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GEV AND NESTED LOGIT MODELS IN THE CONTEXT OF CLASSICAL CONSUMER THEORY

Hisayoshi MORISUGI[1], Takayuki UEDA[2] and Dam Hanh LE[3]

[1] Member of JSCE, D.Eng., Professor, Dept. of Civil Engineering, Gifu University
[2] Member of JSCE, D.Eng., Associate Professor, Dept. of Civil Engineering, University of Tokyo
[3] Member of JSCE, M.Eng., Graduate Student Dept. of Civil Engineering, Gifu University

Following the development of the proposed consumer theory-based demand model that is discriled in the paper entitle "Logit Models and Gravity Model in the Context of Consumer Behavior Theory", this paper seeks the derivations of the generalized extreme value models (GEV) and the nested logit models. By the context of this study, the so-called quasi-GEV model and quasi-nested logit models are introduced.

Key Words *: extended GEV model, quase-GEV model, conditional GEV model, quasi-nested logit model, conditional nested logit model*

1. INTRODUCTION

Utility maximization has emerged as a fundamental behavioral principle of travel demand modelling. According to this principle, an individual's preferences for the travel options he/she faces can be described by a utility function, and each individual chooses the option that maximizes his/her utility. Logit model is one of the typical applications that is based on this principle and is mathematically derived within the framework of the random utility theory wherein the Gumble distribution of the random part is assumed. Traditionally, the application of the logit model to demand behavior, however, often assumes that the total travel volume of transport services is fixed exogenously. Though this assumption proved significant in most of the practices, especially in regards, to commuting trips in urban areas, it is not always satisfied as far as the generated trips are concerned. Again based on the theory of continuous consumer again and following the same idea as demonstrated in the study "Logit model and Gravity model in the Context of Consumer Behavior Theory", (Morisugi and Le[13]), this paper aims at providing a different derivation of the demand models wherein the share models are mathematical resemblances to the generalized extreme value models (GEV), and the nested logit models (NL). These models permit us to deal with the generated demand.

In the paper of H. Morisugi, and Le D.H.,[13], we have proposed a different derivation of the logit models and the gravity models within the framework of the consumer behavior theory. As the results showed, without loss of generality, first, we can endogenously calculate the total demand of a certain group of commodities/services rather than assuming a fixed level as is usual with logit models. And because of this, second, the model enable us to extensively analyze the generated demand. Third, since the demand model is formulated as the product of two separate-but-consistent components: the total travel volume and the share, the model provided a different travel demand forecasting technique that is quite the reverse process as compared to the conventional one. By the proposed forecasting technique the influences of the commodity other than transport service can be considered and the absolute quantities demanded for these composite goods can also be calculated by using only the observed transport data.

This paper is an attempt at extending the proposed consumer theory-based logit demand model for the further derivation of the generalized extreme value models (GEV), and the nested logit models (NL). The basic difference in the development of this model as compared with the random -based model is that this model depends on the specification of the systematic part of utility function. Thus, by different specifications of the indirect utility function, firstly, the derivations of the looks-like-GEV models are introduced such as the Extend GEV Model, the Quasi-GEV Model and the Conditional GEV Model. Secondly, following the idea of Mc.Fadden (1979) in deriving the nested logit models within the framework of the GEV model that is consistent with random utility maximize (RUM), derivations of the Quasi-Nested Logit Model and Conditional Nested Logit Model are derived, though, in the framework of the proposed conceptual continuous-based-GEV model. Finally, some conclusions are added.

2. SOME IMPLICATIONS OF CONSUMER BEHAVIOR THEORY

Consumer Behavior

In this paper, we derive various probability models that are mathematical resemblances to the GEV models and nested logit models by utilizing the microeconomic theory of consumer behavior in depicting the household's decision problem which can be transformed into a demand function. The representative consumer demand function expresses the action of a consumer. To situate our derivations, we first briefly review the consumer theory before introducing our models.

It is assumed that the consumer maximizes his/her direct utility under a given budget constraint

$$\underset{x}{\text{Max}}\, U(X_1, X_2, \ldots, X_n), \quad \text{s.t.} \sum_{j=1}^{n} p_j X_j = I \tag{1}$$

where U (.) : direct utility function ; X_i : the demand of commodity/service j that is generally assumed to be a non-negative continuous variable ; P_j : price of commodity/service j ; I : income of an individual ; $j \in \Omega 1, 2, \cdots, n\}$ a label of commodities/services.

Then the solution which gives optimal amounts of X_j to the above maximization problem (1) is as follows

$$X_j^* = X_j(P, I) \tag{2}$$

Equation (2) known as the demand function for the commodity i, describes the choice of consumer with respect to the consumption level of commodity i for a given price vector $\boldsymbol{p} = (p_1, p_2, \cdots, p_n)$ and income I. The demand functions which provide an expression for the optimal consumption of commodities/services can now be substituted into the given direct utility function to obtain the maximum utility level that can be achieved under the given price vector and income. This is known as the indirect utility function V $(\boldsymbol{p}, I)$ which is defined by the following

$$V(P, I) = U(X_1^*, \ldots, X_1^*) \equiv \underset{x}{\text{Max}}[U(X), \sum_{j=1}^{n} p_j X_j = I], \tag{3}$$

where X is the consumption vector, $X = (X_1, \cdots, X_n)$.

In many applications it is convenient to deal with the normalized indirect utility function. The normalized indirect utility function is an indirect utility function where prices are divided by income so that the expenditure is identical to one. It is given as

$$V(q) \equiv \underset{x}{\text{Max}}, U(X), \text{s.t.} \sum_{j=1}^{n} q_j X_j = 1 \tag{4}$$

Where q_j is called normalized price, $q_j = p_j / I$. Note that all the four properties of the indirect utility function as specified by Varian (1992) are held w. r. t. the normalized price q vector. One of the important identities of the indirect utility function which will be used as main access to this research work is Roy's identity. Roy's identity shows that the demand function can be expressed as the function of the normalized indirect utility function. It is given as (see Varian (1992), p. 155).

$$X_i(q) = \frac{\partial V(q)/\partial q_j}{\sum_{j'=1}^{n} q_{j'} \partial V(q)/\partial q_{j'}} \tag{5}$$

One may by more familiar with Roy's law wherein the demand function is expressed by a system of partial derivatives of V (p, I) than the one showed in equation (5). Potentially, they are the same and we can derive one from the other. For convenience, Eq. (5) is preferred throughout this study.

3. EXTENDED GEV MODEL

Generalized extreme value model (GEV) is a large class of models which includes the multinomial logit model (MNL) and nested logit model (NL), (Ben-Akiva and Lerman (1985), pp. 126). This model was first derived by McFadden (1976) from within the random utility maximization framework. In this section, our aim is to show how the GEV models can be derived within the framework of consumer utility maximization by extending the proposed conceptual model that was obtained in the paper by Morisugi and Le (1993).

Suppose that the indirect utility function is given as

$$V(.) = \int_q^{\infty} \sum_{i \in \Omega} \Big[\frac{\partial}{\partial y_i} G(y_1(s_1), \ldots, y_n(s_n)) \Big(\frac{-dy_i(s_i)}{ds_i} \Big) ds_i \Big]. \tag{6}$$

Where y_i is the non-increasing function of q_i, and G is μ homogeneous function of a vector associated with y_i. We assume that G is a function with the following properties: (Ben-Akiva and Lermann (1985, pp. 126)).

1. G is non-negative.
2. G is homogeneous of degree $\mu > 0$.
3. $\lim_{y_i \to \infty} G\ (y_1, \cdots, y_n) = \infty$
4. the lth partial derivative of G with respect to any combination of l distinct y_j's, $i = 1, \cdots, n$, is non-negative if l is odd, and nonpositive if l is even.

With these properties, the above line integral function (6) satisfies the following integrability condition (appendix)

$$\frac{\partial}{\partial q_j}\Big[\Big(\frac{\partial}{\partial y_i}G(y_1, \ldots, y_n)\Big)\Big(\frac{dy_i}{dq_i}\Big)\Big] = \frac{\partial}{\partial q_i}\Big[\Big(\frac{\partial}{\partial y_j}G(y_1, \ldots, y_n)\Big)\Big(\frac{dy_j}{dq_j}\Big)\Big]. \tag{7}$$

From Roy's identity (5), by a simple modification, the corresponding total demand for commodities i, $j \in J$, is therefore

$$X_j(q) = \frac{\dfrac{\partial V}{\partial q_j}}{\sum_i \dfrac{\partial V}{\partial q_i}} = \frac{\sum_{j' \in J} \dfrac{\partial V}{\partial q_{j'}}}{\sum_{i \in \Omega} q_i \dfrac{\partial V}{\partial q_i}}, \frac{\dfrac{\partial V}{\partial q_{j'}}}{\sum_{j' \in J} \dfrac{\partial V}{\partial q_{j'}}} \tag{8}$$

Where j is a subset of Ω and its element labels transport services. Note that the demand for commodities i, $X_i\ (q)$, is formulated as the product of the total demand of group commodities J, $N_J\ (q)$, times the share in the group j, $x_j\ (q)$.As an illustrating behavior for this, we can give an example of shopping trip or recreation trip. We can interprete total demand $N_J\ (q)$ as total number of trips and $x_i\ (q)$ as the share of trips to the shopping area or resort place i.

$$\frac{\partial V(q)}{\partial q_i} = \Big[\frac{\partial}{\partial y_i}G(y_1(q_1), \ldots, y_n(q_n))\Big]\Big(\frac{dy_i(q_i)}{dq_i}\Big) \tag{9}$$

For convenience, let us set

$$\frac{\partial}{\partial y_i}G(y_1, \ldots, y_n) = G_{yi}$$
$$\frac{dy_i}{dq_i} = y_{i_{q_i}} \tag{10}$$
$$\frac{\partial V(q)V}{\partial q_i} = G_{yi} \cdot y_{i_{q_i}}$$

Now from equations (9) and (10), the demand for commodity i is obtained as

$$X_i(q) = \frac{\sum_{j' \in J} G_{yj'}\, y_{jqj'}}{\sum_{i \in \Omega} G_{y_j} y_{jq_j} q_j}, \frac{G_{yi} y_{iq_i}}{\sum_{j' \in J} G_{y_i} y_{jqj'}} \tag{11}$$

It is worth to point out that the first term of the right hand side (RHS) of equation (11), mentioned as the total demand of a group commodities J is endogenous and it depends on the relative price of the outside commodities $j \in J$. The second term of the RHS of (11) is the share of commodity j, $j \in J$. Although this general share model is similarly formulated as the GEV model, it has the "interruption" of the partial derivative components showed as Y_{jqj}. Due to this, it is called the *Extended GEV Model* because of its mathematical similarity.

In the GEV model an independent variable y_i *seems to represent the attractiveness of each commodity/services, and* G (.) does the interdependency in stochastic choice. Definition (6) is an interpretation of such a structure of the GEV model in the context of classical consumer theory, and (11) gives us the determination of demand derived from it. Based on the Extended GEV Model, next let us show its variants.

4. QUASI GEV MODEL

In the above section, we have derived the so-called extend GEV model by specifying the indirect utility function as given in Eq. (6). Now let us relax the partial derivative terms in equation (11) by specifying the function of y (.) as follows

$$y_j = \int_{q_i}^{\infty} \exp(-k_i(s_i))ds_i \tag{12}$$

Where $k_i\ (q_i)$ is the non-increasing function. Thus

$$y_{iqi} = -\exp(-k_i(q_i))k_{iqi} \tag{13}$$

Then the demand function of commodity i is obtained as

$$X_i(q) = N_J(q), x_i(q_J) \tag{14}$$

where the logistic components of equation (14) are given as follows

$$N(q) = \frac{\sum_{j \in J} G_{yj} y_{jqj}}{\sum_j G_{y_i} y_{jq_j} \cdot q_j} \tag{15}$$

and

$$x_i(q_J) = \frac{G_{yi}\exp(-k_i(q_i)) \cdot k_{iqi}}{\sum_{j \in J} G_{y_j}\exp(-k_j(q_j)) \cdot k_{jq_j}} \tag{16}$$

Equation (16) looks like the GEV model. But it is not exactly the same due to the existence of an extra term, k_{jqj}, that is the first derivative of function $k_j\ (q_j)$ w.r.t. q_j. For this extra component we prefer to name the model given in equation (16) as the *Quasi GEV Model.*

5. CONDITIONAL GEV MODEL

More specifications of equation (6) in terms of the functional form of y_j (k_j) and k_j (q_j) will be demonstrated in this section. Suppose that functions y_j (.) and k_j (.) are given as

$$y_j=\int_j^{\infty} \exp(-k_j(s_j))ds \tag{17}$$

where in

$$k_j(s_j)=a_j+bs_j \tag{18}$$

Then

$$\begin{aligned} y_j&=-\frac{1}{b}\exp(-a_j-bq_j) \\ y_{jq_j}&=\exp(-a_j-bq_j) \\ k_{jq_j}&=b \end{aligned} \tag{19}$$

Recalling that G is a differentiable function that is homogeneous of degree μ. We specify G (.) as an additively separable junction,

$$G(.)=H(y_1,\cdots,y_{j-1})+\bar{G}(y_j,\cdots,y_{j-1}) \tag{20}$$

Where $1,\cdots, j-1\in$ and $j,\cdots n\in J$. Thus by Euler's law, equation (16) becomes

$$\begin{aligned} x_i(q)&=\frac{G_{y_i}\exp(-a_i-bq_i)}{\sum_{j\in J}\bar{G}_{yi}\exp(-a_j-bq_j)} \\ &=\frac{G_{yi}e^{-k_j(qi)}}{\mu\bar{G}[e^{-k_j(q_j)},\ldots,e^{-k_n(q_n)}]} \end{aligned} \tag{21}$$

where define

$$e^{-k_j(q_j)}=e^{-a_j-bq_j}$$

Equation (21) is exactly the so-called GEV model in the context of Ben-Akiva and Francois (1983) where in the context of this paper, function V_j that is the systematic component of the random utility function, is replaced by the so-called sub-utility function k_j (q_j) assuming as linear form. Due to this conditional derivation, the model as shown in (21) is called a *Conditional GEV Model.*

However, it is noteworthly the conditional GEV model is consistent with endogenous total demand for a group of commodities/services J, while it has the exactly same form of share expression as the GEV model.

6. QUASI NESTED LOGIT MODEL

McFadden (1978) has shown that the generalized extreme value model is consistent with the random utility maximization, and from that he has provided a generalization of nested logit models. We have derived the GEV models from within the ordinary consumer behavior theory. The following is an attempt to derive the nested logit models within the framework of the proposed conceptual GEV model (21). In equation (21), suppose that given function G (.) as

$$G(q)=\sum_{m=1}^{M}(\sum_{j\in D_m} y_j)^{\mu} \tag{22}$$

where

$$y_j=\int_{q_j}^{\infty} \exp(-k_j(s_j))ds_i \tag{23}$$

Where $m\in M=\{1,\cdots,h\}$ and $h\leq n$, label a subgoup of commodities/services. D_m is subset of commodities/services coresponding to m. It is needless to say that a subgroup means destination and its element is the mode available in travelling to the destination. Therefor, subscript j labeling a commodities/services in group J denotes a combination of the destination and the mode. Then, $D_m \subset J$ and $D_m \cap D_m' \equiv \phi$. With these specifications, equation (20) can now be given as

$$x_{im}(q)=\frac{e^{-k_i(q_1)}(\sum_{j\in D_m} y_j)^{\mu-1}}{\sum_{m\in M_n}[\sum_{j\in D_m} q_j\cdot e^{-k_j(q_j)}(\sum_{j\in D_m} y_j)^{\mu-1}]} \tag{24}$$

Purposely, equation (24) is rewritten and given in the following equation.

$$X_{im}=\left[\frac{\sum_{m\in M_n}[\sum_{j\in D_m} e^{-k_j(q_j)}(\sum_{j\in D_m} y_j)^{\mu-1}]}{\sum_{m\in M_n}[\sum_{j\in D_m} q_j e^{-k_j(q_j)}(\sum_{j\in D_m} y_j)^{\mu-1}]}\right]* \left[\frac{e^{-k_i(q_1)}(\sum_{j\in D_m} y_j)^{\mu-1}\sum_{j\in D_m} e^{-k_j(q_j)}}{\sum_{j\in D_m} e^{-k_j(q_j)}\sum_{m\in D_n}[(\sum_{j\in D_m} y_j)^{\mu-1}\sum_{j\in D_m} e^{-k_j(q_j)}]}\right] \tag{25}$$

It is recognized that the first term of the right hand side (RHS) of Eq. (25) presents the total demand of a group of commodities/services, e. g. total demand for transport service, is denoted as N_{DM} ($\boldsymbol{q}$). Obviously, N_{DM} ($\boldsymbol{q}$) is not an exogenous variable, rather it can be endogenously computed. This result concurs with the one we first obtained in Eq. (11). Now our interest turns to what the second term of the RHS of (25) means. To be more visible, the second term can be rewritten and defined as

$$x(i,m)=x(i|m),\, x(m)$$

$$=\frac{e^{-k_j(q_i)}}{\sum_{j\in D_m} e^{-k_j(q_j)}}\cdot\frac{\sum_{j\in D_m} e^{-k_j(q_j)}(\sum_{j\in D_m} y_i)}{\sum_{m\in M_n}[\sum_{j\in D_m} e^{-k_j(q_j)}(\sum_{j\in D_m} y_j)^{\mu-1}]} \tag{26}$$

x (i, m) is known as a joint probability in the context of random theory, wherein the combination of mode (m) and destination (i) is chosen. It consists of two choice probabilitis; the conditional choice probability, x ($i|m$), and the marginal choice probabilities x (m). The conditional choice probability is the probability that destination i is chosen on condition of the given subgroup m. The formulation

$$x(i|m)=\frac{\exp(-k_i(q_i))}{\sum_{j\in D_m}\exp(-k_j(q_j))} \tag{27}$$

is given by the logit model with the exponential component exp $(-k_i$ (q_i) Which is the separable function w.r.t. q_i. The second term of the RHS of (26)

$$x(m)=\frac{e^{\ln\sum_{j\in D_m} e^{-k_j(q_j)}+(\mu-1)\ln\sum_{j\in D_m} y_j}}{\sum_{m\in M_n} e^{\ln\sum_{j\in D_m} e^{-k_j(q_j)}+(\mu-1)\ln\sum_{j\in D_m} y_j}} \tag{28}$$

is the so-called marginal choice probability by the context of random utility wherein transport mode m is chosen. Though this model looks like the "original" logit model, it is not exactly the same due to the extra term of ln $\Sigma_{j\in Dm}$ y_j. Due to this extra problem, the nested probability as derived in equation (26) is not a perfect nested logit model. Rather, in the context of this paper, we prefet to name it *the Quasi-Nested Logit Model.* Moreover, if the scale parameter μ is normalized to 1, the marginal choice probability given in Eq. (28) becomes the logit mode. Thus, the nested probabilities (26) has become the multinomial logit model (MNL) with the normalized scale parameter $\mu=1$.

Clearly, the demand of the mode-destination choice, X_{im}, is again generally formulated as the product of the two separated-but-consistent components; the total demand of transport services and the nested probability wherein the combination (i, m) is chosen. The only difference is that in this model, the probability model is given by the so-called quasi nested logit model with its logistic components a depicted in Eqs, (27) and (28). Noticeably, all the merits of this general demand formula with regards to the generated demand that have been discussed in the paper by Morisugi and Le (1993), are still faithful to this model.

As the result shows, though we may not be able to derive the general nested logit model directly from the theory of consumer behavior, we have succeeded in generating the general probability model, as well as the multinomial logit model (MNL) within the framework of the continuous behavior theory.

7. CONDITIONAL NESTED LOGIT MODEL

Though we still have to derive the nested logit model, we have already derived the so-called quasi nested logit model due to the extra term as shown in equation (28). This problem itself has hinted that we can derive the "exact" nested logit models by specifying the suitable functions of y_j (.) and k_j (.). Suppose that with the specifications y_j (q_j) & k_j (q_j) as given in Eqs. (17) & (18), equation (28) now becomes

$$x(m)=\frac{e^{\ln\sum_{j\in D_m} e^{-a_j-bq_j}+(\mu-1)\ln\sum_{j\in D_m} e^{-a_j-bq_j}}}{\sum_{m\in M_n} e^{\ln\sum_{j\in D_m} e^{-a_j-bq_j}+(\mu-1)\ln\sum_{j\in D_m} e^{-a_j-bq_j}}}$$

$$=\frac{e^{\mu\ln\sum_{j\in D_m} e^{-aj-bq_j}}}{\sum_{m\in M} e^{\mu\ln\sum_{j\in D_m} e^{-aj-bq_j}}} \tag{29}$$

With this logit-like marginal choice model, the joint probability choice, x (i, m), as given in (26) becomes

$$x(i, m)=x(i|m)*x(m)$$

$$=\frac{e^{-a_j-bq_i}}{\sum_{j\in D_m} e^{-a_j-bq_j}}*\frac{e^{\mu\ln\sum_{j\in D_m} e^{-aj-bq_j}}}{\sum_{m\in M_n} e^{\mu\ln\sum_{j\in D_m} e-a_j-bq_j}} \tag{30}$$

Equation (30) is the formula of the nested logit model with the scale parameter μ. The exponential component in (30) appears as the linear one variable function. The looks-like-nested logit model (30) is derived from conditions on the functions of $y_i(q_j)$ and k_j (q_j), therefore it is named the *Conditional Nested Logit Model.*The conditional Nested Logit Model has exactly the same form as the nested Logit Model that has been a stylized model in transport analysis. However, let us again emphasize that the model proposed here has been derived from the classical consumer theory.

8. CONCLUSION

Within the context of the "ordinary" consumer behavior theory, though we have not yet be able to obtain the general framework from which to derive the "exact" GEV models and the nested logit models as well, the study has shown several different derivations of the likeness-models such as the Extended GEV Model, the Quasi-GEV Model, the Conditional GEV Model, the Quasi-Nested Logit Model and the Conditional Nested Logit Model. In spite of the involvement of different density functions in the conventional derivations, e.g. normal distribution, Gumble distribution, etc, these derivations are depending on the functional form of the indirect utility function as additive and separable functions. While retaining the distinguished properties that are obtained in the paper by Morisugi and Le (1993), the potential of the proposed model in deriving the alikeness-GEV models and nested logit models gains three implications. Firstly, it grants another alternative of deriving the GEV models and nested logit models possible with the random theory–based model. Secondly, with the derived demand models, the study provides a more consistent demand forecasting model wherein the models of different alternatives (refering to the trip generation and mode choices) are separable but consistent with regard to its formulation, parameters, and variables. Thirdly, the study gives more significant ground to generate an entire general framework from which the demand models and the evaluation measures of a transport system can be carried out consistently and accurately with regard to the users behavior preferences.

ANNEX

As previously mentioned, this is the expands on a previous one entitled. "Logit Model and Gravity Model in the Context of Consumer Behavior Theory". To help the readers follow the extended development of this study more easily, a brief summary of the results and comments that were obtained in the first paper are mentioned here.

(1) General logit model

As mentioned above, suppose that given any indirect utility function V ($\boldsymbol{q}$), the resulting consistent demand function with that indirect utility function is given by Roy's identity formula (as shown in Eq. 5 of this paper). Now by multiplying the numerator and denominator of (5) by $\sum_{j\in J} \partial V(\boldsymbol{q})/\partial q_j$, where J is a set of commodities/services, and obtaining the total demand for commodity i as

$$X_i(q)=N_J(q)*x_{iJ}(q), i\in J \qquad (1')$$

where

-N_J ($\boldsymbol{q}$) is the total consumption of a given group of commodities/services J, that is explicitly obtained as

$$N_J(q)=\frac{\sum_{j\in J}\partial V(q)/\partial q_j}{\sum_{j=1}^{n} q_j\cdot\partial V(q)/\partial q_j} \qquad (2')$$

-x_{iJ} ($\boldsymbol{q}$) denotes the frequency (probability) of purchasing commodity i within group J. It is formulated in the form of a share model

$$x_{iJ}(q)=\frac{\partial V(q)/\partial q_i}{\sum_{j\in J}\partial V(q)/\partial q_j}, i\in J \qquad (3')$$

It is clearly recognized that the total demand for commodity i, X_i ($\boldsymbol{q}$), as shown in Eq. (1') is formulated as the product of the total consumption of a group of commodities/services J, N_J (q) and the share of the commodity i in the total consumption of the focused group of commodities/services J, x_J ($\boldsymbol{q}_J$). This share model takes the form of a logit model and satisfies the requirement that $\sum_i x_i=1$, $i\in J$.

(2) A specific Indirect Utility Function

In practice, it is necessary to specify the indirect utility functions in all applications. There are a number of different functional forms of V (.) from which to choose, depending on the purposes/requirements of the analysis. The following specific indirect utility function is one of the most significant functions which is chosen to verify the proposed model and to further discuss the interpretation of the share model under the framework of this study. Suppose, the indirect utility function is given in the additive and separable form as

$$V(q)=F[\sum_{j=1}^{n}\int_{q_j}^{\infty}\exp(k_j(s_j))ds_j] \qquad (4')$$

Where, F is the increasing monotonous function of $\sum_j \int \exp k_j(s_j)\, ds_j$, and k_j (.) are arbitrary functions such that the equation (4') satisfies the characteristic properties of an indirect utility function, with $q_j=(p_j/I)$. In the context of this study, function k_j (q_j) can be considered as the subutility function. Obviously, from (4') the explicit equations for the total consumption N_J (q), and the choice probability of commodity i, x_j (q_J), are easily obtained. Here, we place more emphasis on the study of travel forecasting through the share type of logit model. For more convenience, the discussion is restricted on three goods case, $i=1, 2, 3$, where the subscript $i=1$ denotes the composite goods and $i=2, 3$ are transport modes. Following the proposed framework, the total demand of the transport mode 2 is computed by using Roy's identity. The result is given as

$$X_2(q)=\frac{\exp(k_2(q_2))}{\sum_{j=1}^{3}q_j\exp(-k_j(q_j))} \qquad (5')$$

By multiplying the nominator and denominator of (5') by $\sum_{j=2}^{3}\exp(-k_j(q_j))$, the demand for transport mode 2 now becomes

$$X_2(q)=\frac{\sum_{j=2}^{3}e^{(-k_j(q_j))}}{\sum_{j=1}^{3}q_je^{(-k_j(q_j))}}\cdot\frac{e^{(-k_2(q_2))}}{\sum_{j=2}^{3}e^{(-k_j(q_j))}} \qquad (6')$$

Equation (6') is the product of the two components. The first component of RHS of (6') is

$$N(q_1, q_2, q_3) = \frac{\sum_{j=2}^{3} \exp(-k_j(q_j))}{\sum_{j=1}^{3} q_j \exp(-k_j(q_j))} \tag{7'}$$

which precents the total demand of transport service that is the function of not only the normalized prices of the transport goods q_2, q_3 but also the outside goods (q_1). The second component of RHS of (6') is the share of the transport mode 2 in the total transport demand. It is the function of q_2 and q_3 and can be formulated as follows

$$x_2(q_2, q_3) = \frac{\exp(-k_2(q_2))}{\exp(-k_2(q_2)) + \exp(-k_3(q_3))} \tag{8'}$$

From equation (6'), the demand for transport mode 2 is obtained as

$$X_2(q_1, q_2, q_3) = N(q_1, q_2, q_3) * x_2(q_2, q_3) \tag{9'}$$

Though this formula is identical with (1'), the difference is that the exponential component exp k_j (q_j) is more specified as a separable function in terms of q_j. Moreover, since the formulation (7') includes the characters of composite goods, $i=1$, the travel demand as given in equation (9') has the possibility to consider the related influences of the outside-factors into the travel forecasting. This merit provides a comprehensive technique to the transport demand forecasting, and through this, the demand for the composite goods can also be easily computed. The proposed demand forecasting technique is the inverse process as compared with the usual calculation procedure. While the usual operation starts with the estimation of the total volume and do estimate the modal share, both are carried in different experiments, this technique consistently estimates the modal share first, and then obtains the relative total volumes.

Note that, due to being formulas of N (.) and x_j (.) as shown in Eq. (6') being separate and computable, all the consequences of the demand forecasting process do not only result in percentage figures (%), that are usual for the share model, but also result in numerical quantities, which, of course, include the generated travel demand.

There are four implications that emerged from these results. Firstly, with any given indirect utility function, we can derive the logit-like model as shown in Eq. (3'). Secondly, the total consumption level of a focused group of commodities/services N (q), is an endogenous variable that changes according to the variation of the contributed factors of both the inside group J goods and also the outside goods (as refers to $j=1$). Thirdly, because of this distinguished merit, the proposed model can be used to easily and explicitly analyze the generated demand. Having this property, we have overcome the restriction of the conventional approach. Fourthly, the model provides a consistent demand forecasting technique that is the inverse of the conventional four-step method. While this estimation seems to retain the computational advantages of the previous technique, it is accomplished with regards to consistency and generated demand.

There still several implications of the proposed model in regards to the derivations of the conditional logit model, the gravity models, and etc,. For more detail, refer to the original paper.

* *

APPENDIX

The specified integral indirect utility function as given in equation (6) satisfies the integrability condition (7) because:

$$\frac{\partial}{\partial q_j}\Big[\Big(\frac{\partial}{\partial y_i}G(y_1, \ldots, y_n)\Big)\Big(\frac{dy_i}{dq_i}\Big)\Big]$$

$$=\Big(\frac{dy_i}{dq_i}\Big)\Big[\frac{\partial^2}{\partial y_i \partial y_j}G(y_1, \ldots, y_n)\Big]\Big(\frac{dy_j}{dq_j}\Big)$$

$$=\Big[\frac{\partial^2}{\partial y_i \partial y_j}G(y_1, \ldots, y_n)\Big]\Big(\frac{dy_i}{dq_i}\Big)\Big(\frac{dy_j}{dq_j}\Big)$$

and the symmetry of cross-partial derivatives.

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古典的消費者行動理論におけるGEVモデルとNested Logitモデル

森杉壽芳・上田孝行・LE Dam Hanh

著者は，既に発表した論文で，古典的な消費者行動理論にもとづいてLogitモデルと動モデルを誘導できることを示した．本研究では上記の論文と同じく古典的消費者行動モデルにもとづいて，GEVモデルとNested Logitモデルの誘導を試み，より一般的な拡張GEV (Nested Logit)，準GEV (Nested Logit) および線型GEV (Nested Logit) モデルの誘導に成功した．