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# CONTINUUM FRACTURE IN CONCRETE NONLINEARITY UNDER TRIAXIAL CONFINEMENT

Koichi MAEKAWA*, Jun-ichi TAKEMURA**, Paulus IRAWAN*** and Masa-aki IRIE****

Under triaxial stresses, the continuum fracture defined as the damaged elasticity of concrete was experimentally extracted. The volumetric elasticity was found not to be mechanically damaged even though the concrete subjected to lower confinement comes up to the higher nonlimearity in appearance. On the contrary, it was verified that the elasticity of concrete in shear is seriously deteriorated according to the internal shear stress intensity expressed by the elastic strains. The concept of the internal stress, which is directly proportional to the elasicity, was adopted for evaluation of the stress intensity applied to non-damaged volume which retains the capacity to absorb the elastic strain energy. The fracture parameter, which indicates the loss of elastic strain energy in the mode of shear, was formulated in terms of the hysteresis of the internal hydrostatic stress to represent non-localized defects in space.

***Keywords*** : *fracture, confinement, constitutive law, damage mechanics*

## 1. INTRODUCTION

The nonlinearity of concrete is classified into plasticity and continuum fracture defined hereafter as the damaged elasticity caused by the assembly of spatially dispersed defects with no distinguished localization[1]. In the frame of elasto-plastic and fracture concept, the empirical formula is available for fracturing of concrete under biaxial stresses[1]. This paper aims at the general constitutive law to predict the continuum fracture appearing in concrete nonlinearity under three dimensional (3D) generic stresses.

The 3D confinement effect on the strength and ductility has been reported worldwide, where the overall nonlinearity has been of main interest. The key of this research is the extraction of the continuum fracture from the whole nonlinearity occurring under triaxial stresses. It is convinced that the purely extracted mechanics will be simply described by the constitutive law even though the 3D nonlinearity of concrete regarded as the combined plasticity and fracture seems complex as a whole[1]. In other words, the total energy consumption by concrete should be estimated with regard to the plastic and frictional energy converted to the heat as well as the fracture energy which is consumed by the generation of cracks and defects.

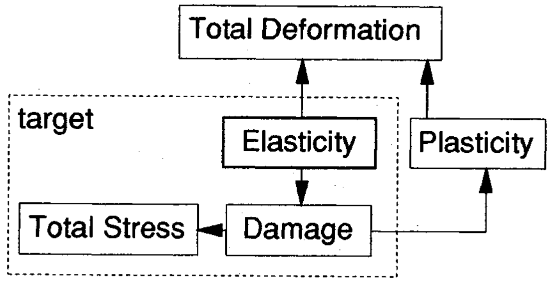


**Fig.1** Scheme of formulation based on elasto-plastic and fracture concept.

In this study, the volumetric (mean) and deviator components of damaged elasticity are individually discussed. The quantification of the deteriorated elastic energy absorption is sought under triaxial stress states. The so called "confinement effect" vaguely used in the past was defined as the effect of the hydrostatic component of the triaxial internal stress on the damaged elasticity.

The continuum fracture model discussed herafter is thought to serve as a component of the system of complete elasto-plastic and fracture constitutive laws as shown in **Fig.1**. Thus, the total stress has to be mathematically related to the elasticity which represents the intensity of the internal stress[1] and the damage condition predicted in terms of the continuum fracture. The plasticity of damaged continuum must be incorporated with the continuum fracture into the entire scheme of formulation as illustrated in **Fig.1**. As far as the plasticity appearing in 3D nonlinearity is concerned, much attention will be paid in another paper.

* Member of JSCE, Dr. Eng., Associate Professor, Dept. of Civil Engineering, The University of Tokyo.
(〒113 Hongo 7-3-1, Bunkyo, Tokyo, Japan).
** Member of JSCE, Ms. Eng., Ministry of Transport. Former Graduate Student of The University of Tokyo.
*** Graduate Student, Ms. Eng., Dept. of Civil Engineering, The University of Tokyo.
**** Member of JSCE, Ms. Eng., Civil Engineer, Nikken Sekkei, Former Graduate Student of The University of Tokyo.

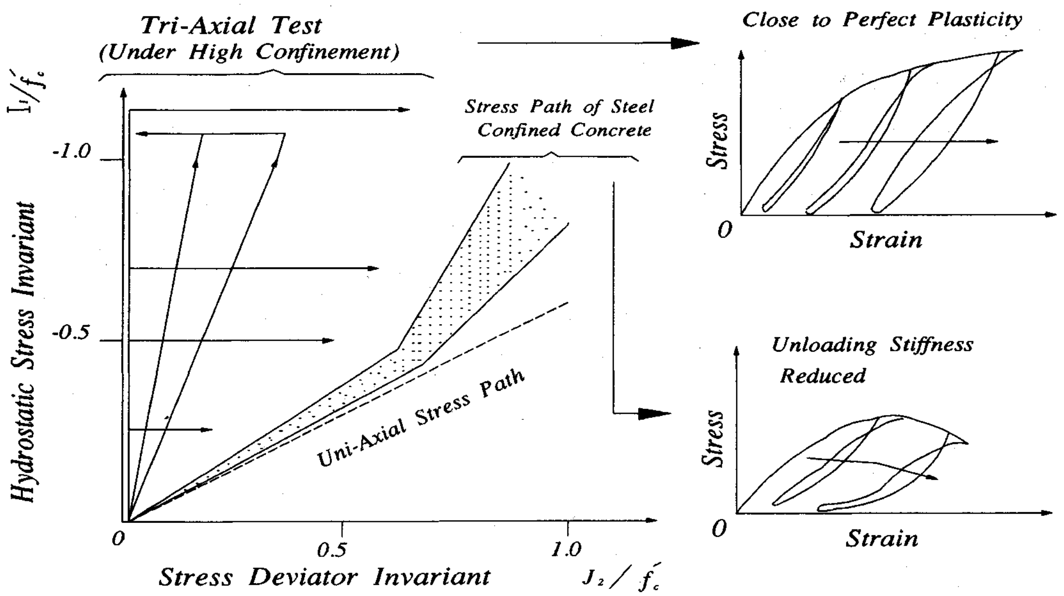


**Fig.2** Stress Path to be Covered and Corresponding Nonlinearity.

## 2. COVERAGE OF LOADING PATH

Typical paths of 3D loading tests conducted by researchers were summarized by Mang, et al.[8] in terms of the hydrostatic and deviator invariants of total stresses denoted by $I_1$ and $J_2$ (See **Fig.2**) as,

$$I_1=\frac{1}{3}\sigma_{ii} \quad \cdots\cdots (1a)$$

$$J_2=\sqrt{\frac{1}{2}s_{ij}s_{ij}} \quad \cdots\cdots (1b)$$

$$s_{ij}=\sigma_{ij}-\delta_{ij}I_1 \quad \cdots\cdots (1c)$$

where, $\sigma_{ij}$ and $s_{ij}$ are total stress and stress deviator tensors, respectively, and positive values express tension. If superscript ['] (prime) would be put, the rule of sign is changed, i.e., compression is positive defined. This inverse rule holds for strains and its invariants too. The compressive strength of concrete is denoted by $f_c'$ which is positive definite.

In general, the hydrostatic invariant $I_1$ as the indicator of confinement was much greater than the deviator one in comparison of the loading condition adopted in specimen based tests with the typical stress states occurring in RC columns with lateral steel. In **Fig.2** where typical loading paths of tri-axial tests conducted in the past are shown, it should be noted that the confinement happens to be so greater that the unloading stiffness remains unchanged. This means that the elasticity seems not to be deteriorated. On the contrary, the smaller the hydrostatic invariant is, the more remarkable reduction of unloading stiffness is proceeded irreversibly[5]. The reduction of the unloading stiffness, which is specified to be the damaged elasticity, is the mark of continuum fracture. It can be qualitatively said that the fracture is associated with the confinement.

Since the lateral reinforcement of columns gives rise to the lesser hydrostatic invariant in the core concrete, the quantification of the continuum fracture as well as the plasticity is supposed indispensable for the purpose of structural analyses on the capacity and ductility gain. The authors concentrated on the continuum fracture of elasticity applicable to relatively higher deviatoric and lower hydrostatic stress states.

## 3. SEPARATION OF ELASTICITY FROM DAMAGED CONCRETE

**Fig.3** shows the axial stress strain relation of cylinder specimens confined by steel rings, which supply the lateral normal stress in concrete by self-equilibrium. Since rings were uniformly distributed, the lateral stress when the steel comes up to yield is equal to $pf_y/2$ where $p$ is the volume ratio of steel and $f_y$ is the yield strength of the steel rings. In the test, the lateral stress at the yield of steel ranges from 0MPa to 12MPa. The axial load was applied through teflon sheets to reduce the frictional confinement by the testing machine[5),10)].

The authors define the plastic strain tensor $\varepsilon_{pij}$ as the residual total one when the stress is completely removed as shown in **Fig.4**. The rest of the total strain associated with acting stresses is defined as the elastic strain $\varepsilon_{eij}$ of interest to us. For example, when we have the complete unloading from the point A or B in **Fig.4**, the axial and lateral elastic strains denoted by $\varepsilon_{ezz}'$, $\varepsilon_{exx}'$, and $\varepsilon_{eyy}'$ can be

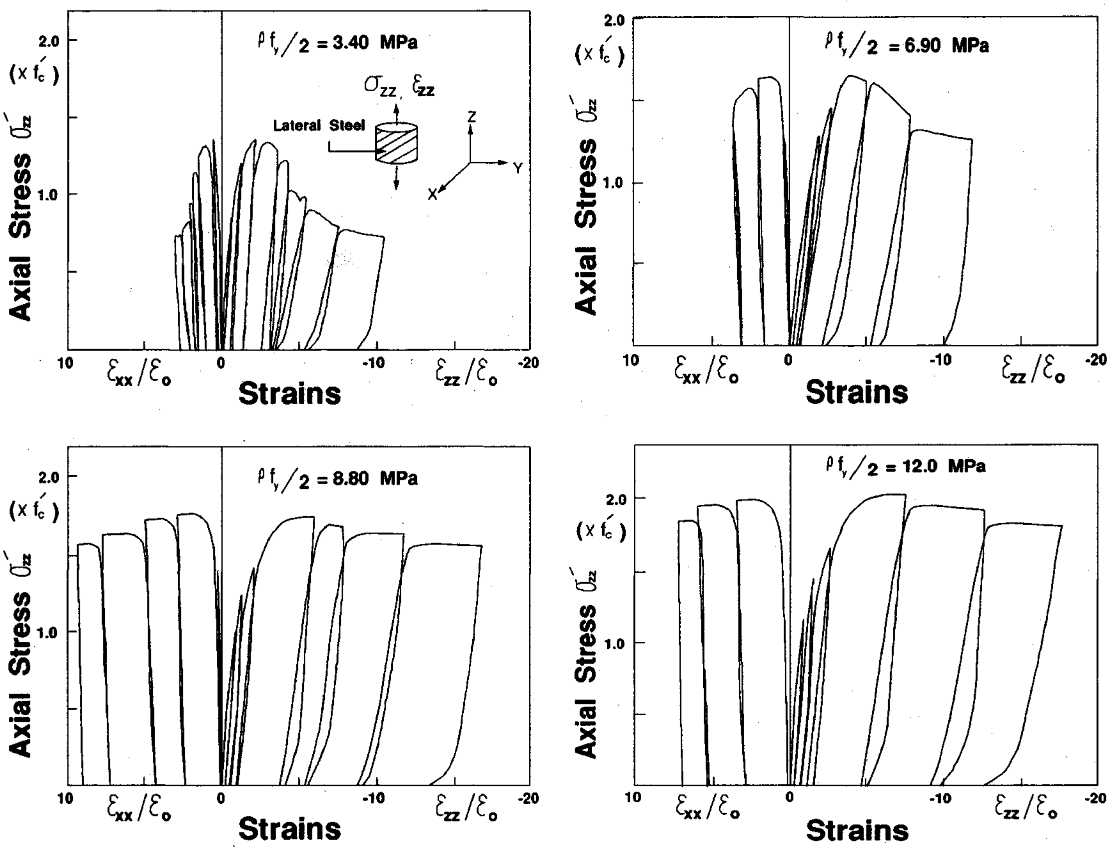


**Fig.3** Stress-strain Relation of Steel Confined Cylinder : $f'_c$=29MPa, $\varepsilon_0$=0.0025.

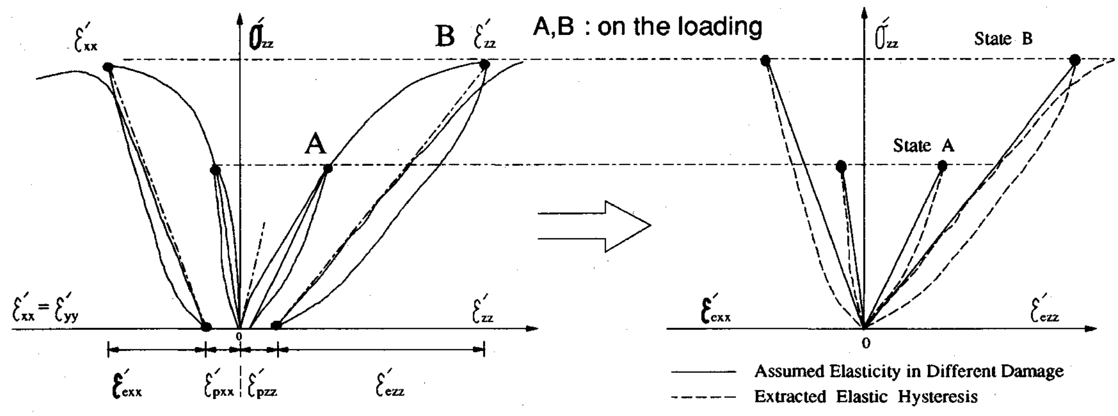


**Fig.4** Separation of Elasticity and Plasticity from Experiment.

obtained. At the same time, by using Eq.(1), we can compute the stress invariants corresponding to the point A or B concerned. Thus, it is possible to get several data sets of elasticity and the updated stress at different intensity when we conduct the cyclic 3D loading tests of concrete.

$$\varepsilon_{eij}=\varepsilon_{ij}-\varepsilon_{pij} \quad \cdots\cdots (2)$$

The elasto-volumetric first invariant, and the deviator second and third invariants $<I_{1e}, J_{2e}, J_{3e}>$ can be defined with the same manner as the stresses on the loading envelopes (See **Fig.4**) as,

$$I_{1e}=\frac{1}{3}\varepsilon_{eii} \quad \cdots\cdots (3a)$$

$$J_{2e}=\sqrt{\frac{1}{2}e_{eij}e_{eij}} \quad \cdots\cdots (3b)$$

$$J_{3e}=\sqrt[3]{\frac{1}{3}e_{eij}e_{ejk}e_{eki}} \quad \cdots\cdots (3c)$$

where, $e_{eij}(=\varepsilon_{eij}-\delta_{ij}I_{1e})$ is the elastic deviator tensor. Tension is also defined positive and the superscript prime ['] means the inverse of the rule of sign.

The damage mechanics proposed in the past was mainly coupled to the total strain and stress relation[6]. The residual strain when the complete stress release would be performed is computed in terms of the inelastic stresses[11)~13]. However, the authors attempt to relate the elasticity of damaged concrete with the path of total stresses. It is expected that the combination of plasticity described independently with the continuum fracture concerned will finalize the full constitutive laws[1]. The elasticity defined above is still cyclic stress-path dependent as well as plasticity (See **Fig.4**). In this paper, we try first to discuss the continuum fracture occurring in elasticity of the loading condition. Concerning the cyclic nonlinearity, the multi-component modeling of elasto-plastic and fracture made by Song and Maekawa[3] will be applied in future.

So far, few tensorial discussions on the continuum fracture were made on 3D generic states. Mazars[11], Simo[12] and Schreyer[13] proposed the tensorial forms of the damaged continuum. Due to the complete isotropy of the stiffness matrix derived, the Poisson's ratio when unloading is made is automatically the same as the initial one. However, the damaged concrete exhibits the varying Poisson's ratio in appearance under the elastic response[1]. This experimental reality implies the non-associated damage accumulation (anisotropy of fracturing). The authors will pay their special attention of the volumetric and deviatoric aspects of the continuum damage of concrete. If the release rate of the fracture energy would be different (non-associated) between volumetric and deviatoric modes, the apparent Poisson's effect at least differs from the initial value. This will be one of primary discussions in this paper.

## 4. VOLUMETRIC ELASTICITY AND FRACTURE

The stiffness of the separated elastic strain versus stress relation are known to vary according to the loading paths under biaxial stresses[1]. We have the same story also under tri-axial stress state as shown in **Fig.3**[5]. It was first tried to purely pick up the fracture arising in the volumetric component of elasticity.

The hydrostatic stress and the volumetric elastic strain invariants are plotted on **Fig.5**, where different levels of confinement by steel in **Fig.3** are put together. The lateral stress on the loading path acts on the circular section of cylinders isotropically. The results shown in **Fig.5** seem to us meaningful. Regardless of the different confinement denoted by $I_1$ on the loading path, no continuum fracture is observed in the volumetric mode of elasticity when applied hydrostatic stress is less than $0.5fc'$.

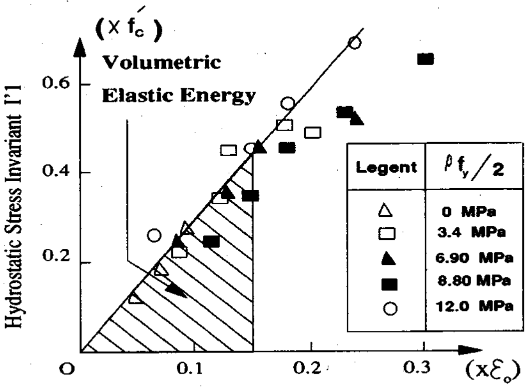


**Fig.5** Relation of Hydrostatic Stress versus Volumetric Elasticity.

The area enclosed by $I_1$ and $I_{1e}$ represents the reversible hydrostatic energy of the damaged concrete including defects and cracks. The density of the microscopic defects might be different in each specimen due to different levels of the lateral stress, but the hydrostatic energy absorption capacity remains unchanged and undamaged when applied hydrostatic stress is less than $0.5fc'$. In other words, the entire volume of concrete is effective to absorb and release the elastic strain energy in the hydrostatic component.

Under the higher hydrostatic stress, the damage in volumetric mode is seen, but the reduction of volumetric stiffness is within 20% of the initial one and the inelastic volumetric strain is less than $0.09\varepsilon_0$. This means that if we neglect the volumetric nonlinearity of fracture, the error of predicting normal strains is around $0.03\varepsilon_0$. Since the authors intend to apply the continuum fracture model to the steel confined concrete in which the stress deviator invariant is prominent (See **Fig.2**), it may be acceptable to assume no fracture in volumetric mode. Therefore, the constitutive law on the volumetric elasticity takes the same form as the linear elasticity as,

$$I_1=3K_0I_{1e} \quad \cdots\cdots (4)$$

The notation $K_0$ is the volumetric elasticity constant equal to $E_0/3(1-2v_0)$, where $E_0$ and $v_0$ are elastic Young's modulus and Poisson's ratio. These values can be obtained from the initial

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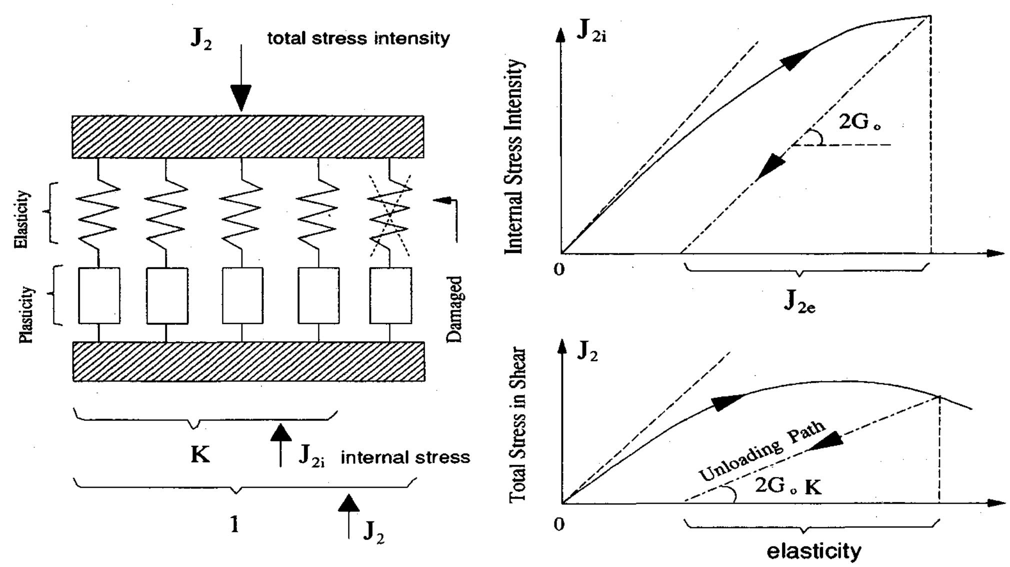


**Fig.7** Schematic System of Elasto-Plastic and Fracture System.

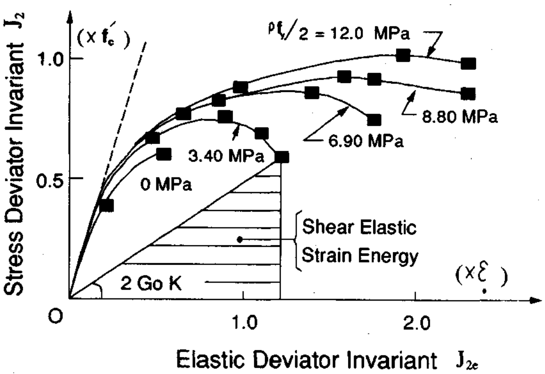


**Fig.6** Relation of Deviator Stress versus Strain Invariants.

stiffness.

## 5. ELASTIC DEVIATOR AND FRACTURE

### (1) Deteriorated Shear Elasticity

**Fig.6** shows the stress and elastic strain deviator invariants of each loading with different levels of lateral confinement. When unloaded, the stress and elastic strain go back to the origin according to the definition. The higher the lateral confinement is placed on concrete, the greater stress deviator invariant is sustained by the damaged concrete. Here, according to the analogy of the elasto-plastic and fracture model[1], the fracture parameter $K$ in terms of the stress and elastic strain deviator invariants is introduced as,

$$J_2 = 2G_0KJ_{2e} \cdots\cdots (5)$$

where, $G_0$ is the elastic shear modulus which coincides with the initial shear stiffness, and equal to $E_0/2(1+v_0)$. Accordingly, the value of $K$ is positive definite not greater than unity.

The area enclosed by the point $\langle J_2, J_{2e} \rangle$ as shown in **Fig.6** corresponds to the shear elastic strain energy stored in concrete. The lower confinement reduces the storage capacity of the elastic strain energy in shear. The area enclosed by the $J_2 - J_{2e}$ curve minus the shear elastic strain energy means the fracture energy consumed by generating defects in concrete. The value of $K$ indicates the ratio of the effective volume of concrete which can absorb and release the shear elastic strain energy, and $(1-K)$ implies the loss of the fictitious volume concerning the elastic shear.

### (2) Fracture parameter

Let us again consider the elasto-plastic and fracture system consisting of parallel springs as shown in **Fig.7**. Since the value of $K$ represents the remaining fictitious volume of concrete as non-damaged in shear elasticity, $K$ indicates the ratio of the remaining springs against shear. From the analogy, we have the internal stress intensity denoted by $J_{2i}$ on active springs in **Fig.7** as,

$$J_{2i} = \frac{J_2}{K} = 2G_0J_{2e} \cdots\cdots (6)$$

For prediction of the fracture, the total stress $J_2$ is not appropriate because $J_2$ does not represent the internal stress intensity arising in the effective constituent element. Since $J_{2e}$ is directly proportional to $J_{2i}$, $J_{2e}$ implicitly represents the internal shear intensity applied to the effective volume. The authors adopted the elasticity as the indicator to

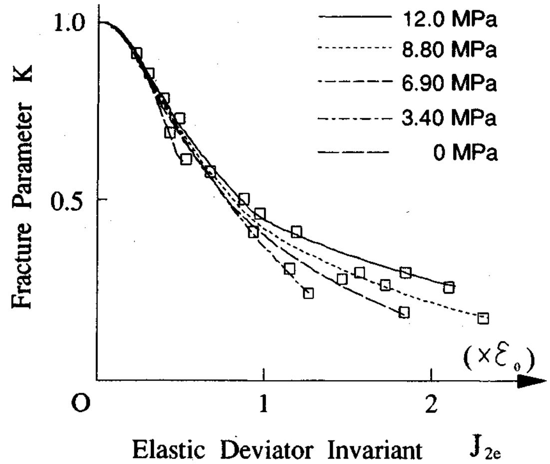


**Fig.8** Fracture Progress under Different Confinement.

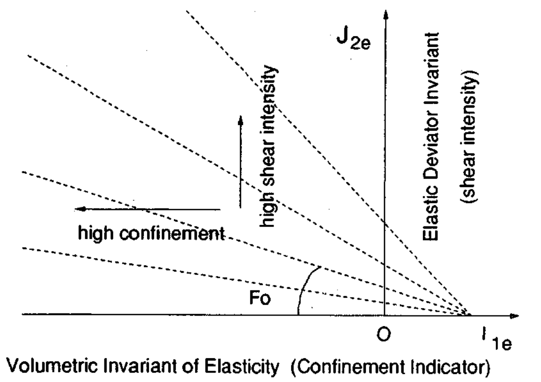


**Fig.9** Influencing Parameters on Continuum Fracture.

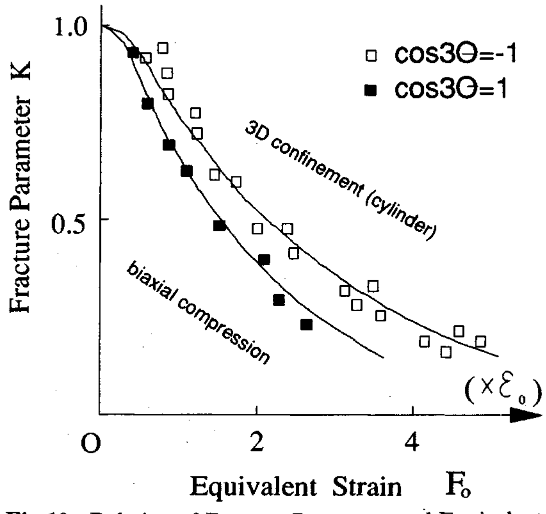


**Fig.10** Relation of Fracture Parameter and Equivalent Strain.

represent the magnitude of internal stress applied to the effective non-damaged part of damaged continuum concerned. The fracturing and plasticity should be described by the internal stress. The internal stress is greater when smaller fracture parameter is assumed even if the smaller total stress would be applied.

The progress in continuum fracture is proved only in the shear mode. According to Eq.(2) and similar to Eq.(6), $I_{1e}$ also represents the confinement denoted by the internal stress intensity, which is expected to restrain the progress of the fracture. The authors adopted the elastic strain invariants as parameters to predict the value of $K$ to indicate the continuum fracture.

**Fig.8** indicates the continuum fracture in progress when the shear elastic intensity increased. The confinement described by the volumetric invariant is not constant during the loading process but different levels in each test. The higher confinement is shown to restrain the proceeding fracture by the increased deviator invariant. From the cyclic loading tests of confined cylinders including Ohshima[5] and Irie et al.[10], we have a number of data set $\langle I_{1e}, J_{2e}, K\rangle$ on the loading path as explained in Chapter 3 (See **Fig.4**).

Let us first discuss the effect of $I_{1e}$ and $J_2$ on the fracture. As shown in **Fig.8**, the higher internal shear intensity advances the progress of fracture but the higher confinement restrains the fracturing. There will be the contour line on the plane of $\langle I_{1e}, J_{2e}\rangle$, which gives rise to the same value of the fracture parameter $K$. The authors assumed the linear contour lines as shown in **Fig.9**, where each line can be specified by its gradient. The equivalent elastic strain denoted by $F_0$ was defined from **Fig.9** so that the fracture criterion would be formed. The value of $F_0$ indicates the equivalent elasticity level which gives the common fracture. We propose the following empirical formula so that the unique relation between the fracture parameter and the value of $F_0$ would be obtained as follows.

$$F_0=\frac{\sqrt{2}J_{2e}}{0.23\varepsilon_0-\sqrt{3}I_{1e}} \quad \cdots\cdots (7a)$$

$$\varepsilon_0=1.6(1+v_0)\frac{f_c'}{E_0} \quad \cdots\cdots (7b)$$

where, $f_c'$ is the uniaxial compressive strength and $\varepsilon_0$ is a material constant obtained so that Eq.(7a) becomes applicable to the normal strength concrete from 15MPa to 50MPa[5,10]. In the tensorial expression[4], the term $\sqrt{2}J_{2e}$ geometrically represents the arch length of $\pi$-plane, and $\sqrt{3}\,I_{1e}$ corresponds to the location of the $\pi$-plane measured from the origin (See **Fig.11**).

The continuum fracture discussed above is conceptually performed within compression. But, in fact, the isotropic tension can be carried by

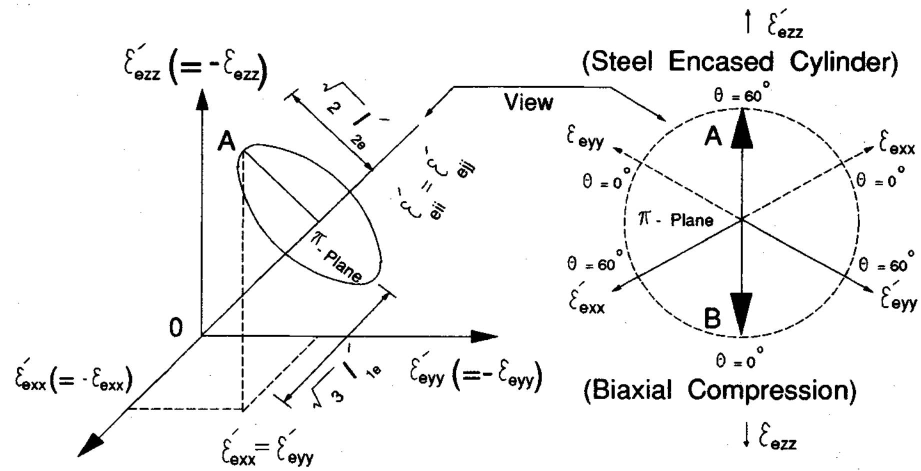

Fig.11 Elastic Strain State on the $\pi$ Plane under Common Hydrostatic and Deviator Invariants.

concrete without any catastrophic fracture when applied shear is small. Thus, concerning the progressive fracture, the bottom term of Eq.(7a) was introduced as an indicator of the effective compressive confinement originated from the critical isotropic tension denoted by material constant $0.23\varepsilon_0$, which also indicates the tensile coordinate of the focal point as shown in **Fig.9** . Eq.(7) was not verified in the case of high strength or lightweight concrete, in which the coarse aggregates are split and damaged sharply in local tension unlike the normal concrete.

As far as the steel encased confinement is concerned, the unique relation between the fracture parameter and the equivalent strain is exhibited regardless of different confinement as shown in **Fig.10**. In order to verify the versatility of the parameter, the authors conducted the isotropic cyclic compression test in accordance to the same procedure specified in reference[1]. However, the fracture parameter can not be well predicted on the same line when the isotropic biaxial compression path is adopted. Here, the source of this discrepancy has to be taken into account.

Let us consider the plane where the values of $I_{1e}$ and $J_{2e}$ are common. In general, this plane is named $\pi$ plane as shown in **Fig.11**. The location corresponding to the updated stress on this plane is specified by the directional angle $\theta$ of the elastic deviator vector on the $\pi$ plane (See **Fig.11**), and computed with the second and third invariants[4] as,

$$\cos 3\theta = \frac{3\sqrt{3}}{2}\left(\frac{J_{3e}}{J_{2e}}\right)^3 \cdots\cdots (8)$$

The equivalent elasticity in the case of steel lateral confinement is to be recognized being under the special condition where the cosine of the angle $3\theta$ on the $\pi$ plane gets minus unity. As other extreme case, the cyclic and isotropic biaxial compression test, in which the direction of the elastic deviator vector on the $\pi$ plane is quite opposite to the steel encased cylinder tests (See **Fig.11**). At this time, the value of cosine of direction $3\theta$ becomes unity. Compared with the isotropic lateral confinement by cylinders, the continuum fracture is much advanced as shown in **Fig.10** when biaxial compression stress state would be realized. This is understood to be the effect of the third invariant. Actually, the failure envelope on the $\pi$ plane of the total stress field is not circular but distorted[4),8)].

When $\cos 3\theta = -1$, the passive lateral confinement is placed and the fracture is supposed to be induced by the maximum principal stress. When $\cos 3\theta = 1$ similar to the biaxial compression, the fracture is mainly induced by the lateral action and the principal direction of the defects will be different. Concerning the intermediate condition on the $\pi$ plane $(-1 < \cos 3\theta < 1)$, the authors simply and linearly assume the equivalent elasticity function $F$ as,

$$F = F(I_{1e}, J_{2e}, J_{3e}) = F_0(I_{1e}, J_{2e})\left(\frac{6+\cos 3\theta}{5}\right) \cdots\cdots (9)$$

When the value of $F$ is adopted, we have the unique relation between the value of $K$ and $F$ as the 3D equivalent elastic strain as shown in **Fig.12**. When $\cos 3\theta = -1$, the value of $F$ coincides with $F_0$. Within the applicability (normal aggregate concrete, $f_c' = 15\text{-}50$MPa), the authors propose the following empirical equation so that it can fit data shown in **Fig.12**.

$$K = \exp\left[-\frac{F}{a}\left\{1-\exp\left(-\frac{F}{b}\right)\right\}\right] \cdots\cdots (10)$$

where, $F$ is implicitly equal to $F_{max}$, i.e., the

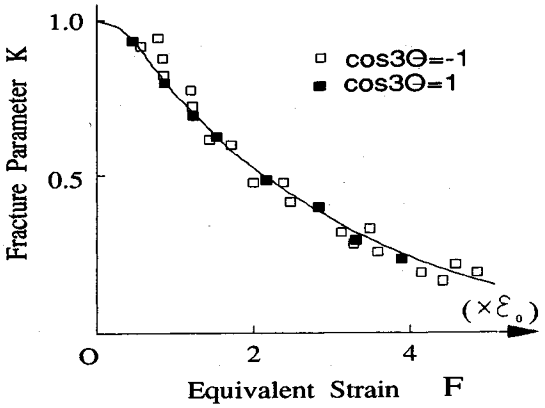


**Fig.12** Fracture Parameter and Generic Equivalent Strain.

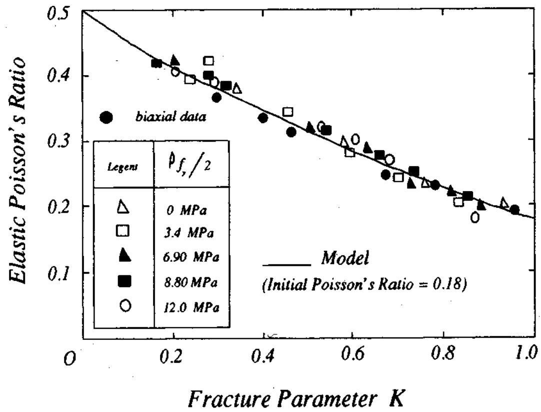


**Fig.13** Elastic Poisson's Ratio of Damaged Concrete.

maximum value of the equivalent elasticity $F$ in the past loading. If not, no fracturing is assumed as damage criterion. The material constants<a, b> are empirically<3.25, 0.8>.

## 6. TENSORIAL EXPRESSION OF FRACTURED ELASTICITY

The fracture parameter is thought to be averaged scalar to exhibit the damage. The anisotropy of the reduced elasticity is the next problem. Li et al.[2] reported that the principal direction of the total stress coincides with that of the elastic strain even under the rotating principal axis of biaxial stresses. If the coincidence is accepted, the generalized expression of the fractured elasticity is to be described as,

$$s_{ij}=2G_0K_{ij}e_{eij} \quad \cdots\cdots (11)$$

Maekawa[1] and Li[2] also experimentally clarified the isotropy of the concrete shear elasticity appearing under biaxial compression-tension stress states. In considering that 3D compressive stresses would make the internal damage further isotropic, the authors assume the following tensorial expression in terms of the fractured elastic body in shear as,

$$K_{ij}=K \quad \cdots\cdots (12)$$

It can be mathematically proved that Eq.(11) and Eq.(12) satisfy Eq.(5). The isotropy in deviator tensors mentioned above holds for fractured elasticity of concrete in shear only. It does not mean the apparent anisotropy of concrete nonlinearity[1] caused by the entire plasticity. By combining the volumetric and deviatoric fracture laws of concerte, the total tensorial expression of the fractured elasticity is obtained as,

$$\sigma_{ij}=\delta_{ij}I_1+s_{ij}=3K_0I_{1e}\delta_{ij}+2G_0K(F)e_{eij} \quad \cdots\cdots (13)$$

If the value of $K$ is unity, Eq.(13) becomes the perfect elasticity equation. When $K$ is equal to 0 as fictitious case, Eq.(13) describes the non-frictional fluid material. Here, we have the fracture criterion where $F_{max}=F$ and $dF>0$. Otherwise, the progress of the fracture is to be zero. Then, the incremental form of the damaging elasticity in progress is formulated as,

$$\begin{aligned} d\sigma_{ij}&=dI_1\delta_{ij}+ds_{ij}\\ &=3K_0\delta_{ij}d(\varepsilon_{ell}/3)+2G_0Kde_{eij}+2G_0e_{eij}dK\\ &=M_{ijkl}d\varepsilon_{ekl} \quad \cdots\cdots (14a)\end{aligned}$$

$$\begin{aligned} M_{ijkl}=&2G_0K\delta_{ik}\delta_{jl}+\frac{1}{3}\Big[(3K_0-2G_0K)\delta_{ij}\\ &+2G_0e_{eij}U_f\left(\frac{\partial K}{\partial F}\right)\left\{\left(\frac{\partial F}{\partial I_{1e}}\right)\right.\\ &\left.-\frac{2}{3}\left(\frac{J_{2e}}{J_{3e}}\right)^2\left(\frac{\partial F}{\partial J_{3e}}\right)\right\}\Big]\delta_{kl}\\ &+2G_0e_{eij}U_f\left(\frac{\partial K}{\partial F}\right)\left[\left(\frac{\partial F}{\partial J_{2e}}\right)\frac{e_{ekl}}{2J_{2e}}\right.\\ &\left.+\left(\frac{\partial F}{\partial J_{3e}}\right)\frac{e_{ekm}e_{elm}}{3J_{3e}^2}\right] \quad \cdots\cdots (14b)\end{aligned}$$

$U_f=1$, when $F=F_{max}$ and $dF>0$

$U_f=0$, otherwise

The propriety of the tensorial expression can be examined with regard to the elastic Poisson's ratio of the fractured continuum. If the fracture in terms of the volumetric elasticity might be definitely associated with the deviatoric one, the elastic Poisson's ratio remains constant and independent on the damage[9]. The elastic Poisson's ratio can derive from the compliance of the stress and elastic strain relation. The total elastic strain is described in turn by the hydrostatic and deviator stress tensors as,

$$\varepsilon_{eij}=I_{1e}\delta_{ij}+e_{eij}=\left(\frac{I_1}{3K_0}\right)\delta_{ij}+\frac{s_{ij}}{(2G_0K)}=C_{ijkl}\sigma_{kl}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(15a)}$$

where,

$$C_{ijkl}=\frac{1}{(2G_0K)}\delta_{ik}\delta_{jl}+\left\{\frac{1}{9K_0}-\frac{1}{2G_0K}\right\}\delta_{ij}\delta_{kl}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(15b)}$$

The apparent elastic Poisson's ratio of the damaged concrete is equal to $-C_{xxyy}/C_{xxxx}$ as the function of the fracture parameter. Here, the fracture parameter remains unchanged. From Eq.(15), we have,

$$v_e=\frac{1+v_0-K(1-2v_0)}{2(1+v_0)+K(1-2v_0)} \cdots\cdots\cdots\cdots \text{(16)}$$

The elastic Poisson's ratio obtained in the unloading or reloading path is shown in **Fig.13**, which involves tri-axial and biaxial loading tests. The precision of Eq.(16) seems reasonable. The shear fracture isotropy in Eq.(12) is practically applicable. The theoretical maximum value of $v_e$ is 0.5 whatever the initial Poisson's ratio is. This prediction was verified also on the biaxial stress paths[1].

## 7. CONCLUSIONS

The nonlinearity appearing in the deformational behavior of concrete under tri-axial stresses was separated into plasticity and elasticity. The continuum fracture was defined as the reduced stiffness and associated loss of the volume which can reserve the elastic strain energy. The elasticity of fractured concrete was investigated in terms of the isotropic (volumetric) and deviator (shear) components as follows.

(1) Within the range where the hydrostatic stress is less than 60% of the uniaxial compressive strength, the capacity of the volumetric elastic strain energy is not deteriorated no matter how mechanical defects such as micro-cracks are induced.

(2) The continuum fracture appears in terms of the elastic strain energy of the deviatoric (shear) mode. The elastic shear stiffness irreversibly decreases when defects are accumulated. the hydrostatic component of stress was found to sensitively affect the rate of fracture when the elastic shear strain is getting ahead.

For predicting the fracture parameter, i.e., the reduction rate of the shear elasticity, the elastic strain invariants are adopted as the representative of the internal stress intensity to govern the microscopic fracture. The tensorially expressed constitutive equation for elasticity of the fractured concrete was derived. The path dependent Poisson's ratio in unloading and reloading was theoretically derived from the constitutive law and experimentally verified.

The fully formulated constitutive equation will be formed by combining the continuum fracture model for elastic component with the plasticity, which will be independently discussed later.

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## 三軸拘束下にあるコンクリートの非線形性に現れる破壊挙動

前川宏一・竹村淳一・Paulus IRAWAN・入江正明

三軸拘束を受けるコンクリートの非線形挙動から，弾性ひずみエネルギーの保存能力を実験的に抽出し，コンクリート連続体に導入される損傷の評価と定式化を行った．弾性エネルギー吸収能の観点からは，1）三軸拘束度の大小に関係なく，体積弾性成分に力学的損傷は現れないこと，2）せん断弾性成分にのみ，弾性エネルギー吸収能の低下が載荷経路に応じて導入されることが判明し，この低下率を弾性ひずみ不変量の履歴に応じて表現することに成功した．