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## REVIEW

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# THE FUZZY SET THEORY APPROACH TO TRANSPORTATION PROBLEMS

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## 1. INTRODUCTION

Many topics in transportation planning and engineering are often characterized by subjective, ill-defined, ambiguous, and vague. Decision making process of transportation policy and investment, traveler's choice of routes and modes, driver's perception and reaction are typical examples of problems involving subjective judgement. As the scope of transportation analysis proliferates and as the consequences of a transportation decision pose far-reaching impacts on many non-transportation aspects, the approach to transportation problems must naturally incorporate analysis of uncertainty, public perception and subjective judgement, which are difficult to measure.

Traditionally, such problems have been placed and dealt in the framework of binary logic; in other words, the answer is assumed to lie only in one of two distinct sets, yes or no, or wrong or right. This logic, which has contributed enormously to the progress of science and engineering for the past centuries, cannot deal effectively with ambiguity, uncertainty and vagueness that many transportation problems demand.

Fuzzy set theory is based on a set theory which allows for sets having the vague boundaries, and thus, it enables the analysis of problems involving ambiguity and uncertainty. Since it is proposed by Zadeh more than a quarter of a century ago, the theory has seen an increasing level of acceptance in engineering in the last decade. It appears that some transportation problems can be dealt with using fuzzy set theory. In fact, a sizable number of attempts have already been made to apply fuzzy set theory to transportation planning and engineering. This paper discusses the application of fuzzy set theory to transportation problems for its potential,

limitations and examples.

## 2. FUZZINESS IN THE NATURE OF TRANSPORTATION PROBLEMS

The following are the characteristics of many of transportation engineering and planning problems.

- An important function of transportation planning process is to predict the consequences of decisions. Unlike other fields of science, the planning process cannot be experimented in a laboratory environment; rather consequences are found only in the future after the decision is implemented in the real world situation.
- Information required for making decisions are often imprecise. Accuracy is often inconsistent among different data sets available, and the lack of data is often supplemented by interpolation based on the available data. Data pertaining to human factor in particular, such as perception and feeling, are difficult to deal with because the information is usually qualitative and descriptive.
- The objective of the decision maker is vague or stated qualitatively so that the target value of the objective is not clear. Further, some plans intend to achieve many objectives without clear priorities among them.
- Reasoning and inference for the decision is based on the approximate logic, in which the implication is ambiguous due to the vagueness of the meaning of words and the lack of precise knowledge.
- The cause-effect relationship is not clear. The relationship itself cannot be defined as a one-to-one relationship in a functional form. A typical such example is the relationship between different socio-economic factors

which are considered to generate trips and the actually generated trips.

As a result, compounding effects of approximations and compounding effects of ambiguous inferences often lead to a conclusion whose validity requires many qualifying statements.

The analysis of transportation engineering and planning needs an approach which allows for a systematic treatment of approximate numbers and vague inference of the decision maker. This need has made fuzzy set theory suitable to some of transportation analysis. Fuzzy set theory provides a tool to quantify the qualitative information.

Given the uncertainty embedded in the environment of the transportation analysis process, value of subjective or qualitative statement and information should be recognized and such information should be captured and preserved as the integral part of the analysis. Traditionally, uncertainty has been treated as the random variable of probability theory. This approach is not necessarily appropriate in some problems.

For example, the travel time estimated by a traveler has both the probabilistic and subjective uncertainty. Delays due to traffic signals, accidents and congestion may be caused by random events. The driver, however, has some controls as to the speed of the vehicle, and choice of route. Furthermore, the estimate may be made conservatively or risky depending on the circumstances and the judgement of the decision maker. Thus, the travel time estimated by an individual can be a combination of random number and fuzzy number. In the past, little effort has been made to analyze the nature of unknown and uncertainty, whether it is due to probabilistic randomness or perception.

### 3. BASIC CONCEPT AND ALGORITHMS OF FUZZY SET THEORY POTENTIALLY APPLICABLE TO TRANSPORTATION PROBLEMS

This section presents the basic concept of fuzzy sets. It is followed by a presentation of different general analysis algorithms of fuzzy set theory.

#### (1) The basic concept of fuzzy set theory

**Fuzzy Sets** A set is a collection of elements with a common characteristic or concept. In the traditional set theory, when a subset is defined in the universal set, then the elements in the universal set can be divided into two groups, the ones belonging to the subset set and the ones not belonging to the subset. This type of subset is called crisp set.

Subjective feeling, such as the expression "late" or "around eight a.m.", cannot be clearly defined. Interpretations of these words are subjective. It

cannot be clearly stated if in fact eight p.m. is late, or if 8 : 15 is considered "around eight a.m.".

A fuzzy set is a set whose boundary is fuzzy so that the degree that an element of a universe belonging to its subset is defined by a degree which takes a value between 0 and 1. This degree is called the membership grade. "Late" or "around eight a.m." can form fuzzy sets and the degree that their elements belong to the sets is expressed by a membership function.

When one states that "A is B", the truth of this statement depends on what kind of set A and B are : A and B are both crisp sets ; one is crisp and the other fuzzy; or both fuzzy sets. Furthermore, what does "is" mean? Does it mean exactly equal to or approximately equal to.

Fuzzy set theory allows treatment of the truth value of the statement for cases other than that A and B are both crisp sets and "is" means exactly equal. Using this basic concept of fuzzy set many of the traditional mathematical algorithms can be modified to incorporate the notion of fuzziness.

**Fuzzy Relations** The relationship between two sets can be vague; for example, the similarity of two items, the impact of one phenomenon on the other and the ordering of several items by priority. All of these involve a subjective judgement about the relationship between items. A typical example in transportation problems may be the attraction of traffic volume between two traffic zones, or an O-D table. Such an association of two sets can be represented by the fuzzy relation. The membership function of a fuzzy relation represents the strength of the relationship between the two sets for their pair of elements.

**Fuzzy Measures** A recent development in fuzzy set theory is fuzzy measure. It represents the degree that an outcome of an event belongs to a set, be it fuzzy or crisp. This measure is useful for evaluating alternatives with respect to criteria which are vague; this is often the case in transportation planning process. These measures are based on the idea that the membership function of a fuzzy set represents the possibility distribution, and thus, how much the set belongs to another set (either fuzzy or crisp) can be measured by the way which the two sets intersect. The fuzzy measure can be used for measuring a vague object by a vague scale, predicting vague outcome, estimating weights for evaluation, perception of distance and time. Combining with the technique of the fuzzy integral, it can be used for the multi-criteria evaluation. It can also be used for determining the relationship between two fuzzy numbers.

A typical example of the use of fuzzy measure is the evaluation of highway traffic flow condition

with respect to different levels of service. Because the criteria for each level of service are vague, it is difficult to place a given condition into only one category (one of Levels of Service A through F). The nature of the problem which is conducive to the use of fuzzy measure is when the unknown is to be placed into one of the sets; the sets may be crisp or fuzzy.

Different fuzzy measures have been proposed, including possibility and necessity measures, and belief and plausibility measures. These measures represent the decision maker's view either optimistic or pessimistic. One obvious difference between fuzzy measure and probability measure is that the values of fuzzy measure need not add up to one. Returning to the highway level of service problem, one may say the given condition is Level of service A with 0.8 and Level of Service B with 0.5 using the fuzzy measure.

## (2) Fundamental algorithms of fuzzy set theory

Several algorithms of fuzzy set theory can be potentially applicable dealing with the uncertainty encountered in transportation problems. They are

Fuzzy regression and fuzzy data handling techniques

Fuzzy optimization

Fuzzy logic and inference

Fuzzy system

Each is explained briefly here.

**Handling of Fuzzy Data** To conduct an analysis based on fuzzy data, fuzzy arithmetic operations, fuzzy regression and fuzzy clustering techniques are available.

The extension principle of fuzzy theory facilitates arithmetic operations involving fuzzy numbers. Many operators for the arithmetic operations (such as max-min) have been proposed. The operator should be selected with caution after its meaning is examined closely. The arithmetic operations of fuzzy numbers allow the examination of the compounding effect of approximate numbers. This is important particularly in the travel demand forecasting process in which the approximate numbers used in the trip generation process affect the ranges of outcome in the subsequent phases. The travel forecasting process has been analyzed as purely deterministic process even though the data and functional relationship among the parameters are known to be approximate. It should be interesting to examine the implication of approximate numbers in the travel forecasting process using fuzzy arithmetic operations.

The ordinary regression equation assumes that a functional relationship between the independent and dependent variables exists, and that any

scattering of data points are considered to be the random error of the data. In other words, an underlying deterministic relationship between cause and effect is assumed to exist. The fuzzy regression, on the other hand, takes the position that the relationship itself is fuzzy; and thus, introduces fuzzy numbers for the coefficients and also considers that data points are fuzzy numbers. The scattering of the points are assumed not as the result of the data error but rather inherent to the phenomena.

An extension of fuzzy data handling includes the fuzzy GMDH (Group Method for Data Handling) by which non-linear relationship between parameters are determined using a technique similar to the neural network back-propagation method. Fuzzy clustering analysis is also based on the assumption that the data points cannot be classified between different groups in a crisp manner, and that it designates a fuzzy set for each group and assign a membership grade to each data point for its degree of belonging to individual clusters. Fuzzy clustering technique may also be useful for multi-criteria evaluation of alternatives when matching the desired characteristics of the objectives with the performance of each alternative.

**Fuzzy Optimization** The ordinary optimization process minimizes or maximizes the objective function given a set of constraints. In the fuzzy optimization, both the objective and the constraints are considered vague and that they are expressed by fuzzy sets. The process is to find the set of parameter values which maximizes the satisfaction of both the objective or the constraint sets at the same time. Thus, both the objective and the constraints perform the same purpose of defining the solution set. This formulation allows multiobjective programming.

Fuzzy optimization technique is suited for problems in which both the objective and constraints are not clearly defined; for example, when the objective is to control cost "around" a certain value to meet the budget level, or when the constraints are not known exactly and a certain allowance is acceptable. The fuzzy optimization approach can be used for the strategic level of planning. The trade-off between the costs of obtaining precise information to find the optimum versus the cost of obtaining less accurate data to find somewhat less optimum solution must be evaluated before using fuzzy optimization. In some problem, the objectives and constraints are inherently vague, in this case, fuzzy optimization approach is suited.

The concept of fuzzy optimization has been applied to the traditional optimization algorithms,

such as fuzzy linear programming transportation problem and fuzzy dynamic programming. In the case of fuzzy linear programming transportation problem, supply and demand are known only in fuzzy numbers and it determines the amount to be shipped between each demand-supply node pair.

**Fuzzy Logic and Inference** Fuzzy logic provides foundations to "approximate reasoning". It represents the majority of applications of fuzzy set theory. It deals with not only the truth or false of the proposition but also the indeterminate or unknown conditions of implications. Most applications of fuzzy inference are seen in the form of "fuzzy expert system" in which the preposition and the conclusion are expressed in fuzzy terms; for example, when the knowledge base of the expert system is not deterministic or the rules themselves are fuzzy. A specific example could be the case of driving rule such as "If the speed is high, then keep a long headway".

When combined with an expert system, several rules can be fired at the same time. This allows the combination of different fuzzy rules to derive the consequence. This combination of fuzzy rules seems to represent the human decision process well.

The essence of fuzzy logic is the inference method based on *modus ponens* in other words; given the rule if  $X=A$  then  $Y=B$ , and the input  $X=A$  then  $Y=B$ . In this logic if the premise is vague and the consequence is also vague, then the rule can be expressed by a fuzzy relationship. When the input  $X$  is not exactly  $A$  (let's say  $X-A'$ ), still an inference as to the condition of  $Y$  can be made. The consequence can then be inferred with fuzzy membership grade.

**Fuzzy System** A complex system in which input, output and states are known fuzzy is a fuzzy system. The fuzzy system can be assumed for inferring the effect of a fuzzy input in a multi-stage process. For example, given a vague state, the cause of the state can be inferred using the fuzzy relationship between the cause and effect. Among the applications of the fuzzy system are when diagnosing the cause of an accident, inferring a consequence of a decision in a complex system involving chain reactions.

Modeling of a fuzzy system involves establishing fuzzy relationship between cause and effect first, and chaining the matrices of fuzzy relationships by the composition operation. Analysis of a large system, such as evaluating the environmental impact of a large scale civil engineering project, can be performed by modeling the system's uncertainty in the cause-effect relationship as fuzzy relationship. An advantage of this modeling approach is

that it can identify the weak and strong links in the logic, and also allows the sensitivity analysis of changing some of the link's strength by further research or investment activities.

#### 4. EXAMPLE APPLICATIONS OF FUZZY ALGORITHMS TO TRANSPORTATION PROBLEMS

Application of fuzzy set theory may encompass many types of transportation problems. Some of promising applications are listed under the same titles as above

##### **Fuzzy data handling methods**

- Classification and organization of data, delineation of zone boundaries for traffic demand forecasting.
- Travel demand forecasting, in particular, regression analysis of trip generation models.
- Estimation the cost of a large project when individual costs are known only in approximate terms.

##### **Fuzzy optimization methods**

- Fuzzy linear programming for the strategic level planning of resource allocation, scheduling, and network analysis.
- Fuzzy dynamic programming for network analysis, fuzzy sequential decision making process.
- Multi-objective programming, in particular, determining the design parameters of a multi-purpose urban development project.

##### **Fuzzy Logic and Inference**

- Various control problems, including traffic signal timing and phasing controls, dispatching control of public transportation, air traffic control, land use controls, traffic flow controls.
- Obtaining numerical solution to a non-linear problem.
- Non-linear interpolation based on the available fuzzy data.

##### **Fuzzy System**

- Large scale transportation planning and investment modeling.
- Site and traffic impact analysis.
- Diagnostic analysis: accident analysis, evaluation of the cause of environmental damage.

##### **Fuzzy Measures**

- Evaluation of driver perception of safe separation of vehicles and stopping distance.
- Evaluation of alternative plans involving subjective judgement.
- Evaluation of vehicle design, passenger comfort, safety.
- Highway capacity analysis

To illustrate application of fuzzy theory, a

schedule development method (developed by this author) which uses fuzzy control is presented here. The problem is to develop a set of schedules for different bus lines so that transfer time at a terminal becomes minimum. Each bus line operates buses with constant intervals but different bus lines have different intervals. The stopping time of a bus at the terminal is constant for all the buses of one line but different among the bus lines.

In this problem, a straight-forward method is to enumerate all possible combinations of bus arrival and departure times by shifting the arrival and departure times little by little for each line while keeping the other line stable. The schedule which results in the minimum transfer time (in this example, the sum of transfer time is minimum) will be found eventually. Mathematically this type of problem is known to be a NP-hard combinatorial problem and it is known that no efficient method of computation exists.

In reality, when a scheduler is faced with this problem, perhaps he will not enumerate all possible combinations but rather make a guess as to which routes to shift slightly and see how it will improve the schedule and continues his trial and error process. He may not pursue the "optimum" solution but may be satisfied with "near" optimum solution. The fuzzy control method basically follows this approach.

The proposed fuzzy algorithm consists of several simple if-then rules and emulates the scheduler's judgement. Each rule states that if the transfer time between routes A and B is long, shift route A's arrival time close to the departure time of route B. If, as a result, the transfer time between A and C becomes too long, then do not shift the arrival time of A too close to B, etc. In the rules, terms "close" and "not too close" are defined as fuzzy sets. Experiment was conducted for a terminal where 25 routes meet; buses of each line arrival and depart at different regular intervals and stopping at the terminal different.

Fig.1 compares how the transfer time improves (how the transfer time reduces in percent) with respect to the computation time for the enumeration and fuzzy control methods. From an initial value of transfer time, it is seen that the fuzzy control method improves the schedule at a much faster rate than that of the enumeration method. Fig.2 shows the case when 50 routes are to be coordinated. The comparison of the result is even dramatic. The added advantage of fuzzy control is that it can incorporate many site specific or unique constraints in the rules.

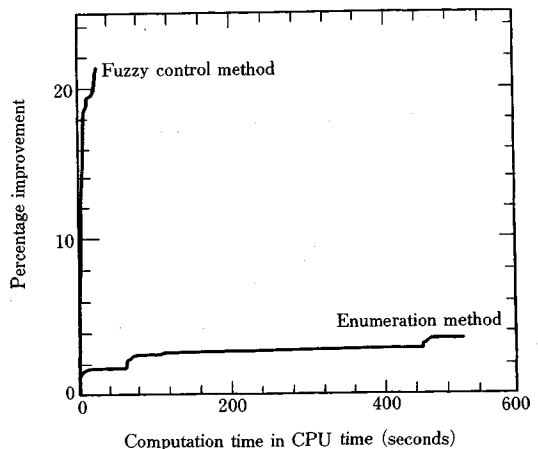


Fig.1 Comparison of fuzzy control and enumeration for the 25 routes example

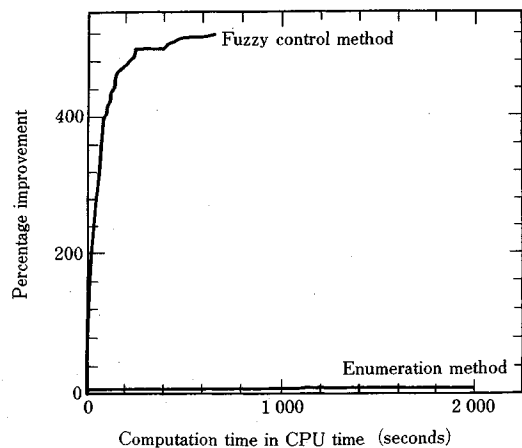


Fig.2 Comparison of fuzzy control and enumeration for the 50 routes example

## 5. CONCLUSION

The essence of fuzzy set theory lies in :

1. Its ability to analyze natural and human related phenomena using multi-valued logic. Measures of feeling and ambiguity can be expressed by a continuous variables. This approach is suited for analyzing problems involving perception and subjective judgement.
2. It can be used within the framework of the traditional mathematics. Therefore, many of the traditional OR techniques can be modeled with fuzzy set theory.
3. It does not explain the reason why the system is fuzzy rather it merely describes the phenomena. The analyst must examine rigorously if in fact the phenomena being

analyzed are fuzzy phenomena. It should be warned that fuzzy theory should be used only when the merits exist; for many transportation problems, the traditional deterministic assumptions or probabilistic approach may suffice. Therefore, it should be used only when the benefits exist and the phenomena being analyzed meets the definition of fuzziness.

Terms such as fuzziness, vagueness, and ambiguity have not been considered "desirable" in the

traditional analytical approach. Definitive crisp statements and logic have been preferred for reasoning. This approach, however, has not been sufficient to allow for the analysis and control and reasoning involving human judgement. Fuzzy set theory offers an alternative to complement the traditional mathematical approach in transportation engineering and planning and perhaps, for the generalization of the traditional methodology.

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