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ANALYSIS OF NONLINEAR SYSTEM USING NARMA MODELS

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The purpose of this paper is to develop a systematic method of time-domain nonlinear system identification using NARMA model. In the beginning the Hilbert transform test was used for nonlinearity, then an orthogonal parameter estimation algorithm is applied to a NARMA model which represents the nonlinear system model. Finally the nonlinear frequency response functions were computed. Application of the methodology to a combined Duffing and Van Der Pol nonlinear system and the seismic response data of Van Nuys building to Whittier and Northridge earthquakes are studied.

Key Words: *System Identification, NARMA Model, Signal Processing*

1. INTRODUCTION

The fact that many structural systems exhibit linear behavior at low levels of excitation allows estimation of modal properties from test data, modelling of structures using constant parameter systems, and correlation of test and analysis results. Unfortunately, as the level of excitation increases, the system may not continue to respond in a linear fashion. Modal properties may differ greatly between low and high level excitation. Many attempts are made to develop systematic procedures to identify the nonlinear system from measurements. One of the approaches from frequency domain identification of a nonlinear system is the reverse dynamic system of spectral analysis.[1~3] This approach considered the displacement response as input, and the force excitation as output. This nonlinear single-defree-of-freedom (SDOF) system viewed in this manner is a multiple-input/single-output system where the number of inputs depends on the nature of the nonlinearity term. To apply this method it is necessary to know the exact form of nonlinear feedback input. As a matter of fact the nonlinear damping-restoring force function of a nonlinear system is generally not known, and the application of this reverse dynamic system is limited.

The successful development of identification procedures for a nonlinear system depends upon the model which is used to represent the system under investigation. Experimental analysis of nonlinear systems may be accomplished by using time domain models. Traditionally the functional series descriptions of Volterra and Wiener have been used.[4] Unfortunately, functional series models require an excessive kernel values to describe even simple nonlinear systems. However, by expanding the system output in terms of past input and output using a nonlinear autoregressive moving average model with exogenous inputs (NARMA) model, a very concise representation for a wide class of nonlinear systems can be obtained.[5,6] An orthogonal parameter estimation algorithm was derived for a stochastic nonlinear system which can be represented by a NARMA model.

The purpose of this paper is to develop a systematic method of time domain nonlinear system identification using NARMA model. The orthogonal estimation algorithm will be used to estimate the modal parameters. Interpretation and properties of the nonlinear frequency response functions are discussed. A simulated nonlinear example (combined Duffings and Van Der Pol model) is included to demonstrate the effectiveness of the algorithm. Finally, application of the identification algorithm to the building seismic response data is examined.

2. NONLINEAR SYSTEM DETECTION USING HILBERT TRANSFORM

The Hilbert Transform is an integral transform that relates the real and imaginary parts by any complex function. The Hilbert Transform of the complex function $f(x)$ of real variable x is defined as:[7]

$$h\Big(f(x)\Big) = H(x) = \frac{1}{i \cdot \pi} \int_{-\infty}^{\infty} \frac{f(y)}{x-y}\, dy \qquad (1)$$

Details of the mathematical development of the Hilbert transform are well documented in the literature. In the case of a general system, the system "frequency response function" will be defined as the ratio of the Fourier transforms of the output and the input signals:

$$\begin{aligned} G(\omega) &= \mathcal{F}\{y(t)\}\Big/\mathcal{F}\{x(t)\} \\ &= \mathrm{Re}[G(\omega)] + i\, \mathrm{Im}[G(\omega)] \end{aligned} \qquad (2)$$

where $\mathcal{F}\{\,\cdot\,\}$ denotes the Fourier transform, $y(t)$ and $x(t)$ are system output and input signal, respectively. If the system is nonlinear, $G(\omega)$ will depend on the input $x(t)$. The Hilbert Transform of the complex function $G(\omega)$ is defined as:

$$H\{G(\omega)\} = H(\omega) = \mathrm{Re}[H(\omega)] + i\, \mathrm{Im}[H(\omega)] \quad (3)$$

where $\mathrm{Re}[H(\omega)]$ and $\mathrm{Im}[H(\omega)]$ represent the real and the imaginary parts of the Hilbert Transform defined by the following discrete form:[7]

$$\mathrm{Re}[H(\omega_j)] = -\frac{2}{\pi} \sum_{k=1}^{n} \frac{\mathrm{Im}[G(\omega_k)]\ \omega_k\ \Delta\omega}{\omega_k^2 - \omega_j^2}$$

$$\mathrm{Im}[H(\omega_j)] = -\frac{2\,\omega_j}{\pi} \sum_{k=1}^{n} \frac{\mathrm{Re}[G(\omega_k)]\ \omega_k\ \Delta\omega}{\omega_k^2 - \omega_j^2} \qquad (4)$$

For a linear system, the response at any time does not depend on the future of the input, and the Hilbert Transform of a linear system is equal to the frequency response function of that system's $G(\omega)$. If the Hilbert Transform does not equal the frequency response function, then the system is nonlinear.

Consider a broad class of SDOF nonlinear dynamic systems. It can be described by the constant coefficient differential equation:

$$m\,\ddot{u} + c\,\dot{u} + k\,u + p(u, \dot{u}, t) = F(t) \qquad (5)$$

where m, c, k denote the system mass, linear viscous damping and linear elastic stiffness, and $p(u, \dot{u}, t)$ represents a general nonlinear damping-restoring force function component. For a combined Duffing and Van Der Pol system, $p(u, \dot{u}, t) = [k_2 + (c_2/3)(d/dt)]u^3$. Use the fact that the Hilbert Transform of a nonlinear system (in this study the combine Duffing and Van Der Pol system is used) will create changes in the Hermitian symmetry property of the original frequency response function. One can implement the use of statistical moments in the identification of system nonlinearities. An index is suggested to compare the result of the Hilbert Transformed data to the original frequency response data:

$$\mathrm{ID}^n = 100\left\{\frac{H_v^n - G_v^n}{G_v^n}\right\} \qquad (6)$$

where ID^0 ($n = 0$) represents the energy ratio and ID' ($n = 1$) represents the frequency ratio, and the moment integral about the vertical axis are defined as:

$$H_v^n = \int_{\omega_1}^{\omega_n} \omega^n\, H(\omega)\, d\omega$$

and
$$G_v^n = \int_{\omega_1}^{\omega_n} \omega^n\, G(\omega)\, d\omega \qquad (7)$$

Figure 1 shows the comparison between the amplitude of the frequency response function and the modulus of Hilbert Transform of the combined Duffing and Van Der Pol SDOF nonlinear system. Table 1 shows the index (ID^0) for different degree of nonlinearity. It is clear that the stronger nonlinearity in the model will make a significant difference in the comparison on the energy ratio between linear & nonlinear model. It is concluded that this method provides a good index for the detection of system nonlinearity.

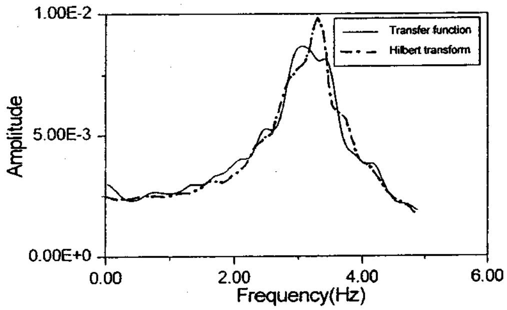


Fig. 1 Comparison between the system transfer function and its Hilbert Transform for the combined Duffing and Van Der Pol nonlinear system ($K_2 = 700$ and $C_2 = 20$).

Table 1 Plot of model parameters (different degree of nonlinearity) with respect to $\max\{||HT[G(\omega)| - |G(\omega)||\}/\max\{|G(\omega)|\}\%$ for the combined Duffing and Van Der Pol model.

Parameters of nonlinear terms		Input PGA=0.3 g
C2=0	K2=0	1.188E−4
C2=3	K2=300	53.4345
C2=12	K2=600	58.6872
C2=48	K2=1200	62.4739
C2=128	K2=2400	64.5676

3. THE NARMA METHODOLOGY

The theory associated with each of the stages in the NARMA methodology is well documented in the literature. A single input single output system takes the form

$$y(t) = F^l\Big[y(t-1),\cdots,y(t-n_y),u(t),\cdots, u(t-n_n),\varepsilon(t-1),\cdot,\varepsilon(t-n_\varepsilon)\Big] + \varepsilon(t) \quad (8)$$

where $y(t)$ and $u(t)$ represent the measured output and input respectively, $\varepsilon(t)$ is the prediction error, and $F^l[\,\cdot\,]$ is some nonlinear function. Several nonlinear models may include nonlinear damping force, F_D, and F_D can be a function of relative displacement and relative velocity. In the model equation of Eq. (8), one can incorporate the velocity term ($\dot{y}(t)$) in the model to represent the nonlinear damping. The NARMA model equation may be represented by the regressional equation

$$y(k) = \sum_i^{n_\theta} p_i(k)\,\theta_i + \varepsilon(k) \quad (9)$$

where $p_i(t)$ represents a term in the NARMA and no $p_i(t)$'s are identical. Rather than estimating the parameter θ_i directly from Eq. (9) the orthogonal algorithm operates on an equivalent auxiliary model

$$y(k) = \sum_{i=1}^{n_\theta} g_i\, w_i(k) + \varepsilon(k) \quad (10)$$

The parameters g_i in Eq. (10) can be estimated by implementing the orthogonal estimator.[9] The algorithm is described as follows:

1. Set $w_1(t) = p_1(t)$ (generally $p_1(t) = y(t-1)$) and
$$\hat{g}_1 = \frac{\sum_{k=1}^N w_1(k)\, y(k)}{\sum_{k=1}^N w_1^2(k)} \quad (11)$$
2. Set $j = 1$ and compute
$$\alpha_{ij} = \frac{\sum_{k=1}^N w_i(k)\, p_j(k)}{\sum_{k=1}^N w_i^2(k)}$$
$$\text{for} \quad i = 1,2,\cdots,j-1 \quad (12)$$
$$w_j(k) = p_j(k) - \sum_{i=1}^{j-1} \alpha_{ij}\, w_i(k) \quad (13)$$
$$\hat{g}_i = \frac{\sum_{i=1}^N w_j(k)\, y(k)}{\sum_{k=1}^N w_j^2(k)} \quad (14)$$
increment j and compute Eqs. (12), (13) and (14).
3. Compute NARMA parameter θ_i backwards using $\hat{\theta}_{n_\theta} = \hat{g}_{n_\theta}$
$$\hat{\theta}_i = g_i - \sum_{j=i+1}^{n_\theta} \alpha_{ij}\,\hat{\theta}_j$$
$$\text{for} \quad i = n_\theta - 1,\ n_\theta - 2,\ \cdots,\ 1 \quad (15)$$

Besides the estimation of model parameter θ_i from Eq. (15), it is necessary to detect which terms should be included within the NARMA model. It can be achieved by computing the error reduction ratio for the i-th terms as

$$\mathrm{ERR}_i = \frac{\hat{g}_i^2 \sum_{k=1}^{N} w_i^2(k)}{\sum_{k=1}^{N} y^2(k)} \times 100 \qquad (16)$$

ERR_i provides a measure of the reduction in mean square error.

Taking an example of a combined Duffing and Van Der Pol model, the initial analysis involved fitting linear models of various orders ($n_y = n_u = 1,2,3,4$) and delays ($d = 0,1,2,3$) and computing the loss function (sum of square errors) of the fitted models using the forward inclusion algorithm with $l = 1$. The result is summarized in Table 2. It shows that the time delay is $n_d = 1$ and that an appropriate model order would be $n_u = n_y = 3$ because the loss function will monotonically decrease with increasing n_u and n_y. Inputting these values into the forward regression estimator[9] and setting the degree of nonlinearity $l = 2$, one can select the terms which are significant from the possible terms. Table 3 shows the final process model of the nonlinear system. Model validation test of the system can be performed by calculating the autocorrelation function of residuals (difference between the recorded output and the output from the model) and the cross-correlation function between input and residuals. Figure 2 shows that there is a peak at zero lag and it demonstrates the residuals are whitenoise. The cross-correlation function between residuals and input is shown in Fig. 2 and the result is within 95% confidence interval. The identified model proves to be correct. Figure 3 shows the comparison of the predicted response data and the simulation data. The normalized error is 0.66% (for input PGA=0.3 g). Based on

Table 2 Linear model loss function for the combined Duffing and Van Der Pol system. $n_y = n_u = 2$ and $n_d = 1$ were selected as a starting point for the nonlinear search.

Log Loss function table for Combined Duffing-Van Der Pol Model				
	d (time delay)			
n (order)	1	2	3	4
1	0.67930	0.65702	0.79378	0.69170
2	−1.08571	−0.57994	−0.00411	−0.63030
3	−1.25249	−0.65131	−0.48574	−0.32759
4	−1.34591	−0.71213	−0.59602	−0.49480

the identified model from input PGA = 0.3 g, by increasing the input PGA intensity, Figure 4 shows the error between predicted and simulated response with respect to input PGA. It shows that the identified nonlinear model can still predict the response quite well (within 5% error) even for input PGA = 0.5 g.

Table 3 Identified NARMA model of the combined Duffing and Van Der Pol model.

Combined Duffing Van Der Pol Model $\ddot{x} + 3.77\dot{x} + 355.3x + 700x^3 + 20x^2\dot{x} = -\ddot{u}_g$ Input: El Centro (PGA=0.2 g)
$y(t) = (1.3337421)\, y(t-1)$ $-(8.3787298\ e^{-1})\, y(t-2)$ $+(7.3635571\ e^{-4})\, u(t-1)$ $+(6.6749601\ e^{-4})\, u(t-2)$ $-(8.0204095\ e^{-1})\, y(t-1)^3$ $-(3.4950845\ e^{-9})\, u(t-1)^3$ $+(2.3680245\ e^{-7})\, u(t-2)\, u(t-3)$

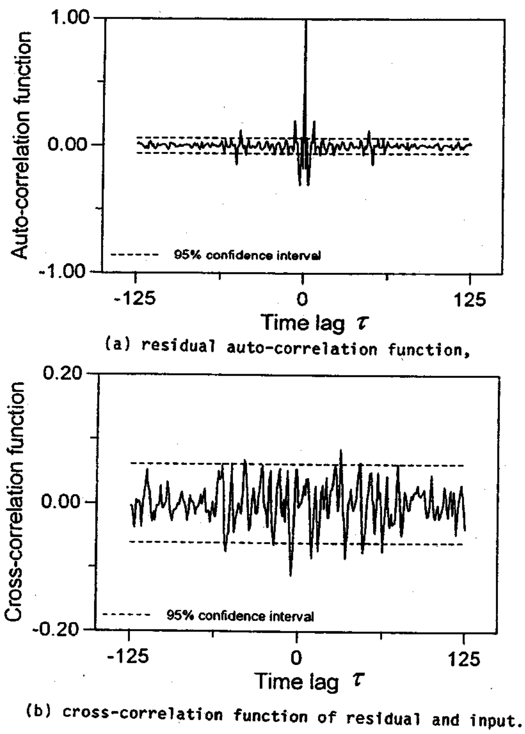


Fig. 2 Model validation test for the combined Duffing and Van Der Pol model.

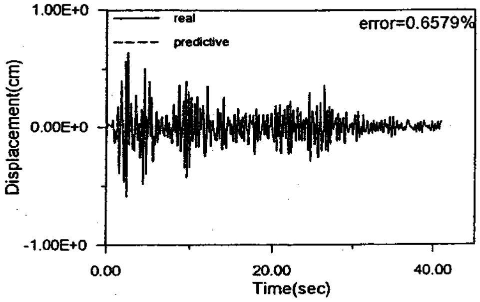


Fig. 3 Comparison between the predicted response and the measured response of the combined Duffing and Van Der Pol model (input PGA=0.3a).

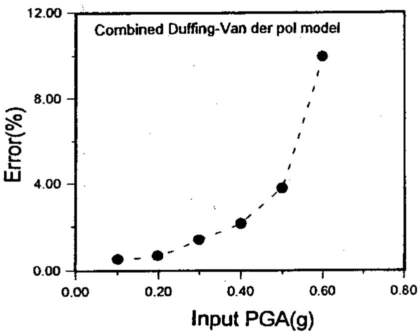


Fig. 4 Plot of prediction (root mean square error) with respect to different intensity of input motion.

4. NONLINEAR FREQUENCY RESPONSE FUNCTIONS

The generalized frequency response functions can be obtained from the estimated NARMA model by discarding the noise model. The probing method[9,10] can be used to obtain the analytical expressions for the frequency response function. This method can be justified by considering the steady-state output of a nonlinear system with several exponential inputs. It was shown that the probing method (or harmonic input method) can be used to determine the symmetries nth order transfer function $H_i(f_1,\cdots,f_2)_{\text{sym}}$ by equating coefficients of $n!\ \exp[i\,2\pi(f_1+f_2+\cdots+f_n)\,t]$ in the system output for an input defined by

$$u(k)=\sum_{k=1}^{K}A_k\ e^{i\,2\pi\,f_k\,t} \tag{17}$$

Based on the probing method, for the combined Duffing and Van Der Pol nonlinear model, this procedure can be applied to find at each step higher order nonlinear frequency response functions in terms of lower order functions. Figure 5 shows the identified transfer functions of $H_1(f_1)$ $H_2(f_1,f_2)$ and $H_3(f_1,f_2,f_3)$ expressed in the following forms:

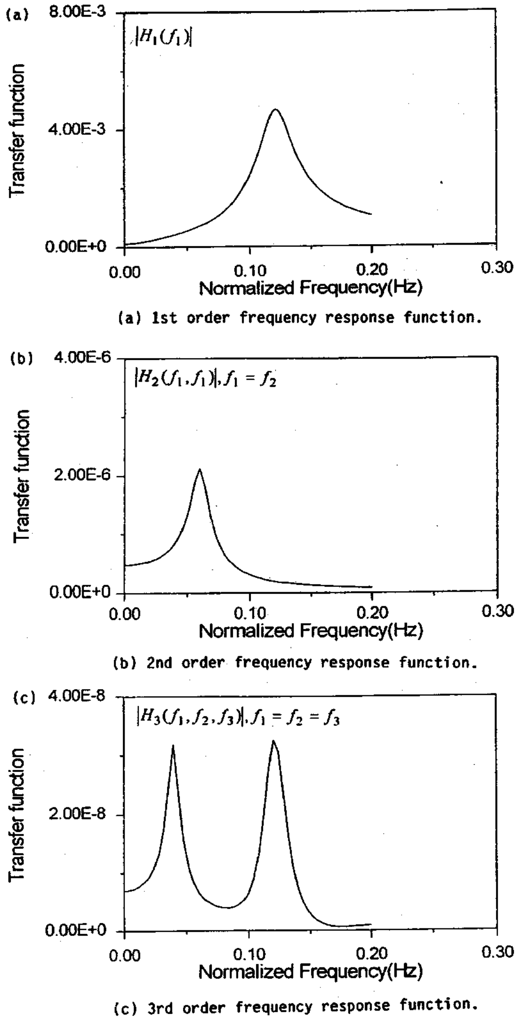


Fig. 5 Estimated higher order frequency response function from the combined Duffing and Van Der Pol model.

$$H_1(f) = \frac{a_3\, e^{-2i\pi f} + a_4\, e^{-i4\pi f}}{1 - a_1\, e^{-2i\pi f} + a_2\, e^{-i4\pi f}} \tag{18}$$

$$H_2(f_1, f_2) =$$

$$\frac{0.5\, a_7 \left[e^{-2i\pi(3f_1+2f_2)} + e^{-i2\pi(2f_1+3f_2)} \right]}{\left[1 - a_1\, e^{-2i\pi(f_1+f_2)} + a_2\, e^{-i4\pi(f_1+f_2)} \right]} \tag{19}$$

$$H_3(f_1, f_2, f_3) =$$

$$\frac{\left[0.5\, H_1(f_1)\, H_2(f_2)\, H_3(f_3) + 6\, a_6 \right] e^{-2i\pi\, f(f_1+f_2+f_3)}}{1 - a_1\, e^{-2i\pi(f_1+f_2+f_3)} - a_2\, e^{-i4\pi(f_1+f_2+f_3)}} \tag{20}$$

The parameters a_1 through a_7 are the identified model parameter of the combined Duffing and Van Der Pol nonlinear model shown in Table 3.

If a system is linear the application of a sinusoidal input will generate a sinusoidal output of the same frequency but with a different gain and phase. It will not generate new frequencies. If a system is nonlinear however new frequency components such as harmonics and intermodulation can be produced together with effects such as gain compression/expansion and desensitization. Each of the nonlinear phenomena will be briefly explained as follows:

1. Harmonics — Harmonics are frequency components which are equal to multiples of the fundamental input frequency.

2. Intermodulation — The process by which two or more signals combined in a nonlinear manner to produce a new frequency component is termed intermodulation.

3. Gain compression or expansion — These terms are used to describe the variation in the gain of a system as the input amplitude is increased. Gain expansion occurs when the system gain is greater than the linear gain and gain compression when the system gain is less than the linear gain.

4. Desensitization — When a system is nonlinear the sinusoidal response at frequency f_1 can be modified by the application of a second sinusoidal signal at another frequency f_2.

Figure 5 shows the nonlinear frequency response functions of $H_1(f_1)$, $|H_2(f_1, f_1)|$ and $|H_3(f_1, f_1, f_1)|$ for the combined Duffing and Van Der Pol nonlinear model. This suggests that the magnitude of the 2nd and 3rd harmonic is proportional to the 2nd and 3rd power of the input magnitude for small input signals. It can be demonstrated by the 2nd order frequency response function of $|H_2(f_1, f_2)|$ along the line $f_1 = f_2$, and 3rd order frequency response function $|H_3(f_1, f_2, f_3)|$ along the line $f_1 = f_2 = f_3$. Also from the 3-dimensional frequency response function (shown in Fig. 6a and 6b), it is observed that peak amplitudes are along two lines $f_1 + f_2 = 0.12$ and $f_1 + f_2 = -0.12$ from $|H_2(f_1, f_2)|$. If the normalized factor (25 Hz) were scaled back, it means that at 3 Hz (i.e., 0.12×25 Hz $= 3$ Hz) there is a significant intermodulation. Also from Fig. 6b, the 3rd order frequency response function $|H_3(f_1, f_2, f_3)|$ shows a peak at $f_1 + f_2 + f_3 = 0.12$ for $f_1 = f_3$ which means that gain compression or expansion is observed.

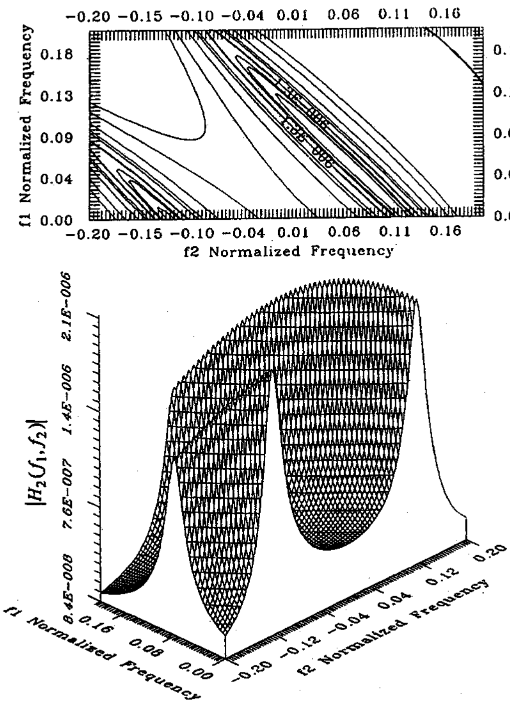


Fig. 6a Estimated nonlinear frequency response function, $|H_2(f_1, f_2)|$.

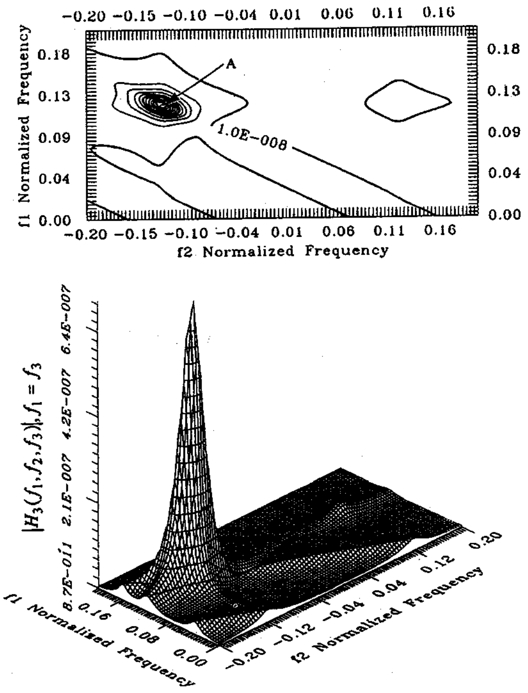


Fig. 6b Estimated nonlinear frequency response function, $|H_3(f_1, f_2, f_3)|$.

5. ANALYSIS ON THE BUILDING SEISMIC RESPONSE DATA

The CSMIP in the Department of Conservation, Division of Mines and Geology, collected two strong-motion records from the Van Nuys building (CSMIP Sta.# 24386). This building is rectangular in plan as shown in Fig. 7. The response data from Whittier earthquake (1 October, 1987) and Northridge earthquake (17 January, 1994) are used for the analysis. Damage was observed in this building from the Northridge earthquake. In this study only the data recorded on the top floor (North-South direction) is used for identification and the roof response data (relative displacement between roof and basement) is selected as output and the basement acceleration is selected as input motion. Following the procedures described in the previous section, the linear model loss function from these two sets of seismic data is summarized in Tables 4 and 6. Inspection of Tables 4 and 6 show that the time delay is $d = 1$ and the appropriate model order would be $n_a = n_y = 2$. The reduction in the loss function does not converge, but for the linear model the selected order and the delay time step are acceptable. To incorporate the nonlinear damping phenomenon in the model the velocity term $\dot{y}(t)$ was considered (i.e., energy dissipation $F_D \approx |\dot{y}^n| \operatorname{sgn}(\dot{y})$, $n = 1$ gives viscous damping), as shown in Table 5 and Table 7. Following the above mentioned procedures (orthogonal estimation algorithm, forward regression algorithm and probing method), the data recorded from the roof response of Van Nuys building was analyzed. The identified NARMA model of the system from the Whittier and Northridge earthquakes are shown in Table 5 and Table 7, respectively. Figure 8a shows the comparison between the predicted response and the recorded data from the analysis of Whittier earthquake data. Through the probing method only the FRF of $H_1(f_1)$ can be identified, as shown in Fig. 8b. The validation test of the model is shown in Figs. 8c and 8d, showing that the results are acceptable. Different from the Whittier earthquake data the results from the analysis of Northridge earthquake is shown in Figs. 9 and 10. Figure 9a shows the comparison between the predicted and the simulated response. The validation test of the model is shown in Fig. 9b and 9c. Because the structure was damaged during the Northridge earthquake, the structural system can not be identified as a time-invariant system. The result of the analysis on these data does not pass the linear test, hence the 2nd order frequency response function was identified as shown in Fig. 10b.

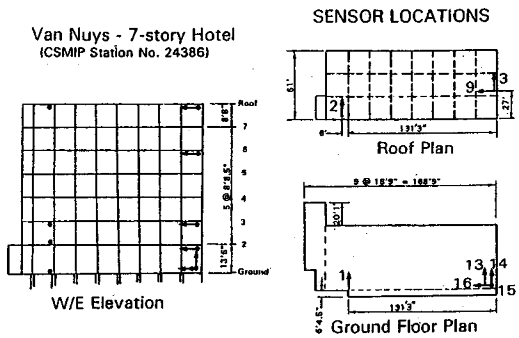


Fig. 7a Plan view of Van Nuys 7-story building and location of accelerometers on ground floor and roof.

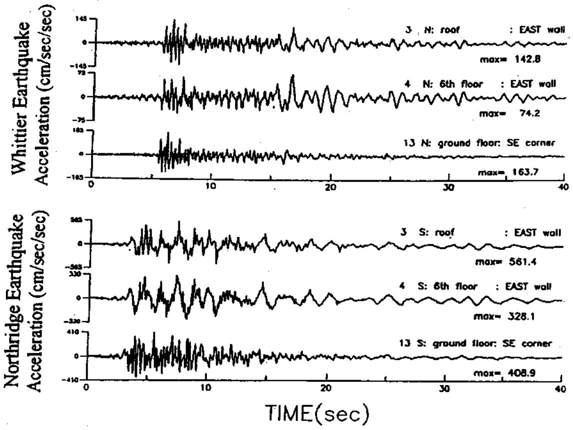


Fig. 7b Recorded acceleration at Chs. 3, 4, and 13 for Whittier and Northridge earthquakes, respectively.

Table 4 Linear model loss function for Van Nuys Building (using Whittier earthquake data). $n_y = n_u = 2$ and $n_d = 1$ were selected as a starting point for the nonlinear search (North-South direction; input: base motion; output: roof response).

Log Loss function table for Van-Nuys hotel (Whittier earthquake)				
	d (time delay)			
n (order)	1	2	3	4
1	−1.90093	−1.92784	−1.96399	−2.00310
2	−4.79163	−4.79056	−4.83380	−4.90353
3	−5.85811	−5.85522	−5.85602	−5.88195
4	−7.46312	−7.46722	−7.45490	−7.41289

Table 5 Identified NARMA model of the Van Nuys building (Whittier earthquake data) using roof response as output only.

Van-Nuys 7-story hotel
Whittier Earthquake (10/1/87)

$$y(t) = (3.6429638\ e^{-2})\ y(t-1) + (3.9172756\ e^{-2})\ \dot{y}(t-1) + (9.634620\ e^{-1})\ y(t-2) + (2.1649173\ e^{-4})\ u(t-1) - (1.9961796\ e^{-4})\ u(t-2)$$

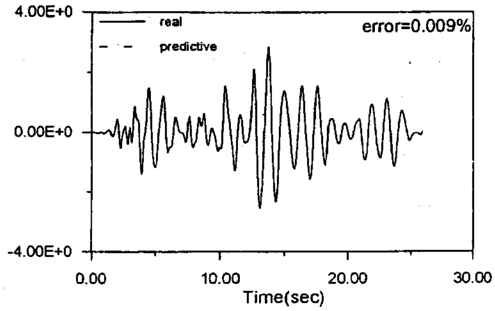
Table 6 Linear model loss function for Van Nuys Building (using Northridge earthquake data). $n_y = n_u = 2$ and $n_d = 1$ were selected as a starting point for the nonlinear search (North-South direction; input: base motion, output: roof response).

Log Loss function table for Van-Nuys hotel (Northridge earthquake)				
	d (time delay)			
n (order)	1	2	3	4
1	0.27979	0.24822	0.20847	0.17465
2	−2.04448	−2.04232	−1.93707	−1.78775
3	−2.71176	−2.64556	−2.55479	−2.52069
4	−3.83703	−3.72995	−3.49421	−3.33059

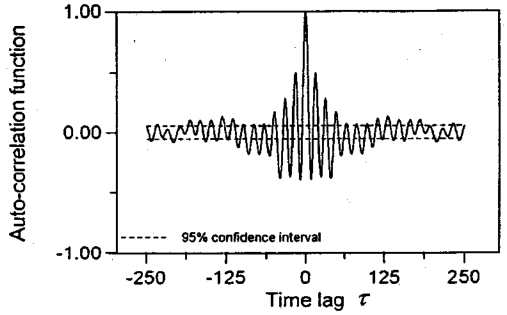
Table 7 Identified NARMA model of the Van Nuys building (Northridge earthquake data) using roof response as output only.

Van-Nuys 7-story hotel
Northridge Earthquake (1/17/94)

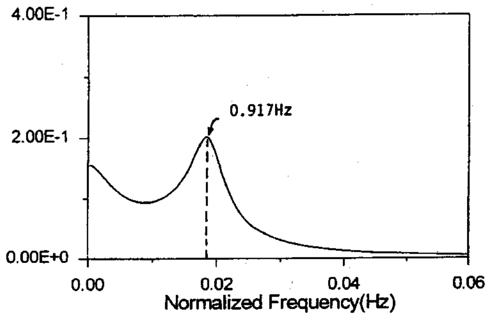
$$y(t) = (1.53113542)\ y(t-1) + (4.5626041\ e^{-2})\ \dot{y}(t-1) - (2.7536559\ e^{-2})\ \dot{y}(t-2) + (5.7779633\ e^{-4})\ u(t-1) - (3.6078692\ e^{-4})\ u(t-2) - (5.3267714\ e^{-1})\ y(t-2) - (7.1361802\ e^{-6})\ u(t-2)\ y(t-2)$$



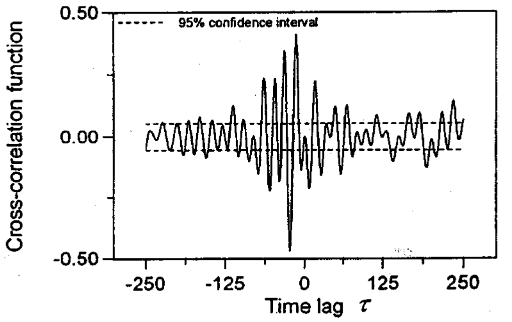
(a) Comparison between predicted and recorded responses.



(c) Residual auto-correlation function.



(b) Identified frequency response function of $H_1(f)$.



(d) Cross-correlation function between residual and input motion (Whittier earthquake).

Fig. 8 Result from the analysis of Van Nuys Building (NS direction between roof and basement motion).

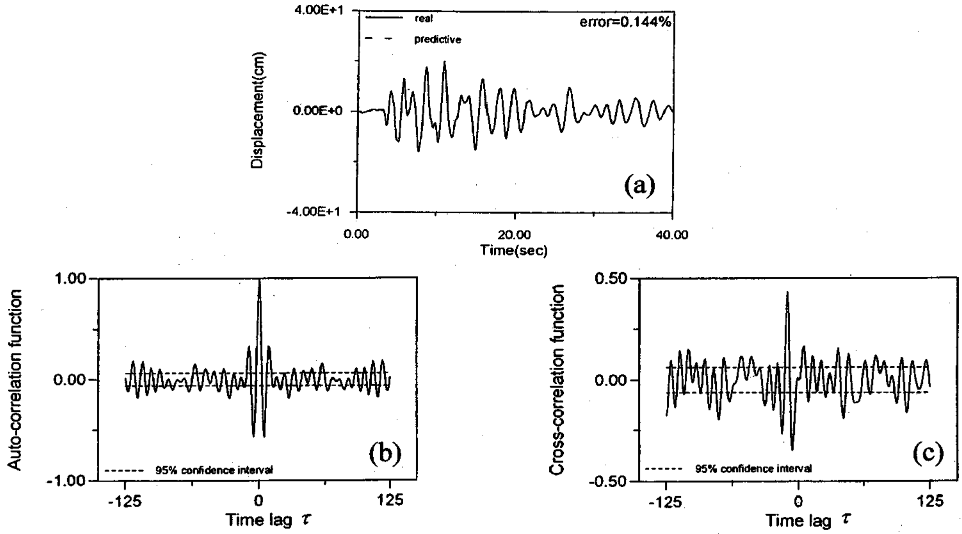
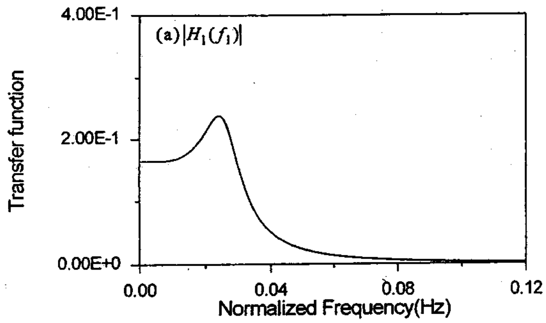
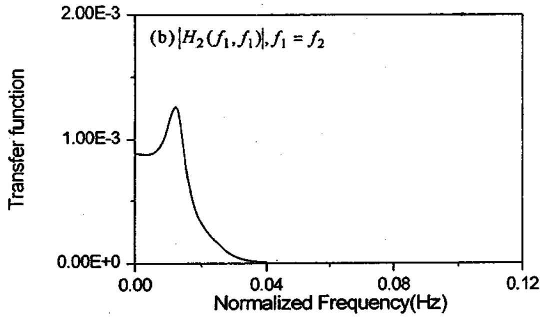


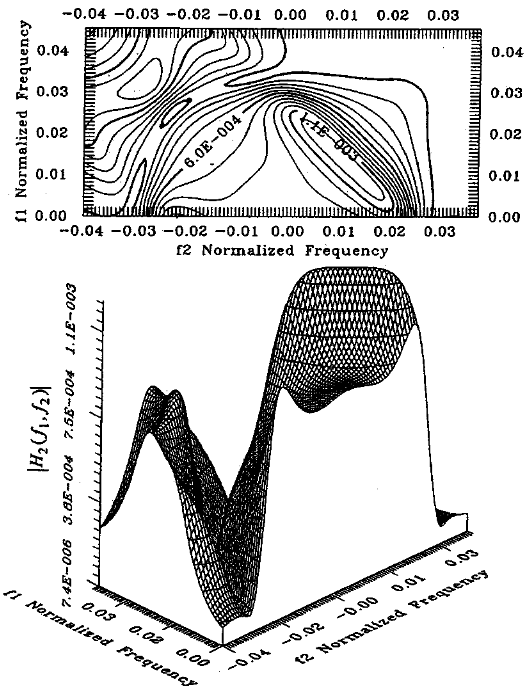
Fig. 9 Result from the identification of Van Nuys Building from Northridge earthquake data.
(a) Comparison between predicted and recorded response.
(b) Residual auto-correlation function.
(c) Cross-correlation function between residual and input motion.



(a) 1st order frequency response function, $|H_1(f_1)|$.



(b) 2nd order frequency response function, $|H_2(f_1,f_1)|, f_1=f_2$.



(c) Estimated nonlinear frequency response function, $|H_2(f_1,f_2)|$.

Fig. 10 Results from the analysis of Van Nuys Building (Northridge earthquake).

6. CONCLUSIONS

The purpose of this paper is to propose an identification scheme for the nonlinear system. The NARMA methodology was applied which includes: orthogonal estimation algorithm and forward regression algorithm was applied to identify the modal parameter of NARMA model and probing method to estimate the higher order Frequency Response Function. Application of the method to a combined Duffing and Van Der Pol nonlinear system was investigated. Finally, based on the seismic response data of the building, identification was performed with the NARMA model. The following conclusions are made:

1. Application of Hilbert transformation can provide a measure of system nonlinearity, from which the NARMA model can be used to present the nonlinear system.

2. The NARMA methodology provides a systematic method to identify the nonlinear system. If the system can pass the validation test (auto-correlation function of residuals and cross-correlation between residuals and inputs), then the proposed model is a good estimate.

3. From the analysis of Whittier and Northridge earthquake data of the Van Nuys Building, significant nonlinearity was observed from the Northridge earthquake data.

4. Incorporating the velocity term, $\dot{y}(t)$, in the NARMA model provides a much better estimate of the nonlinear system, because the velocity damping of the system can be considered.

5. The present study has concentrated on the identification of nonlinear SDOF system. For the multiple DOF nonlinear system, one should convert to the local system identification, so that the present method can be applied.

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