depth h, that is ah^p. If the bottom inclination is balanced by the friction, corresponding to the steady motion, (1) becomes Chezy's formula.

$$u=Ch^{\frac{1}{2}}i^{\frac{1}{2}}$$

Combining this with equation (2), we obtain

$$\frac{\partial h}{\partial t}+\frac{p+\frac{1}{2}}{p}u\frac{\partial h}{\partial x}=0$$

This equation, called Seddon's formula, shows that some phase of h is transmitted by the velocity $\frac{p+\frac{1}{2}}{p}u$. In a rectangular channel, as $p=1$, h is transmitted by the velocity $\frac{3}{2}u$. It can be also discussed by the characteristic curves. In the rising part of the flood, the characteristic curves converge together. In other words, the highest part of the flood runs down more quickly than the frontal part and the former catches up with the latter, then the flood wave becomes a breaker at last. We have never observed such a curious phenomenon. Neglection of $\frac{\partial h}{\partial x}$ in this simplification causes this discrepancy. So the characteristic method is improved. $\frac{\partial u}{\partial t}$, $u\frac{\partial u}{\partial x}$ and $\frac{\partial h}{\partial x}$ are put back again, through numerical integration we elongate segments of the characteristic curves step by step, solving associated equations numerically. This procedure is very troublesome.

Other researches in analytical method are tried. Dr. S. Hayami[2] added the horizontal mixing to the equation of continuity in order to represent diffusion property, and assumed rectangular, uniformly sloped and infinitely long channel. The equation is

$$\frac{\partial h}{\partial t}=-\frac{\partial Q}{\partial x}+\eta\frac{\partial^2 h}{\partial x^2}$$

where $Q=ch^{\frac{3}{2}}\left(i-\frac{\partial h}{\partial x}\right)^{\frac{1}{2}}$, which is derived from the equation of motion without acceleration terms. Next he expanded h to φ_1, φ_2……and collected terms of φ_1, by which he made the following equation

$$\frac{\partial \varphi_1}{\partial t}+\frac{3}{2}U_0\frac{\partial \varphi_1}{\partial x}=\mu_0\frac{\partial^2 \varphi_1}{\partial x^2}$$

where $\mu_0 = \frac{h_0}{2i} + \eta$

and $U_0 = c\sqrt{h_0 i}$

As this is linear, it can be solved analytically with suitable initial and boundary conditions. He obtained the solution of this equation by means of superposition of the simple sine wave or unit pulse function. But the mixing coefficient η is unmeasured.

Dr. T. Hayashi[3] linearized the equations (1) and (2), seperating unsteady parts from steady parts of h, u and Q of the flood, and supposed that the formers were smaller than the latters, so that he obtained the solutions analytically. He assumed that an amplitude of a flood is infinitesimally small.

In both methods there are some restrictions such as channels being rectangular, uniform and infinitely long and their friction coefficients being constant.

As above mentioned analytical ways stood on some hypotheses which are not very adequate to study natures of floods, we hope to develop another new method through which we can investigate them based on more reasonable considerations. If equations can be solved numerically instead of analytically, without simplifications, we can easily obtain the informations about natures of floods with intuitive pictures.

Difficulty in solving these nonlinear equations with the friction term has hindered the development of hydrodynamics. If the numerical approach is able to remove such difficulty as above, it must contribute to not only hydrodynamics but human security very much. Then the author has intended to compute these equations numerically. But in this numerical computation there is an obstacle, that is a computational error which is mainly connected with acceleration terms. The author will show how to deal with them in next chapter.

Chapter 2 On Acceleration Terms

When we are going to solve equations (1) and (2), we find two ways, with and without acceleration terms in equation (1). The way including equation (1) with acceleration terms and equation (2) requires the initial conditions: $h(x)$ and $u(x)$ at $t=t_0$, and the boundary conditions: $h(t)$ and $u(t)$ at $x=x_0$. The way without acceleration terms, on the other hand, requires the initial condition: $h(x)$ at $t=t_0$, and the boundary conditions: $h(t)$ at $x=x_0$, and $h(t)$ at $x=x_1$. Data of mean velocities are required for the initial and boundary conditions, if acceleration terms are not omitted. Nowadays the water-level is measured accurately, easily and frequently, while the mean velocity is not measured accurately nor easily. So we have not plenty of good data of mean velocities for the initial and boundary conditions.

Besides we must have precise data to prevent accumulation of computational error. However as mentioned above, the accurate mean velocity can hardly be measured. Moreover during the flood time, the river flow is so turbulent that the "accurate" velocity is meaningless Consequently we want the method for which mean velocities need not be used as the conditions.

If the acceleration terms are omitted, we can compute the water-levels and the mean velocies defining only the water-levels as the initial and boundary conditions.

Further it is the most important fact that acceleration terms are much smaller than other terms. According to the observations[4] of the experimental flood at Sintakase River $\left|\frac{1}{g}\frac{\partial u}{\partial t}\Big/ i\right|$ and $\left|\frac{u}{g}\frac{\partial u}{\partial x}\Big/ i\right|$ are both smaller than 5%, except only one observation, though this experimental flood was rising more rapidly than actual floods.

Dr. M. Yoneda[4] showed a few examples in actual floods as follows.

	At Tanabe, Kizu River July 1949	At Hirakata, Yodo River June 1935
$\frac{1}{g}\frac{\partial u}{\partial t}\Big/ i$	$+2.6\%\sim-1.2\%$	$+6.9\%\sim-2.0\%$
$\frac{u}{g}\frac{\partial u}{\partial x}\Big/ i$	$+1.6\%\sim-1.0\%$	$+1.3\%\sim-0.2\%$

In the latter chapter the author will express that the water-level computed by his method without acceleration terms and observed records of the water-level coincide with each other very well. This fact means acceleration terms are negligible. Later on he will show the way to solve the equations with acceleration terms and their effects in the model channel.

To be brief, acceleration terms are neglected in most part of the present consideration. Because they are small.

Chapter 3 To Solve Fundamental Equations

supposing that the channel is uniformly rectangular in order to explain simply how to solve equation first, we obtain the following equation by combining equation (1) without acceleration terms with equation (2),

$$\frac{\partial h}{\partial t}+\frac{3}{2}ch^{\frac{1}{2}}\left(i-\frac{\partial h}{\partial x}\right)^{\frac{1}{2}}\frac{\partial h}{\partial x}-\frac{C}{2}h^{\frac{3}{2}}\left(i-\frac{\partial h}{\partial x}\right)^{-\frac{1}{2}}\frac{\partial^2 h}{\partial x^2}=0 \quad\cdots\cdots(3)$$

This is obviously nonlinear, but roughly speaking, consists of diffusion part and transportation part, that is to say, the flood form which is represented by h travels down through the channel, decaying and spreading as diffusion.

Of course, it is a big advantage that we can discuss natures of floods not only in the rectangular channel but also in the channel of arbitrary form with the present method. For the sake of simplicity, the author is going to explain the fundamental relation in equation (3). It consists of h, $\frac{\partial h}{\partial t}$, $\frac{\partial h}{\partial x}$, $\frac{\partial^2 h}{\partial x^2}$, i and C. As i and C are constants in this case, if h, $\frac{\partial h}{\partial t}$, $\frac{\partial h}{\partial x}$ and $\frac{\partial^2 h}{\partial x^2}$ are known at $t=0$, we can derive the increment of h with respect to time, that is $\frac{\partial h}{\partial t}$. Consequently h's at $t=\Delta t$ become known from $\frac{\partial h}{\partial t}\Delta t$ at all mesh points on x-axis. In other words, using the water-levels at $t=0$, equation (3) predicts the water-levels at $t=\Delta t$. In the computational process, the author does not use equation (3) directly because it is not convenient. We had better use its original form (without combining (1) with (2)), so that we can obtain the discharge related with the water-level at every mesh point. Defining q as the discharge per unit width $q=uh$, we obtain equation (4) from (1) without acceleration. Equation (2) is reduced to equation (5) by simple substitution.

$$q=Ch^{\frac{3}{2}}\left(i-\frac{\partial h}{\partial x}\right)^{\frac{1}{2}} \quad\cdots\cdots(4)$$

$$\frac{\partial h}{\partial t}=-\frac{\partial q}{\partial x} \quad\cdots\cdots(5)$$

First we compute the discharge q at every point by the right side of (4) from the water-level, and next the increment of h by the right side of equation (5) from the discharge. Then we know the new water-level at $t=\Delta t$. Repeating this process we can easily obtain the water-level h and the discharge q alternately through equations (4) and (5).

In actual rivers the bottom inclination, the form of the cross section and the friction coefficient vary irregularly with x or with the water-level. In order to apply this method to floods occuring in actual rivers, first the author introduces the absolute height of the water-level above a certain reference level H, replaces $i-\frac{\partial h}{\partial x}$ to $-\frac{\partial H}{\partial x}$, and considers the cross sectional area A to be a function of H at every mesh point as in his previous reports[5),6)].

$$A=A\,(H) \quad\cdots\cdots(6)$$

$$Q=CA\sqrt{\frac{A}{P}\left(-\frac{\partial H}{\partial x}\right)} \quad\cdots\cdots(7)$$

$$\frac{\partial A}{\partial t}=-\frac{\partial Q}{\partial x} \quad\cdots\cdots(8)$$

Equations (6), (7) and (8) are more general forms of (4) and (5), and derived from fundamental equations (1) without acceleration and (2). In this case, while independent variables are t and x, dependent variables are H, A and Q, which are going to be determined successively by this numerical computation for (6), (7) and (8). A functional relation between A and H is known previously by surveying. H's are known at all positions on x-axis at $t=0$ as the initial condition.

Derivertive $\frac{\partial H}{\partial x}$ can be computed by the difference expression as follows.

$$\left(\frac{\partial H}{\partial x}\right)_{x=n}=\frac{H_{n+1}-H_{n-1}}{2\Delta x} \qquad n=1\sim l-1,$$

$$\left(\frac{\partial H}{\partial x}\right)_{x=0}=\frac{4\,H_1-3\,H_0-H_2}{2\Delta x} \qquad \text{at upstream boundary.}$$

$$\left(\frac{\partial H}{\partial x}\right)_{x=l}=\frac{-4\,H_{l-1}+3\,H_l+H_{l-2}}{2\Delta x} \qquad \text{at downstream boundary.}$$

Putting (6) and these difference expressions into equation (7), we derive Q's at all points easily.

Next the derivative $\frac{\partial Q}{\partial x}$ is represented by the difference expression in the similar manner,

$$\left(\frac{\partial Q}{\partial x}\right)_{x=n}=\frac{Q_{n+1}-Q_{n-1}}{2\Delta x} \qquad n=1\sim l-1$$

These transformations are as precise as parabolic approximations with three points. As $\frac{\partial A}{\partial t}$ equals to $-\frac{\partial Q}{\partial x}$, the time increment of the cross sectional area becomes known through equation (8).

Thus the new water-levels $H_1\sim H_{l-1}$ become known by means of the relation (6). They and H_0 and H_l which are given as boundary conditions at upstream and downstream respectively make the form of water surface at $t=\Delta t$. We consider H at $t=\Delta t$ as the initially given value of the water-level, and repeat this work successively. We can consequently obtain H and Q at every time and everywhere.

Chapter 4 Computational Error

In order to prevent the accumulation of the computational error we should examine the relation between mesh sizes Δt and Δx. There is no theory for Δt-Δx relation especially about a nonlinear partial differential equation.

The difference equations corresponding to equations (7) and (8) are

$$Q_{m,n}{}^2=C^2f(H_{m,n})\left[\frac{H_{m,n-1}-H_{m,n+1}}{2\Delta x}\right] \quad\cdots\cdots(9)$$

$$\frac{A_{m+1,n}-A_{m,n}}{\Delta t}=\frac{Q_{m,n-1}-Q_{m,n+1}}{2\Delta x} \quad\cdots\cdots(10)$$

where $f=\frac{A^3}{p}$ and the first of double suffix means the time mesh number and the second the space mesh number.

The error comes from two origins, that is to say, the difference between solutions of the differential equation and of the difference equation and the residue of Taylor's expansion.

Notations to be used are as follows.

	solution of difference eq.	differential eq.	error
water-level	H	η	$e=H-\eta$
cross section	A	α	$\varepsilon=A-\alpha$
discharge	Q	ϕ	$E=Q-\phi$

In this chapter the author decides the relation between Δt and Δx in order to diminish the computational error.

While equation (10) is the equation of continuity in the difference form, the same eqution in the differential form is

$$\frac{\partial \alpha}{\partial t} = -\frac{\partial \phi}{\partial x}$$

By Taylor's expansion of these derivertives, it becomes

$$\frac{\alpha_{m+1,n} - \alpha_{m,n} - T_1}{\varDelta t} = -\frac{\phi_{m,n-1} - \phi_{m,n-1} - T_2}{2\varDelta x} \qquad \begin{aligned} T_1 &= \frac{\varDelta t^2}{2}\alpha_{tt} \\ T_2 &= \frac{\varDelta x^2}{3}\phi_{xxx} \end{aligned} \quad \cdots\cdots(11)$$

where T_1 and T_2 are residues of Taylor's expansions. Subtracting equation (11) from equation (10), we obtain

$$\frac{\varepsilon_{m+1,n} - \varepsilon_{m,n} + T_1}{\varDelta t} = -\frac{E_{m,n+1} - E_{m,n-1} + T_2}{2\varDelta x} \quad \cdots\cdots(12)$$

While equation (9) is the equation of motion in the difference form, the same equation in the differential form modified by Taylor's expansion is

$$\phi_{m,n}{}^2 = C^2 f(\eta_{m,n}) \left[\frac{\eta_{m,n-1} - \eta_{m,n+1} + T_3}{2\varDelta x}\right] \quad \cdots\cdots(13)$$

$$T_3 = \frac{\varDelta x^3}{3}\eta_{xxx}$$

where T_3 is also a residue of Taylor's expansion of the surface slope. Subtracting (13) from (9), and approximating $Q^2 - \phi^2$ to $2EQ$, we make a complicated expression.

$$E_{m,n} = \frac{C^2}{2Q_{m,n}} f(H_{m,n}) \left(\frac{e_{m,n-1} - e_{m,n+1} - T_3}{2\varDelta x}\right) + \frac{f'(H_{m,n})}{2f(H_{m,n})} Q_{m,n} e_{m,n}$$

$$+ \frac{C^2}{2Q_{m,n}} f'(H_{m,n}) \left(\frac{e_{m,n+1} - e_{m,n-1} + T_3}{2\varDelta x}\right) e_{m,n} \quad \cdots\cdots(14)$$

Combining these error relations (12) and (14), we get

$$\frac{\varepsilon_{m+1,n}}{L} = \frac{\varepsilon_{m,n}}{L} - \frac{C^2 f}{4LQ}\frac{\varDelta t}{\varDelta x^2} e_{m,n}$$

$$+ \frac{C^2 f}{8LQ}\frac{\varDelta t}{\varDelta x^2}(e_{m,n+2} + e_{m,n-2} + T_{3,m,n+1} - T_{3,m,n-1})$$

$$+ \frac{f'Q}{4fL}\frac{\varDelta t}{\varDelta x}(-e_{m,n+1} + e_{m,n-1}) - \frac{1}{L}\left(\frac{\varDelta t}{2\varDelta x}T_2 + T_1\right) \quad \cdots\cdots(15)$$

This expression (15) shows that the error of difference and differential equations at $(m+1, n)$ on (t, x) co-ordinates comes from ones at $(m, n-2)$, $(m, n-1)$, (m, n), $(m, n-1)$ and $(m, n-2)$ and residues of Taylor's expansions. Coefficients of $e_{m,n}$ and so on in (15) have to be smaller than 1 in order to prevent the accumulation of errors. At (15) both sides are divided by L which is the width of the channel to compare with them in the same dimension. Thus the author makes restrictions for $\varDelta t$ and $\varDelta x$.

$$\frac{C^2 f}{8LQ}\frac{\varDelta t}{\varDelta x^2} < 1 \qquad \varDelta t < \frac{8LQ}{C^2 f}\varDelta x^2 \quad \cdots\cdots(16)$$

$$\frac{f'Q}{4Lf}\frac{\varDelta t}{\varDelta x} < 1 \qquad \varDelta t < \frac{4fL}{f'Q}\varDelta x \quad \cdots\cdots(17)$$

(16) and (17) are related with the diffusion property and the transportation property of the fundamental equation respectively.

As the relation of residues of Taylor's expansions corresponds to the diffusion equation, the author picks it out the fundamental equation with some simplifications.

Suppose the channel is rectangular,

$$\varDelta t = \frac{2}{3}\frac{i^{\frac{1}{2}}}{Ch^{\frac{3}{2}}}\varDelta x^2 \quad \cdots\cdots(18)$$

Suppose the channel is triangular,

$$\Delta t=\frac{4\sqrt{2}}{3}\frac{i^{\frac{1}{2}}}{Ch^{\frac{3}{2}}}\Delta x \quad \cdots\cdots(19)$$

These relations determine mesh sizes of Δt and Δx from the point of view of computational errors.

Chapter 5 Floods in Actual Rivers

In the case of application of this method for actual rivers, the followings are desirable.

(a) Variations in width, depth, bottom inclination and other quantities of the river should be gradual, becuse the original equations are differential ones.

(b) The river should not be meandering very much, because equations are described as one dimensional phenomena.

(c) There should be neither diversion nor confluence, because of equation of continuity. But if equation of continuity is generalized with the confluence term, this requirement will be removed.

(d) The cross sectional area should well be surveyed, because functional relation between H and A (6) must be known at every mesh point.

(e) The friction coefficient (for example Chezy's coefficient) should be determined.

(f) A number of water-gauges should be installed along the river, because many records of water-levels are used for initial and boundary conditions.

Two rivers, Kinu River and Sintakase River, are chosen as examples. They are sufficient for above-mentioned requirements (a)~(f).

Kinu River rises from mountains in northern part of Totigi Prefecture, runs through Kanto Plain and pours into Tone River. The downstream part of 42 km in length is chosen as the computational zone. The flood occured at Sept. 1st 1949 caused by the typhoon "Kitty".

$\Delta x=$	2 km		6 km	
	rectangular	triangular	rectangular	triangular
(16) $\Delta t<$	1 920 sec	1 920 sec	17 280 sec	17 280 sec
(17) $\Delta t<$	1 100	640	3 200	1 920
(18) & (19) $\Delta t=$	133	370	1 200	3 340

This table shows the relation between Δt and Δx accerding to (16)~(19), in which numerical values are taken as follows,

$h=4$ m $\quad Q=3\,000$ m^3/sec $\quad L=300$ m

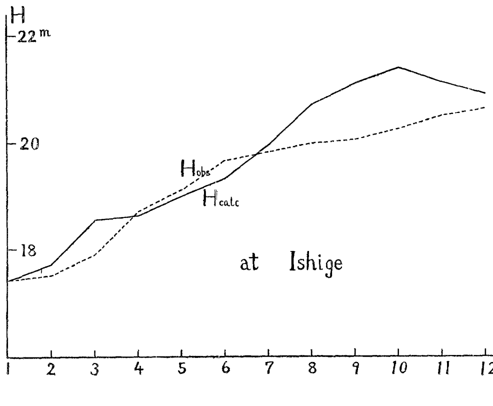
$i=1/2\,500 \quad C=50$ m$^{\frac{1}{2}}$ sec^{-1}

Then Δt and Δx are determined to be 400 sec and 2 km respectively. In this case $t=0$ is taken at 1 a.m. Sept. 1st 1949, when the water surface began to swell up at the upstream boundary.

Comparison between observed hydrographs and calculated hydrographs are illustrated in **Figs. 1** and **2**. The former is at Ishige where the watergauge is set on the mesh point. The latter is at Mizukaido where the water-gauge is out of the mesh point. These curves coincide well with each other.

It was said previously that acceleration terms were small. The author wants to check their effect in computation process. **Fig. 3** shows that the difference between two solutions with

Fig. 1 Calculated and observed hydrographs at Ishige, Kinu River.



and without acceleration terms. In this case the discharge Q_a with acceleration terms is derived from the water-level and accelerations expressed in terms of u_b

$$Q_a=CA\sqrt{\frac{A}{P}\left(-\frac{\partial H}{\partial x}-\frac{1}{g}\frac{\partial u_b}{\partial t}-\frac{u_b}{g}\frac{\partial u_b}{\partial x}\right)}$$

where $u_b=C\sqrt{\frac{A}{P}\left(-\frac{\partial H}{\partial x}\right)}$, that is the velocity without acceleration terms. **Fig. 3** suggests that the acceleration does not deform the solution remarkably, and we can discuss characters of the flood-flow excluding acceleration with good approximation.

Kinu River is a natural river and its width and other quantities vary over a wide range. Moreover, as water-gauges are not precisely set on mesh points of the difference operation, we cannot compare observed water-levels with calculated ones in detail. And it is important for practical use to smooth up irregularities of channel form, friction coefficient and so on. Therefore the author used the data which Dr. M. Yoneda had obtained at Sintakanse River. It is not a natural river but made for drainage. The flood was caused by operating the gate for experimental purpose. Water-gauges were established at every 100 m and observed at every 2 minutes by naked eyes.

Fig. 2 Calculated and observed hydrographs at Mizukaido, Kinu River.

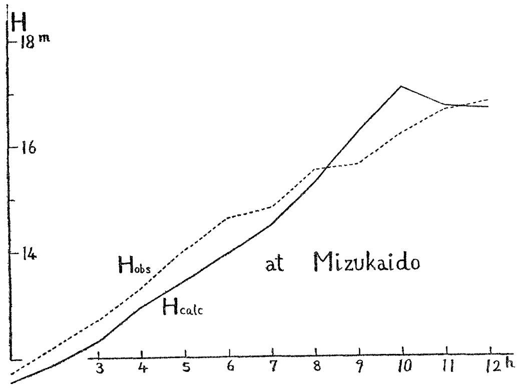
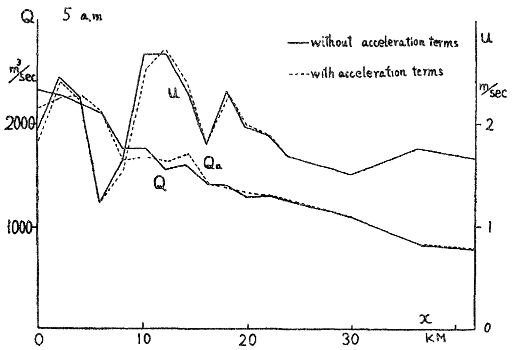


Fig. 3 Correction for acceleration terms. full line : Q, dotted line : Q_a



The initial depth (steady flow) is about 30 cm, the maximum depth is about 1 m and of course, the flood is a phenomenon of finite amplitude. Its duration is about 2 hours. Parameters are defined as follows,

$$i=8.5\times10^{-4} \qquad C=28\ \mathrm{m}^{\frac{1}{2}}\mathrm{sec}^{-1}$$

$$\Delta x=100\ \mathrm{m}$$

Using these values, we make the restriction for Δt based on relations (16)~(18).

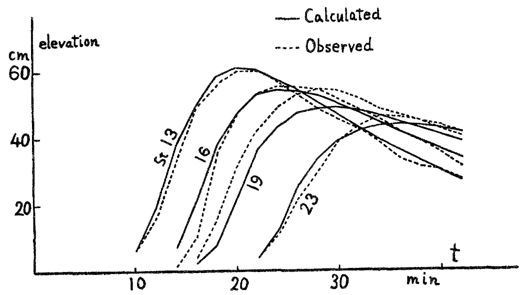
$$\Delta t<120\ \mathrm{sec}, \qquad \Delta t<133\ \mathrm{sec}, \qquad \Delta t=20\ \mathrm{sec}$$

Consequently Δt is determined to be 40 sec.

Chezy's coefficient is supposed to be consnant and the channel is supposed to be uniformly rectangular and infinitely long for the sake of simplicity of this computation.

The calculated hydrographs agree well with the observed ones as we see in **Fig. 4,** though we assume a crude model of the river. In this diagram an elevation means a residue of subtracting the initial depth from the instant depth. Even if the river is made for drainage, the bottom inclination is irregular. The initial depth

Fig. 4 Calculated and ocserved hydrographs at some stations, Sintakase River.

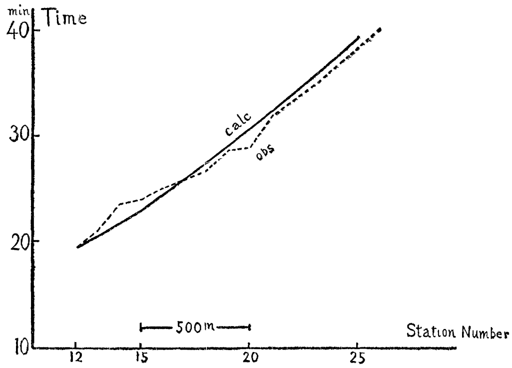


is not so accurate that we see a little discrepancy in water-levels.

The time-distance curves for the flood crest are drawn in **Fig. 5.** They say that above-mentioned simplifications are very reasonable this computation, at least to predict the travel of the flood.

Even though the mathematical treatment of nonlinear partial differential equations is very difficult, their solution obtained by numerical method agrees to the actual phenomenon. The author intends to deduce characters of flood-flows by this useful method and to show the interesting results of the deduction in the next report.

Fig. 5 Time-distance curves for crests, Sintakase River.



Summary

Equations (1) and (2), which are called equations of motion and continuity expressed in terms of the mean flow of warer, can be solved numerically. For this computation we do not need assumptions such that the channel is rectangular, the friction coefficient is constant, the flood has an infinitesimal amplitude and so on.

Two actual floods, in Kinu River and Sintakase River, are computed. Agreements between observed and calculated curves may justify the validity of this method and prove its applicability to practical use.

Acknowledgement

The author thanks heartily Prof. K. Hidaka, Prof. K. Aki, Prof. K. Yoshida, Prof. S. Inokuti and Mr. Keiiti Aki who gave him kind advices and significant suggestions. He expresses his gratitudes to River Bureau, Ministry of construction which offered him useful data, and to Science and Technics Agency which supported him to compute various examples. He wase helped high-speed relay computors FACOM 128 which was made in Japan. He thanks deeply Yurin Electrical Co. which allowed him to operate FACOM 128 at any time on occasion.

References

1) Inokuti, S , and A. Hamamori, unpublished. Preliminary report: Experiment of Steady Flow in Narrow Channel, Seisan-kenkyu, Vol. 9 No. 7
2) Hayami, S , On the Propagation of Flood Waves, Bull. Disaster Prev. Res. Inst., Kyoto Univ. No. 1 (1951).
3) Hayashi, T., Mathematical Study of the Motion of Intumescences in Open Channels of Uniform Slope, Trans, Jap. Soc. Civil Eng. No. 11 (1951)
4) Yoneda, M., The Treatise of Flood Characters (Kozuitokusei-ron) Dobokuzassisya. (in Japanese)
5) Kinoshita, T., A Study on Flood Flow, Resources Council, Prime Minister's Ofice, 1955. (in Japanese)
6) Kinoshita, T.,Hydrodynamical Study on the Flood Flow. Intern. Assoc. Hydro., Symposia Darcy. 1956.

(昭. 33. 11. 3)