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THE UNIVERSAL MODEL FOR STRESS TRANSFER ACROSS CRACKS IN CONCRETE

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The model for stress transfer across cracks in concrete with universal applicability is proposed. It is a physical model based upon numerical computational scheme of the original contact density model developed by B.Li & K.Maekawa. Main part of the model which is significant for model non-linearity is the contact stress formulation. The formulation integrates important characteristics of concrete stress transfer behaviors, i.e., significant effect of microscopic friction, anisotropic plasticity of contact stress, and contact fracturing of contact units of a crack plane. Each characteristic is represented by microscopic model which is combined together into the same model framework culminating in the final universal model.

Keywords : *stress transfer, crack, shear, anisotropy*

1. INTRODUCTION

Basic mechanisms of stress transfer across concrete cracks were thoroughly examined using the original Contact Density Model as a basic analytical tool in the previous published paper of the authors[1]. In the examination, complex mechanisms of concrete stress transfer were divided into a few main ones, i.e., non-normality, anisotropic plasticity and contact fracturing characteristics of contact stress. They were successfully modeled by introducing new basic assumptions representing each of them into framework of the contact density model independently. Besides shading more light on the complicated nature of concrete stress transfer, the studies also confirmed that the Contact Density Model was very compatible to the nature of stress transfer and thus can be used as a firm foundation on which a more applicable model is based. The more applicable model can be achieved by integrating all the characteristics independently examined before into a unified model which can take into account the non-linear inter-relatedness of all the mechanisms. Finally, the ultimate aim is to achieve the physical universal model which realistically represents the complex mechanisms of stress transfer across cracks in concrete.

2. GEOMETRICAL FORMULATION

Basic formulation of the universal stress transfer model may be divided into geometrical and contact forces formulations.

The geometrical formulation of a crack surface is fundamentally the same as the contact density concept as explained in the original model[2]. However it should be aware that, for universal application of a stress transfer model, configuration of a concrete crack surface can not be definitely represented by a single contact density function $\Omega(\theta)$ which denotes the distribution of the contact area dA_θ in each direction θ[2),1)]. Different concrete crack shapes must be represented by different contact density functions as shown in **Fig.1.** Here, an extreme case may be the crack of high strength concrete which was observed to be much flatter than that of normal concrete[1].

The other aspect of geometrical formulation that must be well aware of is the fact that maximum roughness of a crack surface in the function for effective ratio of contact can not be regarded as one half of coarse aggregate size as originally proposed[2),1)]. The maximum roughness must be appropriately decided from the real crack configuration. However, this factor is not so significant when the real crack opening is much smaller than the roughness which are the cases mostly encountered in practical concrete stress transfer problems.

3. CONTACT FORCES FORMULATION

The total transferred shear and compressive stresses across a crack in concrete are derived from contact forces of all constituent contact units of the crack plane. Hence, modeling of the contact forces is very important and any discrepancy of the model from physical reality will amount to deviation of the whole stress transfer model from the real behavior.

The contact forces on contact points can be

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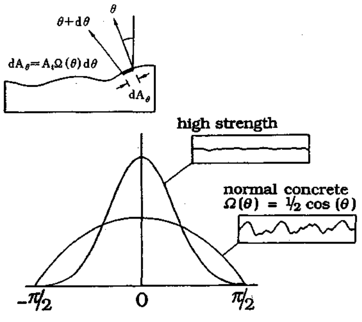


Fig.1 Different shapes of concrete crack represented by different contact density functions.

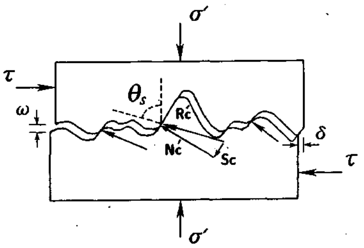


Fig.2 Vector summation of normal contact force N_c' and tangential contact force S_c into the resultant contact force R_c' acting on a contact unit.

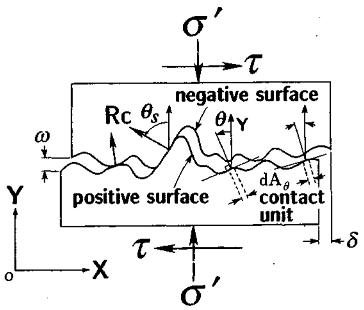


Fig.3 Idealization of a crack plane.

divided into two components, i.e. the normal contact and the tangential ones. The contact force in normal direction $N_c'(\omega,\delta,\theta)\mathrm{d}\theta$ of a contact unit with an inclination within infinitely small interval between θ to $\theta+\mathrm{d}\theta$ is the product of normal contact stress $\sigma_c'(\theta)$ and effective contact area as

$$N_c'(\omega,\delta,\theta)\mathrm{d}\theta=\sigma_c'(\theta)K(\omega)\,\mathrm{d}A_\theta \cdots\cdots\cdots\cdots\cdots (1)$$

where $K(\omega)$ is effective ratio of contact, $\mathrm{d}A_\theta$ is area of the contact unit as shown in **Fig.1**, and thus $K(\omega)\ \mathrm{d}A_\theta$ is effective "contacted" area of the unit. Every term related to crack geometrical modeling of a crack surface is explained in Ref.2) or briefly mentioned in Ref.1).

Similarly, the contact force in tangential direction $S_c(\omega,\delta,\theta)\mathrm{d}\theta$ is the product of tangential contact stress $\tau_c(\theta)$ and the effective contact area as follows.

$$S_c(\omega,\delta,\theta)\mathrm{d}\theta=\tau_c(\theta)K(\omega)\mathrm{d}A_\theta \cdots\cdots\cdots\cdots\cdots (2)$$

The resultant contact force R_c' is the norm of vector summation of the contact force in the normal direction N_c' and the contact force in the tangential direction to the contact unit S_c as

$$\vec{R_c'}=\vec{N_c'}+\vec{S_c} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

and it is acting upon a contact unit at the resultant contact stress angle θ_s as indicated in **Fig.2**.

Finally, contact forces between two sides of a unit area of the crack plane must be balanced by the external shear and compressive stresses applied on the crack plane. The integrals of X and Y components of R_c' in **Fig.3** must be balanced by shear and normal stresses transferred in the plane according to the equilibrium condition

$$\tau=\int_{-\pi/2}^{\pi/2} R_c'(\omega,\delta,\theta)\sin\theta_s\mathrm{d}\theta \cdots\cdots\cdots\cdots\cdots (4)$$

$$\sigma'=\int_{-\pi/2}^{\pi/2} R_c'(\omega,\delta,\theta)\cos\theta_s\mathrm{d}\theta \cdots\cdots\cdots\cdots\cdots (5)$$

Sign convention of the original contact density model[2] is adopted here in which compressive stress σ' and the opening displacement ω are defined as positive while the shear stress and the shear displacement are defined to be positive when the negative side of concrete crack plane [**Fig.3**] moves towards the positive direction of X- axis.

(1) The Tangential Contact Stress Formulation

Deformation of a concrete crack along the path in which there is crack opening or closing with shear slip happening at the same time (hereafter, the "specialized deformational path") induces relatively significant effect of frictional force on contact units compared with normal contact reaction force caused by the units continuous deformation. Therefore the tangential contact force derived from the tangential contact stress was introduced as a constituting component of the resultant contact force [Eq.3]. The tangential contact stress on each contact unit is governed by the path dependent frictional model proposed as

$$\mathrm{d}\tau_c=\begin{cases} G_s\cdot\mathrm{d}\delta_e \text{ when } \tau_c\cdot\mathrm{d}\delta_e\geq 0 \\ 3\,G_s\cdot\mathrm{d}\delta_e \text{ when } \tau_c\cdot\mathrm{d}\delta_e<0 \end{cases} \cdots\cdots\cdots\cdots (6)$$

$$\tau_c=\int_{Path}\mathrm{d}\tau_c \text{ and } \tau_c\leq\mu\sigma_c' \cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

in which τ_c is the tangential contact stress in MPa, G_s is the tangential contact stiffness=21.3 MPa/mm, μ is the frictional coefficient=0.4, σ_c' is the normal contact stress in MPa and $\mathrm{d}\delta_e$ is the incremental effective frictional slip in mm which is supposed to be discontinuous displacement happening on the contact unit.

The incremental effective frictional slip $\mathrm{d}\delta_e$ in Eq.6 is perceived to be a tangential component of deformation of a contact unit as visualized in **Fig.4** (a) and can be modeled by

$$\mathrm{d}\delta_e=K_f\cdot\mathrm{d}\delta_f \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

where K_f is introduced as the frictional contact unit

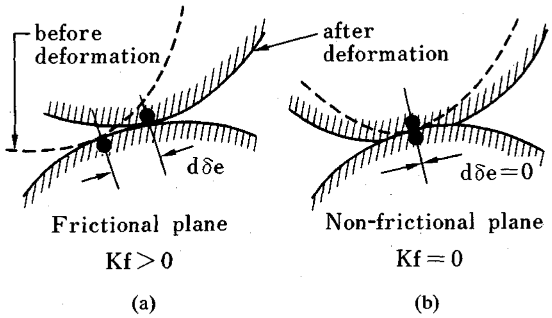


Fig.4 Physical visualization of related terms defining deformational behavior of contact units in two extreme cases of deformational mode.

factor and $d\delta_f$ is the incremental "apparent frictional slip" which is the geometrically computed local displacement in tangential direction that can be derived from

$$d\delta_f=\begin{cases} d\delta_\theta \text{ when } \omega'_\theta>0 \\ 0 \quad \text{when } \omega'_\theta\leq 0 \end{cases} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

in which ω'_θ is the component of local displacement in normal direction computed from the geometrical compatibility condition between the local and global displacements originally proposed in Ref.2) as

$$\omega'_\theta=\delta\sin\theta-\omega\cos\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

The term $d\delta_\theta$ in Eq.9 is the incremental displacement in tangential direction also computed from the geometrical compatibility condition in the incremental form of

$$d\delta_\theta=d\delta\cos\theta+d\omega\sin\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

The frictional contact unit factor K_f in Eq.8 is introduced to quantitatively define deformational behavior of a contact unit whether it will be a frictional unit, a non-frictional one associated with inelastic continuous deformation around the contact unit or a combination of both. The frictional contact plane shown in **Fig.4** (a) with $K_f>0$ is the plane of which the tangential mode of deformation is predominant and thus induces relatively significant effect of friction which causes the contact resultant force on the plane to deviate from the normal direction to the plane, the so-called non-normality condition[1]. On the other hand, the non-frictional contact plane shown in **Fig.4** (b) with $K_f=0$ is the plane of which the deformation in normal direction associated with the plane plastic deformation is predominant offsetting the effect of friction and thus keeps the resultant contact force in the normal direction to the plane, the so-called normality condition[1]. The K_f factor is proposed to be

$$K_f=Q\cdot g(|\gamma|) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

The above equation postulates that deformational behavior of a contact unit represented by the factor K_f is affected by the confinement degree around the unit reflected by the confinement parameter Q and by the direction of external loading relative to the unit reflected by the $g(|\gamma|)$ function. The notation γ is designated as "kinematic direction of local displacement" which can be computed from

$$\gamma=\frac{d\omega'_\theta}{d\delta_\theta} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

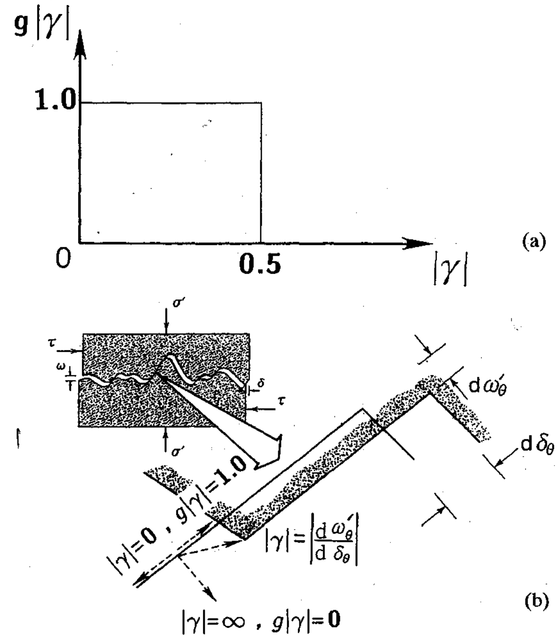


Fig.5 The function $g(|\gamma|)$ representing effect of external loading direction on contact unit deformational behavior. (a) The proposed function. (b) Physical definition.

The terms $d\delta_\theta$ and $d\omega'_\theta$ are infinitesimal local displacement in the tangential direction [Eq.11] and in the normal direction, respectively. The component $d\omega'_\theta$ is computed from

$$d\omega'_\theta=d\delta\sin\theta-d\omega\cos\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

Fig.5 shows the proposed simple model for $g(|\gamma|)$ function as well as the physical visualization of the function.

As for the confinement parameter Q, it is proposed by considering the fact that a contact unit subjected to a specialized deformational path behaves as a frictional contact unit under low confinement state but rather behaves as a deformational one under a high confinement. **Fig.6** shows the quantitative model for the parameter Q as the function of confinement ratio according to the following :

$$Q=q(C_r)=q\left(\frac{\sigma'^*_c}{f'^*_y}\right) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

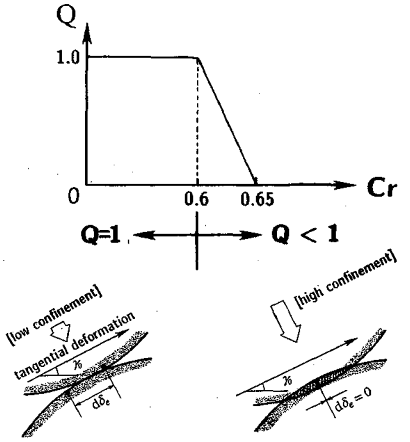


Fig.6 The proposed model for confinement parameter Q and its physical implication in the lower figure showing that a contact unit will behave differently under different confinement level even when subjected to the same tangential displacement caused by a specialized path.

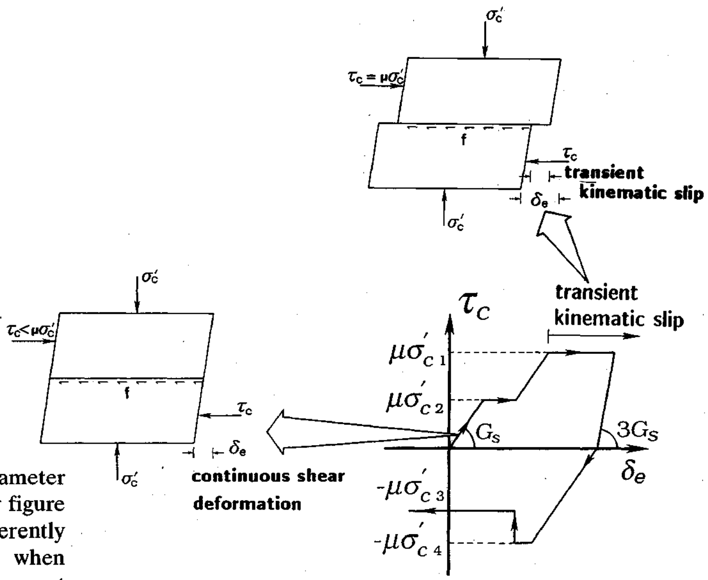


Fig.7 A deformational history of the tangential contact stress τ_c of a contact unit governed by path dependent frictional model under an arbitrary loading path.

where C_r is a term defined as the confinement ratio which is the ratio of $\sigma_c'^*$, the contact stress of the contact unit $\theta-d\theta$ relative to the current θ unit and $f_y'^*$, the yielding level normal to the contact unit of which value will be specified in the next section. The path dependent frictional model in Eq.6 and Eq.7 governs frictional deformation of each contact unit of a crack surface as illustrated in **Fig.7** which is a history of tangential contact stress τ_c of a contact unit subjected to an arbitrary loading path. In the history starting from the origin of the diagram. the unit will linearly deform with friction acting as a medium to transfer tangential force along the unit. The continuous shear deformation will continue up to the frictional capacity $\mu\sigma_c'$ with the current value being $\mu\sigma_{c2}'$. Then it will be in the transient kinematic mode of deformation and will slip at the constant frictional capacity. The frictional capacity $\mu\sigma_c'$ may fluctuate owing to the spontaneous loading history and the unit will response accordingly rendering path-dependent characteristic to the model. There may be reversal of shear deformation and the τ_c will be in the negative region of the diagram.

(2) The Normal Contact Stress Formulation

As previously examined in Ref.1), stress transfer across cracks in concrete is characterized by the so-called anisotropic plasticity and the contact-fracturing of contact units. Qualitative trial models for both factors were separately proposed in Ref.1) in which the anisotropic property was reflected in the anisotropic plasticity parameter K_r which represents the different yielding levels in the different stress direction on the contact unit and the contact fracturing property was considered in the contact fracturing parameter K_c which denotes the loss of elasticity owing to the defect around aggregates[3].

Product of the two parameters is introduced into the contact stress model framework of the original contact density model and the governing model for the normal contact stress σ_c' [Eq.1] on a contact unit corresponding to the stress history in **Fig.8** can be obtained in the form

$$\sigma_c'(\theta)\begin{cases} =K_rK_cR_s(\omega_\theta'-\omega_{\theta p}') & \text{for } \omega_\theta'\geq\omega_{\theta p}' \\ =0 & \text{for } \omega_\theta'<\omega_{\theta p}' \end{cases}\cdots\cdots(16)$$

where R_s is the elastic rigidity per length, K_r is the non-dimensional anisotropic plasticity parameter at $\omega_\theta'=\omega_{\max}'$, and K_c is the path-dependent contact fracturing parameter which varies when the contact displacement exceeds the past maximum one at $\omega_\theta'=\omega'_{\max}$. The following conditions are assumed for the deformational components.

$$\omega_{\theta p}'\begin{cases} =\omega_{\max}'-\omega_{\lim}' & \text{for } \omega_{\max}'>\omega_{\lim}' \\ =0 & \text{for } \omega_{\max}'\leq\omega_{\lim}' \end{cases}\cdots\cdots\cdots\cdots(17)$$

where $\omega'_{\max}$ is the maximum compressive local deformation of the contact unit in θ direction and defined to be positive for compression and $\omega_{\lim}$ is the elastic limit. In this assumption, a general form of contact yield strength f_y' will be

$$f_y'=K_rK_cR_s\omega_{\lim}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

Here, the yield strength $f_y'^*$ appearing in Eq.15 is defined as the contact yield strength f_y' in the above equation when $K_r=K_c=1$.

The diagram shown in **Fig.8** is a typical contact

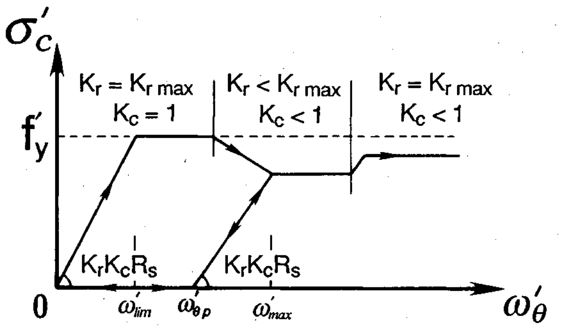


Fig.8 A typical contact stress history of a contact unit subjected to an arbitrary loading path causing anisotropic plasticity and contact fracturing reflected by the parameters K_r and K_c, respectively.

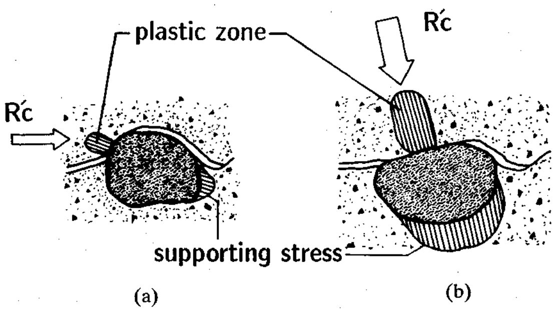


Fig.9 Intrinsic anisotropy of contact units. A steep contact unit in (a) has smaller plastic zone and supporting stress distribution than a flat one in (b).

stress history of a contact unit subjected to an arbitrary loading history causing variation of both parameters K_c and K_r. The combined fluctuation of both parameters renders path dependent characteristic to the universal stress transfer model. Note the decrease in both K_r and K_c around the middle portion of the history in the figure. The decreases cause lowering of contact yielding level f'_y and also the elastic rigidity which is the product of rigidity R_s and the parameters K_r, K_c. At later portion of the history, the contact stress σ'_c rebounds up due to the recovery of the anisotropic parameter K_r caused by the change of principal stress direction but still not up to the initial level owing to the irrecoverable decrease of K_c caused by irreversible damage around the contact unit.

The equations for contact stress σ'_c in Eqs.16, 17 and 18 are the main governing equations. As a general expression of a path-dependent model, we have

$$d\sigma'_c=\frac{\partial\sigma'_c(\theta)}{\partial\omega'_\theta}\cdot d\omega'_\theta+\frac{\partial\sigma'_c(\theta)}{\partial\delta_\theta}\cdot d\delta_\theta$$

$$\text{when } d\sigma'_c\cdot d\omega'_\theta\geq 0 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (19)$$

As far as the stress relaxation on the loading is concerned, we simply formulate the following.

$$d\sigma'_c=-0.1\,R_s\,d\omega'_\theta \text{ when } d\sigma'_c\cdot d\omega'_\theta<0 \quad\cdots\cdots (20)$$

a) Anisotropic Plasticity Modeling

It has been explained in the previous qualitative study[1] that concrete stress transfer exhibits anisotropic property as to the magnitude and stiffness of transferred stresses due to a change in external loading direction on a crack plane. The property was said to be "explicit" because it could be clearly observed from macroscopic stress transfer experiments. However, there are some indirect observations indicating that there still exists another category of anisotropy which has an "intrinsic" nature, that is, changing of contact yielding stress level of a contact unit is due to inherent inclination of the unit.

Steeper contact units, when loaded, will bring about smaller plastic zone and smaller supporting stress distribution [**Fig.9** (a)] corresponding to a lower yielding level in the contact stress model. On the other hand, the flatter ones under loading will have higher yielding level caused by bigger plastic zone and bigger supporting stress distribution [**Fig.9** (b)]. An ideal anisotropic plasticity model must take into account and unify the two characteristics in a meaningful way.

All in all, the unified model has to take into account three concepts of contacting mechanism. Firstly, the concept that magnitude of contact force R'_c on a contact unit changes according to external direction of crack deformation, the explicit anisotropic characteristic. Secondly, the concept that contact force R'_c on contact units of different inclinations behave distinctly due to a different contact stress-local deformation relation. This is the "intrinsic" anisotropic characteristic. Finally, the concept that contact units which manifest the "explicit" anisotropy must be the frictional contact units [**Fig.4** (a)]. In other words, the anisotropic plasticity parameter K_r must always be constant, that is, "isotropic" for "non-frictional" contact units because the resultant contact force R'_c direction should be kept constant conceptually due to the combined balancing effect of local frictional component and local deformation of contact area[2]. The anisotropy should be taken into account only in the case of "frictional" contact units where the resultant contact force direction is not constant but changing according to a spontaneous loading path.

Taking into full account the above considerations, a unified K_r model was proposed to be a function of θ^*_s, a variant of the resultant contact angle θ_s [**Fig.2**] which is defined as

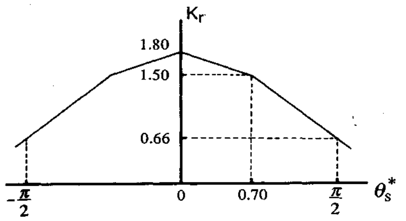


Fig.10 Symmetrical bi-linear function of K_r model.

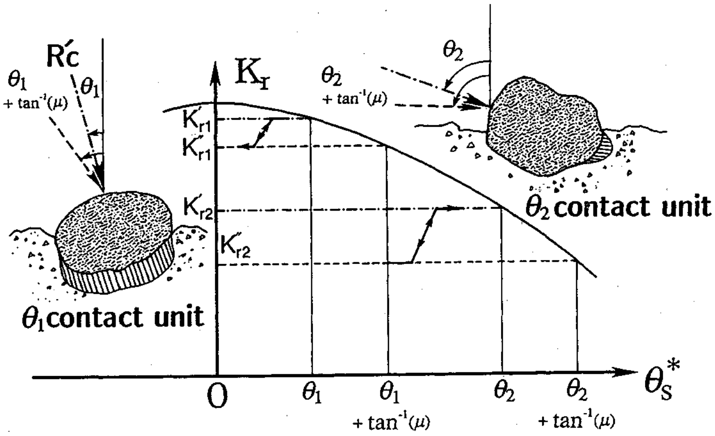


Fig.11 Mechanism of the unified K_r model.

$$\theta_s^* = \begin{cases} \theta + \tan^{-1}(\tau_c/\sigma_c') & \text{when } K_f > 0 \\ \theta & \text{when } K_f = 0 \end{cases} \quad \cdots\cdots\cdots (21)$$

in which θ is the inclination of the contact unit, τ_c is the tangential contact stress, σ_c' is the normal contact stress acting on the unit θ, and K_f is the frictional contact unit factor calculated from Eq.12.

Fig.10 shows the proposed anisotropic model as the function of θ_s^*. The anisotropic plasticity parameter K_r is maximum equaling 1.8 when θ_s^* is 0 which means that the flattest contact unit has the highest plastic yielding level. On the other hand, the value decreases as θ_s^* increases implying that the steeper a contact unit the lower is the plastic yielding level.

Let us take a look in more detail the feature of this unified model illustrated in **Fig.11**. Consider a contact unit with an inclination θ_1 shown in the figure, a resultant contact force R_c' is assumed to act upon the contact unit at a possible angle ranging from θ_1 to $\theta_1 + \tan^{-1}(\mu)$ depending on spontaneous loading at the instant. R_c' will act at the angle of θ_1 if the contact unit is being subjected to a deformation in such a way that it is defined as a "non-frictional" contact unit and will have an anisotropic parameter value K_{r1}'. If this contact unit happens to be a "frictional" contact unit, it may have a possible minimum anisotropic parameter value K_{r1}'' corresponding to the maximum resultant contact force angle $\theta_1 + \tan^{-1}(\mu)$. The value of anisotropic parameter may fluctuate between the higher and lower value depending on the resultant angle in the explained manner and this will take care of the first and the third points mentioned above. Regarding the second point that the steeper contact units have lower yielding level, one can see in the figure that the value of the anisotropic parameter of contact unit θ_2 having steeper inclination than the unit θ_1 falls in the lower range of the diagram and its anisotropic parameter will be fluctuating between K_{r2}' and K_{r2}''. This will thereby take into account the second point of the unified anisotropic plasticity model which incorporates the explicit and intrinsic anisotropy.

b) Contact Fracturing Modeling

The contact fracturing modeling takes on the identical form as previously proposed in the qualitative study[1]. The fracturing mechanism is associated with the tangential deformation of contact units and thus the contact fracture parameter K_c is designated as a function of accumulated effective frictional slip $\int |d\delta_e^*|$ in the form of

$$K_c = g\left(\int_{Path} |d\delta_e^*|\right) \cdots\cdots\cdots (22)$$

The term $d\delta_e^*$ is a variant of the effective frictional slip $d\delta_e$ and is defined as

$$d\delta_e^* = \begin{cases} d\delta_e & \text{when } d\omega_\theta' > 0 \\ 0 & \text{when } d\omega_\theta' \leq 0 \end{cases} \cdots\cdots\cdots (23)$$

Here, the infinitesimal local displacement $d\omega_\theta'$ can be computed from Eq.14 and the incremental effective frictional slip $d\delta_e$ from Eq.8. **Fig.12** shows the proposed model for the parameter K_c which is irrecoverable and thus causes a permanent reduction of contact yielding stress level as well as the contact elasticity.

4. MODELING ALGORITHM

The universal stress transfer model was built upon the basic contact density model[2),1)]. Main numerical integration algorithm is also based upon that of the original contact density proposal illustrated in **Fig.13**. The path dependent stress transfer across cracks in concrete can be accurately predicted by the physical universal model which idealizes a concrete crack asperity using suitable contact distribution density function and the highly

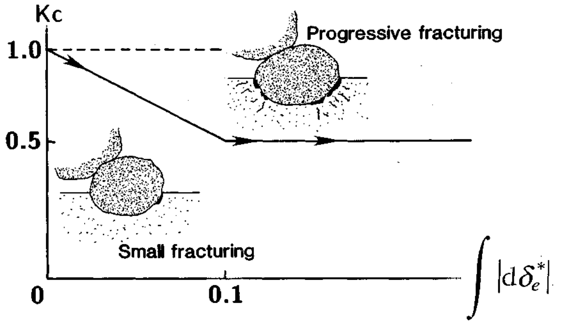


Fig.12 Proposed model for contact fracture paramenter K_c.

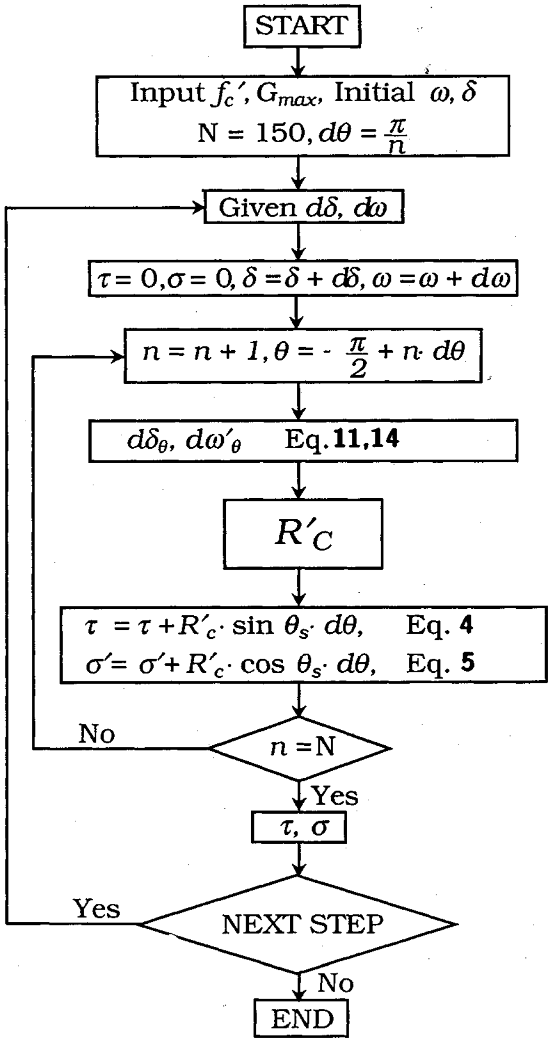


Fig.13 Main numerical integration algorithm of the basic contact density concept. The steps to calculate highly non-linear contact forces denoted by the "R'_c" box are shown in the following Fig.14.

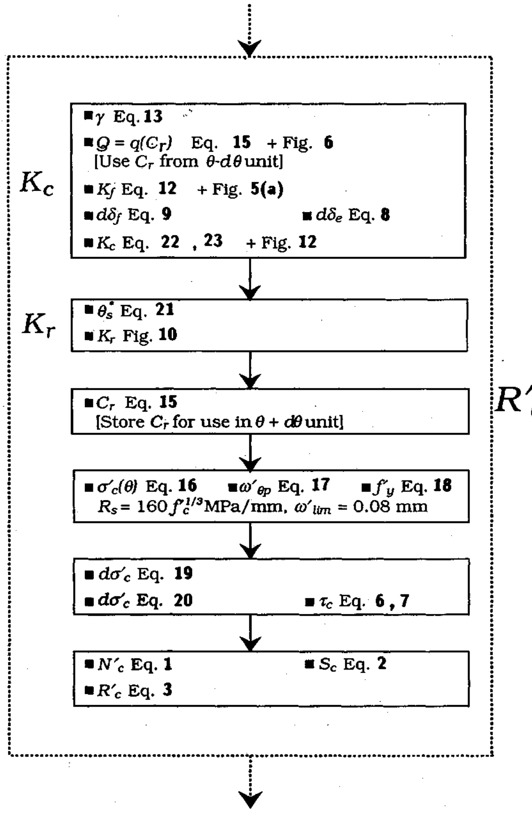


Fig.14 Calculation algorithm of the non-linear contact forces R'_c.

non-linear contact forces model depicted as the R'_c box in the main algorithm in **Fig.13** and further shown in detail in **Fig.14**.

For a given displacement, crack opening and shear displacement, the stress components are obtained by conducting numerical integration of Eqs.4 and 5 with respect to the direction angle θ. Algorithms in **Figs.13** and **14** are the successful schemes to calculate transferred stresses along an arbitrary crack deformational path.

As shown in **Fig.13** too, the entire system of the proposed universal model consists of local-to-global equilibrium denoted by Eqs.4 and 5, local-to-global compatibility specified by Eqs.11, 14 and the local path-dependent constitutive law for R'_c summarized in **Fig.14**. The simultaneous solution of all equations above is the prediction of the universal model.

5. CONFIRMATORY ANALYSES

The complete formulation of the universal model for stress transfer explained above includes three newly introduced elements apart from those already introduced and qualitatively examined in Ref.1), i.e., the path-dependent frictional model

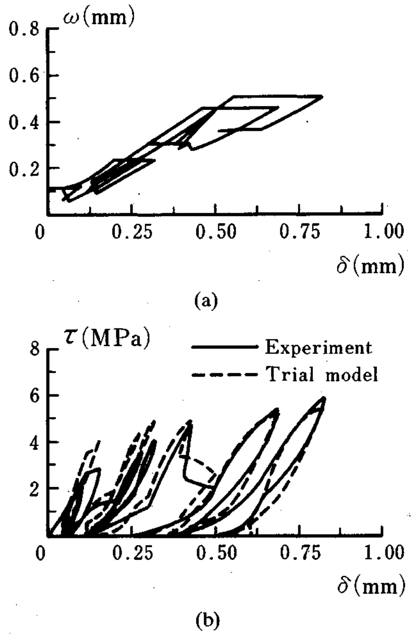
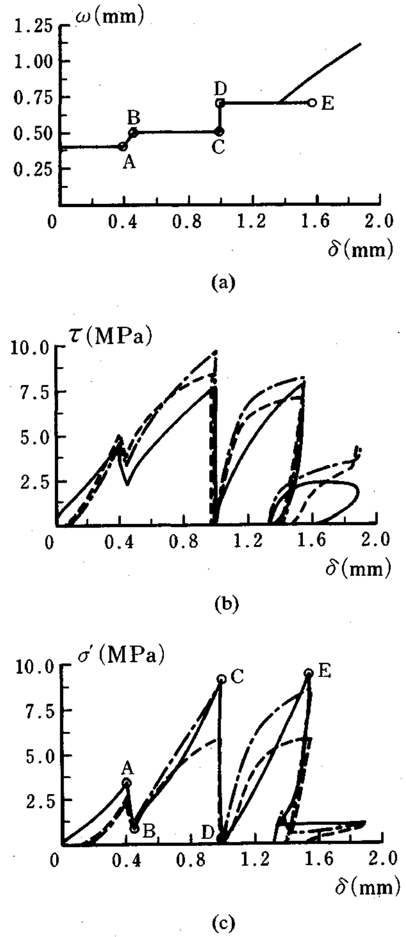


Fig.15 Successful application of trial model adopting the explicit plasticity model on a specialized path, $f_c'=21.7$ MPa.



——Experiment
----Analytical results from the model adopting the explicit anisotropic model
—·—From the universal model adopting the unified anisotropic concept

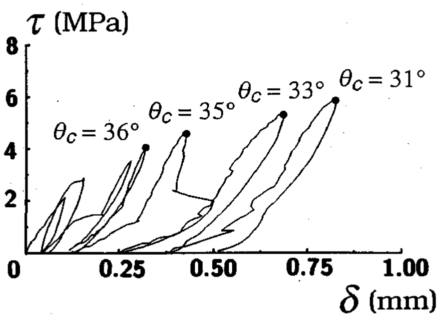
Fig.16 Comparative stress transfer responses of step-type crack deformational path, $f_c'=22.1$ MPa, to confirm applicability of the unified K_r model.

[Eqs.6,7], the unified anisotropic plasticity model denoted by K_r [**Fig.10**] and the effect of confinement [Eqs.12,15]. The path-dependent frictional model differs from the simple model for frictional effect introduced in Ref.1) by being added in its formulation the unloading with higher stiffness (Eq.6, when $\tau_c \cdot d\delta_e < 0$) as well as the upper limit of frictional capacity [Eq.7]. The two items complement the attempt to model the frictional behavior of stress transfer which was experimentally observed as reported in Ref.1). The latter elements, the unified K_r model and the effect of confinement, are indispensable for perfecting the model and their roles in the framework of universal model will be examined in the following sections using some relevant experimental results.

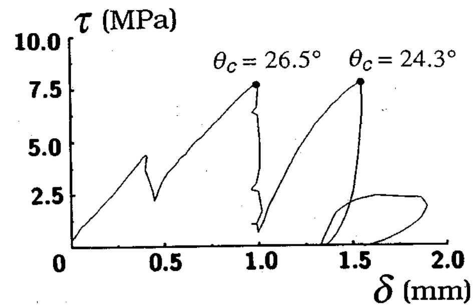
(1) Confirmatory Analysis of the Unified Anisotropic Plasticity Model

Experimental result which is relevant to this confirmatory analysis is the one involving step-type loading path conducted by Li[2] and was originally done to examine and verify the path dependent sensitiveness of the original stress transfer model. Fortunately, the loading path also included effect of some contact fracturing and effect of high deformation of crack plane and thus will be useful for improving applicability of the stress transfer model.

A trial model was devised being composed of three microscopic models, i.e., the path-dependent frictional model [Eqs.6, 7], the explicit anisotropic plasticity model which does not consider the intrinsic anisotropy[1], and the contact fracturing model [Eq.22]. The trial model was successfully applied to stress transfer responses from a specialized loading path [**Fig.15**] but was inapplicable to the step-type loading path [**Fig.16**]. It can be noted in **Fig.16** that the trial model adopting only the explicit anisotropic model failed to anticipate the experimental stress transfer responses especially the transferred compressive stress in **Fig.16** (c) in which the analytical response is lower than the experimental one. This deviation indicated that the trial model was still inadequate and needed further



(a)



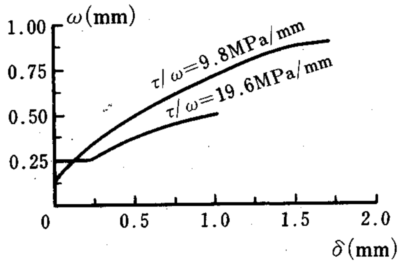
(b)

Fig.17 "Contact inclination θ_c" at certain points of interest. (a) The specialized case from Fig.15 (b). (b) The step-type loading case from Fig.16 (b).

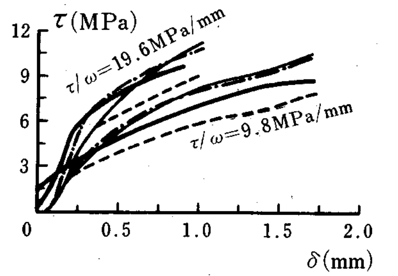
improvement. To relevantly improve the model it is necessary to fully understand nature of the experimental case used as the basis for improvement. In this respect, first of all, it is desirable to review that as a crack plane deforms along an arbitrary loading path the 90° contact unit will come into contact first and then the flatter ones in descending order. For a given crack deformation it is possible to calculate which contact unit has just come into contact by using the geometrical compatibility in Eq.10 upon the condition that the perpendicular contact deformation ω'_θ of that unit must be just zero at the deformation. By rearranging Eq.10, we can obtain

$$\theta_c = \tan^{-1}\left(\frac{\omega}{\delta}\right) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (24)$$

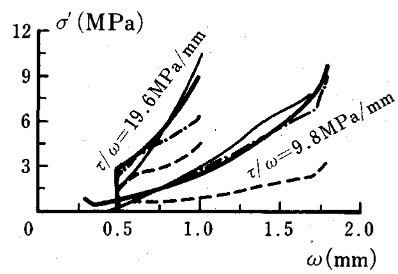
in which θ_c is "Contact inclination", the inclination of the unit which just comes into contact. Using Eq.24, one can compute θ_c of a few points of interest at certain crack deformation in the experimental stress transfer results from **Figs.15** (b), **16** (b) and the calculated θ_c are shown in **Figs.17** (a), (b) beside the points of interest in the figures. It can be seen that the θ_c of the specialized case [**Fig.17** (a)] in which the trial model was successfully applied are in the range of 30° while those of the problematic step-type case [**Fig.17** (b)] are in a range of 20°. From these, one can easily recognize that more amount of flatter contact units are under contact in the step-type case than those in the specialized case and the trial model failed to anticipate stress transfer responses in the step-type high deformation path in **Fig.16** because it did not take into account the effect of intrinsically stiffer contact units which are the flatter ones. This effect of contact-unit-inclination-oriented stiffness is the so-called "intrinsic anisotropic plasticity" mentioned previously and is modeled by the unified K_r model shown in **Fig.10**. Successful



(a)



(b)



(c)

——Experiment
······Model which excludes effect of confinement (Q=1)
——Model on contact isotropy (K_r=0)
—·—Universal model

Fig.18 Experimental and analytical results of a stress transfer experiment in which τ/ω ratio is kept constant and involves specialized paths at rather high confinement, f_c'=38.0 MPa. (a) Deformational paths of two tests having τ/ω=9.8 and 19.6 MPa/mm. (b), (c) Transferred stress responses.

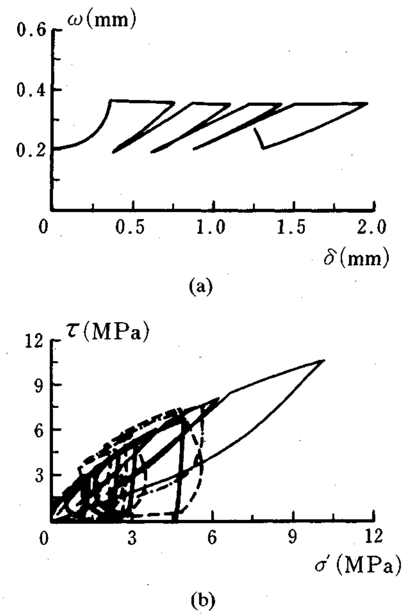


Fig.19 Comparative analytical results of a specialized crack deformational path $f_c'=28.2$ MPa, to examine the effect of confinement on deformational behavior of contact units.
"(the same line legends as in Fig.18)"

analytical results of the model after having adopted the unified K_r model are also shown in **Fig.16.**

(2) Confirmatory Analysis of the Effect of Confinement on Deformational Behavior of Contact Units

Concrete crack deformational path which is specifically sensitive to the effect of confinement on deformational behavior of contact units is the path in which rate of deformation $d\omega/d\delta$ is roughly less than 1. Available experimental results of which deformational paths having this characteristic are the ones also done by Li[2] in which τ/ω ratio are kept constant. Two cases of the experimental results are used as shown in **Fig.18** along with their deformational paths.

For a purpose of clear comparison, an analysis on a specialized deformational path was done using complete universal model adopting the path-dependent frictional model, the unified anisotropic plasticity model and the contact fracturing model except that effect of the confinement parameter Q in Eq.12 was neutralized by assigning a value 1 for the parameter Q. The analytical results are shown in **Fig.19** indicating that the model was applicable to the experimental case. The same model without the effect of confinement was applied to the constant τ/ω paths in **Fig.18** indicating by the analytical results in the figure that the model without effect of confinement, though very well applicable to the case of specialized path in **Fig.19**, is still inadequate for the case in **Fig.18** which is the specialized paths under rather high confinement.

The analytical result in **Fig.18(b)** which is the transferred shear stress agrees very well with the experimental one while the analytical transferred compressive stress in **Fig.18** (c) is quite different. This suggests that there is some inadequacy in the model pertaining the ratio of transferred shear to compressive stress. In the present model, the ratio is mainly dependent on the contact density function $\Omega(\theta)$ and the non-normality assumption. Since there is no significant change of crack surface geometry in the controlled experiment, the effect of contact density function $\Omega(\theta)$ can be ruled out. Consequently, it can be deduced that the rather high confinement around contact units of crack plane in the experiment affects the non-normality assumption which is related to deformational nature of the units.

It can be hypothesized that a contact unit subjected to a specialized deformational path behaves as a frictional unit under low confinement state but rather behaves as a deformational one under a high confinement. This suggests that the frictional contact unit factor K_f described by Eq.12 should be nearly 0 if the unit deforms under high confinement state even if it is undergoing specialized path deformation. Thus the frictional contact unit factor K_f in the trial model was set to be 0 and then the modified model was applied to the experimental case and the successful analytical results are also shown in **Fig.18**. However, when the same trial model with $K_f=0$ was applied to the former specialized case successfully predicted in **Fig.19**, the analytical result came out to be short of good prediction as shown in the figure.

The qualitative analysis done above indicated that there was a need to legitimately consider effect of the confinement level on the deformational nature of contact units. This means that the frictional contact unit parameter K_f must be a function of confinement level ; thus leading to the introduction of confinement parameter Q into the definition resulting in a final version of K_f function as indicated by Eq.12. The confinement parameter Q itself is a function of confinement level as indicated by the simple model shown in **Fig.6**.

The universal model considering the effect of confinement yields satisfactory prediction for both cases of specialized-path deformation under high confinement shown in **Fig.18** and that under low confinement in **Fig.19**.

6. CONCLUDING REMARKS

The complex behavior of stress transfer across cracks in concrete is thoroughly understood and quantitatively modeled by the authors' universal stress transfer model. The universal model is devised based upon the framework of the contact density model proposed by Li & Maekawa[2] which assumes a few simple assumptions. These are the assumption of perfect elasto-plastic of contact stress without any anisotropy or deterioration due to contact fracturing and the assumption of normality of contact force. Under high confinement condition these assumptions make the model to be very simple with fairly good applicability which is the main virtue of the model. However, the concrete stress transfer behavior is highly non-linear and the simple assumptions are unable to cover the whole range of the non-linearity. In order to fully understand the whole realm of the behavior the universal model is devised based upon the framework of the contact density model but with more realistic assumption of anisotropic property of contact stress and deterioration of contact forces and the assumption of non-normality condition due to the significant effect of friction on contact forces. The model is a kind of physical model based upon a numerical scheme which is supposed to be able to anticipate any kind of concrete crack deformational paths. The universality of the model will be verified by extensive experimental results and will be presented in the next paper by the authors.

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コンクリートひびわれ面での応力伝達に関する一般化モデル

ブジャタム ブシャ・前川宏一

本研究は，李らによって提案された接触面密度モデルに立脚した，ひびわれ面における一般化応力伝達に関する物理モデルを提案するものである．巨視的な非線形性に大きく関与している，ひびわれ面での接触力モデルの高精度化を主として図ったものであり，李モデルでは陽な形でモデル化されていなかった接触点での摩擦・異方塑性・破壊を，ひびわれ面の接触点近傍の変形モデルとして採用し，適用範囲の一層の向上を図った．