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BACK ANALYSIS OF TENSION-SOFTENING RELATIONSHIP OF CONCRETE

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It is known that bridging is a dominant mechanism in the fracture of concrete. The material property related to bridging is the tension-softening relationship which is the relationship between the transmitted tensile stress across the crack surface and the crack opening displacement. In this paper, a method to indirectly obtain the tension-softening curve from the bending test of a notched beam is proposed. The method is applied to reported results, and agreement is observed between the tension-softening curves by the proposed method and the modified J-integral-based technique. The method is also applied to bending tests of SFRC beams, and results are compared with those of direct tension tests.

Key Words: *back analysis, tension-softening relationship, bending test, bridging, fracture process zone*

1. INTRODUCTION

It is commonly accepted that the Linear Elastic Fracture Mechanics (LEFM) approach is not directly applicable to quasi-brittle materials such as concrete, rock, and ceramics because of a large fracture process zone ahead of the crack tip. The fracture process zone consists of a microcracking zone and a bridging zone (see **Fig.1**). The microcracking zone is a zone where the initiation of microcracks and their growth are dominant. Microcracks in these materials may be initiated by various mechanisms depending on the materials. In concrete and rock, microcracking is initiated by the concentrated stress near a macrocrack. Microcracking is understood as an ensemble of microevents such as the extension of existing defects and pores, and the debonding at the interfaces between aggregates and the cement matrix rather than the actual cracking. The bridging zone is a part of a macrocrack along which the stress is transmitted by partial matrix, aggregates or fibers. Under mode I loading, the bridging behavior is represented by the tension-softening relationship which is the relationship between the transmitted tensile stress and the crack opening displacement. In different materials, the roles of bridging and microcracking may be different, i.e., either bridging or microcracking may be the dominant mechanism or both may share comparable roles.

Until now, many models have been proposed to model the fracture process zone. In the field of fracture mechanics, the model to model the nonlinear zone ahead of the crack tip in metallic materials was proposed by Dugdale[1)]. In his model, the nonlinear zone is modeled as an extension of the actual crack and perfectly plastic behavior is assumed along this crack extension. In a slightly different way, Barenblatt[2)] considered molecular forces of cohesion acting on the edge region of the crack. Barenblatt limited the analysis to cases where the size of the edge region with cohesion is small compared to the size of the whole crack. Hillerborg et al.[3)] proposed the Fictitious Crack Model (FCM) for predicting crack growth behavior of concrete. In the model, the fictitious crack ahead of the actual crack is assumed, and the transmitted stress along this fictitious crack is considered (see **Fig.2**). When the fictitious crack opens, the stress is assumed to decrease with increasing crack opening displacement. The transmitted stress is determined from a tension-softening curve corresponding to the post-peak stress-displacement relationship of the uniaxial tension test. The validity of this model should be examined by comparing with the experimental observation since, in the model, not only

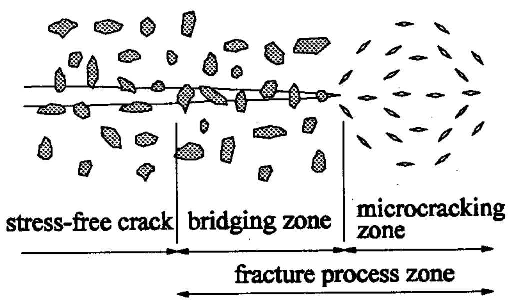

Fig.1 Schematic illustration of fracture process zone.

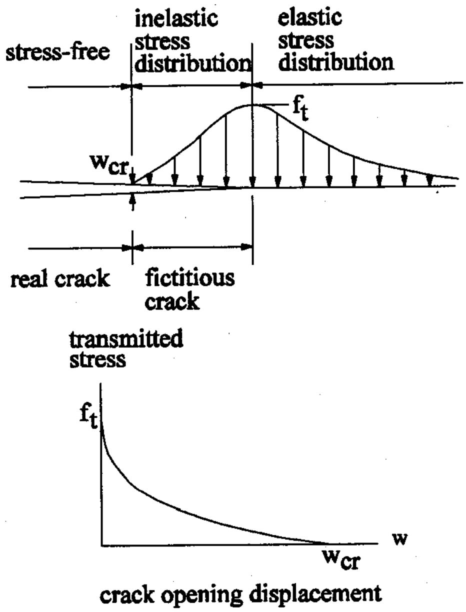

Fig.2 Fictitious crack model with a tension-softening relationship.

the macrocrack with stress transmission but also the two-dimensionally extended microcracking zone is modeled by the extended part of the crack.

Unlike the fictitious crack model which considers the whole nonlinear zone as only an extension of the actual crack with transmitted stress, the model proposed by Nirmalendran and Horii[4] considers the effects from microcracking and bridging separately. The two mechanisms are separately modeled, the Dugdale-Barenblatt type model being used to model bridging alone. The interaction effect between both mechanisms is considered through the interaction between the macrocrack and microcracks. In the model, bridging obeys a tension-softening curve while microcracking obeys a microcracking law. The model is applied to concrete, and the result shows that the effect of microcracking on the toughness of concrete is small, and the toughness induced by bridging is dominant. Therefore, bridging can be considered as the governing mechanism of crack growth in concrete.

From above literature review, we know that bridging is a dominant mechanism in the fracture of concrete. As mentioned before, under mode I, the material property related to bridging is represented by the tension-softening curve. The tension-softening curve can be obtained from the post-peak stress-displacement relationship of the direct tension test. However, the direct tension test is difficult to perform because the behavior after the peak is unstable unless a very stiff machine with a feedback system is used. Nevertheless, in recent years, complete post-peak stress-displacement relationships from direct tension tests using stiff, closed loop feedback-controlled loading machines have been successfully reported by Petersson[5], Gopalaratnam and Shah[6] and Reinhardt et al.[7]. Unfortunately, this kind of machine is not available in most laboratories and, for this reason, the tension-softening curve is usually assumed as linear or bilinear for simplicity[5),8),9)].

To avoid the difficulties in the direct tension test, new testing methods or indirect testing methods have been proposed. Rokugo et al.[10] investigated a new tube tension test to determine the tension-softening relationship. This test needs only a conventional compression testing machine. The geometry of the specimen is designed in such a way that by applying compression on specified boundaries, the tensile stress field is developed in a certain zone. Rokugo et al. observed, by the acoustic emission source location technique, that the distribution of damages during the test was not uniform even along the final crack line. This may be because the stress field in the specimen is not uniform even if the material is homogeneous. Therefore, the obtained tension-softening curve is questionable. Li and Ward[11] proposed a technique based on the J-integral to experimentally obtain the tension-softening curve. In the experiment, two pre-notched specimens with slightly different notch lengths are used. The J-integral is then calculated from the area between the load-load point displacement curves from the two specimens. The test methods employed in the paper were the compact tension test and the four-point bending test. Because two load-load point displacement curves from two specimens are used in the method, any error in the measured curves results in a magnified error in the difference between the two curves. Rokugo et al.[12] proposed a modified J-integral-based technique to

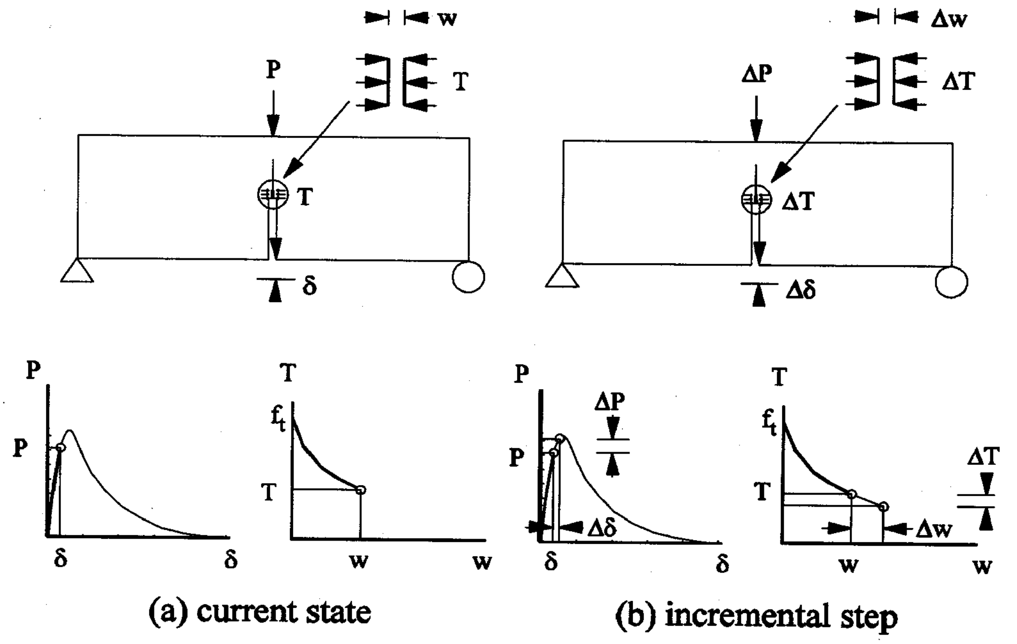

Fig.3 Back analysis method.

obtain the tension-softening curve from one notched specimen.

In the present study, a back analysis method to obtain the tension-softening curve is proposed; the basic idea was presented in Horii and Nanakorn[13]. In this method, the tension-softening curve is computed from a given load-deflection curve of a bending test. Because only a load-deflection curve from a single bending test is used in the calculation, the magnified error from employing two experimental curves in one calculation does not exist. FEM with a cracked element[14] which is an element with embedded displacement discontinuity is employed in the analysis. The analysis is performed incrementally without iteration, and the piecewise-linear tension-softening curve is obtained. The tension-softening curve obtained by the proposed method reproduces the original load-deflection curve if it is used in the forward analysis.

A similar back analysis method to obtain the tension-softening curve from a bending test was proposed independently by Kitsutaka et al.[15]. Though the basic ideas of the two methods are almost the same, the details are different, and will be discussed afterwards. The authors of this paper were not aware of the existence of the work by Kitsutaka et al.[15] during the development of this back analysis method[13].

In this study, the proposed method is applied to the steel-fiber-reinforced concrete. Although approximated or simplified tension-softening relationship may be adequate in many cases, the complete tension-softening relationship is necessary when in-depth studies on the fracture behavior of concrete are concerned. Tasks such as establishment of a new design provision for steel-fiber-reinforced concrete tunnel linings and development of new cementitious materials require this kind of studies. In fact, the proposed back analysis method, as one of necessary tools, is used in a study to establish a design provision for steel-fiber-reinforced concrete tunnel linings by Nanakorn and Horii[16].

2. BACK ANALYSIS METHOD

To obtain the tension-softening relationship by using a load-deflection curve from a bending test, we consider a three-point bending problem with a notched beam as shown in **Fig.3**. In fact, it is possible to use other types of testing as long as there is only a single propagating crack and the starting point of the crack propagation is known. The other important requirement is that the crack opening displacement at the starting point of the crack propagation must always be maximum, compared with the other part of the propagating crack. In the three-point bending test with an initial notch, it is reasonable to assume that a crack, propagates from the tip of the notch, is straight and is at the middle of the beam. After the crack propagates from the tip of the notch, stress is transmitted across the crack surface. The

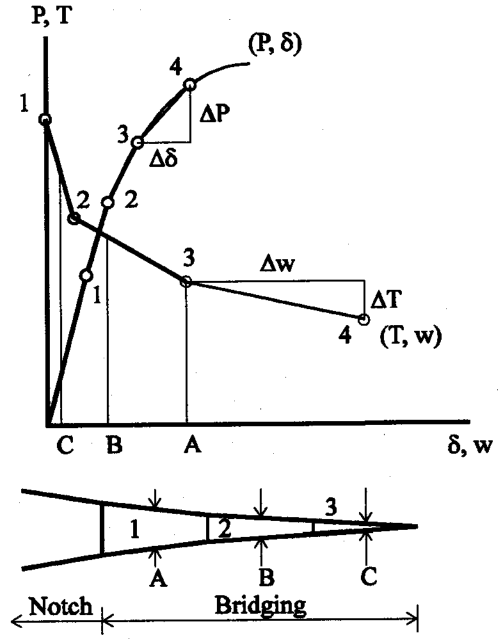

Fig.4 Example of an incremental step.

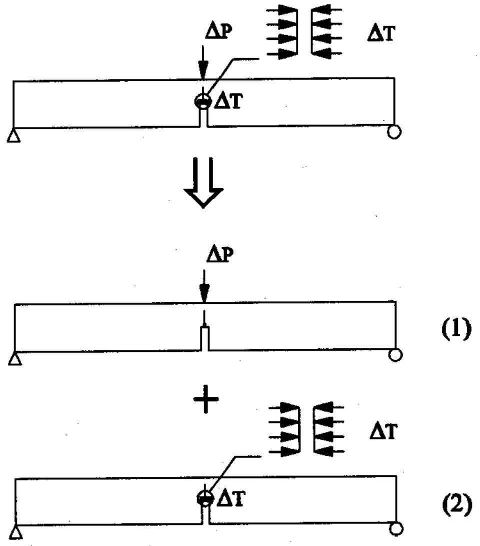

Fig.5 Decomposition of the original problem.

crack just ahead of the tip of the notch will have the maximum crack opening displacement, compared with other part of the propagating crack. If the relationship between the transmitted tensile stress and the crack opening displacement at the crack just ahead of the notch tip is determined, the obtained relationship, which is the tension-softening relationship of the material, can be used for the crack surface farther away from the notch tip because the crack opening displacement is smaller than the one just ahead of the notch tip.

Based on this fact, the relationship between the transmitted tensile stress and the crack opening displacement at the crack just ahead of the notch tip will be determined from a given load-deflection curve from a bending test. The analysis will be done incrementally, and the tension-softening curve will be obtained as a piecewise-linear function.

One of the material properties to be used in the analysis is the tensile strength of the material, denoted in this study by f_t. This f_t will be used as a starting point of the tension-softening curve as shown in **Fig.3**. This means that the tensile strength f_t is used as a cracking criterion; if the tensile stress reaches f_t, the material is cracked, and the crack propagates.

First, we consider a state of loading called a current state as shown in **Fig.3**(a). Imagine that we have already obtained the tension-softening curve until this state by the back analysis. At this current state, the applied load is P and the deflection of the beam is δ. Here, T and w denote the tensile stress and the crack opening displacement at the crack just ahead of the notch tip, respectively. As noted before, w is the maximum crack opening displacement along the crack. Therefore, it corresponds to the end of the obtained tension-softening curve (see **Fig.3**(a)). From this current state, an incremental force ΔP is applied as shown in **Fig.3**(b) and **Fig.4**. The associated incremental deflection is denoted by Δδ. The incremental crack opening displacement and incremental transmitted tensile stress at the crack just ahead of the notch tip are denoted by Δw and ΔT, respectively. From the given load-deflection curve, ΔP/Δδ is known for any ΔP (see **Fig.3**(b) and **Fig.4**). The tangential slope of the tension-softening curve ΔT/Δw for this incremental step is determined so that the value of ΔP/Δδ by the analysis equals to the value from the experimental load-deflection curve.

In the computation, the problem is decomposed into two sub-problems as shown in **Fig.5**. The first sub-problem is a problem with only the incremental load ΔP applied at the middle of the beam, and no incremental force is applied on the crack surface just ahead of the notch tip. The second sub-problem is a problem with only the incremental tension ΔT applied on the crack surface just ahead of the notch tip. In both sub-problems, the obtained tension-softening relationship is incrementally satisfied on the surface farther away from the notch tip. For example, in **Fig.4**, the crack has already propagated for few steps, and there are, at the current stage, three segments of the crack surface in which the crack opening displacements are represented by the values at the center of the segments, i.e., A, B and C. From the figure, in the segment 2, where the crack opening displacement is equal to B, the incremental tension-softening relationship represented by the slope of the line 2-3 of the piecewise-linearly obtained tension-softening curve will be satisfied. At the same time, in the segment 3, where the crack opening displacement is equal to C, the incremental tension-softening relationship represented by the slope of the line 1-2 will be satisfied. The incremental tension ΔT in the second sub-problem will be applied on the segment 1 which is the notch tip segment. Note that the crack opening displacement of this segment, denoted by A, corresponds to the end of the obtained tension-softening curve.

From the decomposition, the resultant incremental crack opening displacement and incremental deflection can be written as

$$\Delta w = \Delta w_P \Delta P + \Delta w_T \Delta T \quad (1a)$$

$$\Delta \delta = \Delta \delta_P \Delta P + \Delta \delta_T \Delta T \quad (1b)$$

where subscripts P and T denote the solutions when unit forces ΔP=1 and ΔT=1 are applied, respectively.

From Eq.(1b), we express ΔT as

$$\Delta T = \Delta \delta \left(\frac{1 - \Delta \delta_P \dfrac{\Delta P}{\Delta \delta}}{\Delta \delta_T} \right). \quad (2)$$

Substituting Eq.(2) into Eq.(1a), we get

$$\Delta w = \Delta w_P \Delta \delta \frac{\Delta P}{\Delta \delta} + \Delta w_T \Delta \delta \left(\frac{1 - \Delta \delta_P \dfrac{\Delta P}{\Delta \delta}}{\Delta \delta_T} \right). \quad (3)$$

Finally, from Eqs.(2) and (3), we derive

$$\frac{\Delta T}{\Delta w} = \frac{\left(\dfrac{1 - \Delta \delta_P \dfrac{\Delta P}{\Delta \delta}}{\Delta \delta_T} \right)}{\Delta w_P \dfrac{\Delta P}{\Delta \delta} + \Delta w_T \left(\dfrac{1 - \Delta \delta_P \dfrac{\Delta P}{\Delta \delta}}{\Delta \delta_T} \right)}. \quad (4)$$

From Eq.(4), it is seen that for a given slope $\Delta P/\Delta \delta$ from the load-deflection curve, one can obtain the corresponding slope of the tension-softening curve $\Delta T/\Delta w$ for this incremental step. In the analysis, FEM with a cracked element introduced in the next section is employed, and the tension-softening curve is obtained as a piecewise-linear function (see **Fig.4**). The magnitude of ΔP is determined as an absolute-minimum value for the next element to be cracked or the slope of the tension-softening relationship at any cracked element changes. For example, in **Fig.4**, when the incremental load ΔP is applied, the transmitted stress at the segment 1 follows the line 1-2 of the obtained tension-softening relationship. If the crack opening displacement at this segment goes beyond the point 2 of the obtained tension-softening curve, the slope of the line 2-3 must be used instead. For the segment 2, the same strategy must be held. The analysis is done incrementally. Therefore, it is necessary to limit the magnitude of ΔP to ensure that the incremental tension-softening relationship represented by the slope of the tension-softening curve does not change within the step.

As already explained, by employing Eq.(4), one can directly obtain the slope of the tension-softening curve $\Delta T/\Delta w$ for a given slope of the input load-deflection curve $\Delta P/\Delta \delta$. Therefore, there is no iteration required for each incremental step. In contrast to the proposed method, the method proposed by Kitsutaka et al.[15] requires iterations in each step.

After obtaining the increments ΔT and Δw, the tension-softening curve is thereupon extended as shown in **Fig.3(b)** and **Fig.4**. The same process is then repeated, and finally the complete tension-softening curve can be obtained.

3. CRACKED ELEMENT

In the present study, the four-noded quadrilateral element modified to capture the behavior of materials after cracking is used. The element employs an interpolation function derived from the consideration of relative displacement induced by the rigid translation and rotation between two parts separated by the discontinuity. The interpolation function, employed in a principle of virtual work equation, leads to equilibrium of forces and moments of each separate part. In this element, the additional degrees of freedom due to the discontinuity can be statically condensed within the element. Therefore, the effect of the discontinuity is completely represented by only the change in the stiffness matrix of the four-noded quadrilateral element in contrast to remeshing scheme in which the mesh topology is changed. Furthermore, with this cracked element, the

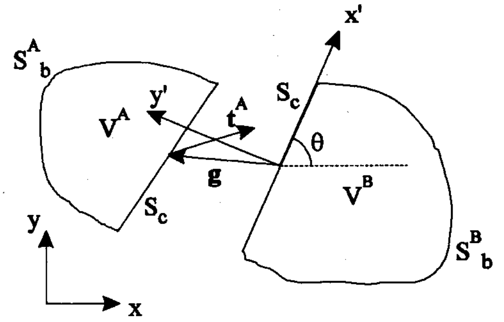

Fig.6 Body with embedded displacement discontinuity.

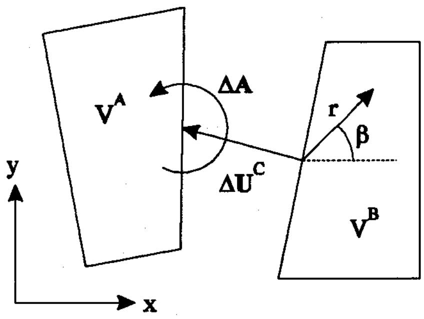

Fig.7 Concept of the displacement interpolation.

constitutive relationship of the discontinuity, which is the relationship between the displacement jump and the transmitted stress, can be taken care of easily and directly within the element. The element is therefore useful even in the case of a straight crack. In this paper, only a brief derivation will be shown. The detail of the element can be found in the work by Nanakorn and Horii[14].

(1) Incremental virtual work for displacement discontinuity

Following Dvorkin et al.[17] and Wan et al.[18], we consider the classical principle of virtual work to derive the incremental equilibrium equations for a body which contains displacement discontinuity. The principle of virtual work for the body shown in **Fig.6** is given in the following matrix form, i.e.,

$$\int_V \delta\ \boldsymbol{\varepsilon}^T\boldsymbol{\sigma}\, dV = \int_V \delta\ \mathbf{u}^T\mathbf{f}\, dV + \int_{S_b} \delta\ \mathbf{u}^T\mathbf{T}\, dS + \int_{S_c} \delta\ \mathbf{g}^T\mathbf{t}^A\, dS \tag{5}$$

The body is separated into two parts called domain A and B. Here **ε** and **σ** are the strain and stress vectors, respectively, **u** is the displacement vector, and **f** and **T** are the body force and boundary surface traction vectors. In addition, $\mathbf{t}^A$ and $\mathbf{t}^B$ are surface traction vectors acting on the internal discontinuity surface S_c of the domain A and B, respectively, and **g** is a displacement jump vector with respect to the domain B. The last term in the equation refers to virtual work done by bridging stress along discontinuity surface. Note that $\mathbf{t}^A=-\mathbf{t}^B$, and this internal traction is determined through a discontinuity condition which is a constitutive relationship of the discontinuity.

Rewriting the virtual work equation for the incremental formulation, we obtain

$$\int_V \delta(\Delta\boldsymbol{\varepsilon}^T)\Delta\boldsymbol{\sigma}\, dV + \int_{S_c} \delta(\Delta\mathbf{g}^T)\Delta\mathbf{t}^B\, dS = \int_V \delta(\Delta\mathbf{u}^T)\Delta\mathbf{f}\, dV + \int_{S_b} \delta(\Delta\mathbf{u}^T)\Delta\mathbf{T}\, dS \tag{6}$$

The discontinuity condition is usually given in a local coordinate system set with respect to the discontinuity plane (see **Fig.6**). Therefore, it is preferable to write the displacement jump and the internal traction in the local coordinate system. The incremental virtual work equation is then rewritten as

$$\int_V \delta(\Delta\boldsymbol{\varepsilon}^T)\Delta\boldsymbol{\sigma}\, dV + \int_{S_c} \delta(\Delta\mathbf{g}_l^T)\Delta\mathbf{t}_l^B\, dS = \int_V \delta(\Delta\mathbf{u}^T)\Delta\mathbf{f}\, dV + \int_{S_b} \delta(\Delta\mathbf{u}^T)\Delta\mathbf{T}\, dS \tag{7}$$

where subscript l denotes the variables in the local coordinate system shown in **Fig.6**.

(2) Interpolation of the displacement field

The incremental displacement interpolation without the discontinuity is given as usual, i.e.,

$$\Delta\mathbf{u}(\mathbf{x}) = \mathbf{N}(\mathbf{x})\Delta\mathbf{U} \tag{8}$$

where **N** denotes an interpolation matrix which contains shape functions. Here **ΔU** denotes the incremental nodal displacement vector.

We now consider the displacement discontinuity which passes through the center of the element as shown in **Fig.7**. The domain A is assumed to undergo relative incremental rigid displacement with respect to the domain B including the incremental rigid translation $\Delta\mathbf{U}^c$ and the incremental rigid rotation **ΔA**. The incremental rigid translation and rotation are referenced

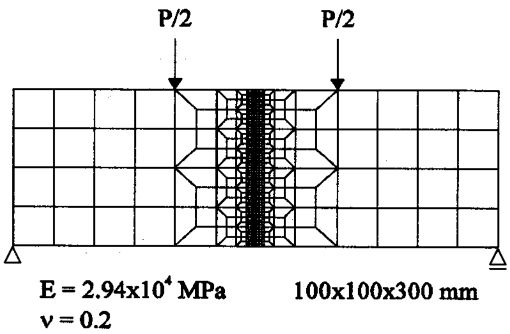

Fig.8 Four-point bending problem.

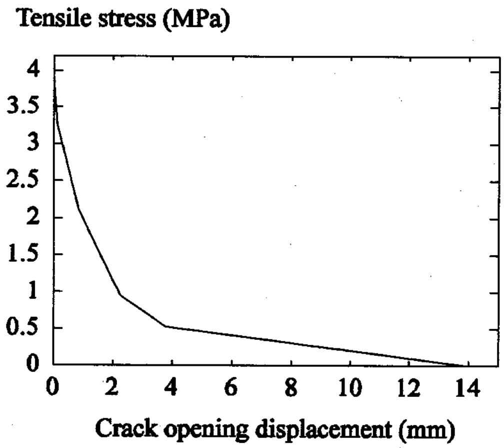

Fig.9 Tension-softening curve.

to the center of the element. We can then express the displacement field as the ordinary continuous displacement field with additional discontinuous displacement field, i.e.,

$$\begin{aligned} \Delta\mathbf{u}(\mathbf{x}) &= \Delta\mathbf{u}_{con}(\mathbf{x}) + \mathbf{R}\Delta\mathbf{U}_l^c + \mathbf{l}(\mathbf{x})\Delta\mathbf{A} \qquad & \mathbf{x} \in V^A \\ &= \Delta\mathbf{u}_{con}(\mathbf{x}) & \mathbf{x} \in V^B \end{aligned} \tag{9}$$

where $\Delta\mathbf{U}_l^c$ denotes the incremental rigid translation in the local coordinates. Symbol **R** is the transformation matrix from the local coordinate system to the global coordinate system (see **Fig.6**). For two-dimensional cases, we have

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{10}$$

where θ is the angle between the two coordinate systems as shown in **Fig.6**. Note that $\mathbf{R}^{-1} = \mathbf{R}^T$.

Here, $\mathbf{l}(\mathbf{x})$ is defined for two-dimensional cases as

$$\mathbf{l}(\mathbf{x}) = \mathbf{l}(r,\beta) = r\begin{Bmatrix} -\sin\beta \\ \cos\beta \end{Bmatrix} \tag{11}$$

in which r is the distance from the center of the element to the point of interest and β is the angle, with respect to the global coordinate system, defined in **Fig.7**.

The continuous part of the displacement field can be simply interpolated using an ordinary interpolation matrix, i.e.,

$$\Delta\mathbf{u}_{con}(\mathbf{x}) = \mathbf{N}(\mathbf{x})(\Delta\mathbf{U} - \mathbf{\Phi}\mathbf{R}\Delta\mathbf{U}_l^c - \mathbf{\Phi}^*\mathbf{L}\Delta\mathbf{A}) \tag{12}$$

where $\mathbf{\Phi}$, for two-dimensional problems, is defined as

$$\mathbf{\Phi}^T = \{\mathbf{\Phi}_1 \quad \mathbf{\Phi}_2 \quad \dots \quad \mathbf{\Phi}_n\} \tag{13}$$

where n is the number of nodes in the element. Each of the submatrices $\mathbf{\Phi}_i$ of dimension 2x2 depends on the position of node i relative to the discontinuity line and is defined as

$$\mathbf{\Phi}_i = \begin{cases} \mathbf{I} & \text{when node } i \in V^A \\ \mathbf{0} & \text{when node } i \in V^B \end{cases}. \tag{14}$$

In addition, $\mathbf{\Phi}^*$ is defined as

$$\mathbf{\Phi}^* = \begin{bmatrix} \mathbf{\Phi}_1 & & & \\ & \mathbf{\Phi}_2 & & \\ & & \dots & \\ & & & \mathbf{\Phi}_n \end{bmatrix}. \tag{15}$$

Here, **L** represents $\mathbf{l}(\mathbf{x})$ at each node, i.e.,

$$\mathbf{L}^T = \{-r_1\sin\beta_1 \quad r_1\cos\beta_1 \quad \dots \quad -r_n\sin\beta_n \quad r_n\cos\beta_n\} \tag{16}$$

Employing Eq.(12) in Eq.(9), we finally have

$$\Delta\mathbf{u}(\mathbf{x}) = \mathbf{N}(\mathbf{x})\Delta\mathbf{U} + \mathbf{N}^c(\mathbf{x})\Delta\mathbf{U}_l^c + \mathbf{N}^r(\mathbf{x})\Delta\mathbf{A} \tag{17}$$

where

$$\begin{aligned} \mathbf{N}^c(\mathbf{x}) &= \mathbf{R} - \mathbf{N}(\mathbf{x})\mathbf{\Phi}\mathbf{R} \qquad & \mathbf{x} \in V^A \\ &= -\mathbf{N}(\mathbf{x})\mathbf{\Phi}\mathbf{R} & \mathbf{x} \in V^B \end{aligned} \tag{18}$$

$$\begin{aligned} \mathbf{N}^r(\mathbf{x}) &= \mathbf{l}(\mathbf{x}) - \mathbf{N}(\mathbf{x})\mathbf{\Phi}^*\mathbf{L} \qquad & \mathbf{x} \in V^A \\ &= -\mathbf{N}(\mathbf{x})\mathbf{\Phi}^*\mathbf{L} & \mathbf{x} \in V^B \end{aligned}. \tag{19}$$

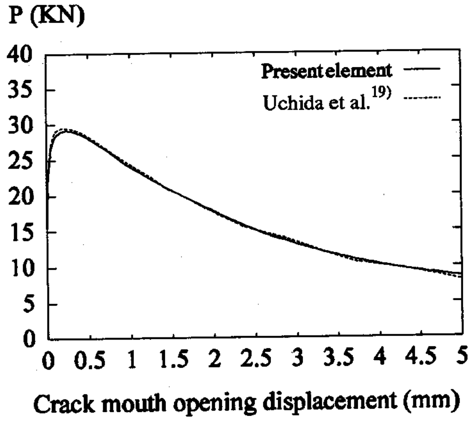

Fig.10 Load-crack mouth opening displacement curve.

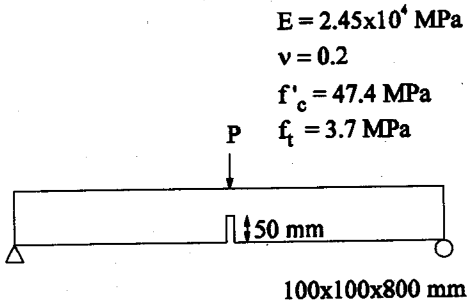

Fig.11 Three-point bending test of a notched beam.

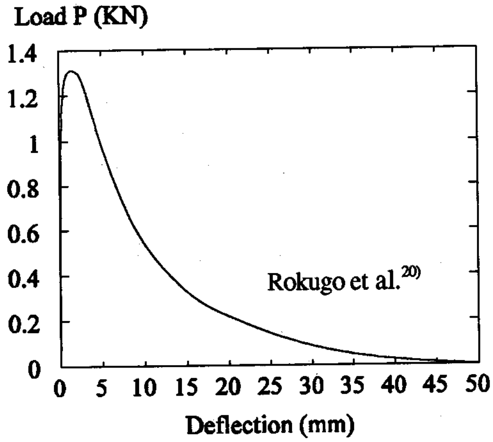

Fig.12 Experimental load-deflection curve of the three-point bending test.

By employing Eq.(17) in Eq.(7) with $\Delta \mathbf{g}_1(\mathbf{x})$ expressed as

$$\Delta \mathbf{g}_1(\mathbf{x}) = \Delta \mathbf{U}_1^c + \mathbf{R}^T \mathbf{l}(\mathbf{x}) \Delta \mathbf{A} \qquad \mathbf{x} \in S_c, \quad (20)$$

the tangential stiffness matrix of the cracked element is obtained following the conventional formulation in FEM[14].

(3) Analysis of the four-point bending of a beam

To show the validity of the element when it is used to solve the straight-crack problem, a four-point bending problem shown in **Fig.8** is analyzed. The cross-section of the beam is 100x100 mm, and the span length is 300 mm. Two point loads are symmetrically applied at 50 mm away from the center of the span. Material constants and a tension-softening curve used in the analysis are given in **Fig.8** and **Fig.9**. In the analysis, a crack is assumed to be initiated and propagate only at the middle of the beam. The result is then compared with the result obtained by Uchida et al.[19] who employed the finite element method that assumes cracking between elements. In this scheme, when the crack extends through a certain node, the node must be split into two in order to allow the crack propagation. **Fig.10** shows the result of the comparison. It can be seen that the results from both analyses agree very well.

4. RESULT

First, the proposed back analysis method is verified numerically. By employing FEM with the cracked element, the forward analysis is done for the three-point bending beam problem with artificial linear and bilinear tension-softening relationships. The analysis is performed by using the assumption that there is only one crack propagating from the notch tip, and the crack is straight. The load-deflection curves obtained from the forward analysis are then used as the input for the back analysis. The objective is to check whether the tension-softening curves obtained from the back analysis differ from the input curves in the forward analysis or not. The results are satisfactory. The obtained tension-softening curves are exact copies of the initial input curves. The compared curves are not shown here because the difference between the input curves and the obtained curves are unnoticeable.

Next, a three-point bending test of a notched beam performed by Rokugo et al.[20] is employed. The cross-section of the beam is 100x100 mm. The span length is 800 mm. A notch with the length of 50 mm is cut at the middle of the beam. The material used in the experiment is the steel-fiber-reinforced concrete with 1% by volume of fibers. The fiber type is the straight steel fiber. The

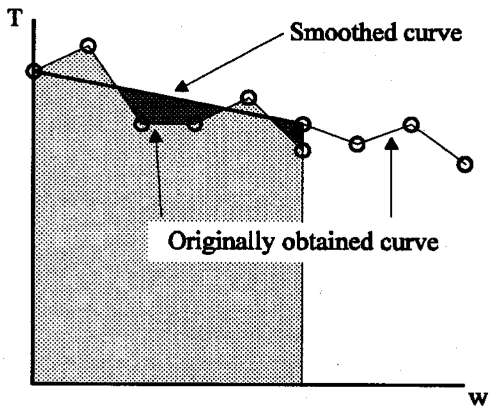

Fig.13 Smoothing scheme.

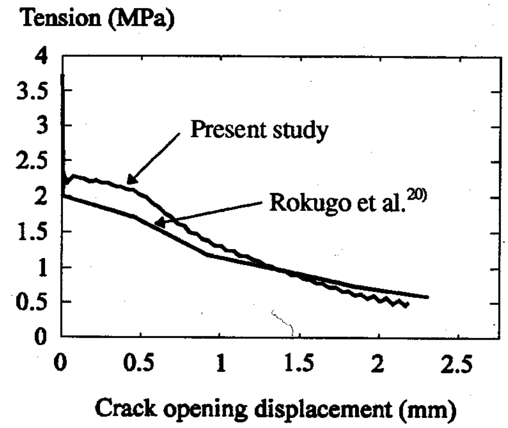

Fig.14 Tension-softening curve from the back analysis of the three-point bending test.

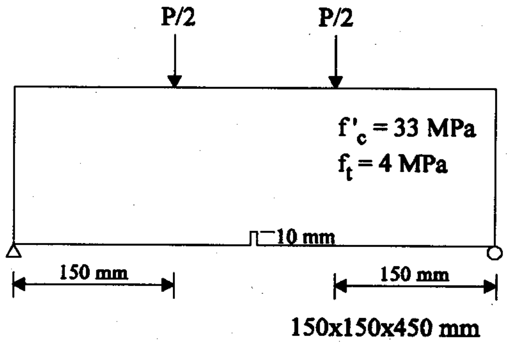

Fig.15 Four-point bending test of a notched beam.

boundary and loading conditions of the problem and the material constants are shown in **Fig.11**. From the experimental load-deflection curve shown in **Fig.12**, the back analysis is performed. Also in this analysis, a crack is assumed to propagate from the notch tip, and its path is assumed to be straight. It is found that the tension-softening curve originally obtained from the method is uneven. This unevenness may subsequently lead to the magnification of the unevenness of the following solution. Hence, the obtained curve is smoothed out during the computation for every few steps. The curve is smoothed in such a way that the area under the smoothed curve is equal to the area under the original curve (see **Fig.13**). It seems that a similar problem resulting from the unevenness of the obtained tension-softening curve also appears in the method by Kitsutaka et al.[15] but the treatment for the problem is not discussed in that paper.

Fig.14 shows the tension-softening curve obtained from the back analysis. The tension-softening curve obtained by Rokugo et al.[20] is also plotted in the same figure. They employed the modified J-integral-based technique[12] to obtain the tension-softening curve. From the comparison, it is seen that the results from the two methods are in agreement. It is also observed that there is a sharp drop in tension at the beginning of the curve, and the back analysis method can capture this behavior well.

The previous example shows that the back analysis used with the load-deflection curve from the three-point bending test with a long notch yields a result that is similar to the result from the modified J-integral-based technique. Next, the method will be tested using the load-deflection curves from the experimental results of the four-point bending test with a short notch. In the test, the cross-section of the beam is 150x150 mm. The span length is 450 mm. A notch with the length of 10 mm is cut at the middle of the beam. The type of the material used is the same as the previous example — the steel-fiber-reinforced concrete with 1% by volume of straight steel fibers. **Fig.15** shows the boundary and loading conditions of the problem as well as the material constants. It should be noted that the tensile strength f_t used in the computation is approximated from the results of direct tension tests. In the real practice, the tensile strength f_t can be determined from the splitting test. The load-deflection curves obtained from the experiments are not smooth; therefore, they are smoothed before used in the analysis. **Fig.16** shows the two smoothed experimental load-deflection curves used as the input for the back analysis as well as some other experimental curves. It can be seen from the figure that the experimental load-deflection curves scatter a lot. The Young's Modulus used in the analysis is determined from the initial slope of the smoothed load-deflection curve.

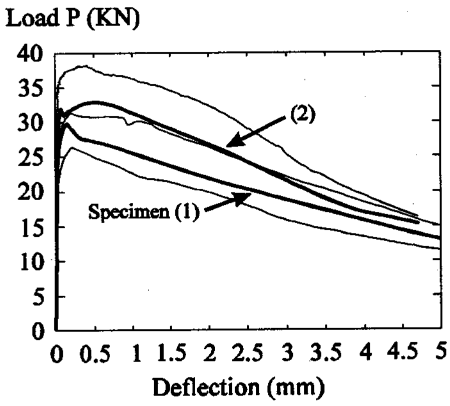

Fig.16 Experimental load-deflection curves of the four-point bending test.

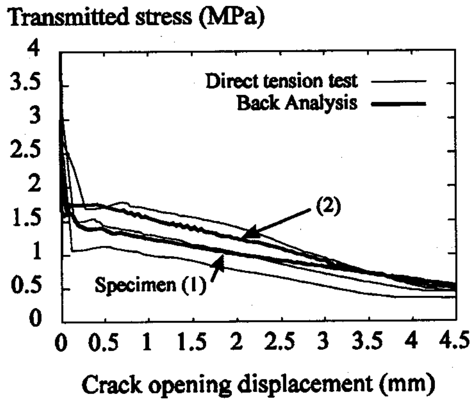

Fig.17 Tension-softening curves from the back analysis of the four-point bending test.

From the observed crack patterns from the experiments, it is found that there is always a single localized crack propagates from the notch tip. Though the localized cracks are not actually straight, their paths do not deviate much from the middle cross-sections of the beams. Therefore, it is reasonable to employ the assumption of having a straight crack at the middle of the beam. The tension-softening curves obtained from the analysis are shown in **Fig.17**. In the same figure, the tension-softening curves for this material obtained from the direct tension tests are also plotted. The direct tension tests are performed by using the displacement control without a feedback system. It can be seen that the tension-softening curves from the proposed back analysis agree well with the experimental results from the direct tension tests. It is interesting to note that the tension-softening curves from the direct tension tests scatter much. The scattering of the tension-softening relationships is expected to be the main reason of the scattering of the experimental load-deflection curves.

5. CONCLUSION

Tension-softening relationship is an important material characteristic controlling the fracture behavior of concrete. The complete tension-softening relationship is necessary when in-depth studies on the fracture behavior of concrete are concerned. The in-depth studies on the fracture behavior of concrete are essential for tasks such as establishment of a design provision for steel-fiber-reinforced concrete tunnel linings[16] and development of new cementitious materials. The tension-softening curve can be obtained from the post-peak stress-displacement relationship of the direct tension test. However, it is difficult to perform a stable direct tension test without using a stiff, closed loop feedback-controlled loading machine. In the present study, a new back analysis method to obtained the tension-softening curve is proposed. The analysis is performed by using a load-deflection curve of the bending test of a notched beam. The method is based on the fact that the magnitude of the crack opening displacement at the crack just ahead of the notch tip is maximum, compared with the other part along the crack. Therefore, if the relationship between the transmitted tensile stress and the crack opening displacement at this position is determined, the obtained relationship can be used for the crack surface farther away from the notch tip because the crack opening displacement is smaller than the one just ahead of the notch tip. The computation employs FEM with the cracked element[14]. In this study, the method is used with load-deflection curves from the three-point bending and four-point bending tests of steel-fiber-reinforced concrete beams. The tension-softening curves obtained from the method show good agreement with the curves from the modified J-integral-based technique as well as the experiments. Because the bending test is simple and does not require so sophisticated loading machine, the proposed method is advantageous. Nevertheless, it is expected that special consideration on the input load-deflection curve data may be necessary if the method is used with normal concrete. It is because, due to stronger instability after the peak in the bending test of the normal concrete beam, it may be difficult to obtain reliable post-peak portions of the load-deflection curves unless very deep notches are used. It is also important to note that, as long as FEM analysis is concerned, the tension-softening curve obtained from the back analysis gives the original experimental load-deflection curve if it is used in the forward analysis.

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コンクリートの引張軟化関係の逆解析

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コンクリートにおける破壊現象の解析を行うためには，適当な材料試験を行い引張軟化曲線を求めることが必要となる．本研究は，曲げ試験により得られる荷重変位曲線から直接引張軟化曲線を逆解析する方法を提案するものである．解析は不連続面を有する要素を用いた有限要素解析法に基づいている．ノッチ先端の要素の開口変位増分と伝達応力増分の関係を決定し，解析をクラックの進展に伴い順次繰り返すことにより，引張軟化曲線を求める．提案する手法を鋼繊維補強コンクリートの曲げ試験結果に適用しその有用性を確認した．