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TOWARDS A MODELLING FRAMEWORK FOR INTERREGIONAL INFRASTRUCTURE PLANNING

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In order to evaluate the economic impacts of sets of interdependent major interregional infrastructure investments, such as transportation, as welll as to allow calibration to data at the regional level, a framework for a probabilistic Spatial Computable General Equilibrium (SCGE) model is introduced. Firstly, the foundations of a probabilistic approach to spatial planning using aggregate data are described. Then, a short-run probabilistic spatial supply model for analysis of multi-sectoral regional production and interregional trade is presented, followed by a sketch of a companion model of spatial household demand. Finally, the short-run spatial supply and demand models are embedded in a long-run sequential dynamic framework, potentially allowing the timing, location and scope of sets of large infrastructure inventments to be better coordinated.

1. INTRODUCTION

Much anecdotal eviednce exists to indicate that major investments in interregional infrastructure, especially transportation, have strong long-term impacts on development of both the interconnected and adjacent regions. Nevertheless, when major interregional infrastructure proposals are evaluated, this in usually on a rather narrow basis, in isolation from similar proposals in competing regions, neglecting the possible existence of strong spatial/economic interdependencies between such projects. Of course, the main reason for this piece-meal approach is that a comprehensive multi-sectoral interregional analysis is extraordinarily difficult, requiring interdisciplinary contributions from transport planning and regional economics.

The art of modelling seems to lie in selection of appropriate levels of detail and complexity, sufficient to characterise the main elements of the system under study and their potential change over time, with this change appropriately decomposed into that caused by both exogenous and endogenous factors. Although, costs have been typically treated as policy variables in infrastructure planning medels, prices have usually been regarded as exogenous. However, as is well known in economics, the evaluation of welfare implications of large infrastructure investment proposals depends critically on both price and quantity (eg. flow) changes, with even small commodity price changes spread over a large region having a potentially large influence on project viability. The only available approach which can consider quantity and price adjustments within a consistent accounting framework is Computable General Equilibrium (CGE), as demonstrated in Shoven and Whalley (1992). In the case of interregional infrastructure planning, this approach needs to be specialised further to Spatial Computable General Equilibrium (SCGE), where, in a bottom-up scheme, regional results can be aggregated up to the national or state level.

The two main countries where the importance of such a comprehensive approach has been recognised are Italy and Japan. In Italy, amid growing concern about future serious congestion in the interregional road freight network, a coalition of civil engineers and economists, with strong funding from the National Research Council, is developing a SCGE model. Early specifications are described in Roson and Vianelli (1993), with later results due for presentation at WCTR'95 in Sydney. In Japan, the main contributors (alphabetically) appear to be Ando and Shibata (1993), Inamura *et. al*. (1994), Mizokami (1995) and Okuda and Hayashi (1994), with Miyagi (1994a) contributing an analysis of a single large region, and several key civil engineers/regional scientists providing further support. In Japan, in particular, where civil engineers appear to be leading the development, there is a major concern to develop models which (i) use readily available data and (ii) are capable of calibration for potential use in forecasting. In the classical economics approach (Intriligator, 1971), whilst production functions, transforming commodity and factor inputs into efficient output, are usually subject to calibration, the profit-maximising input demand and production process is usually medelled deterministically, with the implicit assumption of identical representative firms in each region and conditions of perfect information. On the other hand, random utility or entropy theory, as used conventionally in travel demand analysis, can be applied in enhanced models of the behaviour of the firm and behaviour of the household, introducing a probabilistic structure enabling calibration to observed states of the system, followed by application to forecasting. Although individual choice data from special surveys may sometimes be available to model demand behaviour of households, confidentiality reasons usually prevent the acquisition of individual firm data, with often only average regional statistics of firms being obtainable. This implies that an aggregate probabilistic theory, such as entropy maximisation (Wilson, 1970), is a appropriate. However, the development of a fully probabilistic SCGE framework, for both short-run and long-run adjustments, is a step-by step process. This paper attempts to trace out a few of these steps.

The first section demonstrates the relevance of entropy maximisation techniques in spatial economic planning where aggregate data must be relied upon. In the next section, elements of a short-run probabilistic spatial supply model at the interregional level (Roy, 1995) are introduced. In the case of short-run household demand, the possible structure of a probabilistic interregional model is sketched. Then, long-run behaviour is formulated, where firms and households can dynamically re-locate, or enter or leave the market/system. In conclusion, some requirements for further work are outlined, as well as guidelines for integration with models of a single region. Whilst agglomeration economies need to be considered within regions, congestion diseconomies must be handled both within and between regions.

2. USE OF ENTROPY MAXIMISATION WITH AGGREGATE SPATIAL ECONOMIC DATA

In the use of probabilistic methods, such as entropy maximisation, the solution procedure may be subdivided into two parts (i) parameter estimation and (ii) application to forecasting. Also, entropy itself is presented in two distinct forms (Wilson, 1970) (i) a discrete form via statistical mechanics and (ii) a continuous form using probability theory. Criteria for the equivalence of these two forms are discussed in Fisk (1985) and Roy and Batten (1985). These various aspects are distinguished in the following discussion.

(1) Estimation of Parameters in Entropy Models

a) Discrete entropy via statistical mechanics

In order to illustrate the general principles, we proceed directly to a commodity flow problem, with a restriction to a single commodity in this initial example.

Consider that N_s firms in regions s demand the commodity from among N_r input firms in regions r. In this short-run case, the number of firms and their locations are taken as fixed. Knowing the total observed flows X_r^0 out of regions r during a certain base period, it is necessary to evaluate the commodity flows x_{rs} from r to s. Consider that the average price of one commodity unit at r is p_r and that the buyer firms at s pay the transport costs (i.e. *mill* pricing) c_{rs}, which at this stage are taken as given ā priori. In the manufacture of the final good at s, for which the commodity is the major material input, define a *linear* regional production function $N_s e_s[(\sum_r x_{rs})/N_s]$ assuming identical firms, where e_s denotes the units of the commodity produced per unit of input. Let the labour plus capital costs be an average value w_s per unit of output and let the unit output price be $\bar{p}_s$. Then, the unit profit π_{rs} on a shipment from r of the commodity produced at s, with N_s cancelling, becomes

$$\pi_{rs}=(\bar{p}_s-w_s)e_s-(p_r+c_{rs}) \quad (1)$$

The first step in the solution is the definition of the entropy objective function, following guidelines established in Roy and Lesse (1981). In the statistical mechanics approach (Wilson, 1970), the most probable aggregate result or *macrostate* (eg. the flow pattern x_{rs}) is to be determined. Within these macrostates, *microstates* are identified, representing potential interactions between individual microlevel entities and their available individual choices which are *perceived* to be *distinguishable* or *heterogeneous*. Assuming that all microstates are ā priori equally probable, the most probable macrostate, the required solution, is that associated with the maximum number of microstates. The hypothesis implies that distinguishable entities organise themselves, most probably, to maximise the *potential* number of individual interactions between them. This seems a plausible ā priori objective for a production/buying system.

In defining the microstates, all heterogeneity of individuals and their choices within the aggregates being modelled is identified, insofar as it is perceived to influence choice. For instance, individual beds in a public hospital, whilst heterogeneous, are not available for free choice by the patients. Thus, in hospital planning models, microstates are not defined about the individual beds - attraction is related to the quality of the treatment specialty as a whole. However, in the above flow/production system, each of the N_s output and N_r input firms is likely to be heterogeneous, with specified prices, costs and production levels just representing regional averages. Also, each individual commodity flow unit (or a multiple thereof) within the aggregate flow x_{rs} can be considered as resulting from a *distinct bilateral contract* between a distinct *firm* at r and a distinct *firm* at s, with microstates being defined to reflect all the potential individual choices.

For a given macrostate, defined as a set of flows x_{rs}, $\forall_{r,s}$, let the total number of microstates be denoted as M. In order to capture the essential *symmetry* in the bilateral contracts between input suppliers and producers, all the contracts $X=\sum_{rs} x_{rs}$ (assuming in this initial analysis one shipment per contract) are notionally collected in a *pool*, from which they are allocated into the bilateral flow pattern x_{rs} between regions, yielding M_1 microstates (reflecting that each contract is distinguishable) in the form

$$M_1 = X!/[\pi_{rs} x_{rs}!] \qquad (2)$$

where π represents products and! is the factorial sign. Assuming that the firms N_r and N_s within each region are heterogeneous, they may not, at the microlevel, each have the same level of production (or cost) *within* each region. Thus, additional microstates need to be defined both at r and s to reflect the allocations of x_{rs} to individual firms. Each flow of x_{rs} distinct contracts is arbitrarily allocated at the origin among N_r distinct firms, yielding the total number of microstates (permutations) M_2 in the form

$$M_2 = \pi_{rs}[N_r^{x_{rs}}] \qquad (3)$$

as illustrated in another context in Roy and Lesse (1981). A corresponding contribution M_3 occurs when the individual contracts are allocated to the producing firms N_s, giving

$$M_3 = \pi_{rs}[N_s^{x_{rs}}] \qquad (4)$$

The total number of microstates M is then $M_1^* M_2^* M_3$. As the log of M is a monotonic transformation, its maximisation, rather than that of M, does not influence the result (Wilson, 1970). Also, when the number of flows x_{rs} is not too small, we can make use of the convenient Stirling approximation $\log x! = x(\log x - 1)$. Upon applying this to M as defined in (2), (3) and (4), and realising that the total flow X is a known value X^0 at calibration, the entropy maximisation objective S is given as

$$S = -\sum_{rs} x_{rs}[\log(x_{rs}/N_r N_s) - 1] \qquad (5)$$

where $(N_r N_s)$ reflects the independency distribution of Theil (1967), expressing the flows x_{rs} between regions r and s à priori as proportional to the product of the number of origin firms N_r and destination firms N_s.

Note that, if we had evidence that there was an average of k shipments per contract, all the x_{rs} values in (2) to (4) would need to be replaced by (x_{rs}/k). The reader can check through the following analysis to verify that this would have no influence whatsoever on the results. However, if there was very large variance in k, reflected by defining it as k_r, k_s, or even k_{rs}, the result would be affected. As shown in Roy and Lesse (1985), one may even have to add a shipment size index k to the analysis, expanding x_{rs} to x_{rsk}.

If we lived in a frictionless, homogeneous world, the maximisation of (5) under the total flow constraint $\sum_{rs} x_{rs} = X^0$ would yield Theil's independence solution as described above. In practice, relevant base period information is added as constraints to the maximisation of (5), ensuring that the values of the constraints with the modelled flows x_{rs} coincide with those where the observed flows x_{rs}^0 are substituted. Thus, the modelled total profit from (1) is set equal to the 'observed' profit $\Pi^0 = \sum_{rs} x_{rs}^0 \pi_{rs}$. In fact, with this *constraint* being equivalent to the *objective* of a deterministic economic model (eg. profit maximisation), the analysis of Evans (1973) demonstrates that, as the influence of the entropy reduces, reflected in a continual increase in the magnitude of the Lagrange multiplier on the economic (eg. profit) constraint, the solution of the entropy problem asymptotically approaches that of the corresponding deterministic model, where the constraint coincides with the objective. Thus, the classical economic models emerge as *special case* of the entropy models, with the objective of the former becoming a constraint on the lattter, as evaluated according to the observed flows and prices. This correspondence in vital in attempts to develop probabilistic spatial economic models. The overall strategy for formulating an entropy-based probabilistic model is to apply the objective of the corresponding deterministic model as a constraint, and to define the entropy objective to maximise the potential interactions between all the micro level actors and their choices which are perceived to be heterogeneous. In other words, potential interactivity is maximised, constrained by the properties of the market and the cost of transport.

From the above, and constraining the model to reproduce the observed flows X_r^0 from each input region r, the entropy maximisation Lagrangian Z is defined as

$$\text{Max } Z = \max_{x_{rs},\alpha,\lambda_r} -\sum_{rs} x_{rs}[\log(x_{rs}/N_r N_s)-1] + \sum_r \lambda_r (X_r^0 - \sum_s x_{rs}) - \alpha(\Pi^o - \sum_{rs} x_{rs}\pi_{rs}) \quad (6)$$

where α and λ_r are unknown Lagrange multipliers and the unit profit π_{rs} is defined in (1). Setting $A_r = \exp - \lambda_r$, the maximisation of (6) yields the following result

$$x_{rs} = N_r N_s A_r \exp(\alpha\pi_{rs}) \quad (7)$$

The unknown Lagrange multipliers α and λ_r are determined via an iterative linear extrapolation approach or via the Newton-Raphson method. The modelled flows x_{rs} can be compared with the observed flows x_{rs}^0 using a goodness-of-fit measure, such as the root-mean-square error. If the fit is poor, remedies can be applied, such as the use of a non-linear production function like Cobb-Douglas or CES. Remember also, as shown by Karlqvist and Marksjö (1971), that if an entropy model such as (7) is estimated via the method of Maximum Likelihood assuming a multinomial distribution, parameters such as α and λ_r will come out as identical in value to those evaluated from (6). This is because the maximum likelihood equations associated with α and λ_r turn out to be identical to the corresponding constraints in (6). This result also implies that all the error measures developed for the maximum likelihod approach are directly applicable to our entropy analysis.

Finally, relation (7) permits the definition of a supply function for the expected output $\tilde{x}_s$ at s in terms of prices. Recalling the specification of a linear production function in (1), (7) yields the aggregate supply function at region s as

$$\tilde{x}_s = e_s N_s \sum_r A_r \exp(\alpha\pi_{rs}) \quad (8)$$

As required, this can be shown from (1) to be a (strictly) increasing function of the output price $\bar{p}_s$ and a (strictly) decreasing function of the input prices p_r.

b) Continuous entropy as a measure of uncertainty

As pointed out in Wilson (1970), entropy may be alternatively regarded as a measure of uncertainty of a probability distribution, as proposed by Shannon in 1949. The maximisation of this uncertainty measure under certain constraints yields the most probable result as that in which the constraint information subtracts the least uncertainty from the probability distribution. This conservative hypothesis attributes the minimum information possible from the constraints. If the probability distribution is properly chosen to represent the main stochastic events in the system, the discussion of Roy and Batten (1985), in response to a paper by Fisk (1985), demonstrates that the discrete and continuous approaches can yield identical results. For instance, in the current problem, we could define $p_{rs} = x_{rs}/X$ as the probability that any randomly chosen shipment occurs between regions r and s. The Shannon entropy uncertainty measure $\bar{S}$ would then be in the classical bivariate form

$$\bar{S} = -\sum_{rs} p_{rs}[\log p_{rs} - 1] \quad (9)$$

However, in comparison with (6), it is seen that the $(N_r N_s)$ bias effect is missing from (9). Upon reconsideration, it is clear that the above probability distribution has neglected the assumed heterogeneity of the firms N_r (or N_s) *within* any region. So, following the main message in Roy and Batten (1985), we expand the probability definition to embrace all forms of heterogeneity identified in the microstate approach. The event $\bar{r}$ represents the probability $p_{\bar{r}} = N_r/N$ that an input firm chosen at random is located in region r, whereas the event $\bar{s}$ represents the probability $p_{\bar{s}} = N_{\bar{r}}/\bar{N}$ that a producing firm chosen at random is located in region s. In this, N is the total number of input firms and $\bar{N}$ the total number of producing firms. Let $p_{(rs)/\overline{rs}} = x_{rs}/[N_r N_s X]$ be the conditional probability, given an input firm exists at r and given a producing firm exists at s, a bilateral contract chosen at random exists between them. The entropy of the joint probability distribution $p_{(rs)/\overline{rs}}$ is decomposed (Theil, 1972), with its variable part

$$\bar{S} = -\sum_{rs} p_{\bar{r}} p_{\bar{s}} p_{(rs)/\overline{rs}}[\log p_{(rs)/\overline{rs}} - 1] \quad (10)$$

becoming the maximisation objective. If probabilities are also used in the constraints of (6), with the right-hand sides given as regional output and profits, expressed as per contract per input firm per producing firm, a relation identical to (7) emerges, as expected.

The reader may consider this probability representation as a digression. However, if individual firm data were available, a logit-based approach would be recommended. As this is also defined in terms of probabilities (at the individual level), the sort of thinking about heterogeneity, which led (9) to be replaced by (10), would be required to define the most appropriate probability distribution. Also, this comes back to what is defined as distinguishable in the microstate approach. Note also, that (9) and (10) both represent an expression for the uncertainty of a probability distribution, which obeys certain axioms. It nevertheless has a sense of arbitrariness, in the same way that an error term with a Gumbel distribution has in random utility logit models. Whilst the use of logit models seems preferable for cases where individual data is available (eg. for households in demand models), there seem little to choose between the entropy and logit models when we are confined to using aggregate data.

(2) Applications to Forecasting

The original presentation of entropy theory and its application to spatial modelling by Wilson (1970) did not properly clarify what should be regarded as endogenous vs. exogenous variables in such models. This omission probably delayed the acceptance of entropy-based models and their expansion into new application areas. However, an important paper by Lesse (1982) [very positively reviewed by Wilson himself] finally formalised this distinction. In the following, Lesse's concepts are applied to yield a forecasting version of the above commodity model. Also, a variation of the commodity model is introduced using ideas from vintage analysis (Johansson, 1991).

a) A forecasting version of the commodity model

In (6), the total base period production X_r^0 of the commodity in each input region r, as well as the total profit Π^0 based on the observed flows, must be provided as calibration input. However, in forecasting, such quantities must emerge as output. The first step in the procedure is to carry out an invariant Legêndre transform on the calibration Lagrangian (6) with respect to the right-hand sides of the constraints as follows

$$\operatorname{Max} Z = \max_{x_{rs}} z = \max_{x_{rs},\,\alpha,\,\lambda_r} z - \sum_r (X_r^0 \partial Z/\partial X_r^0) - \Pi^0 \partial Z/\partial \Pi^0 \tag{11}$$

yielding the relationship

$$\operatorname{Max} Z = \max_{x_{rs}} - \sum_{rs} x_{rs}[\log(x_{rs}/N_r N_s) + \lambda_r - \alpha\pi_{rs} - 1] \tag{12}$$

The theory states that, if (12) is solved for x_{rs} using as *input* the Lagrange multipliers λ_r and α obtained as *output* in the calibration problem (6), the solution is unchanged, and substitution of x_{rs} into the left-hand sides of the constraints in (6) will consistently yield the observed values X_r^0 and Π^0 as *output*. Then, if λ_r and α are treated as forecasting *parameters* in (12), changes in transport costs or prices may change the unit profits to π'_{rs} and long-run location changes of firms may yield new values N'_r and N'_s, which including any new short-run input *capacity* constraints ($\leq X'_r$), plus (optionally) constraints on labour/capital $w_s e_s(\sum_r x_{rs})$, gives a forecasting version of (6) via (12) as

$$\begin{aligned}\operatorname{Max} Z = \max_{x_{rs},\,\lambda'_r} & - \sum_{rs} x_{rs}[\log(x_{rs}/N'_r N'_s) + \lambda_r - \alpha\pi'_{rs} - 1] \\ & + \sum_r \lambda'_r (X'_r - \sum_s x_{rs})\end{aligned} \tag{13}$$

With $A'_r = \exp - \lambda'_r$, the forecasting solution becomes

$$x_{rs} = N'_r N'_s A'_r A_r \exp(\alpha\pi'_{rs}) \tag{14}$$

where $A'_r = 1$ when the input capacity constraint is inactive.

Although the process in (11) *formally* transforms Lagrange multipliers into parameters, each Lagrange multiplier must be examined in turn, to see if it has the required stability properties for use in prediction. As the à priori influence of any change in the number of input firms from N_r to N'_r has already been accounted for in the modification of (7) to (14), the Lagrange multipliers λ_r merely reflect a marginal supply generation correction for region r after the size effect via N_r and the relative profit effects via π_{rs} are included. As such, λ_r can reasonably be used as a 'quality' parameter in short term prediction. As the multiplier α can be demonstrated to be a scaling factor of the price elasticities, it also can be confidently used as a parameter for short term forecasting, with the same assurance as in projecting calibrated elasticities in the classical models.

b) An alternative form of the commodity model

In large regions, there is usually some sort of distribution over the relative cost efficiency of the firms. Even within firms, plants/processes of different vintages may exist (Johansson, 1991). In general, the efficient firms will have very high levels of capacity utilisation compared with the less efficient firms. At the regional level, this will tend to yield supply functions of a generic logistic form, as shown by Hotelling (1932). In such cases, as indicated in Roy and Johansson (1993), models need to be developed based on *heterogeneous* units of input firm *capacity*, with implicitly the more efficient units being preferred. As a constant average scaling of capacity units to output levels does not influence the model (see k in earlier case), we consider that each region r has potential output $\bar{X}_r = n_r N_r$ representing distinguishable capacity units, where n_r is the average short-run output capacity per representative firm N_r in region r. Rather than having, as before, the null hypothesis denoting input levels proportional to the number of firms N_r at r, the alternative model's null hypothesis denotes regions r all having the same relative level of *capacity utilisation*. In contrast to the earlier case (13), where capacity $\bar{X}_r$ merely enters as a constraint, here distinct units of heterogenous capacity yield microstates, and help form the objective function itself.

The number of microstates N_2 in (1) is here replaced by the number of permutations $\tilde{N}_2$ whereby $(\sum_s x_{rs})$ distinct orders may be made from among $\bar{X}_r$ distinct capacity units, yielding

$$\tilde{N}_2 = \pi_r \bar{X}_r ! / (\bar{X}_r - \sum_s x_{rs})! \tag{15}$$

where $(\bar{X}_r - \sum_s x_{rs})$ is the unutilised capacity in region r. If this carried forward into the objective of (6), with N_1 and N_3 remaining unchanged, it is not difficult to show that our model (7), upon replacement of the λ_r constraints by a single constraint on total output $\sum_{rs} x_{rs} = X$ with multiplier Ω, becomes

$$x_{rs} = \bar{X}_r N_s C \exp(\bar{\alpha}\pi_{rs}) / [1 + C \sum_s N_s \exp(\bar{\alpha}\pi_{rs})] \tag{16}$$

which is a logistic form, with C defined as $\exp - \Omega$.

So long as the total capacity $\bar{X}=\sum_r \bar{X}_r$ is greater than the total demand X, this model structure ensures that the capacity $\bar{X}_r$ is never exceeded in any region r. In a future period, the forecasting version of (16), with new capacity $\bar{X}'_r$, becomes

$$x_{rs}=\bar{X}'_r N'_s C \exp(\bar{\alpha}\pi'_{rs})/[1+C\sum_s N'_s \exp(\bar{\alpha}\pi'_{rs})] \quad (17)$$

as the counterpart of (14).

For completeness, the probabilistic version of this model replaces event $\bar{r}$ in the previous model (10) by two events $\tilde{r}$ and k. Event $\tilde{r}$ represents the probability $p_{\tilde{r}}=\bar{X}_r/\bar{X}$ that a supplied input capacity unit chosen at random occurs in region r. The conditional probability $p_{k/\tilde{r}}$ is defined in binary form for $k=0$ and $k=1$, with $p_{1/\tilde{r}}=X_r/\bar{X}_r$ denoting that any randomly chosen capacity unit in r is utilised, and $p_{0/\tilde{r}}=(\bar{X}_r-X_r)/\bar{X}_r$ that it is unutilised, where $X_r=\sum_s x_{rs}$ is the total (unknown) utilised capacity at r.

It is clear that (14) is a completely *separable* modol. On the other hand, model (17) reflects interdependent spatial competition by the producing firms for the most preferred units from regions s, introducing an interesting *non-separable* structure. In contrast with (14), where the Lagrange multiplier A'_r on input capacity $\bar{X}'_r$ has no influence (that is, it is zero) until the capacity constraint becomes active, our logistic term introduces an *elastic* constraint, becoming tighter and tighter as the limit $\bar{X}'_r$ is approached (a 'bottom of the barrel' effect in the assumed heterogeneous supply capacity of the regions r). It is recommended that (17) replace (14) where strong intraregional heterogeneity exists in the input supply capacity.

3. ELEMENTS OF A PROBABILISTIC SCGE FRAMEWORK

In a general equilibrium, the supply behaviour of firms and their demand for sector and factor (eg. labour and capital) inputs, as well as the demand behaviour of households for final goods constrained by their budgets, needs to be modelled. As in the simple case above, short-run models are firstly derived, where the locations of firms and households are taken to be given. Then, in the longer run, firms and households are free to re-locate or to leave or enter the market to eliminate any short-run profit or utility differentials. Particularly for the supply models, a framework is presented in detail following Roy (1995), which allows full calibration to observed behaviour.

(1) A Short-Run Model System

a) A producer supply model with intermediate inputs

The previous model for a single commodity needs to be generalised. Eventually, this should be in terms of a commodity-by-sector framework. However, in this initial formulation, we consider $j=1$ to m output sectors and $i=1, m$ input sectors, with the input factors (eg. labour, capital) denoted as $i=m+1, n$. A further extension is the replacement of the simple linear production function by a non-linear form, allowing input substitution effects to be considered. As the production function represents purely a quantity-to-quantity transformation, yielding the maxmum amount of output achievable from a given vector of inputs, its parameters are calibrated *independently*, prior to the inclusion of the production function in the profit-maximising input demand and supply model. As the CES production function is well tested and reasonably tractable, it is used in the following.

In our persistent use of entropy, even with a calibrated non-linear production function, we are implicitly allowing for aggregation error when combining individual firms, their production technology, costs and prices, both for output and transported inputs, into a set of *identical* representative firms within each region, as well as accounting for lack of perfect information on optimal profit-maximising strategies. A similar idea is used when calibrated non-linear congestion functions are attached to entropy-maximising stochastic traffic assignment models. Note that, the widely used Chenery/Moses approach just considers entropy/gravity effects in the specification of trade coefficients, treating the rest of the profit-maximising process as deterministic. On the other hand, Miyagi (1994b), whose paper influenced this author's work in Roy (1995), confines the entropy influence to the revenue part, in forming an entropy production function. As shown in the previous section, our entropy approach is spread over the entire profit-maximising input demand and production process. A further specialisation is achieved by replacing the profit constraint in (6) by separate revenue and cost constraints, potentially allowing the respective Lagrange multipliers to identify different levels of uncertainty in the two contributions to profit for each producing sector j in each region s. However, to retain the essential 'additivity' of the revenue and cost components, yielding the solution of the classical deterministic profit-maximising problem in the limit (Evans, 1973), we connot use the convenient log form of the CES function. Also, rather than formulating a multi-sector version of the separable input demand model (7), as in Roy (1995), we generalise the interesting non-separable model (16), with its implied heterogeneity over the units of input capacity. As the new objective function is a straightforward generalisation of that for a single commodity, it is stated, rather than derived anew.

The input flows x_{ij}^{rs} of products of sector/factor i in region r to producing sector j in region s are to be determined, as well as the output x_j^s implied by the pre-calibrated production function. Let the revenue $\bar{R}_j^{s0}$ of producing sector j in region s be evaluated as the output price p_j^s times the value of the production function when the observed input flows x_{ij}^{rs0} are substituted (*Note* The latter should be very close to the observed base period output x_j^{s0} if the production function has been properly calibrated). The CES production function F_j^s is defined as

$$F_j^s = N_j^s q_j^s [\sum_{ir} q_{ij}^s (x_{ij}^{rs}/N_j^s)^{-\beta_j s}]^{-(h_j s/\beta_j s)} \qquad (18)$$

where h_j^s is the degree of homogeneity, $1/(1+\beta_j^s)$ the elasticity of substitution and N_j^s the number of representative firms producing sector j in region s. Also, with $g_{irs}=(p_i^r+c_i^{rs})$ being defined as the delivered (cif) price of a unit of sector i from region r to s, the total input costs $(\sum_{ir} g_{irs} x_{ij}^{rs})$ are computed as $\bar{C}_j^{s0}$ when the observed flows x_{ij}^{rs0} are substituted. Upon generalisation of (5) and (15), with X_i^0 the observed total input of sector i, the multi-sector entropy maximisation Lagrangian is given as

$$\begin{aligned} \text{Max} Z = \max_{x_{ij}^{rs}, \omega_{js}, \alpha_{js}, \phi_i} & - \sum_{ijrs} x_{ij}^{rs} [\log (x_{ij}^{rs}/N_j^s) - 1] \\ & - \sum_{ir} (\bar{X}_i^r - \sum_{js} x_{ij}^{rs}) [\log (\bar{X}_i^r - \sum_{js} x_{ij}^{rs}) - 1] \\ & - \sum_{js} \omega_{js} (\bar{R}_j^{s0} - p_j^s F_j^s) \\ & + \sum_{js} \alpha_{js} (\bar{C}_j^{s0} - \sum_{ir} g_{irs} x_{ij}^{rs}) + \sum_i \phi_i (X_i^0 - \sum_{jrs} x_{ij}^{rs}) \end{aligned} \qquad (19)$$

where ϕ_i, ω_{js} and α_{js} are unknown Lagrange multipliers and $\bar{X}_i^r$ is the total short-run input producing capacity for sector i in region r. Denoting $\partial F_j^s/\partial x_{ij}^{rs}$ from (18) as $\tilde{F}(x_{ij}^{rs})$ and setting $\exp -\phi_i = B_i$, the maximisation process yields

$$x_{ij}^{rs} = (\bar{X}_i^r - \sum_{js} x_{ij}^{rs}) B_i N_j^s \exp[\omega_{js} p_j^s \tilde{F}(x_{ij}^{rs}) - \alpha_{js} g_{irs}] \qquad (20)$$

If the term in $[\cdots]$ brackets in (20) is called $\tilde{\pi}_{ijrs}$, summation of both sides of (20) over j, s and subtraction from $\bar{X}_i^r$, yields the more explicit result

$$x_{ij}^{rs} = \bar{X}_i^r B_i N_j^s \exp \tilde{\pi}_{ijrs} / [1 + B_i \sum_{js} N_j^s \exp \tilde{\pi}_{ijrs}] \qquad (21)$$

Note that, as $\tilde{\pi}_{ijrs}$ is also a function of x_{ij}^{rs}, (21) must be solved by successive substitution, at the same time as the ϕ_i, ω_{js} and α_{js} multipliers are obtained via the Newton-Raphson method. Furthrmore, as our entropy objective is *strictly concave*, uniqueness of the solution is guaranteed under the same conditions as for the classical deterministic problem with the CES function [i.e. $h_j^s > 0$, $\beta_j^s \geq -1$].

The use of Legêndre transforms with respect to $\bar{R}_j^{s0}$ and $\bar{C}_j^{s0}$, analogously as in (11), followed by inclusion of any changed values (denoted by a prime) by the forecast period, yields the impact analysis version of (21) as

$$x_{ij}^{rs} = \bar{X}_i^{r'} B_i N_j^{s'} \exp \tilde{\pi}'_{ijrs} / [1 + B_i \sum_{js} N_j^{s'} \exp \tilde{\pi}'_{ijrs}] \qquad (22)$$

As the unknown ω_{js} and α_{js} Lagrange multipliers of (19) are now known parameters in (22), it is solved by a simple process of successive substitution.

It is clear that the calibration of (20) for ω_{js} and α_{js}, as well as the precalibration of the production function in (18), is computationally not trivial. One may end up aggregating on regions and/or sectors. However, this is a persistent problem in interregional analysis. Also, if input sector i also produces some final demand, the limit on its total *capacity* $\bar{X}_i^r$ to produce intermediate inputs will be rather fuzzy. Nevertheless, supplied final demand y_i^r is *endogenous* in this model, given as the output x_i^r from (18), when the input demands from (21) are substituted into the production function, minus the sum of intermediate inputs from i at r, in the form

$$y_i^r = x_i^r - \sum_{js} x_{ij}^{rs} \qquad (23)$$

Thus, in the iterative solution of (21), we may progressively update the capacity $\tilde{X}_i^r$ at r to deliver intermediate inputs as the *total* (exogenous) production capacity $\bar{X}_i^r$ at r minus that required for final demand, in the form

$$\bar{X}_i^r = \tilde{X}_i^r - y_i^r \qquad (24)$$

This removes the 'fuzziness' mentioned above.

Relation (21) represents a probabilistic interregional input demand function, and its substitution into (18) yields the corresponding probabilistic supply function. The model may be assessed using the error measures appropriate for maximum likelihood, as mentioned earlier. As it has been constrained to reproduce the important quantities evaluated from observations, it should, ã priori, be expected to perform better than the classical deterministic models in forecasting, where no attempt is made to be consistent with observations in the profit maximisation process.

b) A sketch of a consumer demand model

Whilst there are many analogies in the deterministic procedure between the theory of the household and the theory of the firm (Theil, 1980), the generalisation to a probabilistic analysis yields significant differences in approach. In fact, that is the main reason why this section is denoted as a 'sketch'. The production function, attached as a constraint to the input demand/production model represents an efficient transformation of a vector of input *quantities* into an output *quantity*. As such, it is reasonable that its parameters are estimated *independently*, prior to its inclusion as a constraint in our probabilistic *profit -maximising* input demand/production model. If, by analogy, a deterministic utility function and an accompanying budget constraint were added as constraints to an entropy-type model representing probabilistic utility maximisation, the Lagrange multipliers of this model and the parameters of the utility function, being both concerned with utility maximisation, would be *interdependent*. This feature, pointed out to the author by Professors Kitabatake and Oum during a recent seminar in Kyoto, severely weakens the analogy. Thus, a modest approach is presented here, which may be able to be extended following ideas in Theil (1980).

In general, the behavioural structure of final demand models within a CGE framework is not so clearly defined as for the supply models. The reason is that wholesalers or large retail chains intervene between the producers and households. In the single commodity trade model in Roy and Johansson (1993), the role of exchange intermediaries was recognised explicitly, at the cost of increased complexity. We try to avoid this complexity by adopting a *phenomenological* approach, where using an overall entropy flow objective for final demand goods, we use *revealed* flow and expenditure data to infer the most probable flow pattern. In most cases, the consumers do not *care* in which region their consumption goods are *produced* (with a few exceptions, such as Hokkaido butter !). However, the wholesalers do care. If, in the future, one wants to develop a more fully behavioural model of household demand (eg. random utility), it is suggested that this be run in tandem with a wholesale trade model in which the wholesalers pay the transport costs.

In the short run, given H_k^s households of income class k in region s, with unit budget b_k^s at the base period, we would like to determine the final demands y_{jk}^{rs} for the available goods j at region s from region r by households of class k. Let $g_{jrs}=p_j^r+c_j^{rs}$ be the delivered price of a unit quantity of good j from r to s. Contracts are made between wholesalers at s and individual firms N_j^r for good j at r, with individual consumers selecting from the aggregate of goods. Similar arguments to those in (2) and (3), but with a rather weaker case for discreteness and behavioural integrity (see above), yield the entropy as

$$S=-\sum_{jkrs} y_{jk}^{rs}[\log(y_{jk}^{rs}/N_j^r)-1] \tag{25}$$

The flows are just estimated from *readily available* data, with total *quantity* flows $\bar{y}_j^{r0}$ being given at the *origins*, and aggregate *expenditure* information at the consumption *destinations*, where it is assumed that the average expenditure b_j^{s0} per household for goods of type j in region s can be provided. The addition of the $\bar{y}_j^{r0}$ flow constraints allows identification of any relative final demand delivery potential of region r for good j beyond that implied by the number of firms N_j^r. The b_j^{s0} constraint, supplemented by the bedget constraint, accounts for any different average relative expenditure on good j in region s not purely explainable by the price and budgets and income mix of its households. Thus, the entropy Lagrangian, including the budget constraints, is written as

$$\begin{aligned}\text{Max}Z=\max_{y_{jk}{}^{rs},\,\phi_{ks},\,\lambda_{jr},\,\alpha_{js}} & -\sum_{jkrs} y_{jk}^{rs}[\log(y_{jk}^{rs}/N_j^r)-1]\\ & +\sum_{jr}\lambda_{jr}[\bar{y}_j^{r0}-\sum_{ks} y_{jk}^{rs}]+\sum_{js}\alpha_{js}[H^s b_j^{s0}\\ & -\sum_{kr} y_{jk}^{rs} g_{jrs}]+\sum_{ks}\phi_{ks}[H_k^s b_k^s-\sum_{jr} y_{jk}^{rs} g_{jrs}]\end{aligned} \tag{26}$$

wher $H^s=\sum_k H_k^s$ is the total number of households in s, and α_{js}, λ_{jr} and ϕ_{ks} are unknown Lagrange multipliers. Upon maximisation of (26), the expenditure allocations e_{jk}^{rs} come out in the most suitable form for forecasting as

$$\begin{aligned}e_{jk}^{rs}&=y_{jk}^{rs}g_{jrs}=\\ &H_k^s b_k^s\frac{N_j^r g_{jrs}\exp-[\lambda_{jr}+(\alpha_{js}+\phi_{ks})g_{jrs}]}{\sum_{jr} N_j^r g_{jrs}\exp-[\lambda_{jr}+(\alpha_{js}+\phi_{ks})g_{jrs}]}\end{aligned} \tag{27}$$

in which the unknown Lagrange multipliers can be evaluated via Newton-Raphson iteration. As expected, the demand y_{jk}^{rs} is a strictly decreasing function of delivered price g_{jrs}. Whilst in forecasting, λ_{jr} and α_{js} are treated as parameters, ϕ_{ks} must be adjusted iteratively if the households H_k^s or their budgets b_k^s change. This complication could be avoided if expenditure flows rather than quantity flows were modelled directly (Theil, 1980). This may be examined in future work.

In contrast to the producer model, where distinct bilateral contracts are treated as the obvious decision units, the above consumer model, with the wholesaler intervention, has not such a natural discrete interpretation. Thus, if the reader is happier with the use of continuous entropy as a measure of uncertainty of a probability distribution, he may be guided by the discussion around (10) in defining this distribution with respect to orders from firms at r from an allocated amount of demand. Nevertheless, the result will be the same as obtained in (27).

The concept of 'utility' has not been used explicitly in (27). The consumers are taken to organise their consumption to maximise their choice potential, constrained by their budget and other observed bivariate origin and destination allocations. Alternatively, the most probable flows come from maximising the uncertainty of the corresponding probability distribution. In structure, (27) resembles a logit model, with size biases $g_{jrs}N_j^r$ and linearly additive terms $[\lambda_{jr}+(\alpha_{js}+\phi_{ks})g_{jrs}]$ within the exponential sign. However, at this stage, the author does not see an obvious way to use entropy to effectively add an error term to the estimation of a more general classical utility model, such as the CES or Translog. The main difficulty, as explained in Theil (1980), is that the number of unknown parameters is sometimes not uniquely defined in terms of known flows. This means that estimation usually requires several sets of flows over successive time periods, introducing questionable assumptions about time invariance of the parameters. Models such as (27) can be estimated anew as fresh data becomes available.

If congestion is expected on the network or if modal split is important, these steps can be added, treating the models above as the trip distribution step. The total flows X^i_{rs} of good i between r and s are the sum of the flows of intermediate inputs plus final demand, yielding

$$X^i_{rs}=\sum_j x^{rs}_{ij}+\sum_k y^{rs}_{ik} \tag{28}$$

The quantity units for each good i can be used to convert X^i_{rs} to vehicle units, which can then be summed over all i.

Finally, for computation of the short-run equilibrium, prices p^r_j must be adjusted such that the supplied final demand y^r_i at r from (23) equilibrates with the demand from (27), in the form

$$\sum_{ks} y^{rs}_{jk}-y^r_j=0 \tag{29}$$

The discussion of existence and uniqueness of the equilibrium is a future task. However, analogies with single commodity models in Roy and Johansson (1993) give grounds for optimism.

(2) Long Run Considerations

It should be noted that the short-run probabilistic supply and demand functions formulated above are not homogeneous of degree zero, as in the classical analysis. This implies that *absolute* rather than *relative* prices are required. Nevertheless, in a general equilibrium context, Walras' law must hold, which introduces one degree of indeterminacy into the prices. However, rather than setting one price arbitrarily to unity, as is usual, we must exogenously specify one price at a realistic level. One obvious choice is the price of capital, which is not normally subject to influence at the interregional level, but is determined by national/international factors. If wages are subject to certain rigidities in relation to union/employer negotiations, it may also be preferable to set them exogenously.

With prices regarded as relative, the classical long-run equilibrium condition for firms, expressing zero profits everywhere, does not truly imply no profits, but constant (unknown) profits. In our case, where prices are market prices in relation to the above exogenous factor prices, the zero profit condition is replaced by a condition of achieving a constant value of real profits everywhere. If this is a level of profits regarded as *viable* for that sector, its use as a long-run equilibrium criterion allows firms not only to relocate, but to enter or leave the market in certain regions, so that this specified profit level is attained everywhere.

In a very important paper, Harris and Wilson (1978) presented a long-run sequential dynamic equilibrium model for retail floorspace, where with a very large number of independent operators at each retail centre, a locational Nash equilibrium was achieved when the profits per unit floorspace became constant in each centre. In the profit-maximising input demand/production model formulated previously, an equilibrium distribution N^{s*}_j of the producing firms of type j in region s would occur when the unit profits π_{js}, expressed after *substitution* of the input demand and supply functions, attained a chosen constant viable level $\bar{\pi}_j$ in each region s, taken to be a reasonable expectation in sector j, as follows

$$\pi_{js}=\bar{\pi}_j \ \forall_s \tag{30}$$

This long-run result is to be achieved via a logistic dynamic adjustment process. The iterative solution of (30) will involve considerable mathematical complexity, with multiple equilibria and possible bifurcations being possible (eg. between a more concentrated vs. a more dispersed pattern of interregional activity). In practice, it is suggested that the long-run equilibrium adjustment process be *interrupted*, say at annual intervals, to allow the timing of proposed infrastructure investments to be tested according to different scenarios, as well as allowing other expected changes to be included. Short-run equilibrium must then be reestabished and (30) re-specified, successively altering the path towards long-run equilibrium. Once the time horizon for the latest infrastructure proposal has been reached in this step-by-step process, where we only move *towards equilibrium* at a plausible rate, the model can be run out to a long run equilibrium. This advice is easy to give, but the computational implications are quite daunting. Also, long-run locational equilibrium for the households needs to be achieved simultaneously. From this, the reader will understand why, at the outset of this paper, it was mentioned that only a few possible steps *towards* our final goal were to be covered in detail here!

Although probabilistic supply and demand models have been formulated in this paper, criterion (30) relates to *average* profits rather that *expected* profits or producer surplus, resulting from the distribution over the choice by firms. During a recent seminar, Professor Kiyoshi Kobayashi suggested that equalisation of *expected* profits may be a more consistent long-run criterion than (30) for probabilistic production models. It is difficult to disagree with this suggestion. A similar approach should be applied for the demand models, where consumer surplus rather than the average utility level should be equalised. Future probabilistic models should adopt this idea, and the Harris/Wilson retail equilibrium model be re-formulated accordingly. However, each model should be examined separately, to ensure if conditions for path independence of the surplus integrals are satisfied (Champernowne, Williams and Coelho, 1976). For instance, in (16), the fact that the symmetry condition $\partial x_{rs}/\partial \bar{p}_v = \partial x_{rv}/\partial \bar{p}_s = -\bar{\alpha} e_s e_v x_{rs} x_{rv}/\bar{X}_r$ is satisfied implies path independence and a uniquely defined value for the producer surplus. In such cases, and where the result comes from an extremum problem as in (13), no actual integration is required - the surplus is a constant term plus $(1/\bar{\alpha})$ times the value of the entropy objective (13) when the optimal result (16) is substituted (a type of 'indirect' entropy function). However, in some interesting cases with global interdependency constraints, the above symmetry condition does not hold, and approximate methods must be adopted (Champernowne, *et. al.*, 1976). The user will have to use some judgement to decide whether such global constraint information should be discarded in the cause of theoretical elegance.

4. FINAL DISCUSSION

In models of the short-run, where the locations of firms and households are taken as fixed, a consistent probabilistic framework for formulation of interregional supply and demand functions has been presented. Further work is required, both on specifying more general demand models and establishing conditions for existence and uniqueness of the short-run equilibria. Because infastructure investments are very durable, the long run locational adjustments of firms and households cannot be neglected. Furthermore, if the model is to provide advice on priorities for timing of major projects, the short-run equilibria must be nested a sequential dynamic scheme, in which the path towards long-run equilibrium is systematically updated. For overall consistency within a probabilistic framework, the criteria for long-run equilibria should relate to equalising expected profits (supply) and expected utility (demand). It is clear that this paper and its predecessors, both in Italy and Japan, have only followed through on some of the steps required. Further progress is an important challenge.

The main message from the section on entropy is that when one is trying to model the behaviour of microlevel entities using just macrolevel data, it is vital to define the macrolevel events to identify the major heterogeneity present at the microlevel [eg. between firms N_r in (3) *versus* between units $\bar{X}_r$ of their productive capacity in (15)]. This forms the basis of the microstate descriptions in the discrete case and for the choice of an appropriate probability distribution in the continuous case. In the spatial supply models, where the objective of the corresponding deterministic model is attached as a constraint, with its value computed from observations, the entropy objective of maximising potential interactivity between the heterogenous microlevel units purports to account for the 'dispersion' of the observations from those which would emerge from the deterministic model. However, the spatial demand model, as given in (26) and (27), has no obvious relationship or asymptotic correspondence to any deterministic utility model. However, in some cases, as identified for individual choice models by Le Dam Hanh (1995), it is possible to 'invert' Roy's identity, and move from the demand function, through the indirect utility function to the direct utility function itself. At the same time, the interpretations for the above aggregate demand analysis may not be so clear. The model merely considers sets of heterogeneous final demand firms N_j^r supplying goods in a pool to wholesalers who distribute them to customers via retailers in regions s. As available calibration information, we have the average budgets b_k^s of income groups k in s, the total quantity $\bar{y}_j^{r0}$ of goods j produced in r for final demand and the average expenditure b_j^{s0} per household on goods j in s. The model is phenomenological in that it says - here is a probabilistic interaction law, there is some budget information and some easily obtainable data on origin quantities and destination expenditure - find the most probable consumption pattern. The resulting model has all the properties of a classical demand function, but doesn't use utility! However, it can still be used to obtain consumer surplus. This will certainly be perceived as 'sacrilege' by several readers. Nevertheless, unless we can come to grips with the apparently excessive number of parameters required to estimate some of the non-linear utility functions (Theil, 1980), we either accept a model along the principles of (26)-(27), or use a logit type analysis with a budget constraint.

One of the most interesting current topics in economics is understanding the role of the production and application of knowledge in determining levels of comparative advantage of countries, as well as regions within those countries. In the latter case, agglomeration effects in regions with large populations need to be introduced into regional production functions. After all, it is widely believed that trade-offs between agglomeration economies and congestion diseconomies in the largest cities have strongly influenced rates of regional growth. In the JSCE, recent work by Kobayashi, Sunao and Yoshikawa

(1993) and by Mun and Hutchinson (1995) are yielding insights into this area. As work continues, we may soon be in the position of enhancing the production functions, without which, any proposed SCGE planning model eliminates the key elements contributing to differential growth of regions. With regard to congestion diseconomies, a model for a single region, such as that developed by Miyagi (1994a), could provide changed congestion and land price levels to the household demand and transportation network analysis of a SCGE model. As the interregional locations of households and firms change in the longer run, these changes would need to be fed back into the model of a single region, and modified network conditions and urban land prices fed forward again to the SCGE model.

Finally, as with all model applications, a key constraint on model generality is data availability. This constraint has strongly influenced the structure of the models mentioned previously, such as Ando and Shibata (1993), Mizokami (1995) and Okuda and Hayashi (1994). Of course, this does not mean that we should passively accept poor data. For instance, if we are confronted with the 'ecological paradox', whereby the variance of key data items as measured *within* regions or sectors starts to approach the same order as that *between* regions or sectors, we must either disaggregate or abandon serious analysis. Another key element is flexibility - complexity should not be introduced for its own sake. For instance, Inamura uses a CES production function to account for factor substitution between labour and capital, whilst using simpler input-output between sectors. This all implies that a blueprint for a universal SCGE model does not (and should not) exist - this paper least of all. It has merely tried to illuminate some issues and sketch some possible steps on a path towards a probabilistic model framework with some degree of internal consistency and some potential relevance for empirical application to evaluation of sets of major infrastructure investments.

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REFERENCES

1) Ando, A. and Shibata, T.: A multi-regional model of price and quantity equilibrium for developing economies: an application to evaluating the railroad improvements in China, *Proceedings of International Conference on Regional Science for Development*, Beijing, 1993.

2) Champernowne, A.F., Williams, H.W.C.L. and Coelho, J.D.: Some comments on urban travel demand analysis and economic evaluation of transport plans, *Journal of Transport Economics and Policy*, 10: 276-285, 1976.

3) Evans, S.P.: A relationship between the gravity model for trip distribution and the transportation problem in linear programming, *Transportation Research*, 7: 39-61, 1973.

4) Fisk, C: Entropy and information theory: critique, comments and reply, *Environment and Planning A*, 17: 679-687, 708-710, 1985.

5) Hanh, Le Dam: A study on the class of logit models within the consumer behavior theory, *Doctoral Dissertation, Dept. of Civil Engineering, Gifu University*, 1995.

6) Harris, B. and Wilson, A.G.: Equilibrium values and dynamics of attractiveness terms in production-constrained spatial interaction models, *Environment and Planning A*, 10: 371-388, 1978.

7) Hotelling; H.: Edgeworths's taxation paradox and the nature of demand and supply functions, *Journal of Political Economy*, 40: 577-616, 1932.

8) Inamura, H., Srisurapanon, V. and Tokunaga, Y.: Numerical analysis of multiregional freight flow based on rectangular modelling, 1994.

9) Intriligator, M.D.: *Mathematical Optimization and Economic Theory*, Prentice Hall, New Jersey, 1971.

10) Johansson, B.: Regional industrial analysis and vintage dynamics. *Annals of Regional Science*, 25: 1-18, 1991.

11) Karlqvist, A. and Marksjö, B.: Statistical urban models, *Environment and Planning*, 3: 83-98, 1971.

12) Kobayashi, K., Sunao, S. and Yoshikawa, K.: Spatial equilibria of knowledge production with meeting facilities, In A.E.Andersson, D.F. Batten, K. Kobayashi and K. Yoshikawa (eds) *The Cosmo-Creative Society*, Springer-Verlag, Berlin, 1993.

13) Lesse, P.F.: A phenomenological theory of socio-economic systems with spatial interactions, *Environment and Planning A*, 14: 869-888, 1982.

14) Miyagi, T.: Spatial computable general equilibrium approach to generalised land use/transportation modelling, *Proceedings of Seminar Network Infrastructure and the Urban Environment*, Stockholm, 1994a.

15) Miyagi, T.: The entropy production function and its application to the multi-commodity and multi-sector model, *Annals of Regional Science*, 28: 345-367, 1995b.

16) Mizokami, S.: An interregional trade model considering I/O frame and price equilibrium, To be presented at *7th World Conference on Transport Research*, Sydney, 1995.

17) Mun, Se-il and Hutchinson, B.G.: Empirical analysis of office rent and agglomeration economy: A case study of Toronto, To appear *Journal of Regional*

Science, 1995.

18) Okuda, T. and Hayashi, Y.: A probabilistic approach to multiregional general equilibrium modelling. *International Seminar on Transport Planning and Policy in a Network and Price Equilibrium Framework*, CSIRO, Melbourne, 1994.

19) Roson, R., and Vianelli, P.: Developing a multiregional CGE model for freight transport analysis, *Proceedings of International Workshop on Transportation and Spatial CGE Models*, Venice, 1993.

20) Roy, J.R. and Lesse, P.F.: On appropriate microstate descriptions in entropy modelling, *Transportation Research B*, 15: 85-96, 1981.

21) Roy, J.R. and Batten, D.F.: Invited comment on "Entropy and information theory: are we missing something?", *Environment and Planning A*, 17: 704-706, 1985.

22) Roy, J.R. and Lesse, P.F.: Modelling commodity flows under uncertainty, *Environment and Planning A*, 17: 1271-1274, 1985.

23) Roy, J.R. and Johansson, B.: A model of trade flows in differentiated goods, *Annals of Regional Science*, 27: 95-114, 1993.

24) Roy, J.R.: Dispersed spatial input demand functions, To appear in *Annals of Regional Science*, 1995.

25) Shoven, J.B. and Whalley, J.: *Applying General Equilibrium*, Cambridge University Press, Cambridge, 1992.

26) Theil, H.: *Economics and Information Theory*, North Holland, Amsterdam, 1967.

27) Theil, H.: *Statistical Decomposition Analysis*, North Holland, Amsterdam, 1972.

28) Theil, H.: *The System-Wide Approach to Microeconomics*, Basil Blackwell, Oxford, 1980.

29) Wilson, A.G.: *Entoropy in Urban and Regional Modelling*, Pion, London, 1970.

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