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REVIEW

A REVIEW ON SHAPE OPTIMAL DESIGN AND SENSITIVITY ANALYSIS

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ABSTRACT

Presented is a review of shape optimal design and shape design sensitivity analysis with an emphasis on techniques dealing with shape of the boundaries of two- and three-dimensional bodies. Attention is focused on the continuum structural shape optimization based on numerical models by either finite elements or boundary elements. This requires sophisticated design sensitivity analysis techniques and a careful choice of design variables.

1. INTRODUCTION

Engineering design is an iterative process, in which the design is continuously modified until it meets the criteria set by engineers. A traditional design process is carried out by the so called *trial and error* method, in which the designer uses his experience and intuition to lead the design process. This manual design process has the advantage that the designer's knowledge can be utilized in the design. But as the design problem becomes more complex, design modification becomes more difficult, requiring a new tool.

For a particular design problem, there may exist a number of solutions that satisfy given conditions. The optimum design is a rational approach finding a solution which is *optimal* in the sense defined.

One category of problems attaining much attention recently is the optimal shape design. It is one order of magnitude more complex than the more classical size or parameter optimization. Due to variety of difficulties, it is not yet in the stage of practical applications, although theoretical base is well set up now. The importance of shape optimization is evident, since the first thing in a design problem is to determine the shape of the object to be designed when we look at any design problem. For determining optimal shape of elastic bodies in the general case, the main mathematical difficulty lies in that the domain of the governing equations is not specified beforehand, but is to be determined from conditions that the objective functional attains an extremal value possibly under many other constraints. These are so-called *problems with unknown boundaries*. As well as having a direct practical importance, such problems are of great interest from a mathematical point of view: to develop effective tools to such problems is a real challenge.

An accurate shape design sensitivity analysis (SDSA) is considered basic prerequisite to efficient handling of the shape optimization process. This has been a major topic of intensive research. The present paper reviews recent works in shape optimization and shape design sensitivity analysis. It is focused on the shape design sensitivity analysis methods and is limited mostly to design variables that control the boundary of two- and three-dimensional objects. It does not include works on topology optimization and sensitivity analysis of skeletal structures, because these problems belong to another category. However, the interested reader is referred to a survey by Topping[1] and Levy[2] on this topic.

Section 2 deals with procedures for properly defining shape design variables so that the finite element or boundary element mesh provides accurate state variables and accurate sensitivity results. Section 3 deals with shape optimal design problems treated up to now. Section 4 reviews the developments of shape design sensitivity analyses based on variational formulation and boundary integral formulation. Section 5 lists the shape design literature by areas of applications. The last section concludes the surveys.

2. SELECTION OF BOUNDARY REPRESENTATION METHOD AND DESIGN VARIABLES

The representation of the shape to be designed using a set of parameters or design variables is a key step in the process of formulating most shape optimal design problems, and remains one of the major difficulties. The optimum shape is highly dependent on the design parameterization selected. An inappropriate parameterization can lead to unacceptable shapes[3),4)]. And, changing the geometric shape of the design model to reflect successive changes in design parameters is a tedious, complicated, and inefficient process.

In general, structural shape design problems can be classified into three types, in terms of the characteristics of the design boundary. In the first type, the shape of an arbitrary open or closed boundary, such as a fillet[5)~16)] or a dam surface[17)~21)], is to be determined. In the second type of problem, dimensions of pre-defined shapes, e. g., the radius of a circular hole, the major and minor axes of an elliptic hole, dimensions of a slot, length of a rectangular membrane, or radius of a rounded corner, are to be found[22)~28)]. In the third type, locations of the design boundary, e. g., the locations of the center of a circular hole, an elliptic hole, hole of an arbitrary given shape, or slot that has either arbitrary or pre-defined shape relative to the global reference frame, are to be determined[29),30)]. Shape design of the open boundary of a structure has been studied for some time[5),6),20),31)~41)]. However, design problems with a closed boundary, pre-defined shape, and locations of the design boundaries have not yet been extensively treated.

During the past years, a few methods have been used to parameterize structural boundaries for optimal shape design : boundary shape described by coordinates of boundary nodes[20),31)~33)], coefficients of polynomials[6),34)~36)], control points of splines or spline blending functions[5),14),37)~41),53)~56)], and parameters of generic primitive models[23),24),28),29),42),43)]. A selection of a particular parameterization means a restriction on the set of feasible designs that may be different from original intention.

(1) Coordinates of element boundary nodes

The use of coordinates for boundary nodes in the finite element or boundary element model as shape variables is the earliest method[13),20),31)~33),44)~51)]. It is simple and easy. However, severe drawbacks have been reported[3)] : (1) the number of design parameters tends to become very large, which may lead to high computational cost and difficulty; (2) smoothness of the design boundary is not retained across boundary nodes, which may lead to an unacceptable or impractical design; and (3) analysis error due to a selected discretization can be amplified and thus the optimized shape based on this wrong information is meaningless when looked at from the design problem initially set. Such problems can be seen in the typical fillet or hole problems[3),4),40),227)].

(2) Polynomials representation of boundaries

Polynomial representation is a natural choice for describing boundaries. Some early references are[34)~36)]. The total number of shape design parameters can be reduced by using polynomials for shape representation. However, as reported by Ding[3)], using high order polynomials to represent the boundary shape may result in oscillatory boundaries.

A more general approach is to define the boundary as a linear combination of certain shape functions with the coefficients being the design variables. Thus, Kristensen and Madsen[6)] defined the boundary using linear combination of orthogonal functions added to the initial design by treating the coefficients of the functions as design parameters. Dems[52)] also used a set of prescribed shape functions and applied it to the simple case of piecewise linear boundaries.

(3) Spline representation of boundaries

The use of high-order polynomials to describe the boundary can result in an oscillatory boundary shape as mentioned already. Splines eliminate this problem, since they are composed of low-order polynomial segments that are combined to maximize smoothness of the design boundaries. Furthermore, the spline representation has been shown to yield better sensitivity accuracy than a piecewise linear representation of the boundary[37)].

The cubic spline function, which has two continuous derivatives everywhere and possesses minimum mean curvature, is a natural choice for defining the boundary[38),39)]. Braibant et al.[40),41),53),54)] used Bezier and B-spline blending functions to describe design element boundaries. The blending functions provide great flexibility for the geometric description. With the spline formulation[14),50),55),56)], boundary regularity requirements are automatically taken into account and also an analytical formulation of the sensitivity derivatives can be established.

(4) Generic primitive models

Many CAGD (Computer Aided Geometric Design) programs or pre-processors in FEM packages have modules which can generate geometric models defined by parameters of generic primitive model. Some examples of primitives are unit sphere, unit cube, unit cylinder and so on. Sometimes the primitives are defined by B-Rep (Boundary Representation), such as NURB (Non-uniform Rational B-spline).

These primitives are used to define the shape of the solid model or boundary shape. In this case, the design variables are parameters of the primitives which define the model. Some efforts are seen[24),28),42)] to connect the shape optimization technique with those models defined by generic primitives.

(5) **Design element concept**

One way to achieve an adequate finite element model is to use the design element concept that was first introduced by Imam[33)], and is used by several researchers[20),22),40),41),53),54),58)]. In this approach, the structure is divided into a few regions. Each of these regions corresponds to a design element that is described by a set of master nodes that control the geometry of the design element[15)]. Associated with the design element is a set of suitably chosen design variables that specify the locations of the master nodes only. The design element boundary is described by using two- or three-dimensional isoparametric finite element interpolation functions[33),58)], or spline blending functions[40),41),53)].

3. SHAPE OPTIMIZATION

Finding the optimum shape of a structure under some constraints has been one of the most attractive concerns of many great mathematicians for a long time from Galileo Galilei, who presented the problem of finding the shape of a cantilever beam in order to obtain a uniform stress distribution in the 17th century, to many researchers in the 20th century. Euler and Lagrange had set the necessary conditions for optimization problems. Since then there are many analytical tools that contribute to the study of optimum design of structural components. The work of Michell (1904)[59)] was a prevailing basic theory of these analytical optimization works. The conference book edited by Haug and Cea (1981)[60)] provides an extensive review of analytical works in this field. Unfortunately, these analytical approaches have limitations in solving practical problems. However they are very valuable and important because they can provide insight into the design process as well as lay down a foundation for numerical methods.

Numerical optimization methods have seen extensive development over the past thirty years. It was Schmit[61)] who proposed a general approach to structural optimization in 1960, which indicated the feasibility of coupling finite element analysis and non-linear mathematical programming for the optimum structural design. Following Schmit's work, major advances in structural optimization have been made by Kicher, Gallagher, Gellatly *et al.*[62),63)]. However the major interest of their effort was the member sizing problem where design variables are the cross-sectional area or thickness.

(1) **Shape Optimization using FEM**

One of the first treatments of shape optimization was presented by Zienkiewicz and Campbell[31)]. They formulated the shape optimization problem using an FE model and treated the boundary nodes of the FE model as design variables. They obtained derivatives by directly differentiating the discretized equations and employed a sequential linear programming method for numerical solution. Francavilla *et al*[12)], Schnack[64)], and Oda[65)] employed the finite element method for a fillet optimization to minimize stress concentration. Similar but more extensive methods to minimize stress concentrations are presented by Tvergaard[66)], Kristensen and Madsen[6)] with a sequential linear programming method.

Bhavikatti and Ramakrishnan[34)] presented a refinement of the formulation of [31)] and Ramakrishnan and Francavilla[67)] employed a similar finite element formulation, but they used a penalty function method for numerical optimization. A function space gradient projection method for two-dimensional elastic bodies was presented by Chun and Haug[68)], using the design sensitivity analysis methods similar to those presented by Rousselet and Haug[69)].

Optimality criteria have been developed for selected classes of shape optimal design problems. Banichuk[70)] formulated a general problem of selecting the optimum shape of the cross section of a nonhomogeneous shaft, to maximize torsional stiffness, with a given amount of material available. He used variations of functional with respect to both the warping function and boundary variation, using material derivatives, and obtained a necessary condition for optimum location of the boundary. Dems and Mroz[17),71)~76)] presented a quite general approach of shape optimal design using both the optimality criterion method and the variational calculus, and the optimality conditions were generated for both the conservative and the nonconservative load system.

Three dimensional shape optimization problems were proposed by Kodiyalam and Vanderplaats[77)] using the forced approximation technique, in which the approximate stresses were obtained through linearization of nodal forces instead of direct linearization of the stresses. Applications of three-dimensional shape optimization are found in[5),23),28),33),78),79)].

Bendsoe and Kikuchi[225)] and coworkers[81)~83)] proposed a topological shape optimization method, called homogenization method that utilizes infinitely many microscale holes in a design domain. For a skeleton topology optimization readers are referred to the two papers by Topping[1)] and Levy[2)].

(2) Shape optimization using BEM

Because of convenience in remeshing compared to FEM as well as relatively good accuracy of the solutions at the boundary, the BEM has become an attractive analysis method in shape optimization.

The use of the BEM in shape optimization started from the 1980's. One of the first formulation of shape optimal design was presented by Futagami[84),85)]. He coupled the BEM with the linear programming to optimize systems governed by partial differential equations Barone and Caulk[30)] optimized position, size and surface temperature of circular holes inside a two-dimensional heat conductor to produce the minimum variation in surface temperature. They employed a special boundary integral method and a conjugate gradient method. Mota Soares *et al.*[44),45)] presented a model for optimization of the geometry of solid and hollow shafts in terms of boundary elements and the nonlinear programming. They extended the formulation to two-dimensional elasticity problems[13),46)]. The determination of the optimum shape under displacements and geometrical constraints was presented by Zochowski and Mizukami[86)] with the objective of minimum area.

Meric[87)~89)] applied the BEM to solve the necessary conditions for optimality of performance index derived by the calculus of variations using a Lagrange multiplier technique. He extended his method for non-linear anisotropic heat conduction problems and applied his method to obtain an optimal outer boundary profile of an orthotropic solid body[90)]. Optimal cross-sectional shapes for minimum viscous drag for fully developed magnetohydrodynamic channel flow are investigated by using a similar method[51)]. Kobelev[91)] also used the BEM for the best shape of an elastic bar in torsion. On the other hand, Burczynski and Adamczk[92),93)] started with integral optimality conditions and used boundary element method. The resulting nonlinear system was solved by the Newton-Raphson method.

Sandgren and Wu[55)] obtained the optimal shape of ladle hook with substructuring method. They have shown that the subregion approach can reduce the computing time significantly. Carter *et al*[47)]. described an iterative numerical optimization procedure for generating the cryosurgical probe tip geometry to produce the desired lethal temperature envelope for a steady state axisymmetric system. They used the Kirchhoff transformation to include the nonlinear effect of variable thermal conductivity at cryogenic temperatures. Gracia and Doblare[49)] obtained the solution of the shape optimization problem for simply and multiply connected orthotropic sections under Saint Venant torsion. Espiga *et al.*[94)] used the BEM for two-dimensional elastic orthotropic solids.

Saigal and Chandra[48)] adopted the implicit differentiation of discretized boundary integral equations for the shape optimization of heat conduction problem. A boundary element formulation for acoustic shape sensitivity analysis is formulated by Kane *et al.*[160)]. Shape optimization of structures to minimize stress concentration is formulated as a sequential linear programming problem with an adaptive move limit by Xu and Yu[95)]. A method for automated grid refinement and grid adaptation of boundary elements is introduced by Hajela and Jih[15)] to interface with the optimum shape design problem. They used a predefined control function in a variational formulation and master node concept to obtain an optimal node distribution.

Kwak and Choi[96)] developed a general procedure and formulas for the SDSA based on the BIE formulation for a potential problem and applied it to a seepage problem. They extended the formulation to plane elasticity problems and studied a fillet and an elastic ring design problem[50)]. Lee and Kwak[97),98)] extended the adjoint method of Choi and Kwak to two dimensional thermoelasticity problems and considered a shape optimal design to minimize the weight of a turbine disc under stress constraints. Lee and Kwak[56)] also extended the approach to transient diffusion problem and applied to a shape optimization problem of a glass forming plunger to minimize the variation of temperature along the cavity surface.

An optimal design technique for magnetostatic fields is described by Ishiyama *et al.*[99),100)]. They have shown two application examples ; a 1-Tesla superconducting magnet system with a magnetic shielding for magnetic resonance imaging (MRI) and a magnetic levitation system.

Chaudouet *et al.*[101)] applied the BEM to three dimensional optimum design with a growing-reforming technique. A modular approach for shape optimization used in the finite element[28)] is employed for the boundary elements by Yang[16)].

Stochastic shape optimal design problems are investigated by Nakagiri[102)], Tada *et al.*[103)] and Burczynski[104)]. The boundary element SDSA and the optimal design of vibrating structures for the criterion of maximizing the lowest natural frequency are considered by Fedelinski and Burczynski[105)].

The coupling of FEM and BEM is employed by

Kamiya and Kita[106] that takes the advantages of easier remeshing of BEM and the sparse matrix of FEM. They also proposed a numerical approach to search for the design synthesis of the optimal shape of a spring wire under a minimum weight restriction and stress criterion[57].

4. SHAPE DESIGN SENSITIVITY ANALYSIS (SDSA)

There are many special methods for solving the shape design problems, for example, optimality criterion methods, intuitive method (pattern transformation method)[65], experimental techniques employing photoelastic models and so on. Optimality criterion methods for shape optimization consist of the following two steps : 1) the derivation of a set of necessary conditions that must be satisfied at the optimum design ; and 2) the development of an iterative redesign procedure that drives the initial trial design toward a design which satisfies the previously established set of necessary conditions (see, for example, 57), 92), 93)). Pattern transformation method is a technique of scaling up or down the shape of the boundary based on their stress ratio or strain energy ratio. The detail and other methods are explained well in[3),4)].

Most of the work, however, is based on employing mathematical programming methods coupled with finite element method or boundary element method. Most methods in nonlinear programming require gradient information at each iteration.

Design sensitivity analysis, that is, the calculation of quantitative information on how the response of a structure is affected with respect to changes in the variables that define its shape, plays a key role in shape optimization. The first partial derivative of structural response quantities with respect to shape design variables provides the essential information required to couple mathematical optimization and structural analysis procedures. This problem of shape design sensitivity analysis has been addressed over about the past 20 years. There are two popular baseds : variational formulation and boundary integral equation (BIE) formulation.

(1) SDSA based on Variational Formulation

The dominantly used analysis method based on variational formulation is the Finite Element Method (FEM). There are two main approaches to calculate the shape design sensitivities in this context ; the discretized approach and the continuum approach.

a) Discretized Approach

The discretized method uses a discretized model to carry out the sensitivity analysis, which includes three methods : Finite Differentiation Method (FDM), analytical method and semi-analytical method.

The simplest method for obtaining the partial derivatives is actually calculating the increments using FDM[22),58)]. The FDM is to disturb the design variables one by one, and using finite difference formula to approximate the derivatives. FDM has the advantage of being simple in concept, and easy in implementation, but it has two disadvantages. First, changes in shape can lead to a distortion of the finite elements[22),58)], and so the accuracy often depends upon the size of perturbation step. Second, the computation cost is comparatively high especially when the number of design variables is larger than that of constraints.

An analytical method is to differentiate the system equation directly with respect to the design variables[20),107)~109)]. Analytical shape sensitivity may be obtained through the implicit differentiation approach[110)], which is quite straightforward in terms of mathematical derivation and programming. From the initial efforts of Zienkiewicz and Campbell[31)] and Ramakrishnan and Francavilla[12)] to the more recent work of Braibant and Fleury[40)], and Wang *et al.*[20)], the theory of the implicit differentiation approach has been established. But, unfortunately the stiffness matrix is usually nonlinear with the shape design variables, therefore it is difficult to obtain the derivative of the stiffness matrix analytically.

The semi-analytical method differentiates the system equation first as in the analytical method, then employs the finite difference method to calculate the derivative of the stiffness matrix[111),112)]. This is one of the attractive method in practical problems because of its generality and easy implementation. However many researchers indicated that semi-analytical method could have serious accuracy problem for beam. plate and solid problem[40),58),113)~117)].

b) Continuum Approach

In the continuum approach, the sensitivity formulas are derived for the system before discretization, so there is no approximation involved in the formulation. There are two standard methods for describing the variation of a functional over a varying domain : Material Derivative Method (MDM) and Domain Parameterization Method (DPM).

The first approach, MDM involves the introduction of time-like parameters to describe the evolution of the undeformed geometry into neighboring shapes. The material derivative approach is based on the calculus of variation[118),119)].

The variation is obtained by determining the first order changes in the functional on the moving domain as the time-like parameters are varied[120]. Such an approach forms the basis of the MDM for shape sensitivity analysis[110),121)].

The material derivative approach of structural design sensitivity analysis has been developed over the last ten years from several different points of view[8),9),17),76),110),115),118),122)~127)]. This approach was first proposed by Cea, Zolesio and Rousselet[115),126),127)], and further developed by Haug and Choi *et al.*[110),118)]. The general formulation for elasticity problems was very well summarized in Haug *et al.*[110)].

An efficient approach, referred as the Direct Differentiation Method (DDM), involves implicit differentiation of the elasticity equation to obtain the partial derivatives[110)]. It is expensive for problems with a large number of design variables. Some recent articles on the DDM discusse a rigorous treatment of shape variations[114),128)]. On the other hand, the so called Adjoint Variable Method (AVM) has been derived by direct application of the weak governing equations, often in the form of virtual work, without introducing mutual energy principles. Haug, Choi and their co-workers[110),112),121),134),135)] considered both discrete and continuous systems. Dems and Mroz[76)] presented a similar approach based on the variational method, which included more general boundary conditions. They also identified the physical interpretation of the adjoint field variables as extended set of design variable to include shape, loading and support parameters. Belegundu and Arora[133)] showed that the adjoint variables represent the sensitivity of the cost function and constraint functions with respect to the loading or forcing functions in the design problem. Comparisons of the variational and implicit differentiation approaches have been investigated[37),47),55),74),112)]. Other researchers[69),76),114),118),122)~124),128)~133)] have presented formulations for SDSA of linear structures by introducing adjoint structures from a physical consideration.

Yang and Botkin[112)] demonstrated equivalence of variational and implicit differentiation method for linear problems. This equivalence can also be shown for nonlinear problems when finite element formulations are used[8)]. Several authors have proposed formulations based on boundary integrals and the adjoint method[69),118),123),129),132)]. But there are considerable numerical difficulties with the evaluation of boundary integrals[110),136)]. These problems were avoided by using domain integrals instead of boundary ones[131),137)], but it becomes expensive to calculate the full domain integration, so a boundary layer approach was suggested[10)]. Hou *et al.*[138)] pointed out some discontinuity problems that can rise at the interface between finite elements in the domain method. A problem of accuracy occurs when the adjoint system under a singular load must be solved, especially for a stress sensitivity. Even though a local averaging may be used to smooth the singularity[115)], the problem still exists and remains as an open problem.

The second approach, DPM uses a variable mapping to transform the problem to one with a nonvarying domain[139)~147)]. The geometric coordinates and the usual set of dependent variables are written as functions of parametric coordinates defined on a fixed domain. Functionals are rewritten on the fixed domain using the parametric coordinates as the independent variables, and functional variations are then determined in the usual way. This method forms the basis of the approach for shape sensitivity analysis found in the reference[148)], in which the shape variations are described in terms of a mapping from an independent, fixed reference geometry.

The DPM can be considered as an extension of the isoparametric concept of finite element analysis to the design and optimization problems. Comparison between MDM and DPM is done by Tortorelli *et al.*[143)], and Arora and Cardoso[146)]. Recently, the two approaches have been shown to be theoretically equivalent[147),149)]. However, their numerical implementation can be quite different.

(2) SDSA based on BIE

In the past decade the Boundary Element Method (BEM) has been recognized as an alternative numerical method for engineering problems, especially in the area of shape optimization. The BEM can reduce the two major drawbacks of the FEM : the remeshing problems during iterations and inaccuracy of boundary value evaluation. The principal advantage is that there is no need to discretize the interior of the body. There is also a large reduction in the number of unknowns. These can be seen from the researches in early 1980's such as Futagami[84),85)], Barone and Caulk[30)], Meric[87)~89)], Mota Soares *et al.*[13),44)~46)] and Zochowski and Mizukami[86)].

As in the variational formulation, there are two approaches to perform the SDSA based on boundary integral equations. One is a discretized approach, and the other a continuum approach.

a) Discretized Approach

This approach uses a discretized model to obtain the shape design sensitivity, and could be divided by three categories: Finite Difference Method (FDM), analytical method, semi-analytical method.

The FDM is straightforward and easy to

implement, and many authors[47),55),80),150)] have used it as a reference for comparison or a tool for sensitivity calculation. However, the FDM have a few shortcomings. It can not be exact unless the system is linear in the design variables. Thus, the result is highly dependent upon the size of perturbation. Also the computational cost can be high. Zhao and Adey[151)] presented a different SDSA scheme, which is based on FDM but independent of the perturbation step.

As an analytical approach, Kane and Saigal[152),153)] proposed an implicit direct differentiation method, in which the system matrix discretized from the boundary integral equation is differentiated analytically. Their formulation involves the products of shape functions, fundamental solutions and their derivatives. They introduced a rigid body motion technique of Barone and Yang[154)] to treat the singularity that exists in the derivative of fundamental solutions. While those approachs provide an easy and straightforward process, they have shown some problems such as the computational burden of performing singular integration of new kernels. They and their coworkers have extended the implicit differentiation method to various problems[14),48),155)~165)]. Similar method is used in[49),94),95),102)].

In the semianalytical method, the discretized system matrix is differentiated analytically, but the derivative of the stiffness matrix is calculated by employing the finite difference method. This method has economical and practical advantages but also the disadvantages of the FDM, and reliability problems[15),157),166)].

b) Continuum Approach

The continuum approach uses the material derivative concept of continuum mechanics to represent the variation of responses with respect to a shape change. The first step of this approach is to differentiate the boundary integral equation. No approximation is involved in the expression of the sensitivity, until the derived equations are discretized by boundary elements.

There are basically two methods to perform the SDSA. One is the direct differentiation method and the other the adjoint variable method. In the first method the state boundary integral is directly differentiated with respct to design variables and then a boundary integral equation similar to the original BIE is obtained in terms of derivatives of state variables. In the adjoint variable method, the constraint functional is first differentiated and the state variable dervatives are then replaced by terms calculable from adjoint systems.

Kwak and Choi[50),96),178)] and their followers[56),97),98)] have developed a general method for SDSA using the formal boundary integral equation. They used the material derivative concept and adjoint variable method utilizing a boundary integral identity to obtain an explicit expression for the variation of the performance functional in terms of boundary shape change, and the formulations were applied to thermal and elastostatic problems. Although this approach had aleady proven successful through numerical examples, there are some difficulties to determine the approximate adjoint tractions uniquely. An improved formulation of Kwak and Choi was developed by Zhao and Adey[226)], in which a singularity subtraction technique was employed to model the adjoint problem. Park and Yoo[179),180)] proposed a method employing the material derivative idea and an adjoint variable method in variational form for heat transfer system and axisymmetric elastic problem. They transformed the variational adjoint equation into an equivalent partial differential equation, and solved it by boundary integral equation method. Meric[51),90),181),182)] used the material derivative and adjoint variable method by means of an augmented functional method using the Lagrange multiplier. While his work throughout their derivation was independent of the boundary element formulation, he proposed the BEM for the solution of the original and the adjoint system. The same procedure is investigated by Aithal *et al.*[183)] and Kobelev[91)].

Barone and Yang[154),167)] and Yang[16)] developed a direct differentiation method, that is based on a direct application of the material derivative concept to the conventional boundary integral equations for displacements and stresses in an elastic solid. They employed a rigid body motion to remove high-order singularities that arise when taking derivatives of the basic Kelvin kernels in the displacement sensitivity. Zhang and Mukherjee[168)~171)] and Mukherjee and Chandra[172),173)] used the same concept of Barone and Yang but they used another boundary integral equation, derivative boundary integral equation, in which the basic unknowns are the tractions and the tangential derivatives of the displacements. Chandra and Chan[174),175)] utilized these for a steady state conduction-convection problem. Choi and Choi[176)] obtained the design derivatives directly by solving a new BIE, which is obtained by taking the material derivative of the boundary integral identity. The Authors[177),229)] presented formulas that consider changes in boundary conditions.

In the direct differentiation method we need one SDSA for each design variable, whereas we need one for each active constraint in the adjoint method. Therefore one can be computationally more efficient that the other depending on the number of design variables versus the number of active constraints. Ignoring the computational burden of complexities of singular kernels, the direct method may be more advantageous than the adjoint method, since the concentrated adjoint loads occurring in some cases are not suitable for the usual boundary element anlysis. Neither the adjoint variable nor direct method, however, can provide the most efficient computation if used alone. There may be some efficient hybrid methods. Choi and Kwak[184] proposed a unified approach for SDSA in the BIE formulation, which covers both the adjoint variable and direct method.

Table 1 Sensitivity Application Fields

Application Fields	References
Linear elastic	9, 11, 13, 14, 15, 32, 50, 55, 75, 76, 86, 92, 94, 95, 118, 122, 130, 132, 139, 152～157, 163, 167～170, 176～178, 180, 183, 186, 187
Nonlinear elastic	8, 140, 144, 145, 171～173, 188～191
Unilateral plane elasticity	191
Plate/Shell	58, 71～73, 108, 188, 190, 192～197
Nonsmooth boundary	18, 71～73, 76
Thermal	30, 47, 48, 56, 85, 87~90, 96, 141, 159, 161, 162, 165, 174, 175, 179, 182, 188, 198～200
Thermoelasticity	79, 97, 98, 143, 158, 181, 201～204
Thermoviscoelasticity	205
Elastodynamic	142
Frequency/Eigenvalue	105, 135, 189, 195, 197, 206, 207
Dynamic	84, 123, 137, 206, 207
Acoustics	160
Magnetostatics	51, 99, 208
Stochastic structure	102～104

Table 2 Optimization literature by specific problem

Application Problems	References
Elastic bar/Beam	28, 33, 44, 45, 49, 52, 70, 91, 93, 106, 124, 138, 201, 209, 210
Disks	32, 34, 38, 40, 41, 52, 72
Plate/Shell with a hole	8, 11, 24, 50, 53, 54, 58, 64, 97, 119, 192～194, 211～214
Plane arch/Arch dam	17～21
Fillet/Weld surface	5～16, 46, 50, 54, 97, 215, 216
Torque arm	7, 53, 58, 107, 217
Control arm	23, 43, 209, 216
Engine connecting rod	12, 22, 27, 28, 35, 42, 79, 94, 209
Engine bearing cap	5, 28, 79
Steering knuckle	23
Chain link	26, 119
Cable crimping device	107
Culvert	26, 217
Bracket	53, 214
Pressure vessel	218
Penstock stiffener plate	219
Tire	25
Hook	55
Helical spring	57
Piezoelectric structure	220
Magnetic pole/electrode	99, 100, 208, 221
MHD channel section	51
Cryosurgical probe tip	47
Die/mold	30, 56, 222
Photo cell	223
Speciman for shear test	224
Furnace hearth	48
Obstacle in an Eulerian flow	225

5. APPLICATIONS OF SDSA AND OPTIMIZATION

Test probelms and application cases appearing in the literature may be grouped in terms of SDSA formulation and optimization. It is seen that this area of shape optimal design is still in the growing stage, studying various formulations and testing rather simple problems. Although some of the applications are implemented on commerical softwares in conjunction with the FEM, it may take some more years to see any routine practical applications.

(1) SDSA Formulation

The information on areas of SDSA formulation is summarized in **Table** 1. Refer to Adelman and Hafka[185] for more detailed information about sensitivity analysis application fields.

(2) Specific Problem Applications

References are listed in **Table 2** by area of applications. Refer to Ding[3] and Haftka[4] for more structural shape optimization literature.

6. DISCUSSIONS AND CONCLUSION

An approach for the rapid creation of design and analysis model which is based on the integration of parameterized surface models, called 3-D shape design primitives and fully automatic mesh generation is now under developments. An integrated system for shape optimal design consists of geometric modeling, mesh generation, analysis, and design sensisivity analysis modules. Current trends in structural shape parameterization and optimization use the concept of generating the velocity field for the material derivative method. Many commerical packages have SDSA modules, but their approaches are based on finite difference, or semi-analytic method. The semi-anlytical approach is attractive for its generality and numerical efficiency. However, severe error in sensitivity may cause numerical difficulty and bring in divergence in optimization. In general, analytical sensitivities such as the material derivative approach give relatively good results and better convergence in shape optimal design problems. This suggests that anlytical sensivities should be used whenever available.

It is recognized that the essential and most influential content of a design is the shape, but its determination most difficult. Theories and algorithms for shape optimal design are found available in the literature, although their reliability, efficiency and accuracy remain to be studied more. It is, however, the reviewer's wish to see more realistic and practical application cases. Otherwise the many researchers in this area may spend too much time on trivial improvements in the methods with no attention from potential industry users.

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REFERENCES

1) Topping, B.H.V. : Shape optimization of skeletal structures : a review, *ASCE J. Struct. Engng.*, 109 (8), 1933~1951, 1983.
2) Levy, R. and Lev, O.E. : Recent developments in structural optimization, *ASCE J. Struct. Engng.*, 113 (9), 1939~1962, 1987.
3) Ding, Y. : COMPENDIUM, Shape optimization of structures : a literature survey, *Comp. & Struct.*, 24 (6), 985~1004, 1986.
4) Haftka, R.T. and Grandhi, R.V. : Structural shape optimization -a survey, *Comp. Meth. Appl. Mech. Engng.*, 57, 91~106, 1986.
5) Yao, T.M. and Choi, K.K. : 3-D shape optimal design and automatic finite element regridding, *Int. J. Numer. Meth. Engng.*, 28, 369~384, 1989.
6) Kristensen, E.S. and Madsen, N.F. : On the optimum shape of fillets in plates subjected to multiple in-plane loading cases, *Int. J. Numer. Meth. Engng.*, 10, 1007~1019, 1976.
7) Yang, R.J., Choi, K.K. and Haug, E.J. : Numerical consideration in structural component shape optimization, *ASME J. Mech. Trans. Auto. Des.*, 107, 334~339, 1985.
8) Arora, J.S. and Cardoso, J.E.B. : A design sensitivity analysis principle and its implementation into ADINA, *Comp. & Struct.*, 32 (3/4), 691~705, 1989.
9) Choi, K.K., Hou, J.W. and Yoo, Y.M. : A variational method for shape optimal design of elastic structures, in *New Directions in Optimum Structural Design*, 105~137, 1984.
10) Seong, H. G. and Choi, K. K. : Boundary-Layer approach to shape design sensitvity analysis, *Mech. Struct. & Mach.* 15 (2), 241~263, 1987.
11) Rodrigues, H.C. : Shape optimal design of elastic bodies using a mixed variational formulation, *Comp. Meth. Appl. Engng.*, 69, 29~44, 1988.
12) Francavilla, A., Ramakrishnan, C.V. and Zienkiewicz, O.C. : Optimization of shape to minimize stress concentration, *J. Strain Anal.*, 10 (2), 63~70, 1975.
13) Mota Soares, C.A., Rodrigues, H.C. and Choi, K.K. : Shape optimal structural design using boundary element and minimum compliance techniques, *ASME, J. Mech., Trans. Auto. Des.*, 106 (4), 518~523, 1984.
14) Saigal, S. and Kane, J.H. : Design sensitivity analysis of boundary-element substructuring, *AIAA J.*, 28 (7), 1990.
15) Hajela, P. and Jih, J. : Adaptive grid refinement in a BEM-based optimal shape synthesis, *Int. J. Solids Struct.*, 26 (1),

29～41, 1990.

16) Yang, R.J. : Component shape optimization using BEM, *Comp. & Struct.*, 37 (4), 561～568, 1990.

17) Dems, K. and Mroz, Z. : A variational approach to sensitivity analysis and structural optimization of plane arches, *Mech. Struct. & Mach*, 15 (3), 297～321, 1987.

18) Habbal, A. : Theoretical and numerical study of nonsmooth shape optimization applied to the arch problem, *Mech. Struct. & Mach.*, 20 (1), 93～117, 1992.

19) Zhu, B., Rao, B., Jia, J. and Li, Y. : Shape optimization of arch dams for static and dynamic loads, *ASCE J Struct. Engng.*, 118 (11), 2996～3015, 1992.

20) Wang, S.Y., Sun, Y. and Gallagher, R.H. : Sensitivity analysis in shape optimization of continuum structures, *Comp. & Struct.*, 20 (5), 855～867, 1985.

21) Ricketts, R.E. and Zienkiewicz, O.C. : Shape optimization of continuum structures, in *New Directions in Optimum Structural Design*, 139～166, 1984.

22) Bennet, J.A. and Botkin, M.E. : Structural shape optimization with geometric description and adaptive mesh refinement, *AIAA J.*, 23 (3), 458～464, 1985.

23) Botkin, M.E. : Shape design modeling using fully automatic three-dimensional mesh generation, *Finite Elem. Anal. Des.*, 10, 165～181, 1991.

24) Rossen, D.W. and Grosse, I.R. : A feature based shape optimization technique for the configuration and parametric design of flat plates, *Engng. with Computers*, 8, 81～91, 1992.

25) Noor, A.K., Tanner, J.S. and Peters, J.M. : Sensitivity of tire response to variations in material and geometric parameters, *Finite Elem. Anal. Des.*, 11, 77～86, 1992.

26) Rajan, S.D., Belegundu, A.D. and Budiman, J. : An integrated system for shape optimal design, *Comp. & Struct.*, 30 (1/2), 337～346, 1988.

27) Yang, R.J. : Design modeling consideration in shape optimization of solids, *Comp. & Struct.*, 34 (5), 727～734, 1990.

28) Yang. R.J. and Botkin, M.E. : A modular approach for three-dimensional shape optimization of structures, *AIAA J.*, 25 (3), 492～497, 1986.

29) Chang, K.-H. and Choi, K.K. : A geometry-based parameterization method for shape design of elastic solids, *Mech. Struct. & Mach.*, 20 (2), 215～252, 1992.

30) Barone, M.R. and Caulk, D.A. : Optimal arrangement of holes in a two-dimensional heat conductor by a special boundary integral method, *Int. J. Numer. Meth. Engng.*, 18, 675～685, 1982.

31) Zienkiewicz, O.C. and Campbell, J.S. : Shape optimization and sequential linear programming, in *Optimum Structural Design*, Chap. 7., 109～126, 1973.

32) Cheu, T.C. : Sensitivity analysis and shape optimization of axisymmetric structures, *Int. J. Numer. Meth. Engng.*, 28, 95～108, 1989.

33) Iman, M.H. : Three dimensional shape optimization, *Int. J. Numer. Meth. Engng.*, 18, 661～673, 1982.

34) Bhavikatti, S.S. and Ramakrishnan, C.V. : Optimum shape design of rotating disks, *Comp. & Struct.*, 11, 397～401, 1980.

35) Prasad, B. and Emerson, J.F. : Optimal structural remodeling of multi-objective systems, *Comp. & Struct.*, 18 (4), 619～628, 1984.

36) Pedersen, P. and Laursen, C.L. : Design for minimum stress concentration by finite elements and linear programming, *J. Struct. Mech.*, 10, 375～391, 1982.

37) Yang, R.J. and Choi, K.K. : Accuracy of finite element based design sensitivity analysis, *J. Struct. Mech.*, 13 (2), 223～239, 1985.

38) Luchi, M.L., Poggialini, A. and Persiani, F. : An interactive optimization procedure applied to the design of gas turbine discs, *Comp. & Struct.*, 11, 629～637, 1980.

39) Weck, M. and Steike, P. : An efficient technique in shape optimization, *Struct. Mech.*, 11 (4), 433～449, 1983-4.

40) Braibant, V. and Fleury, C. : Shape optimal design using B-splines, *Comp. Meth. Appl. Mech. Engng.*, 44, 247～267, 1984.

41) Braibant, V. and Sander, G. : Optimization techniques : Synthesis of design and analysis, *Finite Elem. Anal. Des.*, 3, 57～78, 1987.

42) Yang, R.J., Dewhirst, D.L., Allison, J.E. and Lee, A. : Shape optimimization of connecting rod pin end using a generic model, *Finite Elem. Anal. Des.*, 11, 257～264, 1992.

43) Yang, R.J. : A three dimensional shape optimization system-SHOP3D, *Comp. & Struct.*, 31 (6), 881～890, 1989.

44) Mota Soares, C.A., Rodrigues, H.C. and Oliveira Faria, L.M. : Optimization of the shape of solid and hollow shafts using boundary elements, *Boundary Elements V, C.A. Brebbia (ed.)*, Springer-Verlag, 883～889, 1983.

45) Mota Soares, C.A., Rodrigues, H.C., Oliveira Faria, L.M. and Haug, E.J. : Optimization of shafts using boundary elements, *J. Mech., Transm. Autom. Design, ASME*, 106, 199～202, 106.

46) Mota Soares, C.A., Leal, R.P. and Choi, K.K. : Boundary elements in shape optimal design of structural components, *Comp. Aided Opt. Design*, Mota Soares C.A. (ed.), NATO ASI27, 605～631, 1987.

47) Carter, S.M., Barron, R.F. and Warrington. : BEM shape optimization technique applied to cryosugical probe tip deaign, *Numer. Heat Transfer, Part A*, 16, 229～248, 1989.

48) Saigal, S. and Chandra, A. : Shape sensitivities and optimal configurations for heat diffusion problems : a BEM approach, *J. Heat Transfer, ASME*, 113, 287～295, 1991.

49) Gracia, L. and Doblare, M. : Shape optimization of elastic orthotropic shafts under torsion by using boundary elements, *Comp. & Struct.*, 30 (6), 1281～1291, 1988.

50) Choi, J.H. and Kwak, B.M. : Boundary integral equation method for shape optimization of elastic structures, *Int. J. Numer. Meth. Engng*, 26, 1579～1595, 1988.

51) Meric, R.A. : Optimal cross-sectional shapes for MHD channel flows, *Int. J. Numer. Meth. Engng*, 30, 919～929, 1990.

52) Dems, K. : Multiparameter shape optimization of elastic bars in torsion, *Int. J. Numer. Meth. Engng.*, 15, 1517~1539, 1980.
53) Braibant, V. and Fleury, C. : An approximation-concepts approach to shape optimal design, *Comp. Meth. Appl. Mech. Engng.*, 53, 119~148, 1985.
54) Shyy, Y.K. and Fleury, C. : Shape optimal design using high-order elements, *Comp. Meth. Appl. Mech. Engng.*, 71, 99~116, 1988.
55) Sandgren, E. and Wu, S-J. : Shape optimization using the boundary element method with substructuring, *Int. J. Numer. Meth. Engng.*, 26, 1913~1924, 1988.
56) Lee, D.H. and Kwak, B.M. : Shape sensitivity and optimization for transient heat diffusion problems using the BEM, *Int. J. Numer. Meth. Engng*, submitted, 1993.
57) Kamiya, N. and Kita, E. : Boundary element method for quasi-harmonic differential equation with application to stress analysis and shape optimization of helical spring, *Comp. & Struct.*, 37 (1), 81~86, 1990.
58) Botkin, M.E. : Shape optimization of plates and shell structures, *AIAA J.*, 20 (2), 268~273, 1982.
59) Michell, A.G.M. : The limits of economy of materials in frame structures, *Phil. Magazine*, 8 (47), 589~595, 1904.
60) Haug, E.J. and Cea, J. (ed.), *Optimization of Distributed Parameter Structures*, Sijthoff and Noordhoff, Alphen ann den Rijn, Netherland, 1981.
61) Schmit, L.A. : Structural design by systematic synthesis, *Proc. Second ASCE Conf. Electronic Computation*, Pittsburgh, 105~122, 1960.
62) Kicher, T.P. : Structural synthesis of integrally stiffened cylinders, *J. Spacecraft Rockets*, 5, 62~67, 1968.
63) Gellatly, R.A. and Gallagher, R.H. : A procedure for automated minimum weight structural design, Part I - Theoretical basis, Part II -Applications, *Aero Ouart. Part* I, 17, 216~230 and 332~342, 1966.
64) Schnack, E. : An optimization procedure for stress concentrations by the finite element technique, *Int. J. Numer. Meth. Eng.*, 14, 115~124, 1979.
65) Oda, J. : On a technique to obtain an optimum strength shape by the finite element method, *Bulletin of JSME*, 20, 160~167, 1977.
66) Tvergaard, V. : On the optimum shape of a fillet in a flat bar with restrictions, *Optimization in Structural Design*, Sawczuk, A and Mroz, Z (ed.), 181~195, 1975.
67) Ramakrishnan, C.V. and Francavilla, A. : Structutal shape oprimization using penalty functions, *J. Struct. Mech.*, 3 (4), 403~432, 1975.
68) Chun, Y.W. and Haug, E.J. : Two dimensional shape optimal design, *Int J. Numer. Meth. Engng.*, 13 (5), 311~336, 1978.
69) Rousselet, B. and Haug, E.J. : Design sensitivity analysis in structural mechanics. III. Effects of shape variation, *J. Struct. Mech.*, 10 (3), 273~310, 1982-3.
70) Banichuk, N.V. : Optimization of elastic bars in torsion, *Int. J. Solids Struct.*, 12, 275~286, 1976.
71) Dems, K. and Mroz, Z. : Shape sensitivity analysis and optimal design of disks and plates with strong discontinuties of kinematic fields, *Int. J. Solids Struct.*, 29 (4), 437~463, 1992.
72) Dems, K., Mroz, Z. and Szelag. : Optimal design of rib-stiffeners in disks and plates, *Int. J. Solids Struct.*, 25 (9), 973~998, 1989.
73) Dems, K. and Mroz, Z. : Sensitivity of buckling load and vibration frequency with respect to shape of stiffened and unstiffened plates, *Mech. Struct. & Mach.*, 17 (4), 431~457, 1989.
74) Dems, K. and Haftka, R.T. : Two approaches to sensitivity analysis for shape variation of structures, *Mech. Struct. & Mach.*, 16 (4), 501~522, 1989-9.
75) Dems, K. and Mroz, Z. : On a class of conservation rules associated with sensitivity analysis in linear elasticity, *Int. J. Solids Struct.*, 22 (7), 737~758, 1986.
76) Dems, K. and Mroz, Z. : Variational approach by means of adjoint systems to structural optimization and sensitivity analysis-II. Structural shape variation, *Int. J. Solids Struct.*, 20 (6), 527~552, 1984.
77) Kodiyalam, S. and Vanderplaats, G.N. : Shape optimization of three-dimensional continuum structures via forced approximation technique, *Int. J. Numer. Meth. Engng.*, 27 (9), 1256~1263, 1989.
78) Wassermann, K. : Three-dimensional shape optimization of arch dam with prescribed shape functions, *J. Struct. Mech.*, 11, 465~489, 1983.
79) Balasubramanian, B., Svoboda, M. and Bauer, W. : Structural optimization of I.C. engines subjected to mechanical and thermal loads, *Comp. Meth. Appl. Mech. Engng.*, 89, 337~360, 1991.
80) Nguyen, V.U. and Arenicz. : Sensitivity analysis of underground excavation using boundary element method, *BETECH* 85, C.A. Brebbia and B. Noye (ed.), Springer-Verlag, 257~267, 1985.
81) Suzuki, K. and Kikuchi, N. : A homogenization method for shape and topology optimization, *Comp. Meth. Appl. Mech. Engng.*, 93, 291~318, 1991.
82) Bremicker, M., Chirehdast, M., Kikuchi, N. and Papalambros, P.Y. : Integrated topology and shape optimization in structural design, *Mech. Sttuct. & Mach.*, 19 (4), 551~586, 1991.
83) Daiz, A.R. and Kikuchi, N. : Solutions to shape and topology eigenvalue optimization problems using a homogenization method, *Int. J. Numer. Meth. Engng.*, 35, 1487~1502, 1992.
84) Futagami, T. : Boundary element and dynamic programming method in optimization of transient partial differential system, *4th Int. Sem. Boundary Element Meth. Engng.*, C.A. Brebbia (ed), Springer-Verlag, 58~71, 1982.
85) Futagami, T. : Boundary element method-finite element method coupled with linear programming for optimal control of distributed parameter systems, *Boundary Element V*, C. A. Brebbia (ed), Springer-Verlag, 891~900, 1983.
86) Zochowska, A. and Mizukami, K. : A comparison of BEM

and FEM in minimum weight design, *Boundary Elements V*, C.A. Brebbia (ed.), Springer-Verlag, 901~911, 1983.

87) Meric, R.A. : Boundary element method for optimization of distributed parameter systems, *Int. J. Numer. Meth. Engng.*, 20, 1291~1306, 1984.

88) Meric, R.A. : Boundary integral equation and conjugate gradient methods for optimal boundary heating of solids, *Int. J. Heat Mass Transfer*, 26, 261~267, 1983.

89) Meric, R.A. : Boundary elements for static optimal heating of solids, *J. Heat Transfer, ASME*, 106, 876~880, 106.

90) Meric, R.A. : Shape design sensitivity analysis for nonlinear anisotropic heat conducting solids and shape optimization by the BEM, *Int. J. Numer. Meth. Engng.*, 26, 109~120, 1988.

91) Kobelev, V.V. : Numerical method for shape oprimization using BEM, *Comp. & Struct.*, 33 (5), 1223~1227, 1989.

92) Burczynski, T. and Adamczyk, T. : The boundary element formulation for multiparameter structural shape optimization, *Appl. Math. Modelling*, 9, 195~200, 1985.

93) Burczynski, T. and Adamczyk, T. : Boundary element method for shape design synthesis of elastic structures, *Boundary Elements* Ⅶ, Brebbia, C.A. and Maier, G. (ed.), Springer-Verlag, 1985.

94) Espiga, F., Gracia, L. and Doblare, M. : Shape oprimization of elastic homogeneous 2D bodies by the boundary element method, *Comp. & Struct.*, 33 (5), 1233~1241, 1989.

95) Xu, C. and Yu, M. : Shape optimization of structures to minimize stress concentration, *Comp. & Struct.*, 36 (3), 491~497, 1990.

96) Kwak, B.M. and Choi, J.H. : Shape design sensitivity analysis using boundary integral equation for potential problem, in *Comp. Aided Opt. Design*, Mota Soares C.A. (ed.), NATO ASI27, 633~642, 1987.

97) Lee, B.Y. and Kwak, B.M. : Shape Optimization of two-dimensional thermoelastic structures using boundary integral equation formulation, *Comp. & Struct.*, 41 (4), 709~722, 1991.

98) Lee, B.Y. and Kwak, B.M. : Axisymmetric thermoelastic shape sensitivity analysis and its application to turbine disc design, *Int. J. Numer. Meth. Engng*, 33, 2073~2089, 1992.

99) Ishiyama, A., Yokoi, T. and Onuki. : Optimal design technique for magnetostatic fields, *BEM Appl. Meth.*, Tanaka, M. and Cruse, T.A. (ed.), Pergamon Press, 413~422, 1988.

100) Ishiyama, A., Yokoi, T., Takamori, S. and Onuki, T. : An optimal design technique for MRI superconducting magnets using mathematical programming method, *IEEE Trans. Mag.*, 24 (2), 922~925, 1988.

101) Chaudouet, A., Miranda and EI Yafi, F. : Boundary element method applied to 3D optimum design, *Adv. Boundary Elem. Meth.*, Cruse, T.A. (ed.), Springer-Verlag, 101~108, 1988.

102) Nagagiri, S. and Suzuki, K. : Boundary element synthesis for shape modification based on first-oder sensitivities, *BEM Appl. Mech.*, Tanaka, M. and Cruse, T.A. (ed.), Pergamon Press, 383~392, 1988.

103) Tada, Y., Matsumoto, R. and Imamura, O. : Optimum shape design of structures under uncertain loading by boundary element method, *JSME Int. J. Ser. I*, 34 (1), 23~29, 1991.

104) Burczynski, T. : Boundary element method for deterministic and stochastic shape design sensitivity snalysis, *Adv. Boundary Elem. Meth.*, Cruse, T.A. (ed.), Springer-Verlag, 73~80, 1988.

105) Fedelinski, P. and Burxzynski, T. : Shape optimal design of vibrating structures using boundary elements, *ZAMM*, 71 (6), T726~728, 1991.

106) Kamiya, N. and Kita, E. : Structural optimization by an adaptive boundary element method, *BEM Appl. Mech.*, Tanaka, M. and Crus, T.A. (ed.), Pergamon Press, 393~402, 1988.

107) Yatheendhar, M. and Belegundu, D. : Analytical shape sensitivity by implicit differentiation for general velocity fields, *Comp. & Struct.*, 46 (4), 617~623, 1993.

108) Brockman, R.A. and Lung, F.Y. : Sensitivity analysis with plate and shell finite elements, *Int. J. Numer. Meth. Engng.*, 26, 1129~1143, 1988.

109) Brockman, R.A. : Geometric sensitivity analysis with isoparametric finite elements, *Comm. Appl. Mumer. Meth.*, 3, 495~499, 1987.

110) Haug, E.J., Choi, K.K. and Komkov, V., *Design sensitivity analysis of structural systems*, Academic press, New York, 1986.

111) Liu, Z.-S. and Chen, S.-H. : Reanalysis of static response and its design sensitivity of locally modified structures, *Comm. Appl. Numer. Meth.*, 8, 797~800, 1992.

112) Yang, R.J. and Botkin, M.E. : Comparison between the variational and implicit differentiation approaches to design sensitivities, *AIAA J.*, 24 (6), 1027~1032, 1986.

113) Barthelemy, B., Chon, C.T. and Haftka, R.T. : Accuacy problems associated with semianalytical derivatives of static response, *Finite. Anal. Des.*, 4, 249~265, 1988.

114) Braibant, V. and Fleury, C. : Sensitivity analysis in shape optimal design, in *Computer Aided Optimal Design : Structural and Mechanical Systems*, (ed., Mota Soares), Springer, Berlin.

115) Cea, J. : Numerical methods in shape optimal design, in *Optimization of Distributed Parameter Structures* (Ed., Haug, E.J. and Cea, J.), Vol. Ⅱ, 1049~1087, Sijthoff & Noordhof, Netherlands, 1981.

116) Cheng, G., Gu, Y. and Zhou, Y. : Accuracy of semi-analytic sensitivity analysis, *Finite Elem. Anal. Des.*, 6, 113~128, 1989.

117) Fenyes, P.A. and Lust, R.V. : Error analysis of semianalytic displacement derivatives for shape and sizing variables, *AIAA J.*, 29 (2), 271~279, 1991.

118) Choi, K.K. and Haug, E.J. : Shape design sensitivity analysis of elastic structures, *J. Struct. Mech.*, 11 (2), 231~269, 1983.

119) Belegundu, A.D. and Rajan, S.D. : A shape optimization approach based on natural design variables and shape

functions, *Comp. Meth. Appl. Mech. Engng.*, 66, 87～106, 1988.

120) Gelfand, I.M. and Fomin, S.V. : *Calculus of Variations*, Prentice-Hall, 1969.

121) Choi, K.K. : Shape design sensitivity analysis and optimal design of structural systems, in *Computer Aided Optimal Design : Structural and Mechanical Systems*, (ed., Mota Soares), Springer, Berlin.

122) Twu, S.L. and Choi, K.K. : Configuration design sensitivity analysis of built-up structures part I : Theory, *Int. J. Numer. Meth. Engng.*, 35, 1127～1150, 1992.

123) Rousselet, B. : Shape design sensitivity of a membrane, *J. Opt. Theory Appl.*, 40 (4), 595～623, 1983.

124) Zochowski, A. and Mizukami, K. : Minimum weight design with displacements constraints in 2-dimensional elasticity, *Comp. & Struct*, 17 (3), 365～369, 1983.

125) Arora, J.S. : An exposition of the material derivative approach for structural shape sensitivity analysis, *Comp. Meth. Appl. Mech. Engng.*, 105, 41～62, 1993.

126) Zolesio, J.P. : The material derivative (or speed) method for shape optimization, in *Optimization of Distributed Parameter Structures* (Ed., Haug, E.J., and Cea, J.), Vol. II, 1089～1151, Sijthoff & Noordhoff, Netherlands, 1981.

127) Rousselet, B. : Implementation of some methods of shape design, in *Optimization of Distributed Parameter Structures* (Ed., Haug, E.J., and Cea, J.), Vol. II, Sijthoff & Noordhoff, Netherlands, 1969.

128) Wu, C.C. and Arora, J.S. : Design sensitivity analysis and optimization of nonlinear structural response using incremental procedure, *AIAA J.* Aug., 1118～1125, 1987.

129) Hou, J.W. : OPTIMIZATION, Techniques and applications of shape optimum design, *Comp. & Struct*, 20 (1-3), 467～473, 1985.

130) Vigil, A.E.A., Nicieza, C.G. and Mere, J.B.O. : Numerical solution of an optimal shape design problem with elastic solids, *Comp. Meth. Appl. Mech. Engng.*, 99, 147～170, 1992.

131) Choi, K.K. and Seong, H.G. : A domain method for shape design sensitivity analysis of built-up structures, *Comp. Meth. Appl. Mech. Engng.*, 57, 1～15, 1986.

132) Choi, K.K. : Shape design sensitivity analysis of displacement stress constraints, *J. Struct. Mech.*, 13 (1), 27～41, 1985.

133) Belegundu, A.D. and Arora, J.S. : A sensitivity interpretation of adjoint variables in optimal design, *Comp. Meth. Appl. Mech. Engng.*, 48, 81～89, 1985.

134) Haug, E.J. and Arora, J.S. : Design sensitivity analysis of ealstic mechanical systems, *Comp. Meth. Appl. Mech. Engng.*, 15, 35～62, 1978.

135) Haug, E.J. and Rousselet, B. : Design sensitivity analysis in structural mechanics II. eigenvalue variations, *J. Struct. Mech.*, 8 (2), 161～186, 1980.

136) Yang, R.J. and Botkin, M.E. : Accuracy of the domain materical derivative approach to shape design sensitivity, *AIAA J*, 25 (12), 1606～1610, 1987.

137) Meric, R.A. : Shape design sensitivity analysis of dynamic structures, *AIAA J.*, 26 (2), 206～212, 1988.

138) Hou, J.W., Chen, J.L. and Sheen, J.S. : Computational method for optimization of structural shape, *AIAA J.*, 24 (6), 1005～1012, 1986.

139) Phelan, D.G. and Haber, R.B. : Sensitivity analysis of linear sytems using domain parameterization and a mixed mutual energy principle, *Comp. Meth. Appl. Mech. Engng.*, 77, 31～59, 1989.

140) Cardoso, J.B. and Arora, J.S. : Variational method for design sensitivity analysis in nonlinear structural mechanics, *AIAA J.*, 26 (5), 595～603, 1988.

141) Tortorelli, D.A. and Harber, R.B. : First-order design sensitivity for transient conduction problems by an adjoint method, *Int. J. Numer. Meth. Engng.*, 28, 733～752, 1989.

142) Tortorelli, D.A., Lu, S.C.-Y. and Harber, R.B. : Design sensitivity analysis for elastodynamic system, *Mech. Struct. & Mach.*, 18 (1), 77～106, 1990.

143) Tortorelli, D.A., Harber, R.B. and Lu, S.C.-Y. : Adjoint sensitivity analysis for nonlinear dynamic thermoelastic systems, *AIAA J.*, 29 (2), 253～263, 1991.

144) Tsay, J.J., Cardoso, J.E.B. and Arora, J.S. : Nonlinear structural design sensitivity analysis for path dependent problems. Part 2 : Analytical examples, *Comp. Meth. Appl. Mech. Engng.*, 81, 209～228, 1990.

145) Tsaym, J.J. and Arora, J.S. : Nonlinear structural design sensitivity analysis for path dependent problems, Part 1 : General Theory, *Comp. Meth. Appl. Mech. Engng.*, 81, 183～208, 1990.

146) Arora, J.S. and Cardoso, J.B. : Variational principle for shape design sensitivity analysis, *AIAA J.*, 30 (2), 538～547, 1992.

147) Tortorelli, D.A. and Wang, Z. : A systematic approach to shape sensitivity analysis, *Int. J. Solids Struct.*, 30 (9), 1181～1212, 1993.

148) Haber, R.B. : A new variatrional approach to structural shape sensitivity analysis, in *Computer Aided Optimal Design : Structural and Mechanical Systems*, (ed., Mota Soares), Springer, Berlin.

149) Arora, J.S., Lee, T.H. and Cardoso, J.E.B. : Structural shape sensitivity analysis : Relationship between material derivative and control volume approaches, *AIAA J.*, 30, 1638～1648, 1992.

150) Haftka, R.T. and Malkus, D.S. : Calculation of sensitivity derivatives in thermal problems by finite differences, *Int. J. Numer. Meth. Engng.*, 17, 1811～1821, 1981.

151) Zhao, Z. and Adey, R.A. : An alternative approach to shape design sensitivity analysis, *Int. J. Numer. Meth. Engng.*, 35, 1071～1086, 1992.

152) Kane, J.H. and Saigal, S. : Design-sensitivity analysis of solids using BEM, *J. Engng. Mech., ASCE*, 114 (10), 1703～1722, 1988.

153) Saigal, S., Aithal, R. and Kane, J.H. : Conforming boundary elements in plane elasticity for shape design sensitivity, *Int. J. Numer. Meth. Engng.*, 28, 2795～2811, 1989.

154) Barone, M.R. and Yang, R.J. : Boundary integral

equations for recovery of design sensitivity in shape optimization, *AIAA J.*, 26(5), 589~594, 1988.

155) Saigal, S., Borggaard, J.T. and Kane, J.H. : Boundary element implicit differentiation equations for design sensitivities of axisymmetric structures, *Int. J. Solids Struct.*, 25 (5), 527~538, 1989.

156) Saigal, S. : Treatment of body forces in axisymmetric boundary element design sensitivity formulation, *Int. J. Solids Struct.*, 25 (8), 947~959, 1989.

157) Aithal, R., Saigal, S. and Mukherjee, S. : Three dimensional boundary element implicit differentiation formulation for design sensitivity analysis, *Math. Comp. Modelling*, 15, 1~10, 1991.

158) Kane, J.H., Kumar, B.L.K. and Stabinsky, M. : Transient thermoelasticity and other body force effects in boundary element shape sensitivity analysis, *Int. J. Numer. Meth. Engng.*, 1203~1230, 1991.

159) Kane, J.H. and Wang, H. : Boundary-element shape sensitivity analysis for thermal problems with nonlinear boundary conditions, *AIAA J.*, 29 (11), 1978~1989, 1991.

160) Kane, J.H. and Mao, S. : A boundary element formulation for acoustic shape sensitivity analysis, *J. Acoust. Soc. Am.*, 90 (1), 561~573, 1991

161) Kane, J.H. and Wang, H. : Boundary formulations for shape sensitivity of temperature dependent conductivity problems, *Int. J. Numer. Meth. Engng.*, 33, 667~693, 1992.

162) Guru Prasad, K. and Kane, J.H. : Three-dimensional boundary elenment thermal shape sensitivity analysis, *Int. J. Heat Mass Transfer*, 35 (6), 1427~1439, 1992.

163) Kane, J.H., Zhao, G., Wang, H. and Guru Prasad, K. : Boundary formulations for three-dimensional continuum structural shape sensitivity analysis, *J. Appl. Mech., ASME*, 59, 827~834, 1992.

164) Guru Prasad, K. and Kane, J.H. : Boundary formulations for sensitivities of three-dimensional stress invariants, *Comp. & Struct.*, 43 (6), 1165~1174, 1992.

165) Wang, H. Guru Prasad, K. and Kane, J.H. : Three dimensional boundary formulations for nonlinear thermal shape sensitivities, *Comp. Mech.*, 11, 123~139, 1993.

166) Saigal, S., Aithal, R. and Kane, J.H. : Semiananlytical structural sensitivity formulation in boundary elements, *AIAA J.*, 27 (11), 1615~1621, 1989.

167) Barone, M.R. and Yang, R.J. : A boundary element approach for recovery of shape sensitivities in three-dimensional elastic solids, *Comp. Meth. Appl. Mech. Engng.*, 74, 69~82, 1989.

168) Zhang, Q. and Mukherrjee, S. : Design sensitivity coefficients for linear elasticity problems by boundary element methods, *IUTAM/IACM Symposium on Discretization Methods in Structural Mechanics*, Austria, 1989.

169) Zhang, Q. and Mukherjee, S. : Second-order design sensitivity analysis for linear elastic problems by the derivative boundary element method, *Comp. Meth. Appl. Mech. Engng.*, 86, 321~335, 1991.

170) Zhang, Q. and Mukherjee, S. : Design sensitivity coefficeints for linear elastic bodies with zones and corners by the derivative boundary element method, *Int. J. Solids Struct.*, 27 (8), 983~998, 1991.

171) Zhang, Q. and Mukherjee, S. : Design sensitivity coefficients for elasto-viscoplastic problems by boundary element methods, *Int. J. Numer. Meth. Engng.*, 34, 947~966, 1992.

172) Mukherjee, S. and Chandra, A. : A boundary element formulation for design sensitivities in materially nonlinear problems, *Acta Mech.*, 78, 243~253, 1989.

173) Mukherjee, S. and Chandra, A. : A boundary elemnent formulation for design sensitivities in problems involving both geometric and material nonlinearities, *Math. Comput. Modelling*, 15, 245~255, 1991.

174) Chandra, A. and Chan, C.L. : A boundary element method formulation for design sensitivities in steady-state conduction-convection problems, *J. Appl. Mech.*, 59, 182~190, 1992.

175) Chandra, A. and Chan, C.L. : An algorithm for handling corners in the boundary element method : Application to conduction-convection equations, *Appl. Math. Modelling*, 15, 244~255, 1991.

176) Choi, J.H. and Choi, K.K. : Direct differentiation method for shape design sensitivity analysis using boundary integral formulation, *Comp. & Struct.*, 34 (3), 499~508, 1990.

177) Grabacki, J. : Boundary integral equation in sensitivity analysis, *Appl. Math. Modelling*, 15, 170~181, 1991.

178) Choi, J.H. and Kwak, B.M. : Shape design sensitivity analysis of elliptic problems in boundary integral equation formulation, *Meth. Struct. & Mach.*, 16 (2), 147~165, 1988.

179) Park, C.W. and Yoo, Y.M. : Shape design sensitivity analysis of a two-dimensional heat transfer system using the boundary element method, *Comp. & Struct.*, 28 (4), 543~550, 1988.

180) Park, C.W., Yoo, Y.M. and Kwon, K.H. : Shape design sensitivity analysis of an axisymmetric turbine disk using the boundary element method, *Comp. & Struct.*, 33 (1), 7~16, 1989.

181) Meric, R.A. : Boundary elements in shape design sensitivity analysis of the thermoelastic solids, in *Comp. Aided Opt. Design*, Mota Soares C.A. (ed.), NATO ASI27, 643~652, 1987.

182) Meric, R.A. : Shape optimization and identification of solid geometries considering discontinuities, *J. Heat Transfer, ASME*, 110, 544~550, 1988.

183) Aithal, R. and Saigal. S. : Adjoint structural approach for shape sensitivity analysis using BEM, *J. Engng. Mech., ASCE*, 2663~2680, 1990.

184) Choi, J.H. and Kwak, B.M. : A Unified approach for adjoint and direct method in shape design sensitivity analysis using boundary integral formulation, *Eng. Anal. Boundary Ele.*, 7 (1), 39~45, 1990.

185) Adelman, H.M. and Haftka, R.T. : Sensitivity analysis of discrete structural systems, *AIAA J.*, 24 (5), 823~832, 1986.

186) Belegundu, A.D. : Lagrangian approach to design senaitivity analysis, *ASCE J. Engng. Mech.*, 111 (5), 680~695, 1985.

187) Na, M.S., Kikuchi, N. and Taylor, J.E. : Optimal modification of shape for two-dimensional elastic bodies, *J. Struc. Mech.*, 11 (1), 111~135, 1983.

188) Mroz, Z., Kamat, M.P. and Plaut, R.H. : Sensitivity analysis and optimal design of nonlinear beams and plates, *J. Struct. Mech.*, 13 (3 & 4), 245~266, 1985.

189) Park, J.S. and Choi, K.K. : Design sensitivity analysis and optimization of nonlinear structural systems with critical loads, *ASME J. of Mech. Des.*, 114, 305~312, 1992.

190) Dems, K. and Mroz, Z. : Shape sensitivity analysis and optimal design of physically nonlinear plates, *Arch. Mech.*, 41, 481~501, 1989.

191) Haslinger, J., Neittaanmaki, P. and Salmenjoki, K. : Sensitivity analysis for discretized unilateral plane elasticity problem, *Finite Elem. Anal. Des.*, 1992, 12, 13~25, 1992.

192) Hinton, E. and Rao, N.V.R. : Structural shape optimization of shells and folded plates using two-noded finite strips, *Comp. & Struct.*, 46 (6), 1055~1071, 1993.

193) Lee, S.S. and Kwak, B.M. : Shape sensitivity analysis of thin-shell structures, *Finite Elem. Anal. Design*, 10, 293~305, 1992.

194) Lee, M.S., Kikuchi, N. and Scott, A. : Shape optimization in laminated composite plats, *Comp. Meth. Appl. Mech. Engng.*, 72, 29~55, 1989.

195) Pandey, M.D. and Sherbourne, A.N. : Mechanics of shape optimization in plate buckling, *ASCE J. Struct. Engng.*, 118 (6), 1249~1266, 1992.

196) Tripathy, B. and Rao, K.P. : Stiffened composite axisymmetric shells-optimum lay-up for buckling by ranking, *Comp. & Struct.*, 46 (2), 299~309, 1993.

197) Wroblewski, A. : Optimal design of circular plates against creep buckling, *Eng. Opt.*, 20, 111~128, 1992.

198) Haftka, R.T. : Techniques for thermal sensitivity analysis, *Int. J. Numer. Meth. Engng.*, 17, 71~80, 1981.

199) Hou, J.W. and Sheen, J. : Numerical methods for second-order shape sensitivity analysis with applications to heat conduction problems, *Int. J. Numer. Meth. Engng.*, 36, 417~435, 1993.

200) Tortorelli, D.A., Harber, R.B. and Lu, S.C.-Y. : Design sensitivity analysis for nonlinear thermal systems, *Comp. Meth. Appl. Mech. Engng.*, 77, 61~77, 1989.

201) Hou, J.W., Sheen, J.S. and Chuang, C.H. : Shape-sensitivity analysis and design optimization of linear, thermoelastic solids, *AIAA J.*, 30 (2), 528~537, 1992.

202) Meric, R.A. : Simultaneous Material/Load/Shape variation of thermoelastic structures, *AIAA, J.*, 28 (2), 296~302, 1990.

203) Tortorelli, D.A., Subramani, G. and Lu, S.C.Y. : Sensitivity analysis for coupled thermoelastic systems, *Int. J. Solids Struct.*, 27 (12), 1477~1497, 1991.

204) Yang, R.J. : Shape design sensitivity analysis of thermoelasticity problems, *Comp. Meth. Appl. Mech. Engng.*, 102, 41~60, 1993.

205) Poldneff, M. and Arora, J.S. : Design sensitivity analysis of coupled thermoviscoelastic systems, *Int. J. Solids Struct.*, 30 (5), 607~635, 1993.

206) Choi, K.K. and Lee, J.H. : Sizing design sensitivity analysis of dynamic frequency response of vibrating structures, *ASME J. Mech. Des.*, 114, 166~173, 1992.

207) Yang, R.J. : Shape design sensitivity analysis with frequency response, *Comp. & Struct.*, 33 (4), 1089~1093, 1989.

208) Freeman, L.J., Xile, Y. and Jianni, S. : Optimization of electode shape using the boundary element method, *IEEE Trans. Mag.*, 26 (5), 2184~2186, 1990.

209) Yang, R.J. : Shape sensitivity analysis and optimization using NASTRAN, *Mech. Struct. & Mach.*, 19 (3), 281~300, 1991.

210) Tada, Y. and Seguchi, Y. : Shape determination of structures based on the inverse variational principle / The finite element approach, in *New Directions in Optimum Structural Design*, 197~209, 1984.

211) Durelli, A.J. and Rajaiah, K. : Optimum hole shape in finite plates under uniaxial load, *ASME J. Appl. Mech.*, 46, Sep., 691~695, 1979.

212) Tada, Y., Matsumoto, R. and Arinishi, T. : Optimum design of 2-dimensional continuum considering reliability, *JSME Int. J.* Series I, 35 (4), 502~507, 1992.

213) Barthelemy, B., Haftka, R.T., Madapur, U. and Sankaranarayanan, S. : Integrated structural analysis and design using three-dimensional finite elements, *AIAA J.*, 29 (5), 791~797, 1991.

214) Liefooghe, D. and Fleury, C. : An interactive capability for shape optimization, *Finite Elem. Anal. Des.*, 5, 39~55, 1989.

215) Chang, K.J. and Choi, K.K. : An error analysis and mesh adaptation method for shape design of structural components, *Comp. & Struct.*, 44 (6), 1275~1289, 1992.

216) Sluzalec, A. : Shape optimization of weld surface, *Int. J. Solids Struct.*, 25 (1), 23~31, 1989.

217) Rajan, S.D. and Belegundu, A.D. : Shape optimal design using fictitious loads, *AIAA J.*, 27 (1), 102~107, 1989.

218) Younsheng, L. and Ji, L. : Sensitivity in shape optimization design for pressure vessel, *ASME J. Press. Vessl. Tech.*, 114, 428~432, 1992.

219) Zhou, Y. and Brayant, R.H. : The optimal design for an internal stiffener plate in a penstock bifurcation, *ASME J. Press. Vessl. Tech.*, 114, 193~200, 1992.

220) Meric, R.A. and Saigal, S. : Shape sensitivity analysis of piezoelectric structures by the adjoint variable method, *AIAA J.*, 29 (8), 1313~1318, 1991.

221) Kanamori, M. and Ishihara, Y. : Shape optimization of magnetic pole on electromagnetic damper, *JSME Int. J., Series C*, 36 (1), 141~147, 1993.

222) Han, C.S., Grandhi, R.V. and Srinivasan : Optimum design of forging die shape using nonlinear finite element analysis, *AIAA J.*, 31 (4), 774~781, 1993.

223) Achdou, Y. : Numerical optimization of a photocell, *Comp. Meth. Appl. Mech. Engng.*, 102, 89~106, 1993.

224) Albertini, C., Montagnani, M., Zyczkowski, M. and Laczck, S. : Optimal design of a specimen for pure double shear test, *Int. J. Mech. Sci.*, 32 (9). 729~741, 1990.

225) Beux, F. and Dervieux, A. : Exact-gradient shape optimization of a 2-D Euler flow, *Finite Element. Anal. Des.*, 12, 281~302, 1992.

226) Bendsoe, M.P. and Kikuchi, N. : Generating optimal topologies in structural design using a homogenization method, *Comp. Meth. Appl, Mech. Engng.*, 71, 197~224, 1988.

227) Zhao, Z. and Adey, R.A. : The accuraccy of the variational approach to shape design sensitivity analysis, *Proc. Euro BEM Conf.*, Nice, 1990.

228) Kikuchi, N., Chung., K.Y., Torigaki, T. and Taylor, J.E. : Adaptive finite element methods for shape optimization of linearly elastic structures, *Comp. Meth. Appl. Mech. Engng.*, 57, 67~89, 1986.

229) Keum, D.J. and Kwak, B.M. : Energy release rates of crack kinking by boundaty condition sensitivity analysis, *Eng. Frac, Mech.*, 41 (6), 833~841, 1992.

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