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LOGIT MODEL AND GRAVITY MODEL IN THE CONTEXT OF CONSUMER BEHAVIOR THEORY

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Logit model is the simplest and best-known probability choice model that is derived within the framework of the Random utility theory. Since the standard logit model treats the total consumption level as given, there remains the problem of dealing with the generated transport demand. This study proposes a different derivation of the logit type of demand model that is a mathematical resemblance of the logit model in the context of consumer behavior theory. Without loss of generality, the model enables us to explicitly analyze the generated demand and is further used to study the share model and to provide a likely different technique to forecast the absolute quantities of the travel as well as of the other goods. Moreover, a gravity model can also be derived. Finally, the consistency of the proposed model is verified by the derivation of the consistent direct utility functions.
Key Word : *logit model, generated demand, absolute quantities, gravity model, consistent direct utility function, and log-sum function.*

1. INTRODUCTION

Application of the logit model has been given increasing attention in the transport behavior analysis due to its simple formulation. However, the application of the logit model in the demand behavior often assumes that the total consumption of goods/services is fixed exogenously at N. While this assumption is explainable in some context, it is not always completely satisfied (e.g., the number of generated demand due to the introduction of new policy/service is quite a significant element in many economic analyses). Though the independent from irrelevant alternative (IIA) restriction of the conventional logit model has been almost overcome by the development of the nested logit model or the probit model, the generated demand problem is yet to be solved. This paper will present a different derivation of the logit type of demand model that is a mathematical resemblance of the logit model in the context of consumer behavior theory. In this formulation, the linkages between the consumer indirect utility function and the demand function are fully utilized to directly derive the general share model from any given indirect utility function through the application of Roy's identity. Also, the study shows how the share type of logit model and the gravity model, can be derived from the specific additive and separable indirect utility functions. For the main model developed, next it shows that the consumption level of the commodity/service $i, i=1, \ldots, n$, is formulated as the product of two components, the total consumption of all or one group of commodities/services and the share of purchasing commodity/service i among all or the group of commodities/services. The model also shows that since the total consumption of a group of commodities/services can be determined endogenously, it thus enables us to extensively analyze the generated demand. Moreover, the study of the share type of logit model provides a likely different travel forecasting technique, in which, if one specifies the functional form of the demand functions (referred to as g_1, g_2, g_3 in section 3), the absolute quantities demanded for commodities/services other than travel service can also be obtained through only the data of transport service.

According to our classification, the approaches to the theoretical transport behavior analysis can be divided into three approaches ; (1) Continuous choice, (2) Discrete choice, and (3) Continuous and Discrete choice modelling. The first group has the longest microeconomic history with which Niedercorn and Bechdolt (1969), Golob and Beckmann (1971), Beckmann and Golob (1972), and Golob, Gustafson and Beckmann (1972), had successfully derived the gravity model from a specified direct utility function in the context of continuous consumer theory. But they did not derive the logit model nor used duality in consumer theory on which this model fully utilizes. Hausman (1981), in his remarkable study in this field also demonstrated the solution for the system of partial differential equations which express the system of demand functions by using the Roy's identity. This study also follows his idea in obtaining the necessity of the indirect utility function for the given demand function. Around 1970, McFadden, Lerman, Ben-Akiva and their associate group began to develop

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the discrete choice model based on random utility theory (Manski and McFadden (1981), Ben-Akiva and Lerman (1985)). Following this extensive development of discrete choice modelling, Hanemann (1981) and Jong (1990) and others developed the continuous and discrete choice model. Based on the classical consumer modelling, this study aims to develop a reasonable justification to the share model of the logit type with special reference to the absolute amount of demand rather than the share itself.

Evidently, this approach tracked out a new orientation in deriving the logit model in the context of continuous consumer behavior beside the random utility theory. In this respect, we found that our paper has an analogy to the approach of Anderson, Palma, and Thisse (1992)[2]. Their study emphasizes, however, on the functional form of indirect utility function of the consumer representing a discrete choice model. Based on the assumption that an individual chooses only one of the variants offered, they showed that the logit model can be described to be consistent with a representative consumer model through the specification of the linear random utility model, LRUM. The difference with our model is that the logit model developed by them are independent of income as long as all consumers can afford each variant (see Anderson, Palma, Thisse (1992), pp. 73). Also, the restriction on the fixed value of the total demand still existed in their model. While in this approach we place more emphasis on functional form of the travel demand as a logit model. The study which is also based on the continuous model that is developed through the linkages between the logit model and the consumer behavior theory shows how a share type of logit model could be constructed from any given indirect utility function. Furthermore, the study is developed to verify the consistency of the demand model through the derivation of the consistent direct utility function. Finally, the conclusion is added to sum up all the results which we have obtained in the study.

2. SOME IMPLICATIONS OF CONSUMER BEHAVIOR THEORY

(1) Consumer Behavior

In this paper, we study the multinomial logit model by utilizing the microeconomic theory of consumer behavior in depicting the household's decision problem which can be transformed into a demand function. The representative consumer demand function expresses the action of a consumer. It is assumed that the consumer maximizes his/her direct utility under a given budget constraint

$$\max_X . U(X_1, X_2, \dots, X_n),$$

$$\text{s.t.} \sum_{j=1}^{n} p_j X_j = y \quad \cdots\cdots (1)$$

Where $U(.)$: direct utility function ; X_j : the demand of commodity/service j that is generally assumed to be non-negative continuous variable ; p_j : price of commodity/service j ; y : income of an individual ; $j=1, \dots, n$ are commodities/services. Then the solution which gives optimal amounts of X_i to the above maximization problem (1) is as follows

$$X_i^* = X_i(p, y), i=1, 2, \dots, n \quad \cdots\cdots (2)$$

Equation (2) known as the demand function for the commodity t, describes the choice of consumer with respect to the consumption level of the commodity i for a given price vector $\boldsymbol{p}=(p_1, p_2, \dots, p_n)$ and income y. The demand functions which provide an expression for the optimal consumption of commodities/services can now be substituted into the given direct utility function to obtain the maximum utility level that is achievable under the given price vector and income. This is known as the indirect utility function $V(\boldsymbol{p}, y)$ which is defined by the following

$$V(p, y) \equiv$$

$$U(X_1^*, \dots, X_n^*) = \max_X . [U(X), \sum_{j=1}^{n} p_j X_j = y], \quad \cdots\cdots (3)$$

where $\boldsymbol{X}$ is the consumption vector, $\boldsymbol{X}=(X_1, \dots, X_n)$. It is well known that this function has the following four properties for the reasonable consumer theory :

(1) $V(\boldsymbol{p}, y)$ is continuous at all $\boldsymbol{p} \gg 0, y > 0$,
(2) $V(\boldsymbol{p}, y)$ is monotonically increasing in y, and decreasing in price p,
(3) $V(\boldsymbol{p}, y)$ is homogeneous of degree 0 in $(\boldsymbol{p}, y)$,
(4) $V(\boldsymbol{p}, y)$ is convex in $\boldsymbol{p}$.

(2) Roy's identity

One of the important identities of the indirect utility function which will be used as main access to this research work is Roy's identity. Roy's identity which shows the observed market demand function is expressed by a system of the partial derivatives of $V(\boldsymbol{p}, y)$. It is stated that if $\boldsymbol{X}(\boldsymbol{p}, y)$ is the Marshallian demand function, then

$$X_i(p, y) = -\frac{\partial V(p, y)/\partial p_i}{\partial V(p, y)/\partial y}, i=1, 2, \dots, n, \quad \cdots\cdots (4)$$

provided that the right hand side is well defined and that $\boldsymbol{p} \gg 0$, $y > 0$, (see Varian (1992), p.106).

In many applications it is convenient to deal with the normalized indirect utility function. The normalized indirect utility function is an indirect utility function where prices are divided by income so that the expenditure is identical to one. The normalized indirect utility function is given by

$$V(q)=\max_X . U(X), s.t. \sum_{j=1}^{n} q_j X_j=1 \cdots\cdots (5)$$

From this definition and Roy's law, the demand function can be expressed as the function of the normalized indirect utility function (Varian (1992), p.155) as follows

$$X_i(q)=\frac{\partial V(q)/\partial q_i}{\sum_{j=1}^{n} q_j \cdot \partial V(q)/\partial q_j} \cdots\cdots (6)$$

where $q_j=p_j/y$, and $j=1, 2, \ldots, n$.

(3) Duality in Consumption

Once we use the normalized price vector $\boldsymbol{q}$, then we may have quite convenient duality between the direct and indirect utility functions. If we are given the indirect utility function $V(\boldsymbol{q})(Eq. 5)$, we can find the direct utility function $U(\boldsymbol{X})$ by solving the following problem.
(see Varian (1992), p.155~156) (refer to appendix 2)

$$U(X)=\min_q . V(q), s.t. \sum_{j=1}^{n} q_j X_j=1 \cdots\cdots (7)$$

Moreover, the link between the indirect utility function and the demand function shows that if one has a specific functional form for the indirect utility function, the form of the ensuing demand can be arrived at by differentiation and using equation (6). If one has a specific form for a demand function, the indirect utility function which corresponds to it can also be found by integration via equation (6). This identity ensures that we can derive the consistent indirect utility function from a given demand function.(see appendix 1). These properties will be utilized throughout the following sections, specially in the derivation of the consistent direct and indirect utility functions from a given logit model, as well as in the derivation of the logit demand model, and gravity model from the specified indirect utility functions. Note that, though equations (4) and (6) are identified and well defined in the context of the consumer behavior theory, in this study we prefer to use (6) throughout all discussions, for convenience (see section 3 & 4 for further details).

3. A DERIVATION OF LOGIT MODEL

(1) General logit model

In this section, we shall provide a new derivation of the logit model within the framework of the consumer behavior theory which is quite different from the conventional approach of deriving the logit model within the context of discrete choice theory. Following the Roy's identity (6), suppose that given any indirect utility function $V(\boldsymbol{q})$, the resulting demand function consistent with the given indirect utility function is formulated as equation (6). Multiplying the numerator and denominator of (6) by $\sum_{j\in J}\partial V(\boldsymbol{q})/\partial q_j$, where J is a set of commodities / services. Rewriting the total demand for commodity i, equation (6) now becomes

$$X_i(q)=N_J(q)\cdot x_{iJ}(q), i=1, 2, \ldots, n \cdots\cdots (8)$$

where :

$-N_J(\boldsymbol{q})$ is the total consumption of a given group of commodities / services J, that is explicitly obtained as

$$N_J(q)=\frac{\sum_{i\in J}\partial V(q)/\partial q_i}{\sum_{j=1}^{n} q_j \cdot \partial V(q)/\partial q_j}, \cdots\cdots (9)$$

$-x_{iJ}(q)$ denotes the frequency (probability) of purchasing commodity i within the group J. It is formulated in the form of a share model

$$x_{iJ}(q)=\frac{\partial V(q)/\partial q_i}{\sum_{j\in J}\partial V(q)/\partial q_j}, i\in J. \cdots\cdots (10)$$

It is clearly recognized that the total demand for commodity i, $X_i(\boldsymbol{q})$, as shown in Eq. (8) is formulated as the product of the total consumption of a group of commodities/services J, $N_J(\boldsymbol{q})$ and the share of the commodity i in the total consumption of the focused group of commodities / services J, $x_{iJ}(\boldsymbol{q})$. This share model takes the form of a logit model and satisfies the requirement that $\sum_i x_i=1, i\in J$. As a result, it shows that with any given indirect utility function, we can derive the share model as given in equation (10). Having these properties, it is worth pointing out that the total consumption of a focused group of commodities / services J $(N_J(\boldsymbol{q}))$, is not an exogenously given variable any more. Rather it is an endogenous variable that changes according to the variation in the related factors that sum all commodities / services.(e.g. the prices of commodities / services, and all relative parameters). Moreover, the function of $N_J(.)$ as formulated in (9) is defined by $N_J(\boldsymbol{q})=\sum_{j\in J}X_j(\boldsymbol{q})$, Thus, the model can be used to easily and explicitly analyze the generated demand. By that, it has overcome the restriction of the conventional approach. We have derived the share model from any given indirect utility function in the context of consumer behavior theory focusing mainly on the variables q_j, $(q_j=p_j/y, j=1, \ldots, n)$. Obviously, this model can also be developed for the case wherein the time constraint is imposed on each individual, even any characters of goods / services other than q, such as quality, frequency and etc., are also be concerned.

(2) A specific Indirect Utility Function

The results show that, we have constructed the general model for deriving the share model in the context of the consumer behavior theory for any given indirect utility function. In reality, it is necessary to specify the indirect utility functions in all applications. There are a number of different functional forms of $V(.)$ from which to choose, depending on the purposes / requirements of the analysis. The following specific indirect utility function is one of the most significant function which is chosen to verify the proposed model and to further discuss the interpretation of the share model under the framework of this study. Suppose, the indirect utility function is given in the additive and separable form

$$V(q) = F\left[\sum_{j=1}^{n}\int_{q_j}^{\infty} \exp k_j(q_j)dq_j\right] \quad \cdots\cdots\cdots\cdots (11)$$

Where, F is the increasing monotonous function of $\sum_j \int \exp k_j(q_j)dq_j$, and $k_j(.)$: are arbitrary functions such that the equation (11) satisfies the characteristic properties of an indirect utility function, with $q_j = (p_j/y)$. It is described by the characters of separate good / service j, and referred to as the change to italic style in this study. Hence, Eq. (11) is indirect utility of an individual resulting over his/her consumption of goods / services. Obviously, by Eqs. (9) & (10) the explicit values of the total consumption $N_J(\boldsymbol{q})$, and the choice probability of commodity i, x_{iJ}, are easily obtained. In the following discussion, we place more emphasis on the study of travel forecasting through the share type of logit model. For convenience, the discussion is restricted on three goods case, $n=1, 2, 3$, where the subscript $n=1$ denotes the composite goods and $n=2, 3$ are transport modes. Following the proposed framework, the total demand of the transport mode 2 is computed by using formulation (6). The result is given as

$$X_2(q) = \frac{\exp k_2(q_2)}{\sum_{j=1}^{3} q_j \exp k_j(q_j)} \quad \cdots\cdots\cdots\cdots (12)$$

Once again, the multiplication technique that has been used so far, proves to be significant in this discussion. Here, the only difference is that multiplied component relates only with the transport modes. By multiplying the nominator and denominator of (12) by $\sum_{j=2}^{3} \exp k_j(q_j)$, the demand for transport mode 2 now becomes

$$X_2(q) = \frac{\sum_{j=2}^{3} \exp k_j(q_j)}{\sum_{j=1}^{3} q_j \cdot \exp k_j(q_j)} \cdot \frac{\exp k_2(q_2)}{\sum_{j=2}^{3} \exp k_j(q_j)} \quad \cdots\cdots\cdots\cdots (13)$$

Equation (13) is the product of the two components. The first component of the right hand side (RHS) of (13) is

$$N(q_1, q_2, q_3) = \frac{\sum_{j=2}^{3} \exp k_j(q_j)}{\sum_{j=1}^{3} q_j \cdot \exp k_j(q_j)}, \quad \cdots\cdots\cdots\cdots (14)$$

which presents the total demand of transport modes that is the function of prices of the transport goods q_2, q_3, as well as the other commodities q_1. The second component of the RHS of (13) is the share of the transport mode 2 in the total transport demand. It is the function of q_2 and q_3 and can be formulated as follows

$$x_2(q_2, q_3) = \frac{\exp k_2(q_2)}{\exp k_2(q_2) + \exp k_3(q_3)} \quad \cdots\cdots\cdots (15)$$

By equations (14) & (15), the demand for transport mode 2 is obtained as

$$X_2(q_1, q_2, q_3) = N(q_1, q_2, q_3) \cdot X_2(q_2, q_3) \quad \cdots\cdots (16)$$

The formulation (16) implies that the demand function can be expressed by the total demand of transport modes N times the share x_2. Though this formula is identified with (8), the difference is with the exponential component of (15), $\exp k_i(q_i)$ which is a separable function that is characterized by the commodity / service i. Moreover, due to the summation component of (14) that takes all commodities/services, $i = 1, \ldots, n$, equation (16) has the possibility to include the related influences of any factors of other commodities into the travel forecasting. This merit provides a comprehensive technique to the transport demand forecasting, and through this, the demand for the composite goods can be also easily computed. The following steps may cast a clear idea about the above statement.

Step 1 : The functional form of $k_2(.)$ and $k_3(.)$ are specified. Then based on the share model (15), the observable data on the share of transport modes 2, 3, are used to estimate all the parameters of the share model, x_2, x_3.

Step 2 : All the information and results of step 1, now are used as inputs to equation (14). Since all the information and parameters related to the share model are already specified, the only remaining work is the specific functional form of $k_1(.)$. With this specification, by (14) all the relative parameters of $k_1(.)$ are estimated by using the only data of the total transport demand N_J.

Step 3 : If any changes occur, for example, the prices quality and other factors, by using (13), the corresponding absolute figure of the demand for the transport modes and other commodities are easily computed.

Note that, due to the formula (16) where $N_f(.)$ and $x_i(.)$ are separated and computable, all the consequences of the demand forecasting process do not only result in percentage figures (%), that are usual for the share model, but also given in numerical quantities as number of trips which, of course, includes the generated travel demand.

Conditional logit model

A concrete result should be obtained by more specific form of function of $k_j(q_j)$. Suppose that a simple function is given as

$$k_j(q_j)=a_j-c_jq_j, j=1,2,3, \quad \cdots\cdots(17)$$

where a_j, c_j are positive unknown parameters, and $q_j=p_j/y$

Consequently, in this case, by simple substitution of (17) into (11), the indirect utility function now becomes

$$V(q)=F\left[\sum_{j=1}^{3}\left(\frac{1}{c_j}\exp(a_j-c_jq_j)\right)\right] \quad \cdots\cdots(18)$$

which obviously satisfied the properties of indirect utility function (1÷4) as described in section 2. From (14) and (15), the explicit equations of the total transport demand and the share of transport mode 2 are respectively given as

$$N(q)=\frac{\exp(a_2-c_2q_2)+\exp(a_3-c_3q_3)}{\sum_{j=1}^{3}q_j\cdot\exp(a_j-c_jq_j)} \cdots(19)$$

and

$$x_2(q_2,q_3)=\frac{\exp(a_2-c_2q_2)}{\exp(a_2-c_2q_2)+\exp(a_3-c_3q_3)} \quad \cdots\cdots(20)$$

Thus, the demand for transport mode 2 becomes

$$X_2(q)=N(q)\cdot x_2(q_2,q_3) \quad \cdots\cdots(21)$$

We have noticed that, equation (20) is the so-called conditional logit model in the context of random utility theory, of which the exponential component (which is often called as a determinant element of random utility function) is a linear function in terms of the parameters. We have derived the share type of logit model as shown in (20) by assuming the functional form (18). This functional form of the indirect utility function can be said to be a sufficient condition for deriving the conditional logit model. Naturally, the question is whether or not (18) is also a necessary condition for deriving (21) with its logistic components as formulated in (19) & (20). The answer is yes, we can derive the indirect utility function as (18) by solving the simultaneous partial differential equations (19) & (20) *w.r.t.* q_2. The solution is shown in appendix 1, for the three goods case.

The following discussion may give more insight on the specific indirect utility function (18). Since $F(.)$ is an increasing monotonous function, one of the feasible functions that (18) expresses is

$$V(q)=\ln\sum_{j=1}^{n}\left(\frac{1}{c_j}\right)\exp(a_j-c_jq_j)+C \quad \cdots\cdots(22)$$

where the constant C takes any suitable value.

Clearly, the function given in (22) looks like the so-called satisfaction function (or the inclusive value function) in the context of random utility theory. The difference is that in (22), c_j are not constant but variable parameters, as the named alternative specific parameters implies in the context of random utility theory. Evidently, if c_j is constant, then (22) becomes exactly a satisfaction function. Having this result, we come to the conclusion that the satisfaction function itself could be a kind of indirect utility function. In this sense, it contributes one more piece of evidence to the hypothesis that the inclusive value (with regards to the expected value of random utility theory) is superior to the other proposed definitions of benefit measurement. Truly, this function satisfies the basic characteristic of the indirect utility function. Also if we just take a simple derivative of (22) *w.r.t.* q_2, following Roy's identity, we can surely get the consistent logit model as derived in equations (20).

Another significance of the proposed model is increasingly emphasized in the following section by showing another specific function of $k_j(.)$ which can be used to derive the gravity model as well, in the same context of the proposed model.

4. A DERIVATION OF GRAVITY MODEL

As mentioned in the previous sections, with the specified indirect utility function and depending on the objective of the analysis, the problem is to specify a suitable function of $k(.)$ in the specific function (11). The following is another useful implication of the proposed model. Recalling the specified indirect utility function as given in (11), we now consider another function of $k(.)$ that is specified as follows:

$$k_i(q_i)=a_i\ln b_i-c_i\ln q_i, i=1,2,\dots n, \quad \cdots\cdots(23)$$

where a_i, b_i, c_i are unknown positive parameters, and $q_j=p_j/y$.

With this specification, based on (14), (15) and (16), the total consumption of commodity i, results to

$$X_i(q)=N(q)\cdot x_i(q), i=1,2,\dots n, \quad \cdots\cdots(24)$$

where, recalling that $q_j=p_j/y$, the consumption of commodities / services group J, is obtained as

$$N(p,y)=\frac{\sum_{j\in J} b_j^{a_j} y^{(1+c_j)} p_j^{-c_j}}{\sum_{j=1}^{n} b_j^{a_j} y^{c_j} p_j^{(1-c_j)}} \cdots\cdots(25)$$

and

$$x_i(p,y)=\frac{b_i^{a_i} y^{c_i} p_i^{(-c_i)}}{\sum_{j\in J} b_j^{a_j} y^{c_j} p_j^{(-c_j)}},\ i\in J \cdots\cdots(26)$$

The model (26) appears as the share model, by which we can compute the consumption of commodity / service i, $(i\in J)$, taking a portion of the total consumption of commodities / services group J. Clearly, this share model is the same functional form as the modified gravity model in which the living place of the consumer is given, then

- —X_j indicates the absolute volume of trips attracted to zone j from the living zone,
- —$N(.)$ defines the total trips production of the individual
- —the upper part of (26) represents the attractiveness / accessibility characteristics of zone i
- —the lower part of (26) indicates the attractiveness / accessibility characteristics of all zones in the area J.

Note that, in model (26), the exponents of y and p should be identical in order to have a consistent indirect utility function with the characteristic of being homogenerous to degree zero.

Moreover, if c_j is constant at c, which is the usual specification of a gravity model, then (25) & (26) now become

$$N(p,y)=\frac{y\sum_{j\in J} b_j^{a_j} p_j^{(-c)}}{\sum_{j=1}^{n} b_j^{a_j} p_j^{(1-c)}} \cdots\cdots(27)$$

and

$$x_i=\frac{b_i^{a_i} p_i^{(-c)}}{\sum_{j\in J} b_j^{a_j} p_j^{(-c)}},\ i\in J \cdots\cdots(28)$$

Equation (28) is a typical modified gravity model whose summation of all j, $j\in J$, is unity. Thus, the unconstrained gravity model that is the product of (27) & (28) takes the form of

$$X_i=\frac{y b_i^{a_i} p_i^{(-c)}}{\sum_{j=1}^{n} b_j^{a_j} p_j^{(1-c)}},\ j\in J \cdots\cdots(29)$$

which is different from (28) by the multiplication of the two factors, $(1/p_j)$ and y, and includes the composite good (say $j=1$).

There are many possible explanations for each component of the model depending on the objective of each study. For example, in the model (27) b_j can be used to reflect the attractiveness of the zone, such as the size of population in zone j or other socioeconomic characteristics. The element (b_j) can be subdivided into several separable factors, depending on each analysis. Suppose the function $k_i(q_i)$ now is given as follows

$$k_i(q_i)=$$
$$[a_1\ln b_{i1}+a_2\ln b_{i2}+\ldots+a_k\ln b_{ik}-c\ln q_i], \cdots(30)$$

where, b_{ik} with $k=1,\ldots,K$, is the socioeconomic characteristics which express the attractiveness and the accessibility of zone i for the specified living zone, such as population, total employment (associated with a nonnegative value of a_k) or travel time (distance), and other travel conditions (accompanied with a negative sign of a_k). From the specification (30) equation (28) now becomes

$$x_i=\frac{\Pi_k b_{ik}^{a_k} p_i^{-c}}{\sum_j \Pi_k b_{jk}^{a_k} p_j^{-c}},\ i\in J \cdots\cdots(31)$$

The resultant share model (31) is the most frequently used of the modified gravity models that can be factored in a term depending on the travel costs and a term of all other components express by $(b_{ik}^{a_k})$.

5. THE CONSISTENCY WITH THE DIRECT UTILITY FUNCTIONS

We have shown how to recover an indirect utility function from the logit model by solving the system of partial differential equations, (see appendix 1). The study would be more convincing if the direct utility function can also be obtained. The solution to the corresponding direct utility function can be solved by taking into consideration the duality between the direct and indirect utility function shown in equation (7). Recalling that the indirect utility function consistent with the logit model (20) is

$$V(q)=F\left(\sum_{j=1}^{n}\left(\frac{1}{c_j}\right)\exp(a_j-c_jq_j)\right) \cdots\cdots(32)$$

thus we can find the direct utility function by solving the following problem

$$U(X)=\min_q V(q)=\sum_{j=1}^{n}\left(\frac{1}{c_j}\right)\exp(a_j-c_jq_j)$$
$$\text{s.t.} \sum_{j=1}^{n} q_jX_j=1 \cdots\cdots(33)$$

where $q_j=p_j/y$, and $j=1,2,\ldots,n$.

Solving the first-order conditions by the usual calculation, the associated direct utility function is calculated to be (see appendix 2)

$$U(X)=$$
$$\sum_{j=1}^{n}\left(\frac{X_j}{c_j}\right)\exp\left(\frac{-1-\sum_{j=1}^{n}\left(\frac{X_j}{c_j}\right)\ln X_j+\sum_{j=1}^{n}\frac{a_j}{c_j}X_j}{\sum_{j=1}^{n}\frac{X_j}{c_j}}\right)$$
$$\cdots\cdots(34)$$

In a more simple case, where $a_j=c_j=1$, the derived direct utility function becomes a *translong utility* such as :

$$U(X)=\sum_{j=1}^{n}X_j\exp\left(\frac{-1-\sum_{j=1}^{n}X_j\ln X_j+\sum_{j=1}^{n}X_j}{\sum_{j=1}^{n}X_j}\right) \quad \cdots\cdots(35)$$

This function is the same as the representative consumer's utility function which is derived in the study of Anderson, Palma, Thisse. (see reference (6), pp. 78). It has verified that, given the direct utility function (34), the corresponding indirect utility function (18) is optimal value of a objective function of the following maximization problem

$$\max.\ U(X)=$$

$$\sum_{j=1}^{n}\frac{X_j}{c_j}\exp\left(\frac{-1-\sum_{j=1}^{n}\frac{X_j}{c_j}\ln X_j+\sum_{j=1}^{n}\frac{a_j}{c_j}X_j}{\sum_{j=1}^{n}\frac{X_j}{c_j}}\right)$$

$$\text{s.t.}\sum_{j=1}^{n}p_jX_j=y,\ j=1,2,\ldots,n. \quad \cdots\cdots(36)$$

and the associated solution as demand function (21).

6. CONCLUSION

The study shows a different approach to derive the general as well as the specific functional forms of the logit type of share model and the modified gravity model within the framework of the consumer behavior theory. Also, it has demonstrated the consistency of the demand model by constructing the consistent indirect and direct utility functions from the derived demand function. Four significant results that have obtained from this study as follows :

Firstly, the study has shown that from any functional form of an indirect utility function with the normal characteristics, the share model can be derived within the framework of consumer behavior theory. In the context of the proposed framework, the demand function can be expressed in the product form of a total demand model of a group of commodities (e.g. total transportation volume as discussed in section 3) multiplied by the share model that is formed as logit model. Although, the model appears more generalized, the general form of the share model has a restriction on the appearance of the partial derivative components. This restriction is relaxed by assuming the additive and separable forms of the indirect utility function (Eq.11) in deriving the logit type model that is a mathematical resemblance of the logit model with more separable function of the deterministic component, k_i, where $k_i(q_i)$ has no restriction in its functional form except for being a monotonously decreasing convex in q_i.

Secondly, the total consumption of commodities / services which is formulated by this model is no longer fixed at exogenously given value, rather it can be determined endogenously. This result leads to a better general model since we can explicitly deal with the additional demand. In this aspect the model has over-come the limitation of the conventional logit model with fixed total demand.

Thirdly, in the conventional travel forecasting, the effects of factors such as prices of other goods / services to the transport demand behavior are ignored. However, it is clearly shown in this study that, since the total demand model as obtained in (Eq.19) includes the composite goods, (recall $j=1$), the demand functions of the transport modes (Eq.21) are also a function of other commodities. This formulation leads to the possibility of this model in fully considering and obtaining the absolute figure of the transport demand in the relation with other commodities in the process of travel demand forecast.

Fourthly, based on the formulation of the transport demand (Eq.16), this study proposes a new comprehensive parameter estimation technique. By using the parameters that is estimated through the share model (15), the total transport demand function (14) can be obtained by using only the transport related data. It remain for future study, however, to compare our proposed estimation technique with conventional method in term of efficiency, accuracy, and practicality.

Finally, although we have derived the demand model that is mathematically expressed as logit model in the context of consumer behavior theory focusing mainly on the effects of variable q, we can show that our model framework is applicable even to the case where the time constraint is imposed on each individual. In this case, since the level of consumer's activity are also restricted by his / her total available time, consumer will try to allocate his time to maximize his utility level. Thus, the consumer maximization problem becomes

$$\max.\ U(X) \quad (1)$$

$$\text{s.t.}\ pX=y+wl \quad (2)$$

$$l+tX=T \quad (3)$$

where X is a vector of consumption levels for each commodities, $j-1.,,,.n$, l is working hours, p is price vector for X, y is non-working income, w is wage rate, t is the time requirement vector for a unit of consumption level vector X, and T is total available time. Assumming that the working hour is controlable by the individual, (2) and (3) can be transformed into

$$(p+wt)X=wT+y \qquad (4)$$

The components $(p+wt)$ and $(wT+y)$ are known as generalized cost and total expenditure, respectively. If sets $(p+wt)/(wT+y)=q$, then $qX=1$. Therefor, all the results obtained in this study are hold for the time constraint imposed framework.

Moreover, the consistent demand model derived within the context of consumer behavior theory (refer to equation (16) with its logistic components (14) & (15)) can be developed consistently to the study of evaluation measures for both the improvement and the newly introduced transport facility cases. Due to its advantages in dealing with the generated demand that enables us to fully quantify the changes in consumer surplus attributable to changes in price and other qualities. The benefit estimation approach, however, is quite a broad topic and rather different with what we want to discuss in this paper, so it is best not to deal with it in this paper. We hope to do further research on these issues in another paper soon.

APPENDIX 1

Let us consider a derivation of the indirect utility function (18) which corresponds to the demand function given in equation (19). For convenience, the three goods case is chosen. Roy's identity provides that if one is given the demand function, the corresponding indirect utility function can be derived by solving the system of partial differential equations. Thus, the indirect utility function consistent with (19) could be derived from the following problem

$$X_i=\frac{\exp(a_i-c_iq_i)}{\sum_{j=1}^{3}q_j\exp(a_j-c_jq_j)}=\frac{\partial v(q)/\partial q_i}{\sum_{j=1}^{3}q_j\partial v(q)/\partial q_j} \qquad (*)$$

To solve this partial differential equation system (*) for function $v(q)$, first we obtain;

$$(1)\quad \frac{\partial v(q)}{\partial q_1}=\exp(a_1-c_1q_1)r(q_1,q_2,q_3)$$

$$(2)\quad \frac{\partial V(q)}{\partial q_2}=\exp(a_2-c_2q_2)r(q_1,q_2,q_3)$$

$$(3)\quad \frac{\partial V(q)}{\partial q_3}=\exp(a_3-c_3q_3)r(q_1,q_2,q_3)$$

where $r(q_1,q_2,q_3)$ is a certain function. By changing the variable,

$$Y=\left(\frac{1}{c_1}e^{a_1-c_1q_1}\right)+\left(\frac{1}{c_2}e^{a_2-c_2q_2}\right)+\left(\frac{1}{c_3}e^{a_3-c_3q_3}\right)$$

$$Z=\left(\frac{1}{c_1}e^{a_1-c_1q_1}\right)+\left(\frac{1}{c_2}e^{a_2-c_2q_2}\right)$$

$$W=\left(\frac{1}{c_1}e^{a_1-c_1q_1}\right)$$

Then the system of equations (1), (2), (3) becomes

$$(4)\quad \frac{\partial V}{\partial q_1}=\frac{\partial V}{\partial Y}\frac{\partial Y}{\partial q_1}+\frac{\partial V}{\partial Z}\frac{\partial Z}{\partial q_1}+\frac{\partial V}{\partial W}\frac{\partial W}{\partial q_1}$$

$$=\frac{\partial V}{\partial Y}\exp(a_1-c_1q_1)+\frac{\partial V}{\partial Z}\exp(a_1-c_1q_1)+\frac{\partial V}{\partial W}\exp(a_1-c_1q_1)$$

$$=\exp(a_1-c_1q_1)\left(\frac{\partial V}{\partial Y}+\frac{\partial V}{\partial Z}+\frac{\partial V}{\partial W}\right)$$

Similarly for the partial derivative of $V(\boldsymbol{q})$ w.r.t. q_2, q_3 we have:

$$(5)\quad \frac{\partial V}{\partial q_2}=\exp(a_2-c_2q_2)\left(\frac{\partial V}{\partial Y}+\frac{\partial V}{\partial Z}\right)$$

$$(6)\quad \frac{\partial V}{\partial q_3}=\exp(a_3-c_3q_3)\left(\frac{\partial V}{\partial Y}\right)$$

Equation (1)-(3) and (4)-(6) imply that

$$(7)\quad \frac{\partial V}{\partial Y}+\frac{\partial V}{\partial Z}+\frac{\partial V}{\partial W}=\alpha(Y,Z,W)$$

$$(8)\quad \frac{\partial V}{\partial Y}+\frac{\partial V}{\partial Z}=\alpha(Y,Z,W)$$

$$(9)\quad \frac{\partial V}{\partial Y}=\alpha(Y,Z,W)$$

where $\alpha(Y,Z,W)=r(q_1,q_2,q_3)$. The combination of equations (7), (8) and (9), show that the indirect utility function is a function of only variable Y, it means $V=V(Y)$. Recall Y,

$$Y=\left(\frac{1}{c_1}e^{a_1-c_1q_1}\right)+\left(\frac{1}{c_2}e^{a_2-c_2q_2}\right)+\left(\frac{1}{c_3}e^{a_3-c_3q_3}\right)$$

Thus

$$(10)\quad V(q)=F\left[\sum_{i=1}^{3}\frac{1}{c_j}\exp(a_j-c_jq_j)\right]$$

Equation (10) is the same as the specific indirect utility function (18).

APPENDIX 2

According to the duality between direct and indirect utility functions, if we are given the indirect utility function $V(\boldsymbol{q})$ as shown in equation (18), we can find the direct utility function by solving the following problem

$$U(X)=\min_q V(q)=\sum_{j=1}^{n}\frac{1}{c_j}\exp(a_j-c_jq_j)$$

$$\text{s.t.}\ \sum_{j=1}^{n}q_jX_j=1$$

The Lagrangian function for this consumer's minimization problem is written as

$$L(q,\lambda)=\sum_{j=1}^{n}\left(\frac{1}{c_j}\right)\exp(a_j-c_jq_j)+\lambda\left(\sum_{j=1}^{n}q_jX_j-1\right)$$

The necessary conditions for this minimization is that

(1) $\exp(a_i - c_i q_i) = \lambda X_i$

(2) $\sum_{j=1}^{n} q_j X_j = 1$

From (1) we obtain

(3) $q_i = \left(\frac{1}{c_i}\right)(\ln\lambda + \ln X_i - a_i)$

Substituting (3) into (2) we define

(4) $$\ln\lambda = \frac{-1 - \sum_{j=1}^{n}\frac{X_j}{c_j}\ln X_j + \sum_{j=1}^{n}\frac{a_j}{c_j}X_j}{\sum_{j=1}^{n}\left(\frac{X_j}{c_j}\right)}$$

Substituting the combination of (4) & (3) back into the given indirect utility function, the consistent direct utility function results as (34).

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消費者行動理論におけるロジットモデルと重力モデル

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ロジットモデルは総消費量を一定として扱うから，誘発需要を取り扱うことができない．この研究では，近代経済学の消費者行動理論に基づいて，ロジットモデルと数式表現が同じ型となる需要関数の誘導を行う．誘導された需要関数は，総需要量に対象とするサービスの需要比率を乗じた形となっているので，特定のサービスレベルの変化が，その需要比率を変化させるばかりでなく，総需要量およびそれ以外の財サービスの需要をも変化させることを示すことができる．