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# DYNAMICAL SIMILITUDE OF SURGE TANKS

サージタンクの相似律


*By Dr. Eng., Taizo Hayashi, C.E. Member*



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# DYNAMICAL SIMILITUDE OF SURGE TANKS*

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## SYNOPSIS

An extension is presented of the dynamical similitude of model tests of surge tanks for the most general case. Additional conditions to be imposed upon the dynamical similitude and the modification of the similitude for preliminary design purpose of the model installation are examined in detail. Model tests for several selected complicated cases are made for the affirmation of whether the dynamical similitude can still exist in such complicated cases. The surging waves worked out by arithmetical integration of the surging equations are compared with the results interpreted according to the similitude out of the surging waves obtained at the model. The agreement in every case is satisfactorily close, by which the dynamical similitude is affirmed to be always reliable for the model test of any complicated case at will.

## INTRODUCTION

Due to the complex mathematical nature of the equations for surging of the surge tank, it is desirable to carry out the model investigation if it is practical. The application of the dynamical similitude to the surge tank problem was first made by Professor Durand[1]. By his work the possibility of a comprehensive experimental method with the model was presented. Theoretical arguments for the use of models usually follow one of two main lines: the so-called dimensional analysis, and what may be called inspectional analysis[2], where the former is simpler and more widely known, while the dynamical similitude of the surge tanks belongs to the latter. The dynamical similitude presented by him consists in installing the model pipe installation in such a way that the surging equations with respect to model installation equal quantitatively to the surging equations for the prototype installation, term by term of the both corresponding equations. The model may thus, in a sense, be considered as a form of mechanism which solves the equations as applied to the model form, and these results multiplied by suitable relation factors give the corresponding results to be anticipated in field case.

The use of the model in such a connection, however, can only be justified if the scale-effect is so small as to be unimportant. In order to investigate this point, Professor Gibson made an extensive study of the model test of the simple surge tank[3]. However, since no measurement of the surge in prototype is practically available in which the data are sufficiently complete to enable a comparison with model experiments to be made, he worked out by arithmetical integration a number of typical cases covering a wide range of conditions. By these typical examples, the results of the tests on the models, when interpreted according to the laws of dynamical similarity, were in close agreement with

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the results as calculated for the corresponding prototype pipe-line and surge tank, thus the method of the model test with the dynamical similitude being justified.

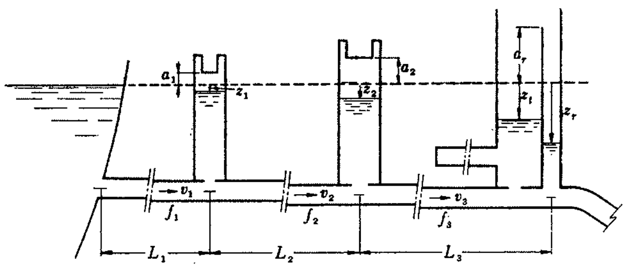
A furthur contribution to the method of the model test was made by Messrs. Bourgeat, Cahuzac and Deullin[4]. They made model tests of a differential surge tank, for which tests they not only examined the solution of the law of dynamical similarity, but also they described many details of the model technique.

This paper deals with complicated cases of surge tanks for the affirmation of whether the dynamical similitute can still exist in such cases. Before giving the results of the tests the author develops the dynamical similitude in the most general form. Since this similitude is nevertheless considered as a mere combination of individual laws of similarity obtained in the previous investigations, main emphasis of this paper is rather on the study of the additional restrictions to be imposed upon the dynamical similitude and on the modification of the similitude for preliminary design purpose of the model installation. The results of the model tests carried out according to the dynamical similitude developed are compared with those calculated for the corresponding prototype, the agreement between them being examined.

## DYNAMICAL SIMILITUDE OF SURGE TANKS

Although we intend to derive the dynamical similitude in the most general case, let us consider tentatively about a series of surge tanks as shown in **Fig. 1**. The main surge

**Fig. 1** A series of surge tanks.



tank is a differential tank with a lower chamber and each auxiliary surge tank has both a restricted orifice and an overflow. Thus, the series is considered to include practically all essential elements of surge tanks. As is seen, the movement of water at this series is represented by the equations as follows:

$$\left.\begin{aligned}
&\frac{L_1}{g}\frac{dv_1}{dt}=z_1\mp\varepsilon_1 v_1^2\pm\eta_1(v_2-v_1)^2\\
&\phi_1\frac{dz_1}{dt}=v_2+C_{s1}-v_1\\
&C_{s1}=\lambda_1(a_1-z_1)^{3/2}\\
&\frac{L_2}{g}\frac{dv_2}{dt}=z_2-z_1\mp\varepsilon_2 v_2^2\pm\eta_2(v_3-v_2)^2\\
&\phi_2\frac{dz_2}{dt}=v_3+C_{s2}-v_2\\
&C_{s2}=\lambda_2(a_2-z_2)^{3/2}
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(1)^*$$

* The notations are referred to in the list at the end of this paper.

$$
\left.\begin{aligned}
&\frac{L_3}{g}\frac{dv_3}{dt}=z_r-z_2\mp\varepsilon_3 v^2{}_3\\
&\phi_r\frac{dz_r}{dt}=C+C_t+C_{sr}-v_3\\
&\phi_t\frac{dz_t}{dt}=-(C_{sr}+C_t)\\
&C_{sr}=\lambda_r(a_r-z_r)^{3/2}\\
&C_t=\pm\gamma(z_t\sim z_r)^{1/2}
\end{aligned}\right\}
$$

On the other hand, at the model installation the movement of water is represented by the equations as follows:

$$
\left.\begin{aligned}
&\frac{L_{1m}}{g}\frac{dv_{1m}}{dt_m}=z_{1m}\mp\varepsilon_{1m}v_{1m}{}^2\pm\eta_{1m}(v_{2m}-v_{1m})^2\\
&\phi_{1m}\frac{dz_{1m}}{dt_m}=v_{2m}+C_{s1m}-v_{1m}\\
&\qquad C_{s1m}=\lambda_{1m}(a_{1m}-z_{1m})^{3/2}\\
&\frac{L_{2m}}{g}\frac{dv_{2m}}{dt_m}=z_{2m}-z_{1m}\mp\varepsilon_{2m}v_{2m}{}^2\pm\eta_{2m}(v_{3m}-v_{2m})^2\\
&\phi_{2m}\frac{dz_{2m}}{dt_m}=v_{3m}+C_{s2m}-v_{2m}\\
&\qquad C_{s2m}=\lambda_{2m}(a_{2m}-z_{2m})^{3/2}\\
&\frac{L_{3m}}{g}\frac{dv_{3m}}{dt_m}=z_{rm}-z_{2m}\mp\varepsilon_{3m}v_{3m}{}^2\\
&\phi_{rm}\frac{dz_{rm}}{dt_m}=C_m+C_{tm}+C_{srm}-v_{3m}\\
&\phi_{tm}\frac{dz_{tm}}{dt_m}=-(C_{srm}+C_{tm})\\
&\qquad C_{srm}=\lambda_{rm}(a_{rm}-z_{rm})^{3/2}\\
&\qquad C_{tm}=\pm\gamma_m(z_{tm}\sim z_{rm})^{1/2}
\end{aligned}\right\}\cdots\cdots\cdots\cdots(2)^*
$$

In order that the model may be dynamically similar to the prototype, it is necessary that all corresponding terms in (2) and (1) should be proportional; that is

$$
\left.\begin{aligned}
&\frac{(L_{1m}/g)(dv_{1m}/dt_m)}{(L_1/g)(dv_1/dt)}=\frac{z_{1m}}{z_1}=\frac{\varepsilon_{1m}v_{1m}{}^2}{\varepsilon_1 v_1{}^2}=\frac{\eta_{1m}(v_{2m}-v_{m1})^2}{\eta_1(v_2-v_1)^2}\\
&\frac{\phi_{1m}(dz_{1m}/dt_m)}{\phi_1(dz_1/dt)}=\frac{v_{2m}}{v_2}=\frac{C_{s1m}}{C_{s1}}=\frac{v_{1m}}{v_1}\\
&\frac{C_{s1m}}{C_{s1}}=\frac{\lambda_{1m}(a_{1m}-z_{1m})^{3/2}}{\lambda_1(a_1-z_1)^{3/2}}\\
&\frac{(L_{2m}/g)(dv_{2m}/dt_m)}{(L_2/g)(dv_2/dt)}=\frac{z_{2m}}{z_2}=\frac{z_{1m}}{z_1}=\frac{\varepsilon_{2m}v_{2m}{}^2}{\varepsilon_2 v_2{}^2}\\
&\qquad\qquad=\frac{\eta_{2m}(v_{3m}-v_{2m})^2}{\eta_2(v_3-v_2)^2}
\end{aligned}\right\}\cdots\cdots\cdots(3)
$$

and so on.

In accordance with the notations by Messrs. Bourgeat, Cahuzac and Deullin, let us now introduce the reduction coefficients as follows:

* The suffix $m$ refers to the quantity of the model.

$$\left.\begin{array}{ll}K_{L_i}=L_{im}/L_i,\ (i=1,\ 2,\ 3) & K_{Csi}=Q_{sim}/Q_{si},\ (i=1,\ 2,\ r)\\ K_{v_i}=v_{im}/v_i,\ (i=1,\ 2,\ 3) & K_t=t_m/t\\ K_{\varepsilon_i}=\varepsilon_{im}/\varepsilon_i,\ (i=1,\ 2,\ 3) & K_{\eta i}=\eta_{im}/\eta_i,\ (i=1,\ 2)\\ K_{zi}=z_{im}/z_i,\ (i=1,\ 2,\ r,\ t) & K_{C_t}=C_{tm}/C_t\\ K_{\phi i}=\phi_{im}/\phi_i,\ (i=1,2,\ r,\ t) & K_{\lambda i}=\lambda_{im}/\lambda_i,\ (i=1,\ 2,\ r)\\ K_{C_i}=C_{im}/C_i,\ (i=1,\ 2,\ r,\ t) & K_\gamma=\gamma_m/\gamma\\ K_{a_i}=a_{im}/a_i,\ (i=1,\ 2,\ r) & \end{array}\right\}\cdots\cdots(4)^*$$

Substituting the reduction coefficients in (3), we obtain

$$\left.\begin{array}{l}K_{L_1}K_{v_1}/K_t=K_{z_1}=K_{\varepsilon_1}K_{v_1}{}^2=K_{\eta_1}K_{v_2}{}^2,\qquad K_{v_2}=K_{v_1}\\ K_{\phi_1}K_{z_1}/K_t=K_{v_2}=K_{Cs_1}=K_{v_1}\\ K_{Cs_1}=K_{\lambda_1}K_{a_1}{}^{3/2},\qquad K_{a_1}=K_{z_1}\\ K_{L_2}K_{v_2}/K_t=K_{z_2}=K_{z_1}=K_{\varepsilon_2}K_{v_2}{}^2=K_{\eta_2}K_{v_3}{}^2,\qquad K_{v_3}=K_{v_2}\\ K_{\phi_2}K_{z_2}/K_t=K_{v_3}=K_{Cs_2}=K_{v_2}\\ K_{Cs_2}=K_{\lambda_2}K_{a_2}{}^{3/2},\qquad K_{a_2}=K_{z_2}\\ K_{L_3}K_{v_3}/K_t=K_{zr}=K_{z_2}=K_{\varepsilon_3}K_{v_3}{}^2\\ K_{\phi_r}K_{z_r}/K_t=K_C=K_{Ct}=K_{Csr}=K_{v_3}\\ K_{\phi t}K_{zt}/K_t=K_{Csr}=K_{Ct}\\ K_{Csr}=K_{\lambda_r}K_{a_r}{}^{3/2},\qquad K_{ar}=K_{zr}\\ K_{Ct}=K_\gamma K_{zt}{}^{1/2},\qquad K_{zt}=K_{zr}\end{array}\right\}\cdots\cdots(5)$$

which are the relations the reduction coefficients should satisfy.

Although the above relations seem at first glance very complicated, they are reducible to a series of much simpler relations. As is easily seen, we have out of the above equations the relations as follows:

$$\left.\begin{array}{ll}K_{z_1}=K_{z_2}=K_{zr}=K_{zt}=K_{a_1}=K_{a_2}=K_{ar} & K_{\varepsilon_1}=K_{\varepsilon_2}=K_{\varepsilon_3}\\ K_{v_1}=K_{v_2}=K_{v_3}=K_{Cs_1}=K_{Cs_2}=K_{Csr}=K_C=K_{Ct} & K_{\lambda_1}=K_{\lambda_2}=K_{\lambda_r}\\ K_{L_1}=K_{L_2}=K_{L_3} & K_{\eta_1}=K_{\eta_2}\\ K_{\phi_1}=K_{\phi_2}=K_{\phi r}=K_{\phi t} & \end{array}\right\}\cdots\cdots(6)$$

These eight relations imply that the reduction coefficients of the same kind of physical quantities should be identical, no matter which portion each physical quantity may concern. Hence, let us now introduce the notations defined as follows:

$$\left.\begin{array}{ll}K_z\equiv K_{z_1}=K_{z_2}=K_{zr}=K_{zt}=K_{a_1}=K_{a_2}=K_{ar} & K_\varepsilon\equiv K_{\varepsilon_1}=K_{\varepsilon_2}=K_{\varepsilon_3}\\ K_v\equiv K_{v_1}=K_{v_2}=K_{v_3}=K_{Cs_1}=K_{Cs_2}=K_{Csr}=K_C=K_{Ct} & K_\lambda\equiv K_{\lambda_1}=K_{\lambda_2}=K_{\lambda_r}\\ K_L\equiv K_{L_1}=K_{L_2}=K_{L_3} & K_\eta\equiv K_{\eta_1}=K_{\eta_2}\\ K_\phi\equiv K_{\phi_1}=K_{\phi_2}=K_{\phi r}=K_{\phi t} & \end{array}\right\}\cdots\cdots(7)$$

Substituting the above notations in eq. (5), we obtain

$$\left.\begin{array}{l}\dfrac{K_LK_v}{K_t}=K_z=K_\varepsilon K_v{}^2=K_\eta K_v{}^2\\ \dfrac{K_\phi K_z}{K_t}=K_v=K_\lambda K_z{}^{3/2}=K_\gamma K_z{}^{1/2}\end{array}\right\}\cdots\cdots(8)$$

which is the dynamical similitude for the case under consideration.

However, it is to be noted, that the above equations consist of the reduction coefficients of basically physical quantities, but that they are not dependent on the configuration

* As is written in the list of notations at the end of this paper, $K$ denotes the reduction coefficient and the suffix of $K$ denotes the quantity to be reduced to the model scale.

of the surge tanks. Hence, although the equations are derived with reference to the series of surge tanks shown in Fig. 1, it is understood with the equations that there may not be any auxiliary surge tank and there may be any more auxiliary surge tanks as well, and that the surge tanks may be of any form at will. Thus, it is understood that the equations practically represent the dynamical similitude of model tests of surge tanks in the most general form.

If the case under consideration does not have any restricted orifice, the term concerned with the restricted orifice drops out of (8), the dynamical similitude for this case reducing to

$$\frac{K_L K_v}{K_t}=K_z=K_\varepsilon K_v^2 \qquad \frac{K_\phi K_z}{K_t}=K_v=K_\lambda K_z^{3/2} \qquad (9)$$

If the series furthur lacks the overflow, the dynamical similitude reduces to

$$\frac{K_L K_v}{K_t}=K_z=K_\varepsilon K_v^2 \qquad \frac{K_\phi K_z}{K_t}=K_v \qquad (10)$$

By (8) nine reduction coefficients, i. e., $K_L$, $K_v$, $K_t$, $K_z$, $K_\varepsilon$, $K_\eta$, $K_\phi$, $K_\lambda$ and $K_\tau$, are combined by six equalities. Accordingly, the values of any three reduction coefficients may be chosen arbitrarily, but the rest are to be determined by (8) as the functions of the three reduction coefficients.

If by either (9) or (10), the number of the coefficients the values of which may be chosen arbitrarily is still the same, that is, three.

## ADDITIONAL RESTRICTIONS TO BE IMPOSED UPON DYNAMICAL SIMILITUDE

### 1. Geometrical similarity of the junction of the surge tank with the conduit.

The model of the surge tank made according to the dynamical smilitude is, in general, not geometrically similar to the corresponding prototype. However, in the case when the minor loss at the junction of the surge tank with the conduit is not negligible compared with the frictional loss of head in the conduit, the geometrical similarity of the junction of the surge tank with the conduit is also desired. In order that the junction is geometrically similar, it is necessary that the reduction coefficient of the cross section of the conduit is equal to that of the surge tank; that is

$$K_\phi=1 \qquad (11)$$

Thus, the dynamical similitude combined with the geometrical similarity is expressed as follows:

$$\left.\begin{array}{l} K_L K_v/K_t=K_z=K_\varepsilon K_v^2=K_\eta K_v^2 \\ K_\phi K_z/K_t=K_v=K_\lambda K_z^{3/2}=K_\tau K_z^{1/2} \\ K_\phi=1 \end{array}\right\} \qquad (12)$$

As compared with eq. (8), one more equation has been added. Hence, the number of the reduction coefficients the values of which may be chosen arbitrarily is reduced by one than at the plain dynamical similitude.

The geometrical similarity is also convenient for the technique of model construction, since the shape of the junction is often more complicated than other portions of the model.

**2. Geometrical similarity of the surge tank.**

If the surge tank has a horizontal chamber, the geometrical similarity of the surge tank is desired, since otherwise the cross section of the chamber would be either so flat or so narrow that the movement of the water would obey a different law from that at the prototype. In order that the surge tank is geometrically similar, it is necessary that the reduction coefficient of the vertical distance and that of the horizontal dimension, both of the surge tank, are equal; that is

$$K_z=\sqrt{K_F} \quad \cdots\cdots(13)$$

Now, by the introduction of a new reduction coefficient $K_F$ in the last equation, one more relation is necessarily to be brought about. The new relation, however, may be expressed only approximately, by the use of the Darcy-Weissbach law for the loss of head of the flow in the conduit, that will being developed subsequently in 'Dynamical Similitude for Preliminary Design Purpose of Model Installation'.

**3. Geometrical similarity both of the surge tank and of the junction of the surge tank with the conduit.**

If the geometrical similarity of both the junction and the surge tank is necessary, both (11) and (13) are to be taken into account, that will being developed also subsequently.

## DYNAMICAL SIMILITUDE FOR PRELIMINARY DESIGN PURPOSE OF MODEL INSTALLATION

**1. Approximate expression of $K_\varepsilon$.**

As is well known, the friction head $h_f$ is expressed by the use of the Darcy-Weissbach formula and furthur the Manning formula as

$$h_f=f'\frac{L}{R}\frac{v^2}{2g}=\frac{2gn^2}{R^{1/3}}\frac{L}{R}\frac{v^2}{2g}$$

Accordingly the term $\varepsilon v^2$ in the previous formulae is expressed as

$$\varepsilon v^2=\left(1+\Sigma f_i+\frac{2gn^2}{R^{1/3}}\frac{L}{R}\right)\frac{v^2}{2g}$$

where $f_i$ is the coefficient of a minor loss. However, here let us either assume approximately that

$$\varepsilon v^2=\frac{2gn^2}{R^{1/3}}\frac{L}{R}\frac{v^2}{2g} \quad \cdots\cdots(14)$$

or assume $n$ to be such a modified coefficient of roughness that it may satisfy exactly the relation (14). Then, according to this relation, we have

$$K_\varepsilon=K_n{}^2K_L/K_R{}^{4/3} \quad \cdots\cdots(15)$$

which is the expression for $K_\varepsilon$ for practical design purpose of model installation.

**2. Plain dynamical similitude.**

As the plain dynamical similitude, it is sometimes more convenient to rewrite the second series of equations of (8) as follows:

$$\frac{K_FK_z}{K_t}=K_fK_v=K_Q=K_PK_z{}^{1/2}=K_OK_z{}^{3/2} \quad \cdots\cdots(16)$$

where

$K_P = K_f K_\gamma$ : reduction coefficient of $C_d\ F_p$ of port

$K_S = K_f K_\lambda$ : reduction coefficient of $CB$ of overflow

Now that two new coefficients, i.e., $K_F$ and $K_f$, have been introduced in place of one, $K_\phi$, a new relation is physically required to be brought in; although $K_\varepsilon$ is independent to $K_\phi$, it is not independent to $K_f$, it being related to $K_f$ through $K_R$ by (15). Furthur, as is obvious, $K_R$ which is to be introduced is related to $K_f$ as

$$K_R^2 = K_f \quad \cdots\cdots (17)$$

Thus, taking (15),(16) and (17) into consideration, (8) is rewritten as follows:

$$\left.\begin{array}{l} \dfrac{K_L K_v}{K_t} = K_z = \dfrac{K_n^2 K_L}{K_R^{4/3}} K_v^2 \\ \dfrac{K_F K_z}{K_t} = K_f K_v = K_Q = K_P K_z^{1/2} = K_O K_z^{3/2} \\ K_R^2 = K_f \end{array}\right\} \quad \cdots\cdots (18)$$

which is the plain dynamical similitude for preliminary design purpose. The number of the reduction coefficients appeared in the above equations is eleven. Since we have seven equalities for these eleven coefficients, any four coefficients may be chosen arbitrarily. If we choose $K_n, K_F, K_R\ K_v$ for the four coefficients, the rests, by (18), are determined as follows:

$$\left.\begin{array}{ll} K_L = K_R^{14/3}/(K_n^4 K_v^2 K_F) & K_Q = K_R^2 K_v \\ K_z = K_R^{10/3}/(K_n^2 K_F) & K_P = K_R^{1/3} K_n K_v K_F^{1/2} \\ K_t = K_R^{4/3}/(K_n^2 K_v) & K_O = K_n^3 K_v K_F^{3/2}/K_R^3 \end{array}\right\} \quad \cdots\cdots (19)$$

## 3. Dynamical similitude combined with the geometrical similitude of the junction of the surge tank with the conduit.

Combining (11) with (18), we obtain

$$\left.\begin{array}{l} K_L K_v/K_t = K_z = (K_n^2 K_L/K_R^{4/3}) K_v^2 \\ K_F K_z/K_t = K_f K_v = K_Q = K_P K_z^{1/2} = K_O K_z^{3/2} \\ K_R^2 = K_f \\ K_f = K_F \end{array}\right\} \quad \cdots\cdots (20)$$

which is the similitude for this case. This similitude has eight equalities for eleven coefficients. Hence, any three coefficients which are independent to each other may be chosen arbitrarily. If we choose $K_n, K_R$ and $K_v$ for the three, the rest coefficients are determined by this similitude as follows:

$$\left.\begin{array}{ll} K_L = K_R^{8/3}/(K_n^4 K_v^2) & K_Q = K_R^2 K_v \\ K_z = K_R^{4/3}/K_n^2 & K_P = K_R^{4/3} K_n K_v \\ K_F = K_R^2 & K_O = K_n^3 K_v \\ K_f = K_R^2 & \\ K_t = K_R^{4/3}/(K_n^2 K_v) & \end{array}\right\} \quad \cdots\cdots (21)$$

## 4. Dynamical similitude combined with the geometrical similitude of the surge tank.

Combining (13) with (18), we obtain the similitude for this case as follows:

$$\left.\begin{array}{l} K_L K_v/K_t = K_z = (K_n^2 K_L/K_R^{4/3}) K_v^2 \\ K_F K_z/K_t K_f K_v = K_Q = K_P K_z^{1/2} = K_O K_z^{3/2} \\ K_R^2 = K_f \\ K_z = K_F^{1/2} \end{array}\right\} \quad \cdots\cdots (22)$$

where the values of any three coefficients which are to each other independent may be chosen arbitrarily. If we choose $K_n$, $K_R\ K_v$ for the three, the rests, are determined by this similitude as follows:

$$\left.\begin{array}{ll} K_L=K_R^{22/9}/(K_n^{8/3}K_v^2) & K_Q=K_R^2K_v \\ K_z=K_R^{10/9}/K_n^{2/3} & K_P=K_R^{13/9}K_n^{1/3}K_v \\ K_F=K_R^{20/9}/K_n^{4/3} & K_O=K_R^{1/3}K_nK_v \\ K_f=K_R^2 & \\ K_t=K_R^{4/3}/(K_n^2K_v) & \end{array}\right\} \cdots\cdots(23)$$

**5. Dynamical similitude combined with the geometrical similitude both of the junction and of the surge tank with the conduit.**

Combining both (11) and (13) with (18), we have as the similitude for this case

$$\left.\begin{array}{l} K_LK_v/K_t=K_z=(K_n^2K_L/K_R^{4/3})K_v^2 \\ K_FK_z/K_t=K_fK_v=K_Q=K_PK_z^{1/2}=K_OK_z^{3/2} \\ K_R^2=K_f \\ K_f=K_F \qquad K_z=K_F^{1/2} \end{array}\right\} \cdots\cdots(24)$$

where the values of any two coefficients which are independent to each other may be chosen arbitrarily. If we choose $K_n$ and $K_v$ for the two, the rest coefficients are determined by this similitude as follows:

$$\left.\begin{array}{ll} K_R=K_n^6 & K_t=K_n^6/K_v \\ K_L=K_n^{12}/K_v^2 & K_Q=K_n^{12}K_v \\ K_z=K_n^6 & K_P=K_n^9K_v \\ K_F=K_n^{12} & K_O=K_n^3K_v \\ K_f=K_n^{12} & \end{array}\right\} \cdots\cdots(25)$$

As is easily seen, the dimensions of the model installation would be too large for practical purpose, since the value of $K_n$ as usual is not necessarily very small compared with unity.

**6. Special notes.**

a) The value of $n$ of the prototype and that of the model, both for the design of the model pipe-line.

For $n$, as is well known, the smallest possible value is assumed at the study of up-surging and the greatest possible value at the study of down-surging. In order that a model pipe-line may work for the experiments of both up-surging and down-surging the head loss at the model pipe-line should not be greater than that required by the dynamical similitude at the case of the smallest possible value of $n$ of the prototype.

On the contrary, by the introduction of concentrated losses by the use of sluice valves in the model pipe-line it is possible to augment the head loss in the model pipe-line by any required amount. Therefore, the model pipe-line should be designed with regards the smallest possible value of $n$ in the prototype.

At the design of the model pipe-line, however, the total loss of head is not definitely determined, since the loss may increase due to rusting even if the inside of the pipe has been completely galvanized. Accordingly, a little greater amount of head loss than estimated for the model pipe-line should deliberately be assumed, the lack of the head loss being adjusted at every test by the introduction of an appropriate amount of head loss by the use of sluice valves inserted in the model pipe-line.

b) Correction by the exact similitude.

By either one of the approximate expressions of similitude developed thus far, a suitable combination of reduction coefficients may be determined for the installation of the model pipe-line. However, since this is based on the approximate expression (15), it should finally be corrected by the exact similitude (8). In that procedure it is often convenient to take the appropriate values for $K_L$, $K_\varepsilon$ and either $K_v$ or $K_R$, according to the values preliminarily determined, and to solve the values of the rest coefficients by (8).

Since $K_\varepsilon$ is related together with $K_v^2$ in (8) and $K_\varepsilon K_v^2$ means the reduction coefficient of head loss which is equal to $K_z$, the value of $K_\varepsilon$ will easily be realized through the adjustment of the stationary drop of the level of water at the surge tank at the determined rate of discharge through the pipe-line by the use of the sluice valves inserted.

According to the similar reason, the values of $K_T$ and $K_\eta$ may easily be realized at the model installation through the adjustment of the area of opening of the orifice so that the required amount of head difference may exist for a definite rate of discharge through the orifice, and $K_\lambda$ through the adjustment of either the overflow length or the coefficient of overflow so that the required amount of rate of overflow may exist at a definite head of overflow.

**7. Illustrative example: Case of single restricted orifice surge tank.**

Let us take for the example an actual case characterized by the following data:

Length of conduit $L=8,000$ m

Conduit section $d=5,000$ m (Circular)

Conduit area $f=19,635$ m$^2$

Tank diameter $D=20$ m

Net area of tank $F=314.16$ m$^2$

Rate of discharge $Q=60$ m$^3$/s

Orifice diameter $D_p=1,800$ m

Coefficient of discharge of orifice $C_d=0.95$ for flow either into tank or out of tank

Drop of level of water between reservoir and surge tank $h_f=8.600$ m at the smallest
$=15.000$ m at the largest

Conduit roughness $n=0.012$ at the smallest
$=0.016$ at the largest

Total height of surge tank $H=30$ m

According to the dynamical similitude combined with the geometrical similitude of the surge tank let us design the model installation.

First, let us make a preliminary design by the use of (23). Referring to the special notes described previously on the value of $n$ for the installation of the model pipe-line, let us assume as follows:

$$n=0.012 \quad \text{and} \quad n_m=0.0123$$

Then

$$K_n=0.0123/0.012=1.025$$

By (23) and this value of $K_n$, the length of pipe-line, total height of the surge tank, the diameter of the orifice, the maximum rate of discharge and its corresponding Reynolds number in the conduit, all of which are of the model, are shown in **Fig. 2**. The coefficient of discharge of the orifice at the model has been assumed 0.65. In order that the Reynolds number of the flow in the pipe-line of the model may be higher, even at the half load, than critical, the Reynolds number at the model at the full load should be greater, say, than $10^4$. However, too great value of $K_v$ results in too much amount of the rate of discharge in the model test, which is impractical. The greater the diameter of the pipe-line of the model is, the longer the pipe-line is, and the greater the height of the surge tank is. The greater the diameter of the pipe is, the more inconvenient for the pipe-line to be installed if it is to be looped back due to its greater radius of bend to be used. It should be checked furthur, whether the dimension of the port is appropriately smaller than the dimension of the surge tank. In view of these and according to the figure a choice is made for $K_v$ and the diameter of the pipe as follows:

$$K_v=1/10 \quad \text{and} \quad d_m=3 \text{ inches}$$

Then, the length of the pipe-line is read as $L_m=31.0$ m, with which the value of $K_L$ is

$$K_L=31.0/8,000=1/258$$

Now that the dimensions of the pipe-line are preliminarily determined, let us make the regular design of the model installation according to the exact similitude (8). For this design let us assume

$$K_L=1/258.0 \quad \text{and} \quad K_v=1/10$$

Then $\quad L_m=8,000/258.0=31.01$ m

and $\quad \varepsilon_m=\left(\dfrac{n^2L}{R^{4/3}}\right)_m$

$$=\frac{0.0123\times 31.01}{(0.0806/4)^{4/3}}=0.8557$$

On the other hand

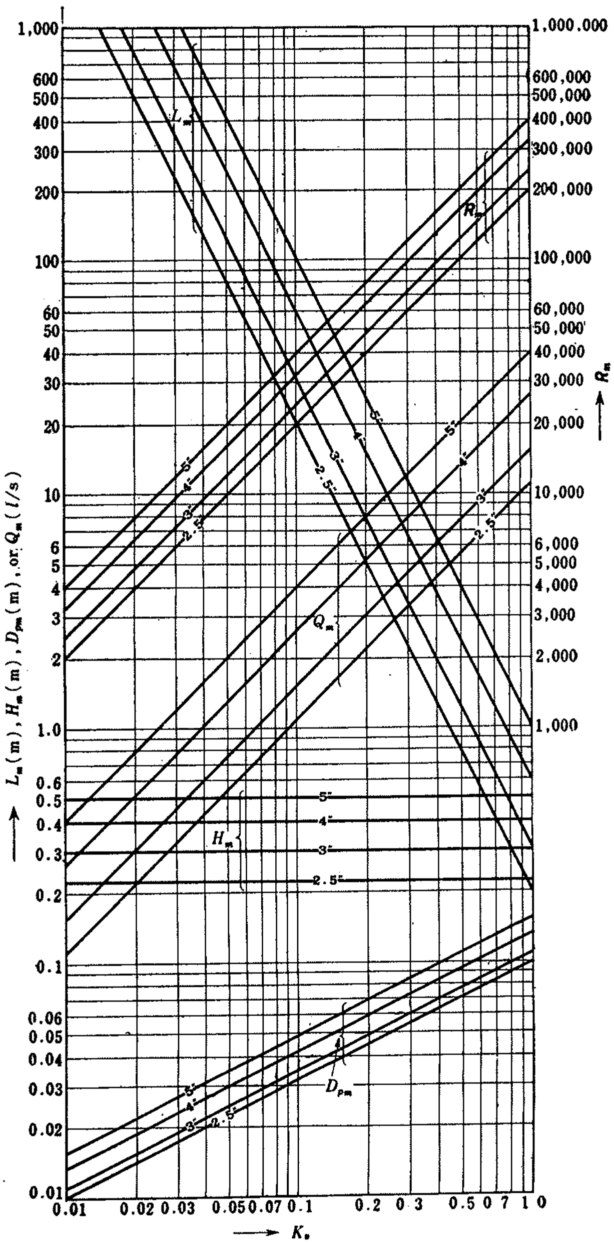
$$\varepsilon=h_f/v^2=8.600/(60/19.635)^2$$
$$=0.9209$$

Hence, $\quad K_\varepsilon=\varepsilon_m/\varepsilon=0.8557/0.9209$
$$=1/1.076$$

With the values of $K_L$, $K_v$ and $K_\varepsilon$ thus chosen, the other reduction coefficients are obtained according to (8) and a few fundamental relations as follows:

$$K_z=1/107.6$$
$$K_t=1/23.98$$
$$K_R=1/2.228$$
$$K_\gamma=1.037$$
$$K_f=(d_m/d)^2=1/3,848$$
$$K_F=K_RK_f=1/8,573$$
$$(\sqrt{K_F}=1/92.59)$$
$$K_P=K_\gamma K_f=1/3,711$$

**Fig. 2** Model dimensions.



## MODEL INSTALLATION

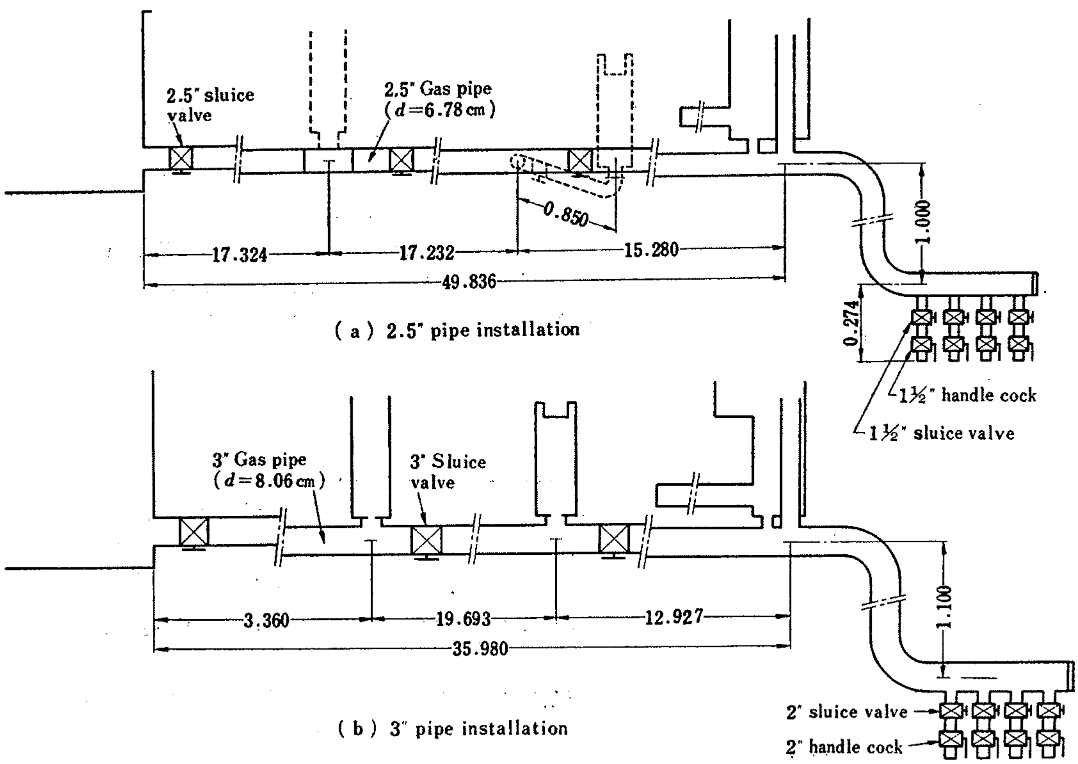
### 1. Pipe-line.

In the model installation at Chuo University two alternative pipe-lines have been provided. (**Fig. 3**). Their dimensions are, respectively, nominally 2.5 inches in diameter (actually d=6.78 cm) 49.836m long and nominally 3 inches in diameter (actually $d$=8.06 cm) 35.980m long, each length being that from the reservoir to the center line of the riser of the main surge tank to be provided. Both pipe-lines consist of galvanized steel pipes. Due to the long length each pipe-line is installed in a looped way. Provision is made for fitting either a surge tank or a branch pipe at the top of which a surge tank is provided, at two points of each pipe-line. A circular sluice gate valve is inserted at each reach between two neiboring such fittings, by the use of which valve the loss of head at each reach is adjusted to the amount required by the dynamical similitude. Air vents at the base of each a sluice gate valve is provided are welded vertically upwards at a few points to the pipe-line to evacuate the air which may remain in the pipe at the beginning of the experiment.

### 2. Outlet valves.

At te end of the main pipe-line four sluice valves each being accompanied by a handle cock are provided. The opening of the sluice valves are adjusted in advance of each test to furnish the required rate of discharge, the change of the rate of discharge being done

**Fig. 3** Experimental installations (lengths in m).



practically instataneously by means of the handle cocks.

**3. Reservoir.**

The reservoir consists of a cylindrical tank of steel 0.90 m deep and 1.00 m in diameter. It has, at its center, a round overflow of adjustable elevation. To the tank another similar pipe-line is connected, which is operated as a pipe-line compensating the change of discharge overflowing the spillway, to keep the level of water in the reservoir always constant.

**4. Port.**

At the model installation the port of the surge tank is provided usually with a metal plate at which a hole of appropriate diameter is drilled unless the drop of head through the port is the main problem of the investigation at the model test. The diameter of the hole is adjusted trially in such a way that the drop of head of flow either into or out of the tank may be equal to that required by the dynamical similitude. The calibration of the port is done for the flow through the port at steady state with piezometric tubes.

**5. Surge tank.**

A number of surge tanks are available with various shapes, all of which are made of transparent plastics.

**6. Recording of surging.**

At the case of multiple surge tanks the changes of water level in surge tanks are recorded simultaneously by an electro-magnetic oscillograph by the use of electrical wave height gage. At the case of the single surge tank the change of the level of water is recorded either with the use of the electrical wave height gage or photographically against a graduated scale with a motion picture camera of a calibrated low speed.

## EXPERIMENTAL RESULTS

The comparison of model results with prototype results is selected as the main aspect to study in detail. As was shown by Professor Gibson the prototype results for comparison with the model results are worked out by arithmetical integration.

**Case 1. Case of a single differential surge tank. (Two steps of load increase)**

Conduit $L=7,973.800$ m

Conduit $d=5.000$ m (Circular)

Conduit area $f=19.635\ \text{m}^2$

Maximum rate of discharge $Q_{max}=60\ \text{m}^3/\text{s}$

Stationary max. conduit vel. $v_{max}=3.056$ m/s

Drop of head $h_f=15.462$ m for $Q=60\ \text{m}^3/\text{s}$

Conduit $\varepsilon=1.6556$

Tank diameter $D_t=17.340$ m

Riser diameter $D_r=4.500$ m

Riser area $F_r=15.904\ \text{m}^2$

Net area of tank $F_t=220.250\ \text{m}^2$

Port diameter $D_p=1.685$ m

Port area $F_p=2.209\ \text{m}^2$

Coefficient of discharge of port $C_d=0.97$ for flow out of tank

As the model installation the 2.5 inch pipe-line is used. The model test is done according to the dynamical similitude combined with the geometrical similitude of the junction of the surge tank with the conduit, i.e. (12). After a preliminary design as shown previously in 'Illustrative example', as the independent coefficients $K_L$ and $K_\varepsilon$ are chosen, the values of which are fixed as follows:

$$K_L=1/160 \quad \text{and} \quad K_\varepsilon=1.150$$

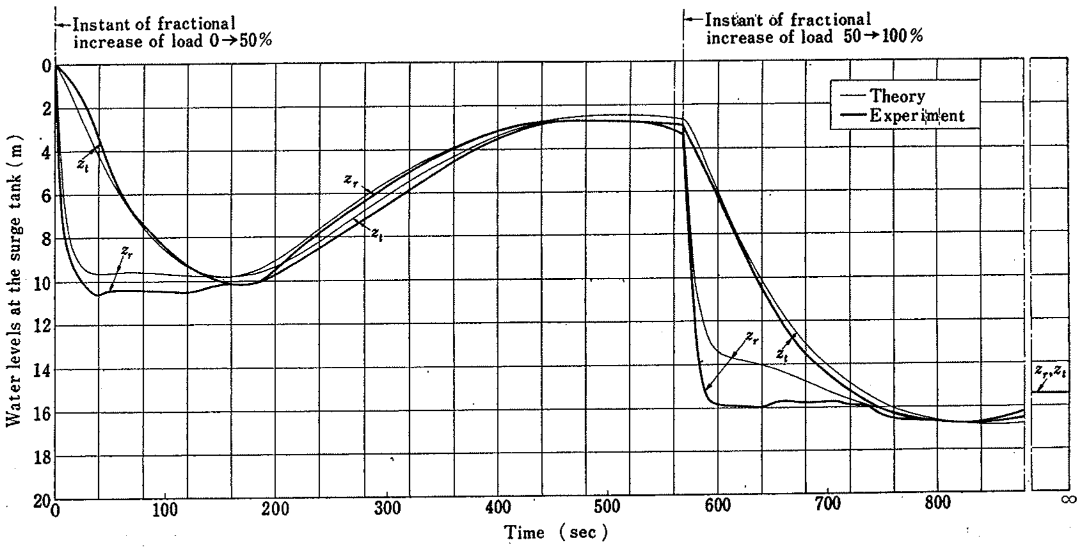
Then, the rest coefficients are determined by (12) as follows:

$K_z = 1/184.1 \quad K_t = 1/12.65 \quad K_v = 1/14.57 \quad K_Q = 1/79,300 \quad K_\gamma = 1/1.071$

The increase of load is made in two steps, from 0% to 50% load and from 50% to 100% load, each increase being assumed to be done practically instataneously.

The model results converted to the prototype dimensions by multiplication by the reciprocals of the reduction coefficients together with the results deduced for the prototype by arithmetical integration are plotted in **Fig. 4**. The results show a close agreement for the

**Fig. 4** Comparison between the theory and the experiment for Case 1.




period of oscillation and for the lowest surge of the tank. However, a certain amount of difference exists for the instataneous descent of the riser at the second step of load increase. Some such difference as this is to be expected, since in the method of arithmetical integration no account is taken of the variation of the value of the coefficient of discharge with respect to the rate of discharge through the port.

An odd value of diameter is chosen at the model test in order to make the value of the instataneous lowest surge of the riser equal to that of the lowest descent of the main tank.

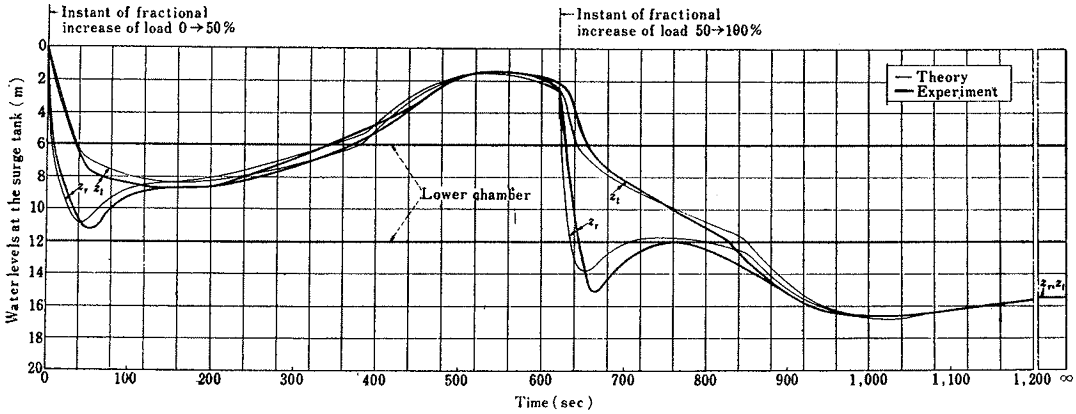
**Case 2. Case of a single differential surge tank with a lower chamber. (Two steps of load increase)**

Conduit $L = 7,973.800$ m
Conduit area $f = 19.635$ m²
Maximum rate of discharge $Q_{max} = 60$ m³/s
Drop of head $h_f = 15.462$ m
(Same as at Case 1.)
Riser diameter $D_r = 4.500$ m
Riser area $F_r = 15.904$ m²
Tank diameter $D_t = 14.000$ m
Net area of tank $F_t = 138.034$ m²

Port diameter $D_p = 1.725$ m
Port area $F_p = 2.315$ m²
Coefficient of discharge of port $C_d = 0.97$ for flow out of tank
Diameter of lower chamber $D_c = 6$ m
Length of lower chamber $L_c = 90$ m
Location of center of gravity of lower chamber = 9 m below L.W.L.

Model tests are done with the same pipe-line and the same reduction coefficients of quantities as in Case 1. The results are shown in **Fig. 5**.

**Fig. 5** Comparison between the theory and the experiment for Case 2.




The agreement is very close as regards the period of oscillation. The agreement for the lowest descent of the main tank at the increase to the full load is also very close, but that at the increase to a half load is less close. That may be attributed to the deviation of law of head loss, due to the relatively small Reynolds number in the model pipeline, from the law of square power of velocity, although in the prototype the square power law is to hold. The agreement with regards the surging wave as a whole is not necessarily very close. But some difference as this is to be expected, since in the method of arithmetical integration no account is taken of the lag of movement of the water in the long lower chamber. Taking this into account the agreement would be also very close.

**Case 3. Case of multiple surge tanks. (Fractional increase from 50% to full load)**

With reference to the arrangement of surge tanks shown in Fig. 3 (a), the fundamental quantities are as follows*:

First conduit $L_1=2{,}850.3\,\text{m}$ $f_1=19.750\,\text{m}^2$
Second conduit $L_2=2{,}772.3\,\text{m}$ $f_2=19.750\,\text{m}^2$
Third conduit $L_3=2{,}455.7\,\text{m}$ $f_3=19.635\,\text{m}^2$ ($d=5$ m circular)
Branch conduit $L_b=136.00\,\text{m}$ $f_b=7.322\,\text{m}^2$
First auxiliary surge tank $D_1=2.800\,\text{m}$ $F_1=6.158\,\text{m}^2$
Second auxiliary surge tank $D_2=4.000\,\text{m}$ $F_2=12.566\,\text{m}^2$
Riser $D_r=4.500\,\text{m}$ $F_r=15.904\,\text{m}^2$
Main tank diameter $D_t=14.000\,\text{m}$
Net area of tank $F_t=138.034\,\text{m}^2$
Port $D_p=1.828\,\text{m}$ $F_p=2.623\,\text{m}^2$
Coefficient of discharge of port $C_d=0.97$ for flow out of tank
Maximum rate of discharge $Q_{max}=60\,\text{m}^3/\text{s}$
First conduit head drop $h_{f_1}=6.148\,\text{m}$ for $Q=60\,\text{m}^3/\text{s}$
Second conduit head drop $h_{f_2}=4.639\,\text{m}$ for $Q=60\,\text{m}^3/\text{s}$
Third conduit head drop $h_{f_3}=4.427\,\text{m}$ for $Q=60\,\text{m}^3/\text{s}$
Total head drop $h_f=15.214\,\text{m}$ for $Q=60\,\text{m}^2/\text{s}$
Branch conduit head drop $h_{f_b}=0.029\,\text{m}$ for $Q=5\,\text{m}^3/\text{s}$
First conduit $\varepsilon_1=0.6661$

* These correspond to the supply conduit of Okuizumi Power Plant [Ref. 6].

Second conduit $\varepsilon_2=0.5026$
Third conduit $\varepsilon_3=0.4741$
Branch conduit $\varepsilon_b=0.062$

Lower chamber $D_c=5,800$ m, $L_c=70$ m
Location of center of gravity of lower chamber = 10 m below L.W.L.

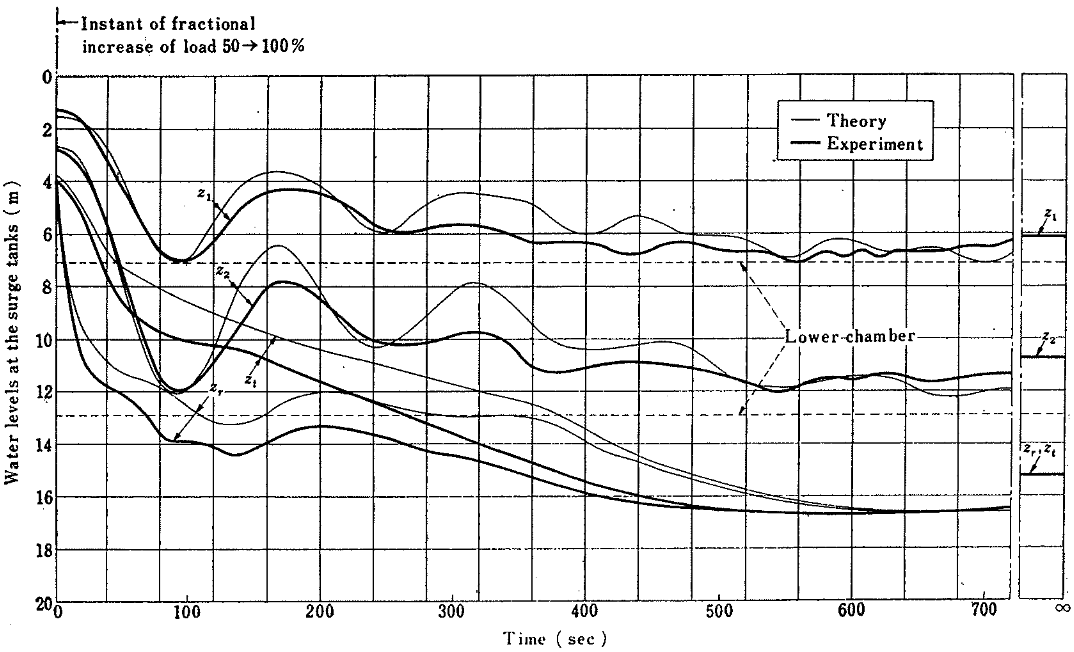
In order to avoid the complication at the model installation that $f_1$ and $f_2$ are not exactly equal to $f_3$, both $f_1$ and $f_2$ are approximated with $f_3$; while, in place of $L_1$ and $L_2$, the equivalent lengths $L_1'$ and $L_2'$, respectively, as follows are taken into consideration:

$$L_1'=L_1\cdot(f_3/f_1)=2,833.7\text{ m} \qquad L_2'=L_2\cdot(f_2/f_1)=2,756.7\text{ m}$$

Model tests are made with the same 2.5 inch pipe-line and the same reduction coefficients of quantities as in Case 1 and Case 2. (see **Fig. 3 (a)**). The length of each reach of the model pipe-line shown in Fig. 3 (a) was not exactly that required by the value of $K_L$, but the error of each length of $L_{1m}$, $L_{2m}$ or $L_{3m}$ is 2.2%, 0.0% or 0.4%, respectively, which is considered not to matter with the result.

The results are shown in **Fig. 6**, which shows the agreement at this case, as a whole, is not necessarily so close. One of the most remarkable reasons for such difference is the

**Fig. 6** Comparison between the theory and the experiment for Case 3.




lag of movement of water in the lower chamber, by which the period of the lowest surge of the main tank is considerably accelerated. However, as far as each lowest descent of the auxiliary surge tanks as well as that of the main tank is concerned, the agreement is still very satisfactory. If the lag of movement of water in the lower chamber were taken into consideration in the surging equations, the agreement would be much closer.

**Case 4. Case of multiple surge tanks. (Fractional increase of load from 50% to full load)**

With reference to Fig. 1, the fundamental quantities are given as follows*:

First conduit $L_1=539.737$ m
Third conduit $L_3=2,085.646$ m
Second conduit $L_2=3,189.933$ m
Total $L=5,815.316$ m

* These and those of Case 5 correspond to the supply conduit of Nishiyama Power Plant.

Conduit area $f=7.069\ \mathrm{m}^2$

First auxiliary surge tank

Diameter $D_1=3.000\ \mathrm{m}$

Port diameter $D_{p_1}=0.600\ \mathrm{m}$

Head drop through port

$\Delta H=0.955\ \mathrm{m}$ for $Q=1\ \mathrm{m^3/s}$

either out of, or into, tank

Height of crest of spillway

$a_1=-\infty$ (e.g., Non-overflow type)

Second auxiliary surge tank

Diameter $D_2=5.000\ \mathrm{m}$

Port diameter $D_{p_2}=0.700\ \mathrm{m}$

Head drop through port

$\Delta H=0.516\ \mathrm{m}$ for $Q=1\ \mathrm{m^3/s}$

either out of, or into, tank

Height of crest of spillway

$a_2=-0.500\ \mathrm{m}$ (e.g. 0.500 m above H.W.L.)

Overflow length $B_2=11.76\ \mathrm{m}$

Riser

$D_r=2.500\ \mathrm{m}$ $F_r=4.909\ \mathrm{m}^2$

$a_r=-5.000\ \mathrm{m}$ (e.g. 5.000 m above H.W.L.)

$B_r=7.20\ \mathrm{m}$

Main tank

$D_t=10.000\ \mathrm{m}$ (above H.W.L.)

$=5.600\ \mathrm{m}$ (below H.W.L.)

Net area

$F_t=73.632\ \mathrm{m}^2$ (above H.W.L.)

$=19.722\ \mathrm{m}^2$ (below H.W.L.)

Lower chamber

$L_c=32.000\ \mathrm{m}$ Section : cf. Fig. 7

Elev. of center of radius $=10.300\ \mathrm{m}$ (below L.W.L.)

Port $D_p=1.050\ \mathrm{m}$ $F_p=0.8659\ \mathrm{m}^2$

The model test is done according to the dynamical similitude added with the geometrical similitude of the surge tank. After a preliminary design as shown previously in 'Illustrative example' $K_L$, $K_v$ and $K_\varepsilon$ are selected as the independent coefficients, the values of which are fixed as follows :

$K_L=1/162.0,\ K_v=1/6,\ K_\varepsilon=1/1.480$

Head drop through port

$\Delta H=8.925\ \mathrm{m}$ for $Q=7.5\ \mathrm{m^3/s}$ out of tank

$\Delta H=37.763\ \mathrm{m}$ for $Q=15\ \mathrm{m^3/s}$ into tank

Maximum rate of discharge $Q_{max}=15\ \mathrm{m^3/s}$

Stationary max. conduit vel. $v_{max}=2.122\ \mathrm{m/s}$

First conduit $h_{f_1}=1.212\ \mathrm{m}$ for $Q=15\ \mathrm{m^3/s}$

Second conduit $h_{f_2}=5.408\ \mathrm{m}$ for $Q=15\ \mathrm{m^3/s}$

Third conduit $h_{f_3}=3.531\ \mathrm{m}$ for $Q=15\ \mathrm{m^3/s}$

Total $h_f=10.151\ \mathrm{m}$ for $Q=15\ \mathrm{m^3/s}$

First conduit $\varepsilon_1=0.2692$

Second conduit $\varepsilon_2=1.2010$

Third conduit $\varepsilon_3=0.7841$

**Fig. 7** Cross section of lower chamber (dimensions in *m*).

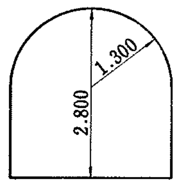
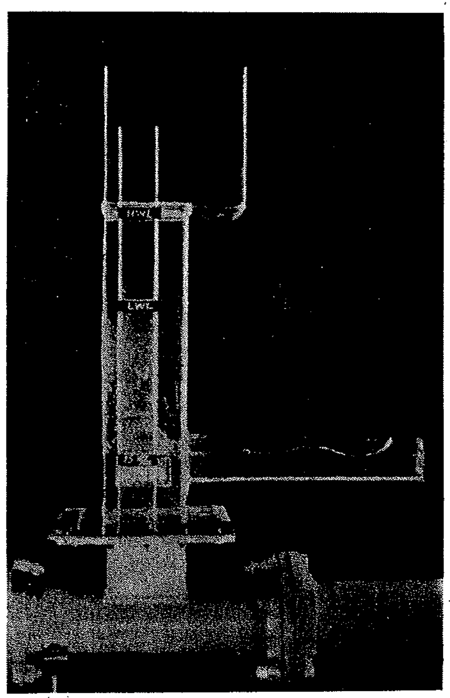



**Photo. 1** Main surge chamber after increase of load.



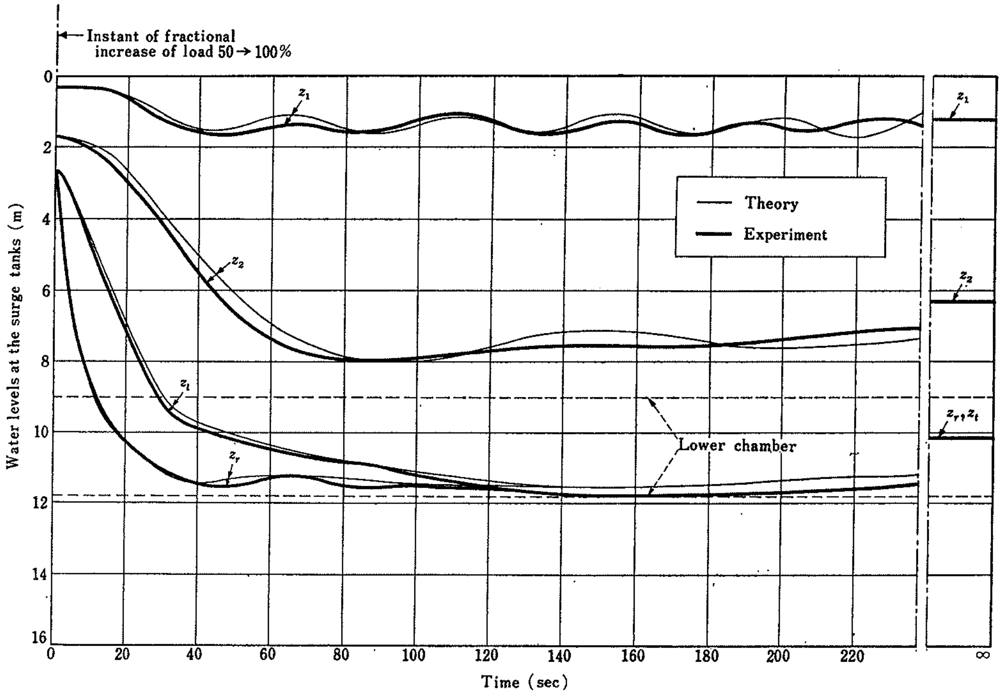
Then, the rest coefficients are determined by (8) as follows :

$K_z=1/53.28,\ K_R=1/2.054,\ K_t=1/18.24,\ K_Q=1/8{,}312,\ K_\Upsilon=1.217$

$K_\lambda=64.84,\ K_\eta=1/1.480$

The model installation thus consists of the 3 inch pipe-line shown in **Fig. 3**. The results are plotted in **Fig. 8**, which shows a satisfactory agreement. It is generally noticed that agreement is improved with 3 inch pipe-line than with 2.5 inch pipe-line. **Photo. 1** shows the main tank at the fractional increase of load from 50% to full.

**Fig. 8** Comparison between the theory and the experiment for Case 4. (The curves by arithmetical integration are by the courtesy of Nippon Kohei Company Ltd.)




**Case 5. Case of multiple surge tanks. (Instataneous total rejection of load)**

The fundamental quantities are the same as at Case 4, except the amount of head loss to be considered. At Case 4, for the amount of head loss we took the largest possible values, since it is for increase of load. On the contrary, at this case we take for the amount of head loss the smallest possible values as follows, since it is for rejection of load:

First conduit $h_{f_1}=0.901$ m for $Q=15$ m$^3$/s, $\varepsilon_1=0.2061$

Second conduit $h_{f_2}=3.571$ m for $Q=15$ m$^3$/s, $\varepsilon_2=0.7930$

Third conduit $h_{f_3}=2.333$ m for $Q=15$ m$^3$/s, $\varepsilon_3=0.5181$

Total $h_f=6.805$ m for $Q=15$ m$^3$/s

The results are shown in **Fig. 9**, which also shows a very close agreement. **Photo. 2** and **Photo. 3** show the main tank and the second auxiliary surge tank, respectively, after the total rejection of load.

## CONCLUSIONS

This investigation leads to the following conclusions:

1. The dynamical similitude of surge tanks at the supply conduit is expressed most generally as (8).

2. The dynamical similitude of surge tanks at the supply conduit for preliminary design purpose of the model installation is expressed by either (18), (20) or (22). (18) is the plain dynamical similitude, (20) is the dynamical similitude combined with the geometrical similitude of the surge tank with the conduit, and (22) is the dynamical similitude combined with the geometrical similitude of the surge tank.

**Photo. 2** Main surge chamber after rejection of load.

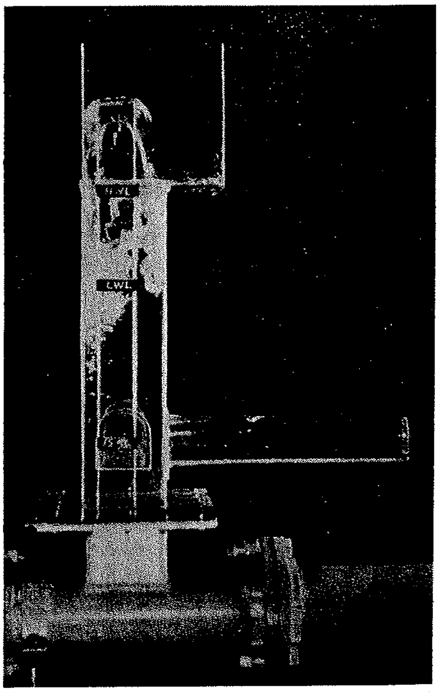


**Photo. 3** Second auxiliary surge tank after rejection of load.

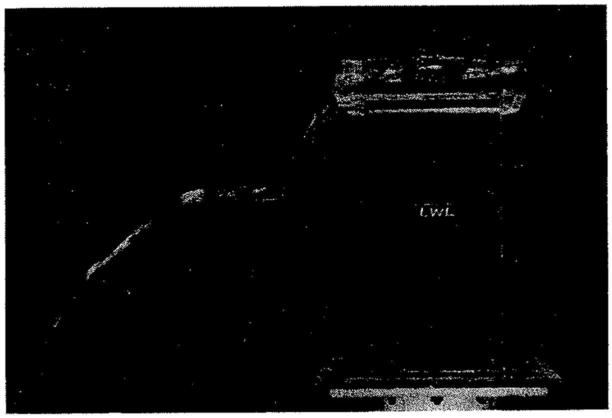
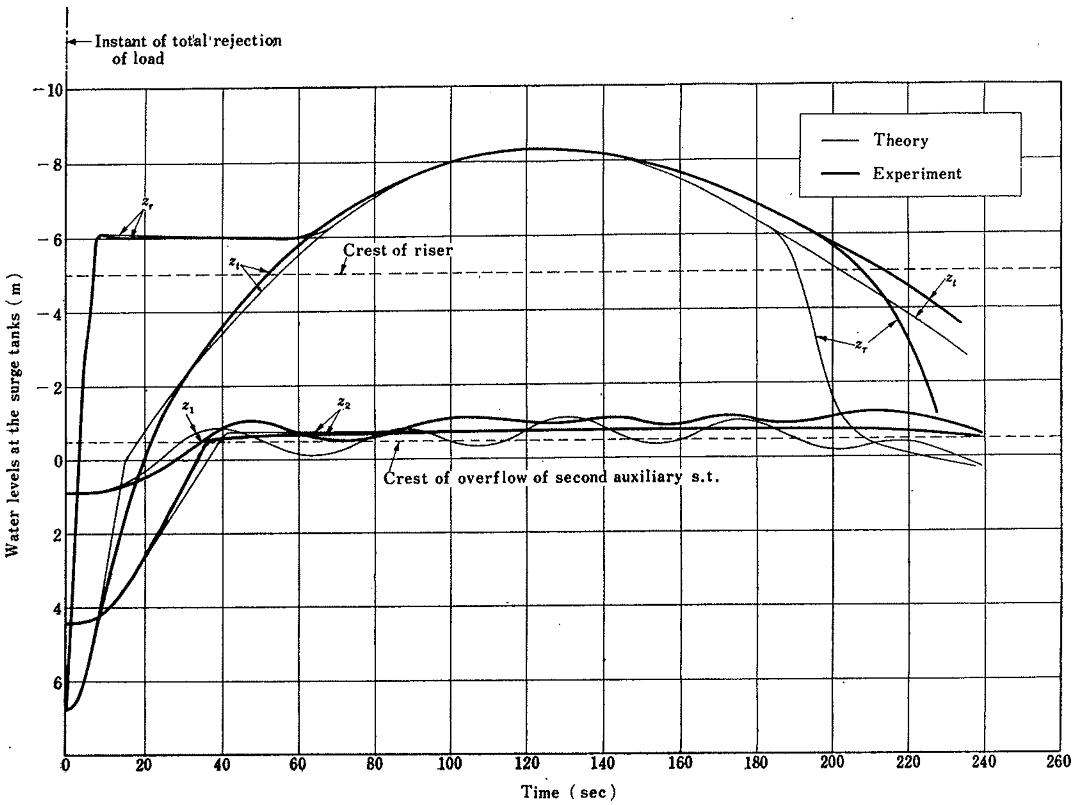


**Fig. 9** Comparison between the theory and the experiment for Case 5. (The curves by arithmetical integration are by the courtesy of Nippon Kohei Company, Ltd.)



3. The results of the model tests conducted with the dynamical similitude for several cases of the complicated type are in every case in a satisfactory agreement with the corresponding prototype results. By the agreement we affirm that the dynamical similitude is always quite reliable for the model test of any complicated case at will.

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## NOTATIONS

$L$ : Length of the conduit

$L_1, L_2, L_3, L_b$: each $L$ of the first reach, the second reach, the third reach and the branch conduit, respectively

$v$ : velocity of flow

$v_1, v_2, v_3$: each $v$ at the first reach, the second reach and the third reach, respectively

$t$ : time

$h_f$: drop of head

$\varepsilon$ : coefficient such that $h_f = \varepsilon v^2$

$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_b$: each $\varepsilon$ of each reach

$z$ : water level at the surge tank measured vertically downwards from the water level of the reservoir

$z_1, z_2, z_r, z_t$: each $z$ of the first auxiliary surge tank, the second auxiliary surge tank, the riser, and the main tank

$f$ : section of the conduit

$f_1, f_2, f_3, f_b$: each $f$ of the each reach of the conduit

$F$ : section of the surge tank

$F_1, F_2, F_r, F_t, F_p$: each $F$ of the first auxiliary surge tank, the second auxiliary surge tank, the riser, main tank, and the port

$\phi$ : ratio of the section of the surge tank to that of the conduit, e.g., $\phi = F/f$

$\phi_1, \phi_2, \phi_r, \phi_t$: each $\phi$ at each surge tank

$\varDelta H$: drop of head through the orifice

$\eta$ : coefficient such that $\varDelta H = \eta \cdot (Q'/f)^2$ $Q'$ = velocity of flow through the orifice

$\eta_1, \eta_2$: each $\eta$ at the auxiliary surge tanks

$Q$ : rate of discharge

$Q_s$: rate of overflow

$Q_{s1}, Q_{s2}, Q_{sr}$: each $Q_s$ of the first auxiliary surge tank, the second auxiliary surge tank, and the riser

$Q_t$: rate of discharge through the port into the main tank of the differential surge tank

$C$ : rate of discharge converted into the dimension of velocity in the conduit, e.g. $C = Q/f$

$C_s$: rate of overflow converted to the dimension of velocity in the conduit, e.g. $C_s = Q_s/f$

$C_{s1}, C_{s2}, C_{sr}$: each $C_s$ of the auxiliary surge tanks and the riser

$C_t$: rate of discharge through the port into the main tank converted into the dimension of velocity in the conduit, e.g. $C_t = Q_t/f$

$B$ : length of overflow

$B_1, B_2, B_r$: each $B$ of the two auxiliary surge tanks and the riser

$k$ : coefficient of overflow

$\lambda = kB/f$

$\lambda_1, \lambda_2, \lambda_r$: each $\lambda$ of the auxiliary surge tanks and the riser

$a$ : vertical distance from the water level of the reservoir to the crest of overflow (positive direction vertically downwards)

$a_1, a_2, a_r$: each $a$ of the auxiliary surge tanks and the riser

$C_d$: coefficient of the rate of discharge

$\gamma = C_d F_p \sqrt{2g}/f$

suffix $m$: suffix which refers to the quantity of the model

$K$ : reduction coefficient

$K_L, K_v, K_t, \cdots\cdots$ : reduction coefficient of $L$, that of v, that of $t, \cdots\cdots$

$D$ : diameter

$D_1, D_2, D_r, D_t, D_p$: each $D$ of the two auxiliary surge tanks, the riser, the main tank, and the port.

$R$ : hydraulic mean depth

$n$ : Kutter's roughness

$P = C_d F_p \sqrt{2g}$

$O = kB$

$\boldsymbol{R}$: Reynolds number

$H$: total height of the surge tank

## 要　　旨

サージタンクの従来の力学的相似律を拡張して，最も一般的な場合における力学的相似律の形を導き，さらにこの力学的相似律に付加されるべきいろいろな条件についての研究を行なつた。また，この相似律を変形して模型実験計画上都合のよい近似的諸式をいろいろな場合について導き，それらによる模型実験計画例を示した。

つぎに，これらの相似律によつて，複雑な型式のサージタンク，あるいはサージタンク列の模型実験を行ない，この模型実験によつて推定される実物についてのサージング曲線と，実物について数値積分により直接に求められた理論的サージング曲線とを比較することにより，これら相似律の信頼性を検討した。その結果によれば，両者の一致の度合はいずれの場合においても，満足しうる程度，あるいは極めて良好，であつて，このことから，これらサージタンクの相似律は，いかなる複雑な形式のサージタンク，あるいはサージタンク列の模型実験に対しても，なお十分な信頼度を保ちうるものであることを確めた。

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