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# THE FINITE ELEMENT WITH INNER LINKAGE RODS IN CAPTURING THE CRACK LOCALIZATION OF CONCRETE STRUCTURES

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A new technique for modeling the crack behaviors of concrete structures is described, where the softening band inside the element is represented by two rod elements whose behaviors are based on the fracture-oriented constitutive relations. This new method, like the discrete model, can reflect the localized nature of cracking , and at the same time can easily be applied without the difficulty of modifying the existing finite element mesh and pre-defining the concrete crack. In this paper, examples are given to show the phenomenon of crack localization and to show the objectivity of this method.

***Key Words :*** *concrete, crack, fracture mechanics, finite element, mesh independence, localization*

## 1. INTRODUCTION

It is generally agreed that softening mechanism in cementitious materials are based on microfracture processes that produce highly localized failure mode at zero and low-confining stress levels. Concrete cracking at low-confined stress levels is a localized phenomenon, which means that in concrete structures, the material inside a crack undergoes loading, while at the same time, the material outside this crack undergoes unloading. This leads to large strain localizing into this major crack without affecting the surrounding material.

Since the major nonlinearities of typical concrete structures are often caused by cracking, in recent years, a lot of research works have been done to develop the constitutive models for the description of crack in not only concrete but also in mortar and rocks, as shown in the review by Bažant[1]. Up to now, to describe the cracking of concrete, two different approaches are available: the discrete approach and the smeared approach.

The discrete approach, first proposed by Ngo and Scordelis[2], is attractive physically, as it reflects the localized nature of cracking, but its numerical implementation is hampered by the need for letting the cracks follow the element boundaries, thereby requiring the introduction of additional nodal points or rearrangement of the original mesh. Even though Ingraffea and Saouma[3] developed a procedure to automatically and locally modify an existing mesh, the discrete model is not easy to apply in the analysis of arbitrary structures.

The smeared approach was first introduced by Rashid[4], in which only the constitutive relation expressed in terms of stresses and strains needs to be modified in the element region when crack occurs. In its original form, the smeared crack approach assumes the slope of the softening branch to be a material property. Computationally, the smeared approach is more practicable than the discrete one, but when the traditional smeared approach is used with a simple strength criterion to detect the initiation and propagation of cracks, the propagation of concrete cracks is dependent on the width of the finite element, and if the width of the element approaches zero, the stress in the element tip approaches infinity, which implies that the propagation of crack may be caused by a small applied load, and the total energy dissipated by cracks approaches zero. This leads the analysis results to unobjectivity as shown by

Bažant and Cedolin[5]. To solve this problem, the crack band model of Bažant and Oh[6] has been proposed, and later, smeared fracture models were proposed by Nilsson and Oldenburg[7], William et al. [8] and Pramono and William[9], which require the proper choice of an internal length normally associated with the element size.

Because of its simplicity in computation, smeared model have been widely used in finite element analysis, but using the smeared models, the localized bandwidth of the concrete crack is represented by the element size. For some problems, as pointed out by Shirai[10] and Rots[11], it is doubtful that the smeared model can suitably simulate a localization of fracture and unloading behaviors in surrounding region due to stress-locking phenomenon resulting from the excessive shear stress transfer along the crack, false compressive strut action parallel to the crack and other reasons.

Other approaches, proposed by Ortiz et al.[12] and Belytschko et al.[13], have been followed to devise more objective failure models starting with the assumption that the active zone of plastic softening is embedded into larger zone of intact behavior. But it is difficult to combine those methods with fracture mechanics to analyze the localized crack of concrete structures.

In view of this, it is worthwhile to develop a way to correctly reflect the localized nature of crack, in which large strain localizes inside the crack without affecting the surrounding material outside the crack, and at the same time to be easily implemented into finite element program to analyze arbitrary concrete structures without pre-defining the concrete crack and modifying the existing mesh.

In this research, an element with inner linkage rods is introduced when crack occurring which is judged by the maximum principle stress at the center of the element and the tensile strength of material. When a single crack occurs, the element with inner linkage rods is composed of two large unloading parts linked with rods which represent the crack and follow fracture-oriented constitutive relations. Actually, the element with inner linkage rods is a substructure.

The fictitious crack model of Hillerborg[14] is extended so that a suitable description of the tension and shear response of a crack plane is achieved. The method proposed considers the existence of multiple cracks, and a consistent approach is obtained for closing and reopening of cracks. To use this method to analyze concrete structures, the only thing to do is first making a subroutine using displacements control method to describe the nonlinear behaviors of the substructure and then implementing this subroutine into the common used finite element programs.

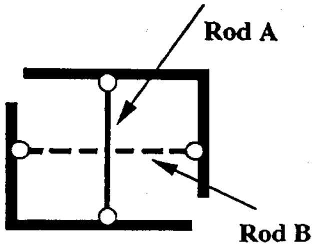


**Fig. 1** The rod linkage element

Since it is most possible for plain concrete structures to fail at the last stage with a major localized crack. In this paper, three kinds of structures of plain concrete (uniaxial tension, pure bending and shearing) are analyzed to show the phenomenon of crack localization. Also, different kinds of meshes are used to show the objectivity of this method.

The problem of rotating-crack is not included in this method, and there are still some issues to be resolved that how to use this method to describe the crushing behavior of concrete.

It is also possible to use this method to analyze the size effect of the reinforced concrete structures, but it is out of the scope of this paper on this stage.

## 2. FINITE ELEMENT WITH INNER LINKAGE RODS

The linkage elements, originally proposed by Ngo and Scordelis[2], has been commonly used for modeling the bond-slip behavior of reinforced concrete structures. In this research, the linkage element is used to represent the localized crack and to link the unloading concretes on two sides of this crack. This linkage element is composed of two rods, which follow the fracture-oriented constitutive relation. Those rod elements have been used by other researchers[15),16] to analyze the size effect problems of reinforced concrete beams with pre-defined cracks.

**Fig.1** shows the rod linkage element. Rod $A$ is the rod for describing the tensile behavior of the crack and Rod $B$ is for the shear slip behavior of the crack. **Fig.2** and **Fig.3** show the finite element with inner linkage rods when a single crack occurs. Points $a$ and $b$, and points $c$ and $d$ have the same coordinates. Using this kind of element makes it possible that the stress inside the crack undergoes loading and that outside the crack un-

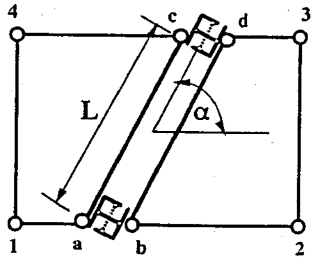


**Fig. 2** The element with inner linkage rods, case 1

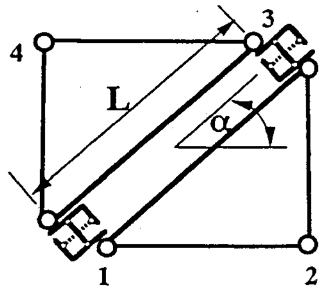


**Fig. 3** The element with inner linkage rods, case 2

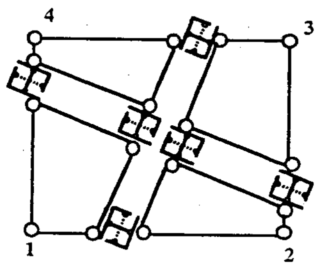


**Fig. 4** The element with inner linkage rods, case 3

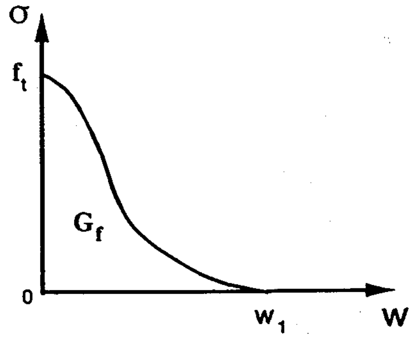


**Fig. 5** The fictitious crack model of Hillerborg

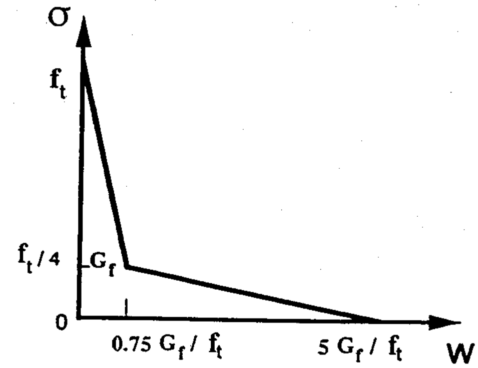


**Fig. 6** $\sigma$–$w$ curve used in the analysis

dergoes unloading.

The procedures to implement this kind of element can be described as follows:

1. For every element in the mesh the maximum principle stress at the center of the element is calculated. The stress state at the center of the element can simply be obtained by averaging the stress at 2×2 Gauss points when 4-node isoparametric elements are used, or directly using one-point integration rule[17]. This calculation is repeated at every step of the solution process until the maximum principle stress is equal to or larger than the tensile strength of the material, that means that a crack occurs through the center of the element.

2. From this point on, the finite element where the crack occurs, is replaced by the finite element with inner linkage rods that is mentioned above. The crack with angle $\alpha$ , shown in **Fig.2** and **Fig.3**, is normal to the maximum principle stress. The crack length $L$ can be calculated out and the section areas of Rod $A$ and Rod $B$ are equal to $Lt/2$, where $t$ is the thickness of this element. The inner freedom related to points $a$, $b$, $c$ and $d$ can be eliminated at element level by means of static condensation[18], by this way the stiffness matrix related with points $1,2,3$ and 4 can be obtained.

When the crack goes through or near the diagonal nodes of the element, the finite element with inner linkage rods shown in **Fig.3** should be used, otherwise the element shown in **Fig.2** should be used to ensure that the element has a good shape and good performance. For the multicrack case which is often observed in the structures of slab and shell, the finite element with inner rods as shown by **Fig.4** can be used. With regard to the examples taken into account in this paper, the multicrack case seldom occurs or it is not the major factor influencing the response of the structures, this kind of element has not yet been implemented into authors' program.

## 3. FICTITIOUS CRACK MODEL

The fictitious crack model was first introduced by Hillerborg, et al.[14] and in its original form, it is a discrete approach. It is of importance that instead of describing the cracking process by a relation between stress and strain, the fictitious crack

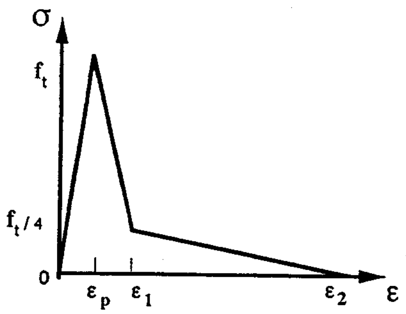


**Fig. 7** The stress-strain curve inner linkage rod $A$

model describes the behavior of the infinitely thin cracked zone by a constitutive relation expressed in terms of normal stress $\sigma$ and the crack width normal to the crack plane. When the crack opens the stress is not assumed to fall to zero at once, but to decrease with increasing width, as shown in **Fig.5**. At the crack width $w_1$ the stress falls to zero. Energy dissipated per unit crack area $D_t$, is related to the area under the $\sigma - w$ curve of **Fig.5**, i.e.,

$$D_t = \int_0^{w_1} \sigma dw = G_f \tag{1}$$

where $G_f$ is the fracture energy, i.e., the energy required to create a fully opened crack plane of unite area, and $w_1$ is shown in **Fig.5**.

In the application of the fictitious crack model, the $\sigma$–$w$ curve may be chosen in different ways. In the analysis of this paper, the curve shown in **Fig.6** was used. Note that **Fig.5** and **Fig.6** bear the same area for $G_f$.

## 4. ROD ELEMENT

In this section, the fracture-oriented constitutive relation of the rod element will be described.

### (1) Rod element for simulating the tension behavior of the crack

In **Fig.1**, Rod $A$ is used to describe the tension behavior of the crack.

#### a) Loading behavior

To describe the loading behavior of Rod $A$, we will adopt the same equations used by Zareen[15], which are based on the fictitious crack model.

Assuming the rod has the unite length, the loading stress-strain relation curve of Rod $A$ can be shown by **Fig.7**. We assume the initial stiffness of the rod element $E_R = 100 \times E_c$, where $E_c$ is the Young's modulus of the concrete, in order to make sure that the difference in displacements at two end of the rod is small enough before the crack occurs.

The initial stiffness of the rod element $E_R$ is necessary in the case when the cracks do not occur at the same time on the two sides of the element, for example, for **Fig.2**, it is possible that the maximum principle stresses at the element center and point $a$ are larger than tensile strength of concrete while at point $c$ it is less than tensile strength of concrete. For the case when the cracks occur at point $a$ and $c$ at the same time, there are not differences between the results obtained with and without the initial stiffness of the rod element. Therefore, for general considerations, $E_R$ is necessary.

The strain, stress and tangential stiffness can be expressed as

$$\varepsilon_p = \frac{f_t}{E_R} \tag{2}$$

$$\varepsilon_1 = 0.75\frac{G_f}{f_t} \tag{3}$$

$$\varepsilon_2 = 5\frac{G_f}{f_t} \tag{4}$$

and

$$\sigma = \begin{cases} E_R\varepsilon & 0 < \varepsilon \le \varepsilon_p \\ f_t - \dfrac{0.75 f_t(\varepsilon - \varepsilon_p)}{\varepsilon_1 - \varepsilon_p} & \varepsilon_p < \varepsilon \le \varepsilon_1 \\ \dfrac{f_t}{4} - \dfrac{f_t(\varepsilon - \varepsilon_1)}{4(\varepsilon_2 - \varepsilon_1)} & \varepsilon_1 < \varepsilon \le \varepsilon_2 \\ 0 & \varepsilon_2 < \varepsilon \end{cases} \tag{5}$$

$$E = \begin{cases} E_R & 0 < \varepsilon \le \varepsilon_p \\ -\dfrac{0.75 f_t}{\varepsilon_1 - \varepsilon_p} & \varepsilon_p < \varepsilon \le \varepsilon_1 \\ -\dfrac{f_t}{4(\varepsilon_2 - \varepsilon_1)} & \varepsilon_1 < \varepsilon \le \varepsilon_2 \\ 0 & \varepsilon_2 < \varepsilon \end{cases} \tag{6}$$

where $f_t$ is the tensile strength and $G_f$ is the fracture energy of concrete.

#### b) Unloading and reloading behavior

The constitutive equations described above apply when the crack are opening. In practice, it is also important to have realistic models for the closing and reopening of cracks, especially when the crack localization phenomenon, which means that in one portions of the concrete structures a major crack will open and the other cracks will close, is concerned. Here the equations proposed by Dahlblom and Ottosen[19] are used.

Assume that at point $(\sigma_u, \varepsilon_u)$, $\varepsilon_p < \varepsilon_u < \varepsilon_2$, the unloading is detected, the path of unloading

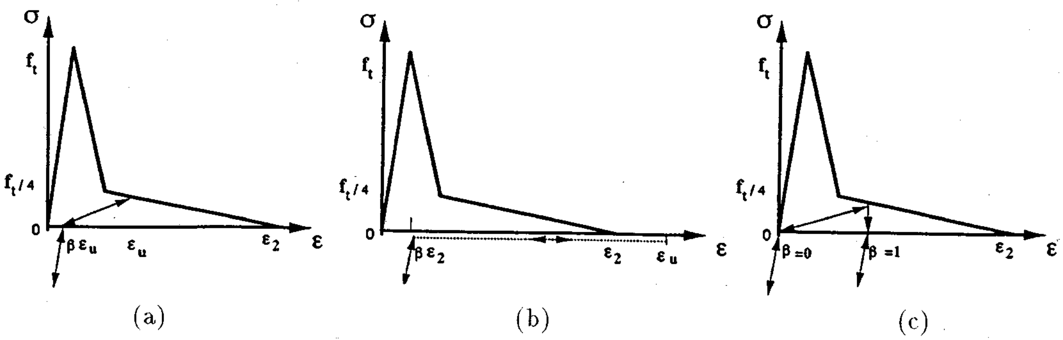


**Fig. 8** The unloading and reloading behaviors of rod $A$

will follow the Eq.(7) as shown by **Fig.8a**.

$$\sigma = \begin{cases} \dfrac{\sigma_u(\varepsilon - \beta\varepsilon_u)}{(1-\beta)\varepsilon_u} & \beta\varepsilon_u \leq \varepsilon \\ E_R(\varepsilon - \beta\varepsilon_u) & \varepsilon < \beta\varepsilon_u \end{cases} \quad (7)$$

where $E_R$ has the same meaning as in Eq.(2). $\beta$ is the material parameter. If $\beta$ is chosen as zero, this corresponds to fully recoverable crack width, whereas, $\beta = 1$ corresponds to total irrecoverable crack width, as shown in **Fig.8c**.

If $\varepsilon_u > \varepsilon_2$ the unloading and reloading path follow **Fig.8b**.

As the main point of this paper is to show the objectivity of this method and that this method can be used to describe the crack localization phenomenon, the effect of changing $\beta$ will not be presented in this paper. In this studies, $\beta$ is chosen to be 0.

### (2) Rod element for simulating the shear slip behavior of the crack

The original fictitious crack model considers only the behavior of a crack loaded normal to the crack plane. In reality, crack planes are often exposed to shear. From the experimental observation[20], when the crack occurs, the tangential crack displacement $w_t$ depends on both the shear stress and the normal crack displacement $w_n$. Note that both the Rod $A$ and Rod $B$ have the unit length, we can write

$$\varepsilon_t = f(\tau, \varepsilon_n) \quad (8)$$

where $\varepsilon_t$ is the strain of Rod $B$, and $\tau$ is the stress of Rod $B$ and $\varepsilon_n$ is the strain of Rod $A$.

For Eq.(8), a simplest form is used as[19]

$$\varepsilon_t = \frac{\varepsilon_n}{G_s}\tau \quad (9)$$

In order to fit with the experiment result[19], $G_s$ is taken as 3.8 MPa.

Note that Eq.(8) has the same feature as the fictitious crack model that it is expressed in the stress-displacement relation form. This promises the analytical result be objective.

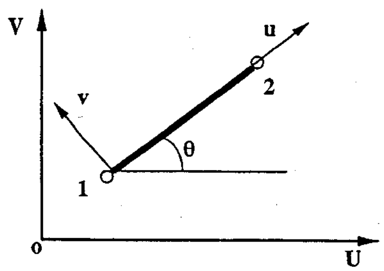


**Fig. 9** Displacement of the rod element

Although experimental date showed a softening effect in the shear stress, we let it fall down to and then remain to be zero when the magnitude of shear stress becomes larger than $f_t$. It is an assumption, however, as the plain concrete fracture is mainly caused by the fracture Mode-I, this assumption will not give rise to significant effect on the analytical results.

## 5. TANGENTIAL STIFFNESS MATRIX FOR ROD ELEMENT

**Fig.9** shows a rod element in global coordinates X-Y, with angle $\theta$ to X axis. The global displacements at two ends are $(U_1, V_1)$ and $(U_2, V_2)$. The displacements of two end nodes with reference to the x-y local coordinates $(u_1, v_1)$ and $(u_2, v_2)$ will be

$$\left\{ \begin{array}{c} u \\ v \end{array} \right\} = \left[ \begin{array}{cc} c & s \\ -s & c \end{array} \right] \left\{ \begin{array}{c} U \\ V \end{array} \right\} \quad (10)$$

where $c = \cos\theta$ and $s = \sin\theta$

The strain along the rod can be written as

$$\varepsilon = \frac{u_1 - u_2}{L_R} \quad (11)$$

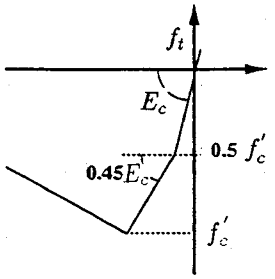


**Fig. 10** The uniaxial stress-strain curve

where $L_R$ is the length of the rod.

With respect to the global displacements, the strain of the rod can be written as

$$\begin{aligned}\varepsilon &= \frac{1}{L_R}[c \;\; s \;\; -c \;\; -s]\,[U_1 \;\; V_1 \;\; U_2 \;\; V_2]^T \\ &= [B]\,\{d\} \qquad (12)\end{aligned}$$

So the stiffness matrix of the rod with reference to the global coordinate will be

$$\begin{aligned}[\mathbf{K}] &= A_s \int_0^{L_R} [\mathbf{B}]^T E\,[\mathbf{B}]\,dx \\ &= \frac{A_s E}{L_R}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ -sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -c^2 & -sc & s^2 \end{bmatrix} \qquad (13)\end{aligned}$$

where E is the Young's modulus of the rod, while $A_s$ is the section area of the rod.

## 6. NUMERICAL EXAMPLE

The finite element used in this analysis is a four-node quadrilateral element. Drucker-Prager type constitutive equation with the uniaxial stress-strain curve in **Fig.10** is employed.

All the problems analyzed below are assumed to be the plane stress problems. The purpose of the calculation is not to compare results with experimental evidence, but rather to show that the suggested method can reflect the localized nature of cracking and is objective in the sense that for decreasing finite element size the total energy dissipated due to cracking approaches the correct value.

For each example, with respect to different load level, the crack patterns, which indicate the orientation and width of the cracks, will be shown. The same magnifying factor will be used for all the crack width in the same example, and when the width of the crack is equal to zero (this means the crack has closed), this crack will not be drawn. From these crack patterns the procedure of opening and closing of the crack can be seen clearly.

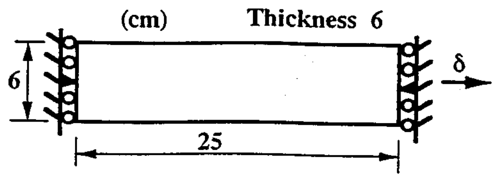


**Fig. 11** Dimension and boundary condition of example 1

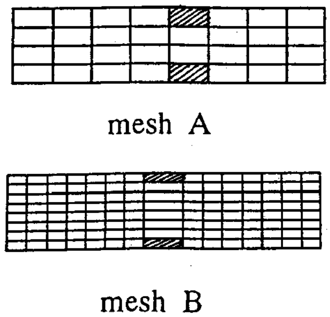


**Fig. 12** Meshes used in example 1

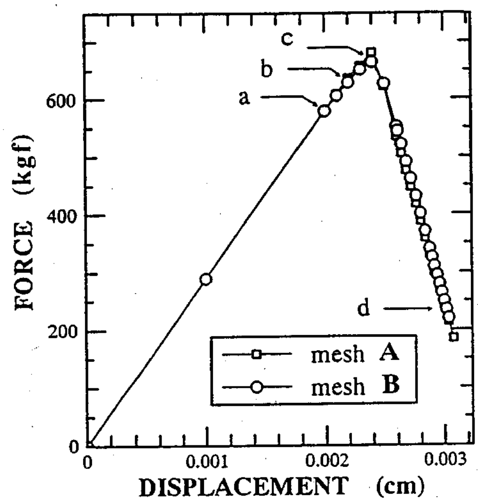


**Fig. 13** Load-displacement curves for example 1

### (1) Uniaxial tension

A plain concrete bar is subjected to prescribed uniform displacements at both ends. The dimension and the supporting condition are shown in **Fig.11**. The material properties are as fol-

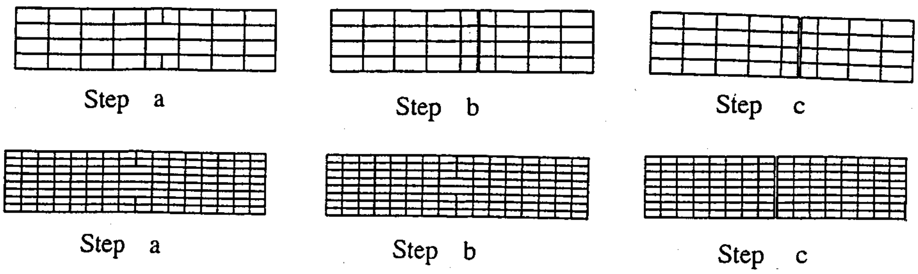


**Fig. 14** Crack pattern for example 1

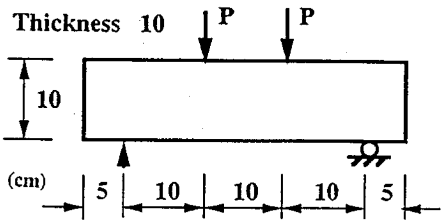


**Fig. 15** Dimension and boundary condition for example 2

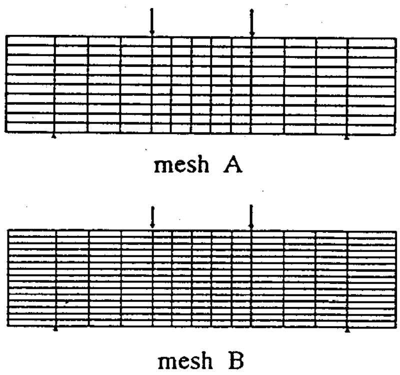


**Fig. 16** Meshes for example 2

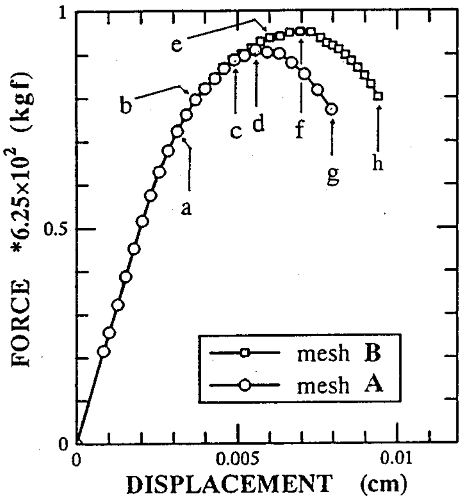


**Fig. 17** Load-displacement for example 2

lows: $E_c = 2.1 \times 10^5$kgf/cm$^2$, $f_t = 26.3$kgf/cm$^2$, $f_c' = 190$kgf/cm$^2$ and $G_f = 0.1$kgf/cm.

Two kinds of mesh $15 \times 8$ and $8 \times 4$ are used to demonstrate the objectivity of this method and shown in **Fig.12**. The imperfection elements, which are shown by the shadow element in **Fig.12** are embedded in the center of two sides of the bar. The imperfection element has the same material properties as the other elements but the thickness is reduced to 3cm.

The load-displacement curves are shown in **Fig.13** with two kinds of mesh used. The results are shown in **Fig.14** depicting the crack patterns at different load levels. The objectivity of this method can be demonstrated by **Fig.13**.

**(2) Pure bending**

The concrete beam geometry is illustrated in **Fig.15**. The portion between the two point loads is subjected to pure bending. The material properties are the same as used in example 1.

Some researchers[10)] have studied the size effect phenomenon of the same plain concrete beam as shown in **Fig.15**, but with different material properties.

Two kinds of mesh, mesh $A$ and mesh $B$, are used and shown in **Fig.16**. **Fig.17** shows the load-displacement curves at the center of the lower side of the beam for mesh $A$ and mesh $B$, respectively. Corresponding to the steps marked in **Fig.17**, the crack patterns are shown in **Fig.18**.

Actually, for this particular problem, the stress

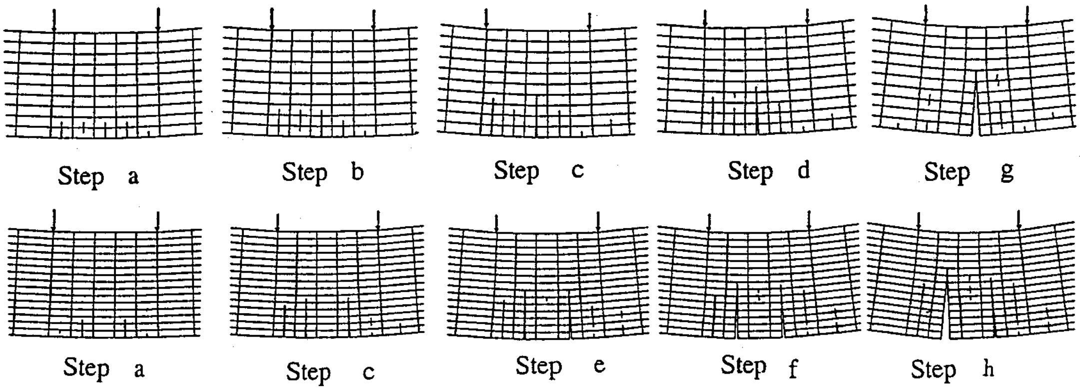


**Fig. 18** Crack patterns for example 2

states at the same depth of the beam between the two point load are the same. Between the two point loads, cracks should occur in all the elements in the bottom side of the beam at the same certain load level. After that the cracks grow towards to the upper element and at the same time the loading (crack opening) and unloading (crack closing) take place in the layers of elements that already cracked. This crack opening and closing procedures take place continuously and the crack opening element can be one element, two elements or more. It is a bifurcation phenomenon. Without any imperfection (like example 1) or small interfering, the bifurcation path, to which the calculation results will go, seem to be determined by factors which are affected by the specimen size, material properties, the mesh, the loading step, supporting conditions, etc. For uniaxial tension problem, the bifurcation path will affect the post-peak response of the specimen causing different post-peak load-displacement curves corresponding to different bifurcation paths, but the values of the peak points are the same. For this particular pure bending case, since the bifurcation phenomenon occurs before the peak point, the bifurcation path affects not only the post-peak response of the specimen, but also the pre-peak response, including the value of the peak point. From **Fig.18**, it can be seen that the failure is initiated by the formation of crack process zone with microcracks in the region of tensile stresses (step $a$). For mesh $A$, at step $d$, which is the peak point, a major crack had already dominated, at step $g$, crack localized with a major crack occurred with others completely or almost completely closed. For mesh $B$, at peak point of step $f$, two major cracks dominated, and at step $h$, it shows the tendency that eventually one major crack would occur. For this example, the supporting condition is not symmetric. The stresses of the elements on the same layer between the two point loads would not absolutely the same. Therefore, one major crack would occur at last, but when the dispersed cracks localize to one crack will have effects on the response of the structure as shown in **Fig.17**.

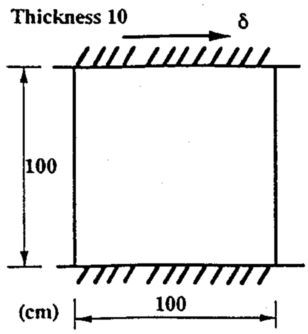


**Fig. 19** Dimension and boundary condition for example 3

This example successfully show the crack localization phenomenon of the pure bending, but for the reason stated above the objectivity of this method can not be shown in this example.

### (3) Shearing

The plain concrete wall is subjected to shear displacement as illustrated in **Fig.19**. A series of computation were carried out with uniformly designed finite element meshes with increasing refinement($4 \times 4$, $6 \times 6$ and $8 \times 8$). The progressive development of cracks with regard to different stages is depicted in **Fig.20**. The inherent

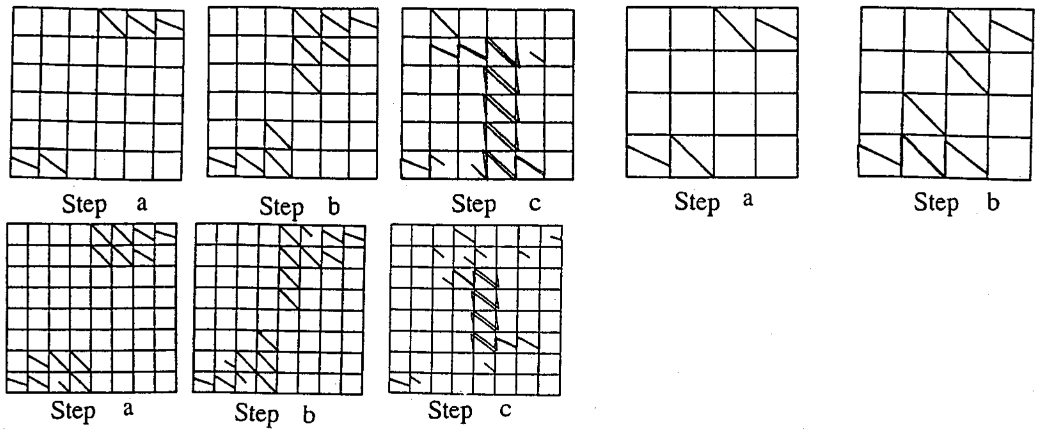


**Fig. 20** Crack patterns for example 3

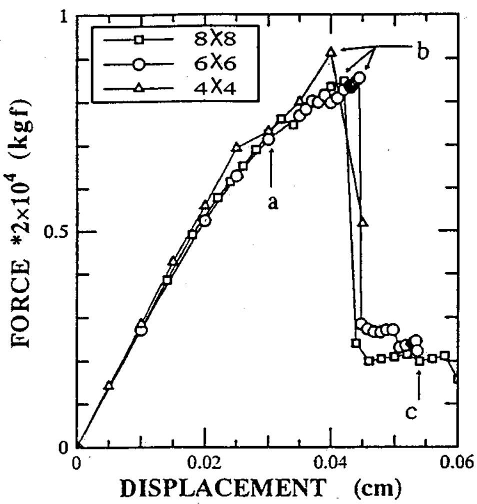


**Fig. 21** Load-displacement curves for example 3

objectivity of the numerical results with respect to the choice of the finite element mesh is reasonably well demonstrated in **Fig.21**

In **Fig.20**, the crack opening and closing procedure is well demonstrated, and at step c the structure finally fails with a major failure zone.

## 7. CONCLUSIONS

In this paper, a new technique for modeling the crack behaviors of concrete structures is presented, where the softening band (crack) inside the element is represented by two rod elements whose behavior is based on the fracture-oriented constitutive relations. In this method, the nonlocal parameters such as internal length measures are not needed. It has been confirmed by the numerical results in this paper that this new method is objective with respect to the choice of the element size, can reflect the localized nature of cracking and can be easily be implemented into the commonly used finite element programs to analyze any arbitrary concrete structures without the difficulties of modifying the existing finite element mesh and pre-defining the concrete crack.

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## コンクリート構造物のクラック局所化に対する内部ロッド要素を含んだ有限要素法

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コンクリート構造物のクラック挙動に対する新しい手法を提案した．そこでは，要素内の軟化域を破壊構成則に従う二つのロッド要素で表した．この新しい手法は，クラック囲りの材料に影響することなく，クラック内部に大きな歪が集中するというクラック局所化の本質を表すことができる．そして同時に，任意のコンクリート構造物を解析する際，汎用有限要素法プログラムに有限要素メッシュの修正等の頻雑なことをする必要が無く，容易に組み込むことが可能である．本論文では，クラック局所化現象を示す為に，3種類の無筋コンクリート構造物（一軸引張，純曲げ，純せん断）に対して解析を行った．