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PREDICTIVE AND DYNAMIC USER OPTIMAL TRAFFIC ASSIGNMENT AS A TIME-DELAY OPTIMAL CONTROL PROBLEM

Hiroshi MATSUI* and Hiep Tuan DAM**

This paper presents a look at dynamic user optimal traffic assignment as a time-delay optimal control problem. The approach proposed here allows us to develop a model for solving the predictive and dynamic user optimal assignment problem which is capable for considering both time-delay effects in the dynamic of link traffic flow and the information providing process to travelers.

Key Words : *dynamic traffic assignment, predictive user optimal, time-delay*

1. INTRODUCTION

It is well-known that dynamic traffic assignment is a very complex problem in travel demand analysis because it involves numerous factors to be considered and because of its large scale dimension. Generally, there have been three approaches for solving the dynamic traffic assignment problem:

(i) computer simulation approach[2),8)].

(ii) mathematical programming approach[1),4)].

(iii) optimal control theoretic approach[5),9),10)].

Among them, the optimal control theoretic approach looks likely most promising because it allows to develop the models capable of solving the dynamic traffic assignment problem in a very general form. However, it is only recent when this approach has become available in the travel demand research world. Regardless of the well-known approaches, almost every existing model represents the dynamics of link flow as the difference between link's entering and exit flows in the same time period. The link's exit flow is usually considered as a function of link's traffic volume during the same time period and this assumption is acceptable if the link flow is supposed to be homogenous in space. In an urban congested network, as many unexpected phenomena may happen, the above assumption on the link's exit flow may be questionable. Thus it seems more realistic to consider that link's exit flow depends on link's traffic volume at some previous time and it naturally leads to the time-delay differential equation for describing such a system. Use of this time-delay is also a way to ensure a more realistic exit function that preserves the first-in-first-out (FIFO) queue discipline. Because at a time instant, cars can exit from a link only when they exist at the endpoint of that link and naturally they must be present on the link at some previous time. We call this delay as time-delay of system dynamic.

Another time-delay factor could be recognized if we look more strictly at the mechanism of how travelers can receive traffic information for making decision on a traffic network. Until now, all existing dynamic traffic assignment models accept the assumption that travelers can get the information about the current state of the traffic network without time delay. This assumption, for example, is clearly stated by Papargeorgious[5)] and Wie et all[9)]. But even on up-to-date technology level, providing information without time delay is not realistic and in practice, travelers can get only the information about the traffic situation at some previous time because of the time-delay effect. When traffic information service is not available, travelers must estimate the current state of the traffic network by themselves and in most cases, they are likely to use their own experiences for that purpose. This situation can be understood as the case when the time delay for acquiring current traffic information is very large.

By the time horizon, there are three kinds of information involving traveler behaviors in their decision making process[11)], that is : historical, current and predictive information about network states. If there is a driver information system, the most desirable purpose of this information system would be to provide all kinds of above-mentioned information to travelers. Anyway, as mentioned by Ben-Akiva *et al.*[11)], only the historical and current

* Member of JSCE, D. Eng., Professor, Dept. of Civil Engineering, Nagoya Institute of Technology.
** Member of JSCE, M. Eng., Graduate Student, Dept. of Civil Engineering, Nagoya Institute of Technology

information are most likely to be provided and travelers have to estimate the predictive information themselves. Because of the time delay effect of providing information process to travelers, the current information about network traffic becomes a historical one when it is available to travelers. Thus, it is more realistic to accept that travelers first will have to update this information in the hope that they can get the true picture of current network traffic and then use this updated information for estimating predictive information. There are some various simulation methods for modelling travelers response (see 11), 12), and 13)) but such an aspect of the problem is still not intensively discussed although the need for studying the time delay effects of information providing process to travelers is mentioned[13].

There is a distinction between two kinds of assumptions about the traveler's route choice behavior, which is clearly defined by Papargeorgious[5]. The first which is called as the reactive user optimal problem is the case when travelers use current traffic conditions in order to estimate the cost criterion for making decision. The second which is called as the predictive user optimal problem is the case when travelers use the future traffic conditions for that purpose. Thus by this context, in the predictive user optimal problem, travelers have to predict the future traffic network condition even if they can get information about the current traffic network condition without time delay.

In this paper, a dynamic traffic assignment model corresponding to the predictive user optimal problem will be formulated. The necessary optimum condition of this model will be also established. Furthermore, the economical interpretation of the optimum condition will be investigated concerning to the traveler's route choice behavior. At the end, for illustration purpose, one simple example will be presented.

2. MODEL FORMULATION AND OPTIMUM CONDITION

In this section, the predictive and dynamic user optimal traffic assignment is formulated. Here we use the following dynamic generalization of Wadrop's user equilibrium principle :

Definition 1. If, at each instant in time, for each pair of network nodes and for network users with same destination, the instantaneous expected unit travel costs for all the paths that are being used are identical and equal to the minimal instantaneous expected unit path cost, the corresponding time-varying flow pattern is said to be user optimal. If the instantaneous expected unit travel cost for all the used paths depends on some future network states which are predicted by network users, the corresponding time-varying pattern is said to be predictive user optimal.

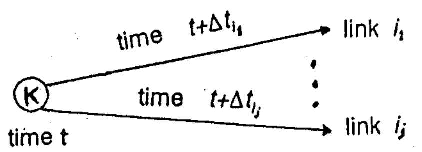

Fig.2.1 Scheme for explaining traveler behavior at node k, time t.
(Δt_{ij} : prediction time interval for link i_j)

Like the definition by Wie *et al.*[9], the above definition does not require equilibration of unit path travel cost actually experienced by drivers, it only requires equilibration of some instantaneous measures of unit travel cost for all the used paths. Anyway, the above definition is stronger than the definition by Wie *et al.*[9] in the sense that it requires the equilibration between any pair of network nodes, not only between origin-destination pairs and thus the instantaneous path unit cost is updated in the sequel using the predicted states of faced paths when travelers are moving downstream arcs of network to desired destination. The switch from use of one path to another at any time is allowable in the above definition.

The model is formulated for a directed multiple *O-D* traffic network with time-varying demand rates using the below notations :

N : set of network nodes.
L : set of network links.
R : set of network origins.
S : set of network destinations.
O_k : set of links whose head node is node k.
I_k : set of links whose tail node is node k.
t_o : initial time of network dynamic.
T : terminal time of network dynamic.
h_x : time-delay of system dynamic.
$t+\Delta t_i$: future time whose traffic network condition of link i ($i \in O_k$) would be predicted at time t by travelers.

The Δt_i (prediction time interval) is supposed to be chosen by travelers when they are at node k ($i \in O_k$), time t and thus they would face a decision making problem for selecting one of faced links as next travel option as shown in **Fig.2.1**.
For that purpose they predict state of faced link i ($i \in O_k$) at a supposed time $t+\Delta t_i$ with a hope that the real link situation would be similar to this predicted state as much as possible when they enter that link. Δt_i is assumed to be prespecified for a link i. This time value can be defined easily in relation with network configuration and concrete

form of prediction function.

h_{pi} : time-delay of acquiring information for link i by travelers.

$q_{ks}(t)$: traffic demand rate at node k to destination s at time t (supposed to be prespecified).

$x_{is}(t)$: amount of traffic on link i with destination s at time t.

$x_{is}^{p}(t+\Delta t_i)$: amount of traffic on link i with destination s at time $t+\Delta t_i$ predicted by travelers at time t.

$u_{is}(t)$: traffic volume entering link i with destination s at time t. $u_{is}(t)$ is assumed to be piecewise continuous function.

$f_{is}^{p}(t)$: prediction function, which is supposed to be unconsciously used by travelers for predicting the future traffic network condition by using the current traffic network condition, that is, $x_{is}^{p}(t+\Delta t_i)=f_{is}^{p}[x_{is}^{e}(t)]$ where $x_{is}^{e}(t)$ is the estimated current traffic state of link i. This prediction function is assumed to be nonnegative, continuous and differentiable.

$f_{is}^{e}(t)$: estimation function, which is supposed to be unconsciously used by travelers for estimating the current traffic network condition by using the past traffic network condition at t-h_{pi} before, that is, $x_{is}^{e}(t)=f_{is}^{e}[x_{is}(t-h_{pi})]$. This estimation function is assumed to be nonnegative, continuous and differentiable.

$g_{is}(t)$: exit flow function from link i at time t. This function is assumed to be nonnegative, continuous, differentiable and depends on $x_{is}(t-h_x)$, that is, $g_{is}(t)=g[x_{is}(t-h_x)]$.

$c_i(x_i^{p}(t+\Delta t_i))$: travel unit cost on link i. This function is assumed to be convex, and separable function of $x_i^{p}(t+\Delta t_i)$, where $x_i^{p}(t+\Delta t_i)$ is the total predicted amount of traffic on link i at time $t+\Delta t_i$, that is, $x_i^{p}(t+\Delta t_i)=\sum x_{is}^{p}(t+\Delta t_i)$.

By using the above notation, the predictive and dynamic user optimal traffic assignment model is formulated as follows :

Find the values of control variables $u_{is}(t)$, ($i\in L, s\in S$) which make the following objective function to be minimal :

$$J=\sum_{i\in L}\int_{t_0}^{T}\int_{0}^{x_i^{p}(t+\Delta t_i)} c_i(w)\,dw\,dt \rightarrow \min! \quad \cdots\cdots(1)$$

subjecting to :

$$\frac{dx_{is}(t)}{dt}=u_{is}(t)-g_{is}[x_{is}(t-h_x)] \quad \cdots\cdots\cdots(2)$$

$$\forall i\varepsilon L, \forall s\varepsilon S$$

$$\sum_{i\in I_k} g_{is}[x_{is}(t-h_x)]-\sum_{i\in O_k} u_{is}(t)+q_{ks}(t)=0 \quad \cdots\cdots(3)$$

$$q_{ks}(t)=\begin{cases}0 & K\notin R, K\notin S\\ >0 & K\in R\\ <0 & K\in S\end{cases} \qquad \forall k\in N$$

$$x_{is}(t)=x_{ois}(t),\ t-h_x\leq t\leq t_0 \quad \cdots\cdots(4)$$

$$x_{is}^{p}(t)=x_{0is}^{p}(t),\ t-h_{pi}\leq t\leq t_0$$

$$\forall i\in L, \forall s\in S$$

$$u_{is}(t)\geq 0, x_{is}(t)\geq 0 \quad \cdots\cdots(5)$$

$$\forall i\in L, \forall s\in S$$

The control variable $u_{is}(t)$ plays the role of traveler's decision and link traffic $x_{is}(t)$ is state variable in our model. The control feed-back law is defined by the fact that travelers use acquired traffic information for making travel decision and their decision in turn will affect network dynamic.

Here we take the assumption that the prediction process of future traffic volume by travelers for their decision making will consist of two stages and is described by the following equations :

$$x_{is}^{p}(t+\Delta t_i)=f_{is}^{p}[x_{is}^{e}(t)] \quad \cdots\cdots(6)$$

where $x_{is}^{e}(t)$ is the estimated current amount of traffic which is supposed to be updated by travelers using the following equation :

$$x_{is}^{e}(t)=f_{is}^{e}[x_{is}(t-h_{pi})] \quad \cdots\cdots(7)$$

Estimation and prediction functions are the tools for planner to model travelers' behavior in their decision making process. The forms of these functions could be designed according to concrete situation as discussed in[15].

Use of two-stage estimation for describing traveler behaviors allows us to study the impacts of traffic information to traveler behaviors in more detail corresponding to the forms of functions used for modelling their behaviors at each stage. The forms of prediction function (6) and estimation function (7) are not crucial for applying Pontryagin principle to our model with the exception they make the objective function discontinuous.

The objective function (1) is intended to represent the sum of individual separate travel cost and it is a natural dynamic generalization of Beckmann type objective function[14] for static user equilibrium case. Although there is not an obvious economical interpretation of this function, it is widely accepted as an adequate criterion for dynamic user optimal traffic assignment models where a traveler is supposed to choose travel option independently in order to minimize the individual travel cost. In our case, the link cost is the travel cost for the future time $t+\Delta t_i$ which is predicted by the equation (6). In our framework, we follows traveler behaviors at each network node for every time instant t, and the generalized Beckmann type objective function (1) is used for describing the predictive feature of traveler behaviors corresponding to the scheme on **Fig.2.1.** Furthermore, by Theorem 2, we can show that the optimal solution of this objective function is user predictive optimal corresponding to Definition 1 with the instantaneous path unit cost defined by Lagrange multipliers. Note that in our model, traveler's behavior is explained using this instantaneous cost measure. Constraints (3) are node conservation conditions. The initial and nonnegative conditions of control variables and state variables are given by the constraints (4) and (5). The prediction equation (6) is intended to describe how travelers will predict the future amount of traffic on links for their decision making by using the current network traffic situations. If the information of current traffic is available, generally with some time-delay h_{pi}, travelers will use first the equation (7) for estimating the current amount of traffic prior to using equation (6) for predicting the future amount of traffic. The initial states $x_{ois}(t)$ and $x^p_{ois}(t)$ must be applied for the interval from $t-h_x$ to t_0 and from $t-h_{pi}$ to t_0 respectively to ensure the $x_{is}(t)$ and $x_{is}(t)$ available at time t_0.

The model formulated above has another advantage that it can include the reactive user optimal problem in its special case putting $h_x=h_{pi}=0, x^p_{is}(t+\Delta t_i)=x^e_{is}(t)$ and $x^e_{is}(t+\Delta t_i)=x_{is}(t-h_{pi})$ that is, when there is no time delay and the traveler will consider that the future traffic network condition is the same as the current one for his/her decision making process.

The necessary optimum condition can be stated in Theorem 1.

Theorem 1 :

The necessary condition for the model (1), (2), (3), (4), (5) and (6) to have an optimal solution is :

$$\frac{d\lambda_{is}(t)}{dt}=\begin{cases} a+b-c & ,t_0\leq t<T-h_x \\ a & ,T-h_x\leq t<T \end{cases}$$

$$\lambda_{is}(T)=0,\ \forall i\in L,\ \forall s\in S \cdots\cdots (8)$$

where

$$a=\frac{-d\left(\int_0^{x^p_i(t+\Delta t_i)} c_i(w)dw\right)}{dx_{is}(t)} \cdots\cdots\cdots\cdots (8.a)$$

$$b=\lambda_{is}(t+h_x)\frac{dg_{is}(x_{is}(t))}{dx_{is}(t)} \cdots\cdots\cdots\cdots (8.b)$$

$$c=\mu_{ks}(t+h_x)\frac{dg_{is}(x_{is}(t))}{dx_{is}(t)} \cdots\cdots\cdots\cdots (8.c)$$

and

$$u_{is}(t)=\begin{cases} 0 & if\ \lambda_{is}(t)>\mu_{ks}(t) \\ 0\leq u_{is}(t)\leq u^{\max}_{is}(t) & if\ \lambda_{is}(t)=\mu_{ks}(t) \\ u^{\max}_{is}(t) & if\ \lambda_{is}(t)<\mu_{ks}(t) \end{cases} \cdots\cdots (9)$$

where

$$u^{\max}_{is}(t)=q_{ks}(t)+\sum_{i\in Ix} g_{is}[x(t-h_x)] \cdots\cdots (9')$$

Proof :

The Hamiltonian H for the system without node conservation constraints (3) is given as follows :

$$H=\sum_{i\in L}\int_0^{x^p_i(t+\Delta t_i)} c_i(w)dw+ +\sum_{i\in L}\sum_{s\in S}\lambda_{is}(t)(u_{is}(t)-g_{is}[x_{is}(t-h_x)]) \cdots\cdots (10)$$

and the Lagrangean function $\bar{L}$ within the node conservation constraints is :

$$\bar{L}=H+\sum_{k\in N}\sum_{s\in S}\mu_{ks}(t)\left(\sum_{i\in I_k} g_{is}[x_{is}(t-h_x)]-\sum_{i\in O_k} u_{is}(t)+q_{ks}(t)\right) \cdots\cdots (11)$$

where, $\lambda_{is}(t)$ is a costate vector and $\mu_{ks}(t)$ are the Lagrange multipliers corresponding to node conservation constraints (3).

By applying the Pontryagin Maximum Principle extended for time-delay system(3),(6) we can get such a costate equation system as :

$$\frac{d\lambda_{is}(t)}{dt}=\begin{cases} -\nabla_{x_{is}(t)}L(.)-\nabla_{xisd}L(.\tau)_{|\tau=t+h_x} & t_0<t<T-h_x \\ -\nabla_{x_{is}(t)}L(.), & T-h_x\leq t<T \end{cases} \cdots\cdots (12)$$

where we put $x_{isd}=x_{is}(t-h_x)$.

For $k\in N$ and $i\in I_k$ the equation (12) will have the same form of the equation (8). Furthermore, we need to show the validity of the condition (9). The derivatives of Lagrangean function $\bar{L}$ with respect to control variables $u_{is}(t)$ can be given as :

$$\frac{\partial\bar{L}}{\partial u_{is}}=\lambda_{is}(t)-\mu_{ks}(t)$$

$$\forall k\in N, \forall i\in O_k \cdots\cdots (13)$$

If the optimal solution happens at a stationary point, the following equations must be satisfied :

$$\lambda_{is}(t)=\mu_{ks}(t) \qquad \forall k\in N, \forall i\in O_k \cdots\cdots (14)$$

Obviously, in this case the Pontryagin Maximum Principle doesn't give us any useful information for finding optimal control variables. Therefore, we must apply the Pontryagin Maximum Principle directly to the Lagrangean function. First, let us rewrite the Lagrangean function $\bar{L}$ as a function of control variables $u_{is}(t)$:

$$\bar{L}(u_{is}(t))$$
$$=f^*+\sum_{i\in L}\sum_{s\in S}\lambda_{is}(t)(u_{is}(t)-g_{is}[x_{is}(t-h_x)])$$
$$+\sum_{k\in N}\sum_{s\in S}\mu_{ks}(t)(\sum_{i\in I_k}g_{is}[x_{is}(t-h_x)]$$
$$-\sum_{i\in O_k}u_{is}(t)+q_{ks}(t))$$

where

$$f^*=\sum_{i\in L}\int_0^{x_i^p(t+\Delta t_i)}c_i(w)dw$$

and f^* is fixed at time t with respect to control variables $u_{is}(t)$.

Because $x_{is}(t)$ and $x_{is}^p(t+\Delta t)$ are fixed with respect to control variables and the Lagrangean function is a separate function of $u_{is}(t)$ and node k, we can further rewrite $\bar{L}$ as :

$$\bar{L}(u_{is}(t))=\sum_{k\in N}\sum_{s\in S}\bar{L}_{ks}(u_{is}(t))+F^* \quad\cdots\cdots\cdots(15)$$

with

$$\bar{L}_{ks}(u_{is}(t))=\sum_{i\in Ok}(\lambda_{is}(t)-\mu_{ks}(t))u_{is}(t)$$
$$\cdots\cdots\cdots(16)$$

where F^* is a component which is fixed over planning horizon T with respect to control variables u_{is}. Thus we have only to take the expression (16) into consideration.

The Pontryagin Maximum Principle for time-delay system[3),6)] requires the Lagrangean function $\bar{L}$ to be minimal with respect to control variables and it follows at the same time that the value of expression (16) also must be minimal for every link i $(i\in O_k)$ whose head node is k $(k\in N)$.

We consider three cases :

-In case of $\lambda_{is}(t)=\mu_{ks}(t)$, the expression (16) has zero value and it means that the Lagrangean function does not depend on the control variables (i.e insensitive to control variables). Thus we don't know how to choose the value of $u_{is}(t)$. This case is called as a singular case in optimal control theory and needs the other criterion than Pontryagin's.

-In case of $\lambda_{is}(t)>\mu_{ks}(t)$, the expression (16) is minimal if and only if control variables are at their lower bound $(u_{is}(t)=0)$.

-Incase of $\lambda_{is}(t)<\mu_{ks}(t)$, the expression (16) is minimal only and only if the control variables are at their upper bound. Considering the node conservation constraint (3), it is the case when :

$$u_{is}(t)=q_{ks}(t)+\sum_{i\in I_k}g_{is}[x_{is}(t-h_x)]$$
$$\forall i\in O_k$$

Because the minimization of expression (16) with respect to control variables subjecting to constraint (3) for node k is a linear programming problem, it is not difficult to show that there is at most only one link i from node k with $u_{is}(t)>O$ and with $\lambda_{is}(t)<\mu_{ks}(t)$.

Thus, the necessary condition for the problem (1), (2), (3), (4), (5) and (6) to have an optimal solution is found.

From Theorem 1, we can make immediately some useful discussion :

-The model formulated above has a bang-bang type control because the Lagrangean function is linear with respect to control variables $u_{is}(t)$. Anyway, one thing should be noted that the bang-bang type could happen only for the inflow with the same destination. The total link inflow, in nature, is not bang-bang type control.

-The necessary optimum condition must be satisfied at each time over the planning horizon T.

-If in the equation (8), we replace $x_{is}^p(t+\Delta t_i)$ by $x_{is}(t)$ and put $h_{pi}=0$, Theorem 1 will be reduced to the reactive user optimal case with time-delay h_x of system dynamic. Furthermore if we put $h_x=0$, the theorem will be reduced to the reactive user optimal case without system time-delay which is until now the main topic of the existing dynamic traffic assignment models.

3. ECONOMICAL INTERPRETATION

In this section, we will show that the solution of problem (1), (2), (3), (4), (5) and (6) will give a time-varying flow pattern which is predictive user optimal by Definition 1. First, we will define an instantaneous path unit cost measure $\Phi_{k1,k2}^{\theta}(t)$ for travelers using path θ at time t to destination s as follows :

$$\Phi_{k1,k2}^{\theta}(t)=\sum_{i\in L_{k1,k2}^{\theta}}(\lambda_{is}(t)+\rho_{is}(t)) \quad\cdots\cdots\cdots(17)$$

where $L_{k1,k2}^{\theta}$ is the set of links on path p from node $k1$ to node $k2$. $\lambda_{is}(t)$ is a costate variable which is calibrated by equation (8). $\rho_{is}(t)$ is a penalty cost for unused link i at time t and defined as :

$$\rho_{is}(t)=\begin{cases}0 & if\ u_{is}(t)>0\\ \infty & if\ u_{is}(t)=0\end{cases}\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

Now, we are ready to show the validity of following theorem :

Theorem 2:

If the optimal solution of the problem (1), (2), (3), (4), (5) and (6) exists, and there is no singular node (i.e no node with $\lambda_{is}(t)=u_{is}(t)$), this optimal solution is predictive user optimal corresponding to Definition 1 with respect to the instantaneous path unit cost measure (17).

Proof :

Let us consider a path between two network nodes $k1$ and $k2$. By the Theorem 1, if the path fully used at time (i.e every link on the path has $u_{is}(t)>0$, then the instantaneous cost measure

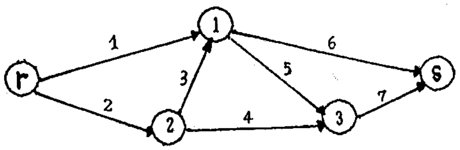

Fig.3.1 Network for explaining traveler route choice behavior.

(17) for the path at time t becomes :

$$\phi^{\theta}_{k1,k2}=\sum_{\substack{i\in L^{\theta}_{k1,k2}\\ i\in Ok}}\lambda_{is}(t)$$

and we have (by Theorem 1) :

$$\phi^{\theta}_{k1,k2}(t)<\sum_{\substack{i\in L^{\theta}_{k1,k2}\\ i\in Ok}}\mu_{ks}(t)$$

or in other words, the instantaneous unit cost measure (17) for the path is bounded above and thus has a finite value. If the path not used (i.e there is some path's links with $u_{is}(t)=O$), the instantaneous cost measure (17) for the path is infinite because the penalty cost $\rho_{is}(t)$ for unused link i on this path is infinite. In consequence, the instantaneous path unit cost of the latter case must be larger then the first case (while all links of path are used). If there is no singular node in the network, only one path between nodes $k1$ and $k2$ has finite instantaneous unit cost measure due to bang-bang type control property and thus the instantaneous path unit cost measure is minimal. Furthermore, the instantaneous path unit cost measure (17) is estimated by costate equation (8) and clearly depends on $x^p_i(t+\Delta t_i)'s$ which are predicted by travelers. Consequently, the solution of problem (1), (2), (3), (4), (5) and (6) is predictive user optimal corresponding to Definition 1 and the validity of Theorem 2 is proved.

It would be useful to show the nature of Theorem 2 in a simple single OD network as given in **Fig.3.1**. The network consists of 5 nodes where r is an origin, s is a destination.

Suppose that at time t_0, a traveler is at origin r and thus he can choose one of the following paths for travelling :1-6, 1-5-7, 2-3-6, 2-4-7 and 2-3-5-7, and suppose that the instantaneous path unit cost of path 2-3-5-7 is minimal. Thus, the Theorem 2 said that he will choose this path as a next travel option. Suppose he arrives at node 2 at certain time instant t_1. At this time instant, he can choose one of 3 available paths for travelling : 4-7, 3-6 and 3-5-7. Suppose at that time, the instantaneous unit cost measure (17) for path 3-6 is minimal, he will choose this path for travelling and arrives at node 1 at time instant t_2. Suppose at that time, between two available paths :6 and 5-7, the path 5-7 has minimal instantaneous unit path cost. By Theorem 2, he will choose path 5-7 and travel by this path to destination s. Thus, the mechanism which governs traveler route choice behavior in our model is as follows : travelers always choose the path with minimal instantaneous path unit cost measure (17) from faced available paths. For that purpose, at every network node, travelers must update the information about current network traffic and use this information to update their knowledge about the future instantaneous unit cost of available faced paths. Clearly, travelers can switch to any available path at any time depending on the instantaneous path unit cost measures of faced paths at the time when they must make a decision for selecting travel option.

The penalty cost $\rho_{is}(t)$ is introduced here only for showing the fact that in the optimal solution, if some link of a path is not used, the instantaneous cost measure can be considered as infinite. The use of this penalty cost allows us to treat the instantaneous path unit cost measure in an unified manner regardless of the fact that the path is fully used or not used.

Here, we also again make a discussion that for travelers with the same destination, the inflow for a link is of bang-bang type put the total link inflow is not a bang-bang type control. We are still unable for studying more the case when there is a singular node in the network but we think that the above mechanism is reasonable for understanding traveler route choice strategy.

4. CALCULATION METHOD

From Theorem 1 we can see that if we know how to estimate the value $\mu_{ks}(t)$, then the solution of the model can be found relatively easily by using the condition (9). It is well-known[(3),(7)] that minimization of the Lagrangean function with respect to control variables is equal to maximization of that with respect to dual variables :

$$\min_{u_{is}(t)}\bar{L}(u_{is}(t),\lambda^*_{is}(t),\mu^*_{ks}(t))=$$

$$\max_{\lambda_{is}(t)\mu_{ks}(t)}\ \min_{u_{is}(t)}\bar{L}(u_{is}(t),\lambda_{is}(t),u_{ks}(t))=$$

$$\max_{\mu_{ks}(t)}\bar{L}(u^*_{is}(t),\lambda^*_{is}(t),\mu_{ks}(t))$$

where, $\mu^*_{is}(t),\lambda^*_{is}(t),\mu^*_{is}(t)$ are optimal values of $u_{is}(t),\lambda_{is}(t),\mu_{is}(t)$.

This property allows us to develop efficient algorithm to solve the model not directly as minimization but maximization of Lagranean function as the dual problem. The algorithm can be briefly described as follows :

Step 0 : assume initial values of $\mu^*_{ks}(t),u^*_{is}(t)$ over the planning horizon T.

Step 1 : solve the state equation (2) to obtain $x_{is}^{*}(t)$ and estimate $x_{is}^{*p}(t+\Delta t_i)$ by the functions (6) and (7).

Step 2 : solve the costate equation (8) to obtain an estimated $\lambda_{is}^{*}(t)$.

Step 3 : update $\mu_{is}^{*}(t)$ by the gradient-type iteration:

$$\mu_{ks}^{*}(t)=\mu_{ks}(t)+\alpha d^{*}(\Delta\mu_{ks}(t)) \quad \forall k\in N, s\in S \quad \cdots\cdots(19)$$

where,

α : step size

$d^{*}(.)$: a function of error

The error terms $\Delta\mu_{ks}(t)$ are calculated as :

$$\Delta\mu_{ks}(t)=\sum_{r\in I_k} g_{is}[x_{i's}^{*}(t-h_x)]-\sum_{i\in O_k} u_{is}^{*}(t)+q_{ks}(t) \quad \cdots\cdots(20)$$

Step 4 : use the rule (9) of Theorem 1 to choose new $u_{is}^{*}(t)$. If $\lambda_{ks}^{*}(t)=u_{ks}^{*}(t)$, choose $u_{is}^{*}(t)$ as a random value of interval $[O,u_{is}^{\max}(t)]$ where $u_{is}^{\max}(t)$ is calculated by formula (9'). The random value can be easily generated by the standard library of many computer programming languages. This case can be considered as a stochastic assignment step.

Step 5 : If $u_{is}^{*}(t)$ does not change, stop the iteration process. Other wise go back to *Step 1* with new estimated $\mu_{ks}^{*}(t), u_{is}^{*}(t)$.

The idea of this iteration process is well-known as a trial-and-error approach[3] for solving the optimal control. Anyway, without Theorem 1, it would be very difficult to solve the above model for a large scale system.

At *Step 3* any type of gradient algorithms can be used but the conjugate gradient method[3] is preferable in this case because of it's efficiency and computer memory saving. By the conjugate gradient method, the update direction $d^{*}(.)$ in *Step 3* is calculated as follows :

$$d^{*}(\Delta\mu_{ks}(t))=\Delta\mu_{ks}^{*}(t)+\xi d(\Delta_{ks}(t)) \quad \cdots\cdots(21)$$

where

$d(\Delta\mu_{ks}(t))$: direction of previous iteration.

$\Delta\mu_{ks}^{*}$: error terms of previous iteration.

and :

$$\xi=\frac{\int_{t0}^{T}(\Delta\mu_{ks}^{*}(t))'(\Delta\mu_{ks}^{*}(t))dt}{\int_{t0}^{T}(\Delta\mu_{ks}(t))'(\Delta\mu_{ks}(t))dt} \quad \cdots\cdots(22)$$

5. EXAMPLE

For illustration purpose, the above model is applied for a network shown in **Fig.5.1**. The network consists of 7 nodes and 10 directed links. The nodes 1 and 5 are origins, and the nodes 4 and 7 are destinations. There are 4 *O-D* pairs between these nodes and the corresponding demand rates are supposed to be prespecified as input data for the problem and are graphically shown in **Fig.5.2**.

The time delays h_x and h_{pi} are supposed to be the same, that is, $h_x=h_{pi}=h$ and the model is applied for 4 cases : $h=0, h=1, h=2$, and $h=3$ with 50 time

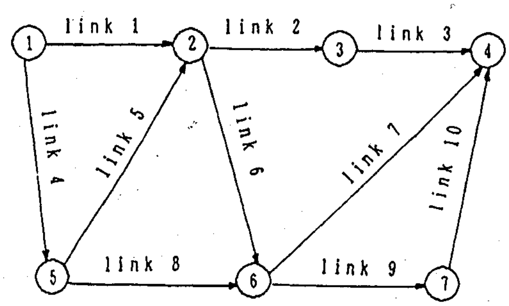

Fig.5.1 Network for an example.

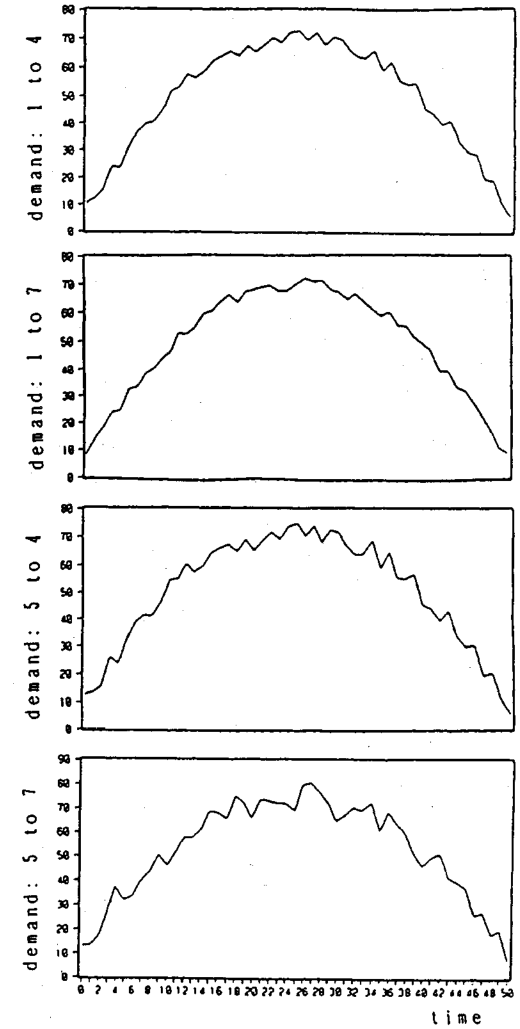

Fig.5.2 Demand rates at origin nodes

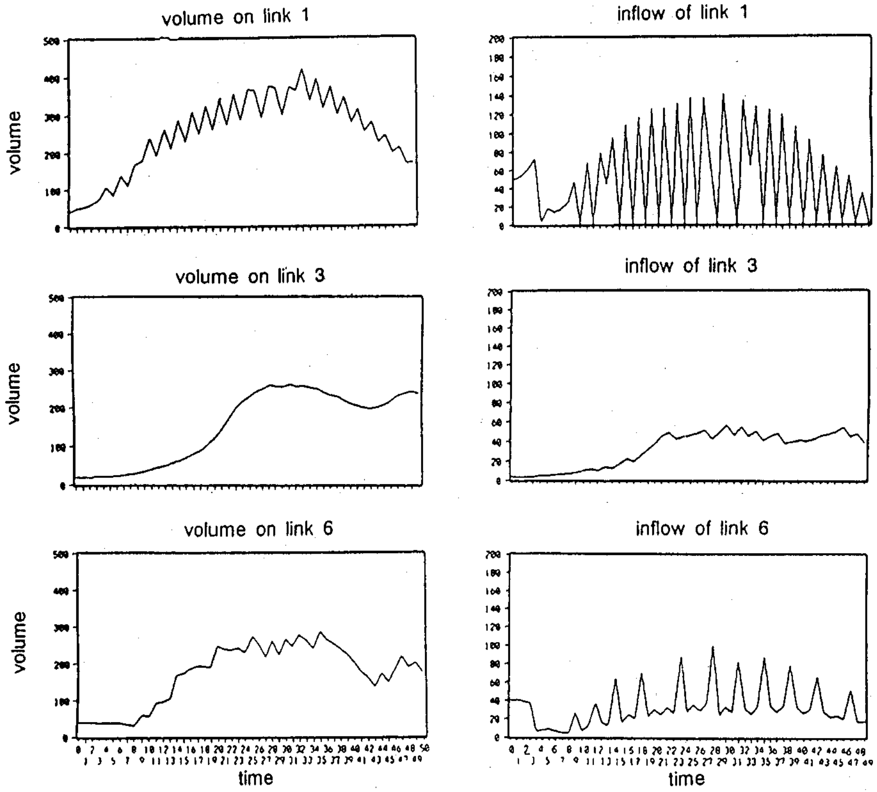

Fig.5.3 Results for the case $h=0$.

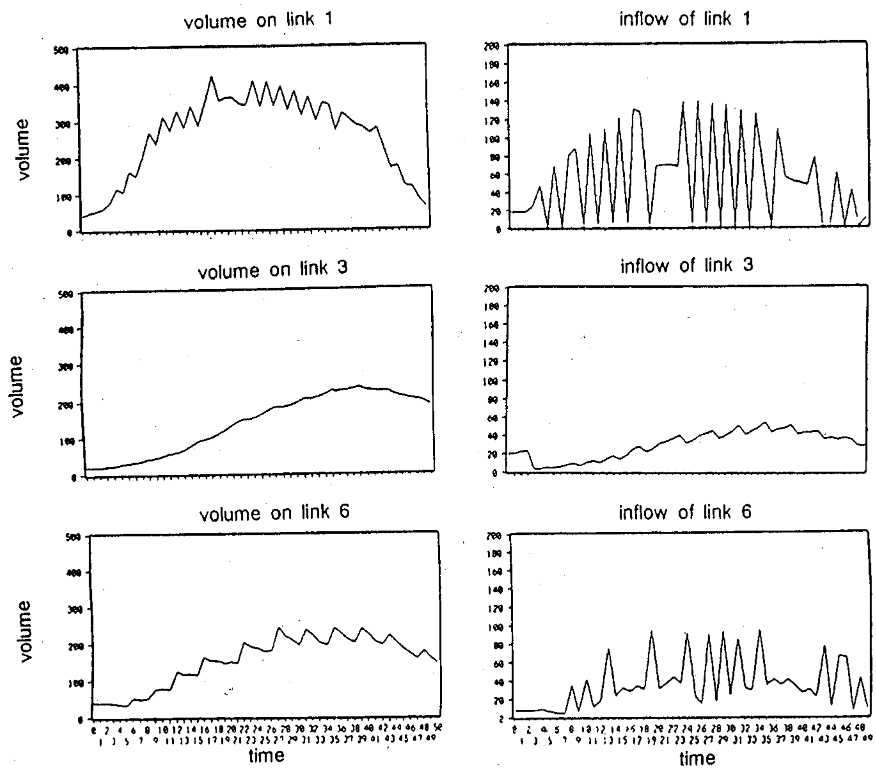

Fig.5.4 Results for the case $h=3$.

instants and with prediction time interval $\Delta t_i = 1$. The prediction function is assumed to have the form : $x_{is}^{p}(t+\Delta t_i) = x_{is}^{e}(t)$ and the estimation function is : $x_{is}^{e}(t) = x_{is}(t-h_{pi})$. The form of the exit function is assumed to be : $g_{is}(t) = 0.2x_{is}(t-h_x)$. Thus the case when $h=0$ will correspond to the reactive user optimal case without time-delay effects. The link travel unit cost function is supposed to be : $c_i[x_i^p(t+\Delta t_i)] = 3x_i^p(t+\Delta t_i)$.

The computation was carried out with the NEC PC-H 98S computer using Microsoft Quick BASIC as a programming language and the computation time is about 10 min. for one case. Because of space limitation, only the total link volumes and inflows of link 1, 6, and 3 are presented on **Fig.5.3** and **Fig.5.4**, analyzed only for the cases with $h=0$ and $h=3$. These links represent typical network situations, that is : link 1 is a link whose head node is origin, link 2 is an intermediate link and link 3 is the last link of a path between a *O-D* pair. From the computation results shown in **Fig.5.3** and **Fig.5.4** we can find some characteristics as follows :

Near the origins (link 1 or link 4, for example), the total link volumes and inflows seem to vibrate most. This can be explained by the fact that on the links near the origin, traffic with various destinations are still not mixed so well and as the traffic moves downstream, they mix up and make link volumes and inflows more stable.

For the case with no time-delay ($h=0$), the total link volumes and inflows seem to vibrate most. The vibration decreases both in amplitude and frequency when the time-delay becomes larger. Note that the inflow is interpreted as travelers' decision in our model. Thus, this fact gives us the reason to think that traffic assignment is sensitive to time-delay effect and the aggressiveness of travelers for changing a travel option also strongly depends on this effect, that is, the less is time-delay for acquiring current traffic network information, the more they are aggressive to find the better travel option as they think.

6. CONCLUSION

It is still early to make any definitive conclusion about the approach solving such a complex problem as the predictive and dynamic user optimal traffic assignment problem but through our research, the following statements may be appeared at least.

(1) Predictive user optimal behavior is an important aspect of dynamic traffic assignment which deserves to research more intensively, especially for urban congested network, where a distinction between predictive and reactive user optimal behavior is obvious.

(2) Time-delay effects, both in system dynamics and information acquisition by travelers are the important and specific features of dynamic traffic assignment. They must be considered as much as possible in urban travel demand analysis especially for predictive user optimal case because it often (although not always) explains the reason why travelers have to predict the future traffic network condition for choosing a travel option.

(3) The model developed in this paper is given in relatively general form. It covers many important phenomena in dynamic traffic assignment such as an information impact, an interaction between the link traffic volumes and it has an adequate economical meaning for describing the traveler's behavior in route choice process. It can also solve the reactive user optimal problem as a special case.

(4) The model, using time-delay factors h_x, h_p as parameters, can give analyst a set of solutions dependent on the values of these time delays. By comparing with some observed network characteristics, we can choose the most suitable solution matching the real network situation.

(5) Because traffic assignment is sensitive to time-delay of providing information to network users and, of course, also to the quality of information, the value of time-delay can be used as a policy-making tool for controlling an urban traffic network system. For example, when traffic jam occurs on an urban congested network, the travel cost is likely to jump up to a very large value drastically. Thus in order to keep link volumes and inflows at a desirable level to avoid undesirable phenomena, the planner may use the time-delay effect of providing current traffic information to network users as a powerful tool. The model proposed here seems to be very useful in this case.

(6) The research of predictive and dynamic user optimal traffic assignment is still at the beginning stage. There are many problems requiring further investigations both from theoretic and practical points of view. The most obvious point is to research on the prediction mechanism which traveler will use for predicting future network condition. This problem seems to be very complex because it involves many unknown factors concerning traveler's behavior. The natural generalization of the model is to consider the value of time-delay as a variable both for time dimension and network configuration. Furthermore, the multiple time-delay (i.e. there is a sequence of time-delay factors for every kind of time-delay effects) and the stochastic time-delay effects should be researched. The effectiveness of the calculation method proposed in this paper also needs considerable efforts to improve, especially for a large scale

network. From a methodological point of view, use of the Maximum Principle is not only approach for solving the abovementioned problems. It is our feeling that Bellman Dynamic Programming Principle may play an important role for many cases. Nevertheless, the successful solution method for these problems is big challenge for travel demand analysis.

REFERENCES

1) Carey, M. : Optimal time-varying flows on congested network, *Operation Res., 35 (1)*, 1987.
2) Ho, J. : Solving the dynamic assignment problem on a hypercube multicomputer, *Transpn. Res., Vol.24 B, No.6*, 1990.
3) Malek-Zavarey, M. and Jamshidi, M. : Time delays systems, *North Holland*, 1986.
4) Merchant, D.K. and Nemhauser, G.L. : A model and an algorithm for dynamic traffic assignment, *Transpn. Sc.*, 12 (3), 1978.
5) Papargeorgious, M. (1990) : Modeling, assignment and route guidance in traffic network, *Transpn. Res., Vol.24 B, No.6*, 1990.
6) Pontryagin, L.S., Boltyanski, K.A., Gramkrelidge, R.V. and Mishenko E.F. : The mathematical theory of optimal process, *Willey, Newyork*, 1962.
7) Saaty and Brawn : Nonlinear mathematics, *Mc Graw-Hill*, 1964.
8) Vythoulkas, P.C.: A dynamic stochastic assignment model for the analysis of general network, *Transpn. Res, Vol.24 B, No.6*, 1990.
9) Wie, B. and Friez, L. : Dynamic user optimal traffic assignment on congested network. *Transpn, Res., Vol.24 B, No.6*, 1990.
10) Ran, B. and Shimazaki, T. : A general model and algorithm for dynamic traffic assignment problems, *Proc. of the 5 th World Conference on Transport Research, Vol.4*, 1989.
11) M. Ben-Akiva, A. de Palma and I. Kaysi : Dynamic network models and driver information systems, *Transpn. Res., Vol.25 A, No.5*, 1991.
12) H.S. Mahmassani and R. Jayarkishnan : System performance and user response under real-time information in a congested traffic corridor, *Transpn. Res., Vol.25 A, No.5*, 1991.
13) R. Arnott, A. de Palma and R. Lindsey : Does providing information to drivers reduce traffic congestion? *Transpn. Res., Vol.25 A, No.5*, 1991.
14) Beckmann, M.J., McGuire, C.B. and Winston, C.B.: Studies in the economic of transportation, *Yale University Press*, 1956.
15) Horowitz, J.L.: The stability of stochastic equilibrium in two-link transportation network, *Transpn. Res. Vol.20, No.1*, 1984.

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時間遅れをもつ最適制御問題としての動的利用者均衡配分

松井　寛・Hiep Tuan DAM

本文は時間遅れをもつ最適制御問題として定式化した動的利用者均衡配分モデルについて考察したものである．すなわち，本モデルでは各リンクの交通流の流入と流出の間にみられるタイムラグや，交通情報提供にみられるタイムラグを考慮に入れた予測的利用者均衡に基づく配分手法を提案し，その均衡解の存在定理を明らかにするとともに，計算例を通してその解法について論じている．