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QUALITATIVE STUDIES ON MECHANISMS OF STRESS TRANSFER ACROSS CRACKS IN CONCRETE

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Mechanisms of concrete stress transfer were examined and thoroughly understood using the contact density model developed by B. Li and K. Maekawa. Qualitative studies were done on high strength concrete and on structural-deformation-like loading paths involving crack closing or opening with shear slip happening at the same time. The studies indicated that stress transfer mechanisms in high strength concrete crack were different from that of normal concrete due to difference in crack surface configuration. They also showed that the mechanisms in the structural-deformatin-like paths much differed from those in classical crack width constant loading paths due to significantly induced effect of friction and due to unconfined nature of the paths which gave rise to anisotropic property and contact fracturing of a crack surface.

Keywords : *crack, stress transfer, shear, contact density*

1. INTRODUCTION

Cracks in concrete are generally divided into two categories, namely smeared crack and discrete crack in finite element analysis. The so-called smeared cracks are the sort of hair-line cracks spreading in myriad number throughout a reinforced concrete entity. On the other hand, the discrete cracks are the kind of large cracks existing in a few number in a concrete structure whose mechanics may be considerably affected by such a few number of the cracks. Stress transfer behaviors of the discrete crack in reinforced concrete are enormously complex and can be understood only when their constituting elements, i.e., the load bearing nature of steel reinforcement, of concrete, and the interaction of the two, are completely comprehended. This study concentrates effort on the understanding of basic mechanisms of stress transfer across crack in the concrete part. For this purpose, the authors conduct systematic simulations using the contact density concept as a basic analytical tool. Through this qualitaitve study, the universal stress transfer model which is linked with various microscopic mechanisms is aimed as the ultimate goal.

2. THE BASIC CONTACT DENSITY MODEL

Investigation into the complex nature of stress transfer in concrete needs analytical tool which is accurate and suitable to fundamental characteristics of the stress transfer. In this aspect, B.Li and K. Maekawa had made a great deal of contribution by thoroughly investigating various models including the ones which were microscopic physical model that simulated stress transfer based on anisotropic crack surface geometry[1)-3)]. They then proposed the original contact density model which is simple and very successful in dealing with nonlinearity, shear dilatancy and path-dependent characteristics of stress transfer problems[4)]. The model is based on a few concepts which may be categorized into fundamental concepts referred to as basic proposals and less fundamental concepts referred to as basic assumptions. There are two of such basic proposals and three of such basic assumptions.

(1) The Basic Proposals[4)]

a) Geometry of crack surface

It was proposed that complicated asperity of a crack surface can be divided into infinitely small pieces defined as "contact units" with various global inclinations θ as shown in **Fig.1**(a). Area of a contact unit dA_θ having an inclination between θ and $\theta+d\theta$ can be reflected by a proposed contact density function $\Omega(\theta)$ in the following form

$$dA_\theta=A_t\Omega(\theta)d\theta \cdots\cdots\cdots (1)$$

in which A_t is whole surface area per unit crack plane and $\Omega(\theta)$ is a stochastic density function representing directional distribution of a crack surface as shown in **Fig.1**(b).

b) Contact stress direction

Contact units of a crack plane provide resistance to external stresses through their contact reactions on the respective units. A contact reaction is composed of normal reaction derived from the unit deformation and tangential reaction derived mainly

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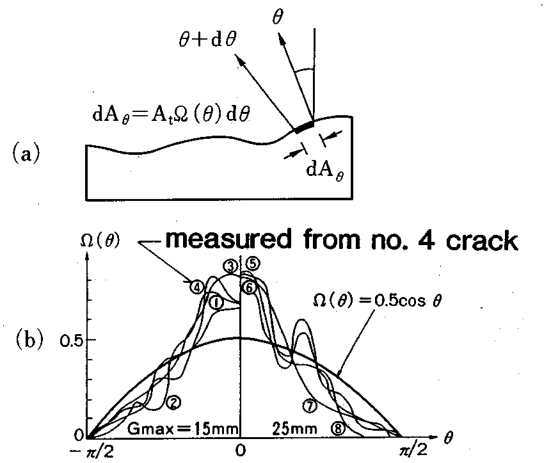


Fig.1 (a) A contact unit with the inclination θ having an infinitesimal area dA_θ. (b) Directional density distributions of many concrete cracks with maximum size of aggregates $G_{\max}=15$ mm and 25 mm along with the proposed contact density function[4].

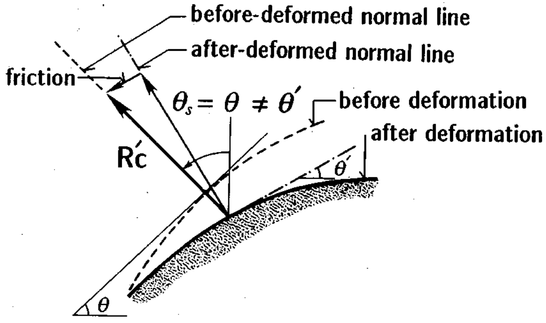


Fig.2 Combined effect of contact plane, deformation and friction which produces normality condition.

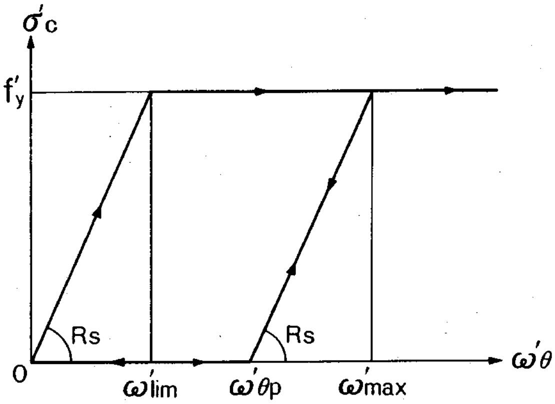


Fig.3 Elasto-perfectly plastic model for contact stress on a contact unit.

from friction and the resultant reaction R_c' is assumed to act on the unit with a resultant contact angle θ_s which is not equal to θ', the inclination of the deformed unit, but is assumed to be equal to θ, the inclination of the unit before subjected to a crack deformation-causing plastic deformation of the unit [see **Fig.2**]. This is the second proposal, namely the normality proposal, which can be written as

$$\theta_s=\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

(2) The Basic Assumptions

a) Contact density function

Li[4] had measured two dimensional projection of many crack surfaces and calculated corresponding density distributions of direction of constituent contact units on the crack surface. The distributions of many measured crack surfaces are extremely difficult to be represented by any simple shape as can be seen in **Fig.1(b)**. However, the figure does reveal that there exist more horizontal contact units and less vertical ones as the distributions concentrate in the region near the direction of 0°(horizontal contact unit has 0°direction). For the sake of simplicity, a trigonometric function to represent the directional distribution of the contact units was assumed as

$$\Omega(\theta)=\frac{1}{2}\cos\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

and this contact density function is supposed to be independent of size and grading of aggregates.

b) Elasto-plastic model for contact stress

Contact stress on a contact unit is related to local deformation of the unit through an elasto-plastic model. The model is elasto-perfectly plastic as shown in **Fig.3** to take into consideration high plasticity of concrete stress transfer behavior. The contact stress σ_c' is assumed to be dependent only on the component in normal direction ω_θ' of local deformation which can be derived from geometrical compatibility condition. The mathematical forms of ω_θ' and σ_c' are

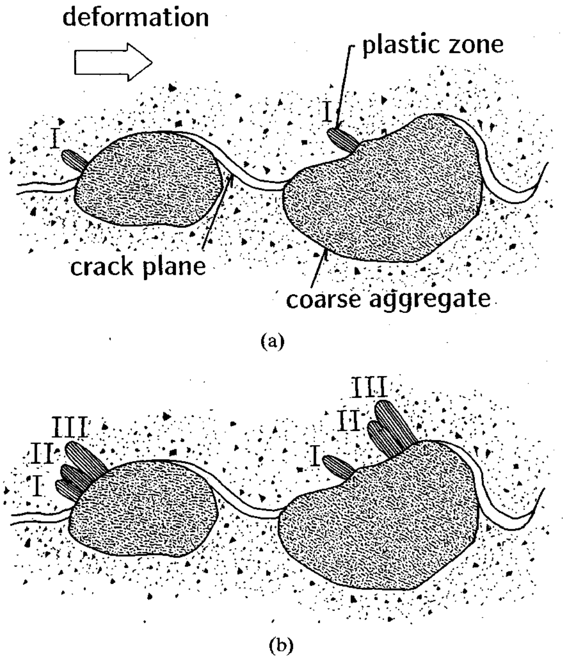
$$\omega_\theta'=\delta\sin\theta-\omega\cos\theta \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

$$\sigma_c'(\theta)\begin{cases} =R_s(\omega_\theta'-\omega_{\theta p}') & \text{for } \omega_\theta'\geq\omega_{\theta p}' \\ =0 & \text{for } \omega_\theta'<\omega_{\theta p}' \end{cases} \cdots\cdots (5)$$

where R_s is the elastic rigidity per length and $\omega_{\theta p}'$ is the local plasticity in θ direction which has path-dependent characteristic.

c) Effective ratio of contat

An effective ratio of contact $K(\omega)$ was introduced to take into account effect of crack roughness size on mechanical behaviors of stress transfer apart from the effect of crack shape expressed by the contact density function $\Omega(\theta)$. The function for $K(\omega)$ was proposed to be ralated to maximum size of coarse aggregate $G_{\max}$ and crack width ω as



(a)

(b)

Fig.4 Conceptual nature of contacting mechanism of contact units. (a) At an initial deformation. (b) After further deformation.

$$K(\omega)=1-\exp\left(1-\frac{0.5\,G_{\max}}{\omega}\right)\geq 0\cdots\cdots(6)$$

Finally, we have the following final path-dependent constitutive equations for the derivation of external compressive stress σ' and shear stress τ,

$$\tau=\int_{-\pi/2}^{\pi/2}\sigma_c' A_t \Omega(\theta) K(\omega)\sin\theta d\theta\cdots\cdots\cdots(7)$$

$$\sigma'=\int_{-\pi/2}^{\pi/2}\sigma_c' A_t \Omega(\theta) K(\omega)\cos\theta d\theta\cdots\cdots(8)$$

All the aforementioned basic proposals, basic assumptions and the constitutive equations constitute the so-called Contact Density Model which has fairly good applicability on various basic stress transfer problems in concrete[4]. Crack deformations in the basic stress transfer are mostly those of crack width constant loading paths including those of reversed cyclic loading.

(3) Conceptual Nature of Contacting Mechanism of Contact Units

It is desirable to construct a conceptual mechanism of contacting since the main point of the basic contact density model is the contacting characteristic of the assumed contact units dA_θ [Eq.1]. From various macroscopic observations and analyses, it is undoubted that plasticity dominates stress transfer mechanism in concrete. The contact stress model in Eq.5 was proposed to deal with the observed plastic characteristics at the level of microscopic modeling. A number of plastic zones would be produced on a crack surface after deformation and there supposed to be a limited number of such zones on the surface. In reality, a number of conceptual small contact units are supposed to gradually combine together to produce single plastic zone as they are plastically deformed during the course of crack deformation.

Fig.4 will explain more vividly the relation of the actual plastic progress and the mechanism conceptually represented by the contact units. **Fig.4**(a) illustrates a state at an initial deformation in which constituent planes with steep inclination will come into contact and become plastic zones first. Two plastic zones on the two aggregates (maybe on another aggregates also in a real concrete crack) constitute one contact unit denoted as I contact unit. As the crack undergoes further deformation, another planes with flatter inclination next to the I contact unit will become contacted, that is, the contact unit II and III respectively [**Fig.4**(b)]. The described mechanism will be used as conceptual framework for later references in the qualitative studies of stress transfer mechanism.

3. THE QUALITATIVE STUDIES ON CRACK SURFACE GEOMETRY

Using the contact density model as the basic analytical tool, qualitative studies on microscopic aspects of concrete stress transfer will be conducted. Basic assumptions will be proposed and independently imposed on the framework of contact density model and results in separate qualitative trial models for different aspects of the respective assumptions. Each qualitative trial model will be verified by selected experimental results which are sensitive to the corresponding assumptions imposed in the model. In this chapter, first, the crack surface geometry is discussed using high strength concrete crack as a case study because of the apparently distinctive features of its crack surface.

A stress transfer experiment was conducted on a high strength concrete specimen of the same size and set up as those used by Li[4]. Compressive strength of the tested concrete was 104 MPa and maximum size of aggregate used was 15 mm. Loading path was that of crack width constant of 0.5 mm with unloadings and re-loadings. **Fig.5** shows transferred shear and compressive stresses versus shear slip for high strength concrete in the experiment. Also shown in the figures are the comparative analytical results from the contact density model. It is very clear that there is a big difference between model predictions and experimental results. The model yields unusually high

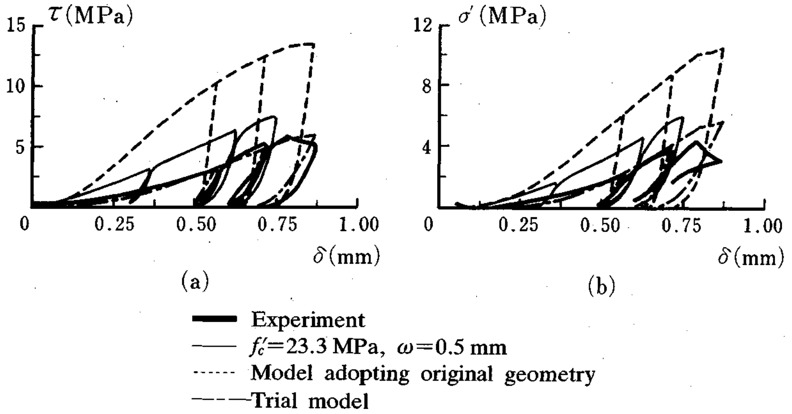


Fig.5 Stress transfer results of high strength concrete, f_c'=104 MPa, w=0.5 mm. Note that stress transfer stiffnesses and magnitudes of normal concrete with f_c'=23.3 MPa with the same crack opening are higher than those of high strength concrete.

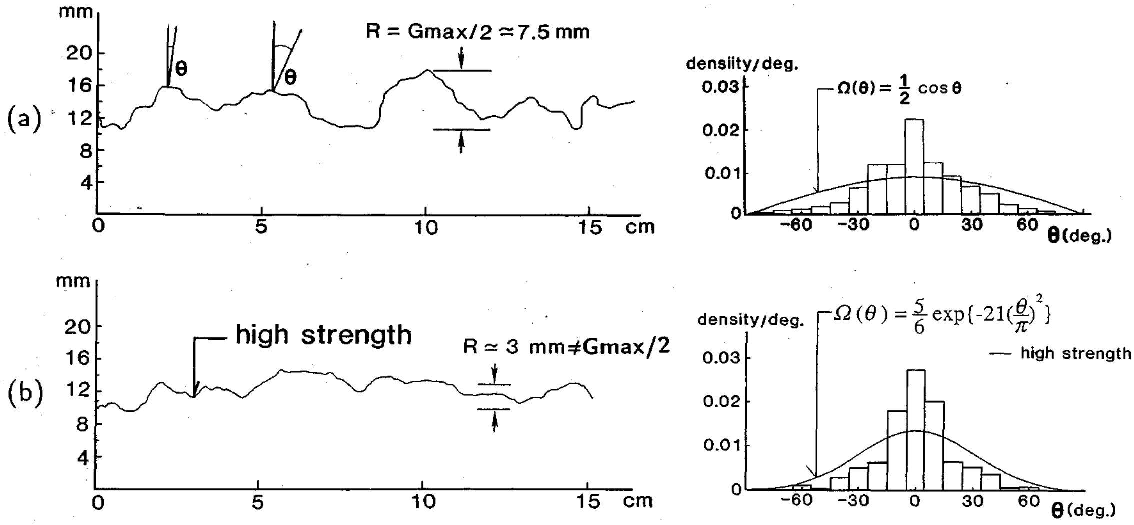


Fig.6 Two dimensional projection of concrete crack surfaces and the corresponding directional density distributions for (a) normal concrete and (b) high strengh concrete.

stiffness and high magnitude of transferred stresses and it should be especially noted that transferred stress stiffnesses of high strength concrete with 104 MPa compressive strength is less than those of normal concrete of just 23.3 MPa in strength as indicated in the figures.

It was noticeable from the outset that crack surface of high strength concrete is much flatter than that of normal concrete. This flatness of high strength concrete crack may be explained by the fact that strength of mortar and bond between mortar and coarse aggregates are so comparatively high that embedding coarse aggregates tend to break apart when subjected to splitting force rather than to get loose from mortar as those in normal concrete[5].

The significant difference in crack geometry is very likely to be the cause of the discrepancy of model predictions. The basic assumptions relevant to crack geometry in the original contact density model are the assumed contact density function $\Omega(\theta)$ and the effective contact area coefficient $K(\omega)$. These two assumptions will be further examined and suitably modified as the followings.

(1) Contact density function $\Omega(\theta)$

The function 0.5 cos (θ) for contact density was originally assumed for the contact density distribution of a normal concrete crack as shown in **Fig.6**(a) which illustrates two dimensional projection of a normal concrete crack along with its

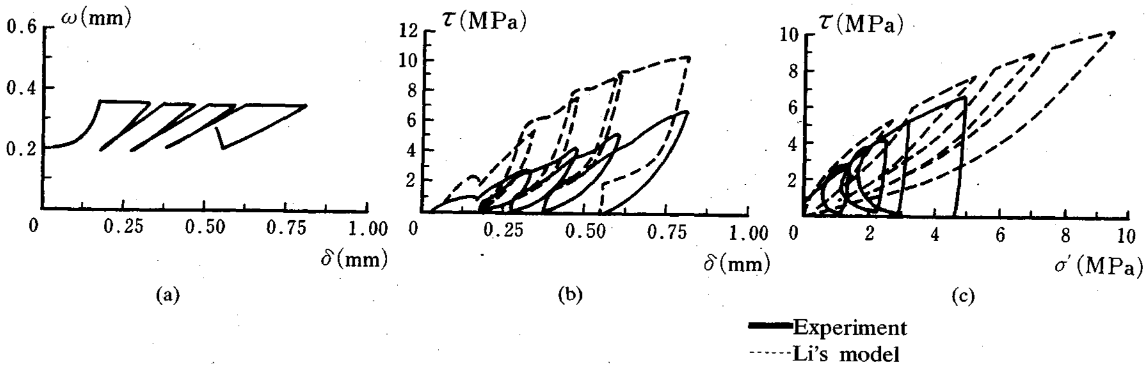


Fig.7 Crack deformation in (a) and stress transfer responses in (b), (c) of a cyclic "mixed mode" deformational path experiment, $f_c'=28.2$ MPa.

directional density distributions indicated by the standing bar charts. It should be noted that the measured directional distribution shape is not perfectly expressed by the assumed contact density function $\Omega(\theta)$. The computed directional distributions portrayed as the bar charts depend significantly on the length of discrete interval adopted for the digitization of crack surfaces. Therefore, the distributions can not be regarded as the real nature of crack surface geometry although they serve very well for qualitative understanding. Projection of a high strength concrete crack is also shown in **Fig.6**(b) as well as its density distribution. It is recognizable by comparing the two figures of projected crack surfaces that the surface of high strength concrete crack is flatter than that of normal concrete. This fact can be further confirmed by examining their corresponding density distributions in which one can see that the density of high strength concrete concentrates more in the region of −20° to +20° than in normal concrete and it can be better represented by a truncated normal distribution curve also shown in **Fig.6**(b) having the following mathematical form,

$$\Omega(\theta)=\frac{5}{6}\exp\{-21(\theta/\pi)^2\} \text{ for } \theta \text{ as radian} \quad \cdots\cdots\cdots\cdots(9)$$

(2) Effective Ratio of Contact $K(\omega)$

The coefficient $K(\omega)$ in the contact density model is provided to take into account the fact that contact area decreases with the increase of crack opening and will be zero if the opening is large enough compared with the maximum roughness R of a crack surface and it is expressed as

$$K(\omega)=1-\exp\left(1-\frac{R}{\omega}\right)\geq 0 \cdots\cdots\cdots\cdots(10)$$

The maximum roughness R of normal concrete crack was proposed to be one half of maximum size of coarse aggregates $G_{\max}$ as indicated by Eq.6. However, by examining **Fig.6**(b), one can figure out that one half of $G_{\max}$ which is 7.5 mm can no more be regarded as the maximum roughness for the high strength concrete crack. Value of the maximum roughness must be suitably decided.

Qualitative trial model customized for high strength concrete crack can be obtained by introducing the modified contact density function from Eq.9 and adopting a value of 3 mm [see **Fig.6**(b)] for the maximum roughness R in Eq.10. Much improved analytical results from the trial model are also shown in **Fig.5** which shows that both transferred shear and compressive stress from the model agree very well with the experimental ones.

4. QUALITATIVE STUDY OF STRESS TRANSFER IN STRUCTURAL-DEFORMATION-LIKE LOADING PATHS

The basic contact density model was developed based mostly on high confined loading paths where crack width remained constant or did not increase so much. However, in some structures, loading paths involve both shear slip and crack opening at the same time. Steel reinforcement crossing a concrete crack provides confining compressive pressure derived from stress due to steel elongation caused by crack opening and, simultaneously, shear stress is provided by the concrete crack under shear deformation. To investigate stress transfer behavior under such deformational paths observed in some structural discrete cracks, series of stress transfer experiments were done on pre-cracked concrete specimens in which crack deformation could be arbitrarily controlled. **Fig.7** shows examples of stress transfer responses from one of the experiments. Note that the cyclic mixed mode deformation, crack closing and opening with shear slipping at the same time, constitutes main part of

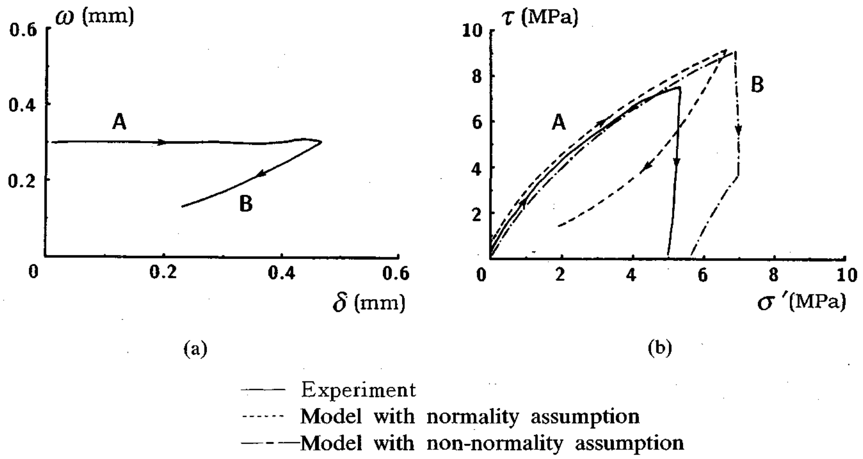


Fig.8 Selected experimental results to illustrate non-normality condition. (a) Deformational path (b) Transferred shear stress response.

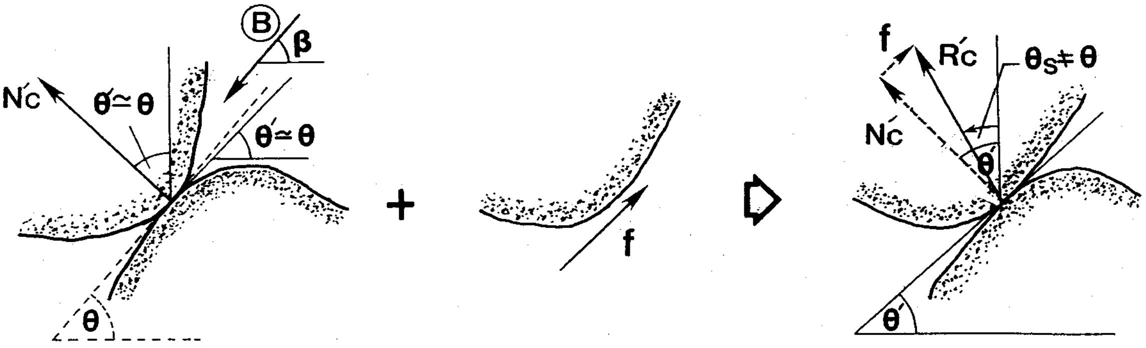


Fig.9 Combination of significant frictional deformation and insignificant plane deformation which produces non-normality condition.

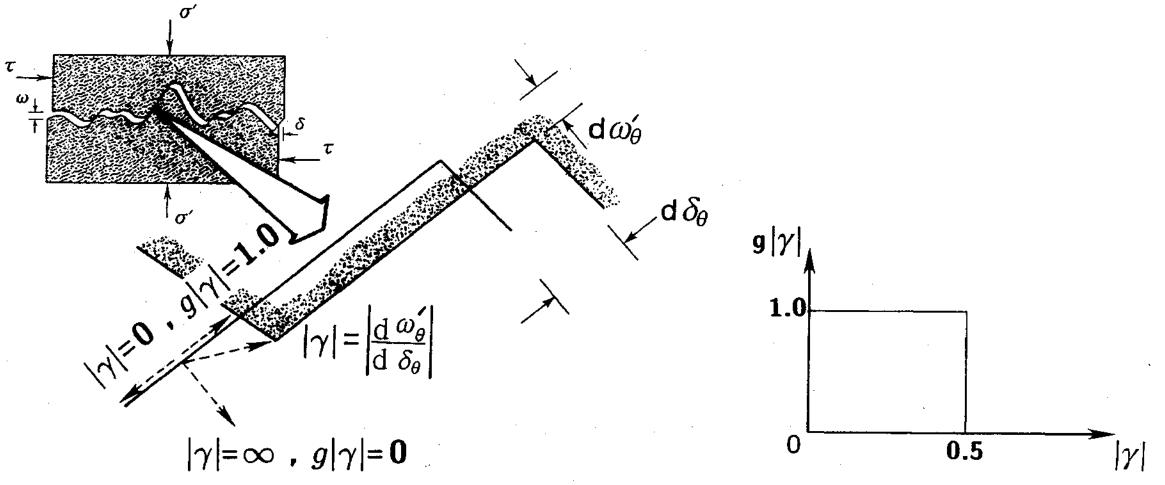
the crack deformational paths. Such cyclic mixed mode crack deformation will be referred to as "specialized path" hereafter.

The analytical results of basic contact density model are also shown in the figure and one can clearly see that the model predictions tend to be of higher values in both shear as well as compressive stresses and the shapes of shear versus compressive stress responses in **Fig.**7(c) are totally different. The significant discrepancy indicates that the "specialized path" deformation involves some aspects of concrete stress transfer which are not taken into account as basic assumptions in the original contact density model. The authors consider that they are the assumptions of non-normality condition, of anisotropic plasticity, and of contact fracturing. The three assumptions will be separately examined using results from separate controlled experiments which are sensitive to the respective assumptions.

(1) The Assumption of Non-Normality

Fig.8 shows a selected experimental case to illustrate the need for non-normality assumption for a universal model. **Fig.**8(a) is crack deformational path in the experiment which involves crack width constant path *A* and then specialized path *B*. It can be obviously seen in **Fig.**8(b) that the model with the normality assumption, the basic contact density model, can be applied very well along path *A* which is the constant crack width path but is very unsatisfactory along path *B*, the specialized path. Predicted compressive stress decreases as shear stress decreases while the experimental compressive stress is practically not decreasing.

This discrepancy can be explained by the fact that, along the specialized path, significant friction *f* is produced on some contact units due to relatively large frictional slip which is induced without significant plastic deformation of the unit as shown in **Fig.9** and this sort of contact unit is termed as "frictional contact unit" to which normality assumption can not be applied. Its



(a) (b)

Fig.10 (a) Kinematic direction γ of local displacement of a contact unit under an infinitesimal displacement $d\omega'_\theta$ and $d\delta_\theta$. (b) Proposed model for the frictional contact unit parameter.

counterpart is termed "deformational contact unit" shown in **Fig.2** on which the frictional effect and plane deformation combine just right to produce the normality condition. The contact friction and normality assumption are sensitive to the relation of the transferred compressive and shear stresses which reflects the direction of resultant transferred stress. The specialized path B in **Fig.8** is suitable for examining the normality assumption since it introduces large frictional slip on comparatively many contact units of the crack plane$^{6)}$.

Normality assumption in the basic contact density model can be modified to be a non-normality one by introducing into the basic model the following two components which govern frictional behavior.

a) Quantitative defintion for deformational and frictional contact units

The trial quantitative definition is expressed by a frictional contact unit factor K_f which is defined as

$$K_f = g(|\gamma|) \cdots\cdots (11)$$

where γ is designated as "kinematic direction of local displacement" which can be computed from

$$\gamma = \frac{d\omega'_\theta}{d\delta_\theta} \cdots\cdots (12)$$

The $d\omega'_\theta$ and $d\delta_\theta$ are respectively infinitesimal local displaement in normal and tangential directions to the unit θ as,

$$d\omega'_\theta = d\delta \sin\theta - d\omega \cos\theta \cdots\cdots (13)$$

$$d\delta_\theta = d\delta \cos\theta + d\omega \sin\theta \cdots\cdots (14)$$

which is the differential form of geometrical compatibility condition proposed in the original contact density model.

The kinematic direction γ represents relative displacement direction for a contact unit at a point of an infinitesimal deformation as illustrated in **Fig.10**(a). The frictional contact unit factor K_f is assumed to be 1.0 when $|\gamma|$ is 0 which means that the companion contact units relatively move along the tangential direction and the unit is a perfect frictional contact unit. The unit will be a perfect deformational contact one when $|\gamma|$ is ∞ (perpendicular direction) and K_f will be assumed as 0. The factor K_f is between 1 and 0 when companion contact units move between the two extremes and the unit will theoretically be a combination of frictional and deformational contact unit.

A simple function for the factor K_f is proposed on trial as shown in **Fig.10**(b). When $|\gamma|$ is more than 0.5, K_f is 0 and the unit is a perfect deformational contact unit. If the parameter $|\gamma|$ is less than the value, K_f is assumed as 1 and the unit will be a perfect frictional contact unit.

Here, a parameter δ_e or "effective frictional slip" is introduced as an important parameter governing frictional behavior on a frictional contact unit and is defined in an incremental form of

$$d\delta_e = K_f \cdot d\delta_f \cdots\cdots (15)$$

The notation $d\delta_f$ is "apparent frictional slip" which is local displacement in tangential direction $d\delta_\theta$ which occurs on a "contacted" unit and can be expressed as

$$d\delta_f = \begin{cases} d\delta_\theta \text{ when } \omega'_\theta > 0 \\ 0 \quad \text{ when } \omega'_\theta \leq 0 \end{cases} \cdots\cdots (16)$$

in which $d\delta_\theta$ is computed from Eq.14 and ω'_θ is the component of local displacement calculated by Eq.4.

b) Simple model for frictional component

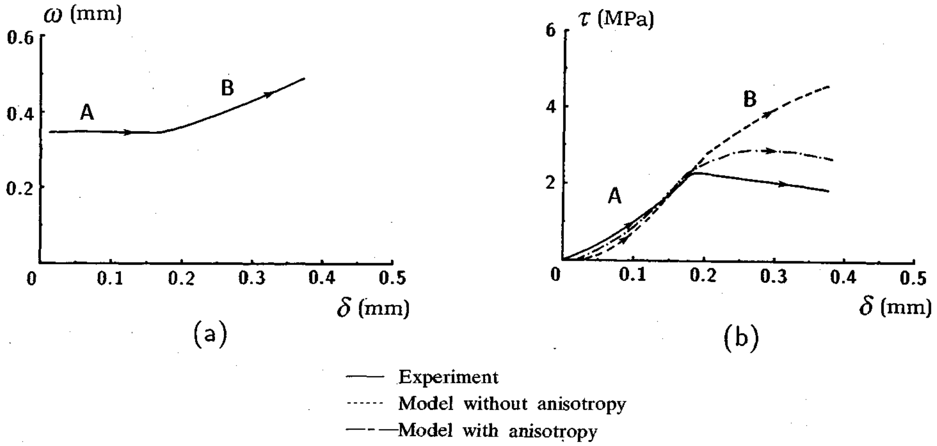


Fig.11 Selected experimental result to illustrate anisotropic property of concrete stress transfer along with the comparative analytical results (a) Deformational path. (b) Transferred shear stress response.

The component of friction on a contact unit can be simply modeled as being derived from tangential contact stress acting on the unit with some stiffness in the direction. A simple model for this purpose will be

$$d\tau_c = G_s \cdot d\delta_e \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (17)$$

in which τ_c is the tangential contact stress in MPa, G_s the tangential stiffness$=32 f_c'^{1/3}$ MPa/mm and $d\delta_e$ is incremental effective frictional slip calculated from Eq.15 in mm.

The two newly introduced elements are incorporated into the basic contact density model to get an enhanced trial model for frictional effect along the specialized path. **Fig.8** shows that the model is much improved and can well anticipate no-decreasing trend of compressive stress along the specialized path *B* in which the frictional slip is reproduced on comparatively many contact units.

(2) The Assumption of Anisotropic Plasticity

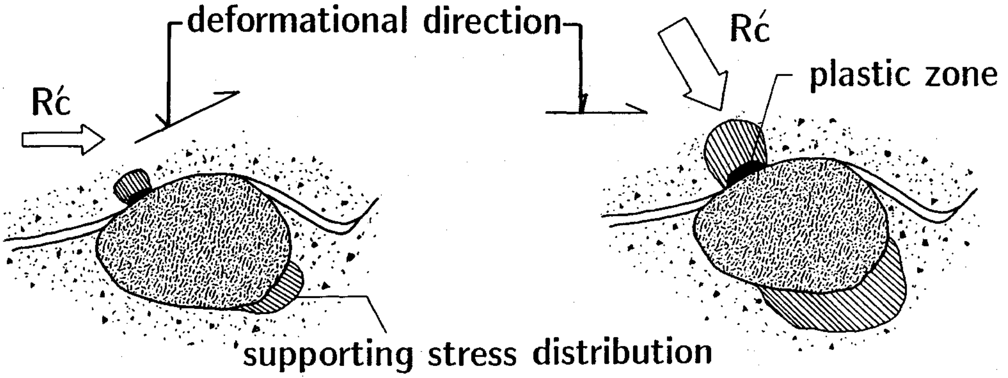
In reality it is very difficult to microscopically observe the anisotropic nature of a contact unit on a crack surface. However, since transferred stresses along a crack plane are derived from the summation of contact stresses on all contact units, macroscopic observation of stress transfer must also imply behavior of constituent contact units at microscopic level. **Fig.11** shows such macroscopic observation from a stress transfer experiment of which crack deformational path consists of constant crack width path *A* and then specialized path *B*. Note that stiffness of transferred shear stress shown in **Fig.11**(b) from the experiment abruptly changes from a higher stiffness to a lower one at the transition point where path *A* turns to path *B* while that of the original contact density model which does not consider anisotropic property apparently does not change at the point.

It can be deduced from the comparison that macroscopic stiffness of transferred stress depends also on direction of loading or, in the microscopic level, contact forces on contact units have anisotropic property. The specialized path in **Fig.11** was intentionally produced in order to create drastic change of contact stress dirction on each contact unit, and the relationship between shear stress and shear slip was supposed to be so affected by the change of contact stress associated with plastic yielding stress level. In the conventional loading paths, the contact stress direction remained stable and thus the contact plastic anisotropy has not been detected nor discussed.

The anisotropic property of contact forces is conceived to be due to changing size of plastic zone and supporting stress distribution as shown in **Fig.12**. It is conceptualized that unconfined state of a specialized path gives rise to small plastic zone and stress distribution [**Fig.12**(a)] while the confined state of crack width constant path causes quite an extensive ones [**Fig.12**(b)]. It is conceived that small plastic zone and supporting stress distribution correspond to a low yielding level f_y' in the contact stress model [**Fig.3**] and the bigger zones to a higher one.

The perfect elasto-plastic model in Eq.5 is modified by introducing an anisotropic plasticity parameter $K_r (0 \leq K_r \leq 1)$ and the modified model will be

$$\sigma_c'(\theta) = \begin{cases} K_r R_s(\omega_\theta' - \omega_{\theta p}') & \text{for } \omega_\theta' \geq \omega_{\theta p}' \\ 0 & \text{for } \omega_\theta' < \omega_{\theta p}' \end{cases} \cdots\cdots (18)$$



(a) (b)

Fig.12 (a) Smaller plastic zone and supporting stress distribution when a contact unit is subjected to a specialized loading path. (b) Bigger ones when it is under a high confined deformational path.

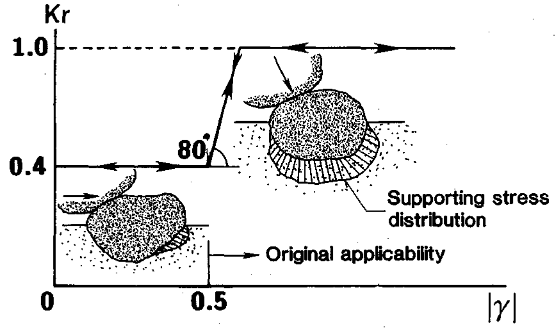
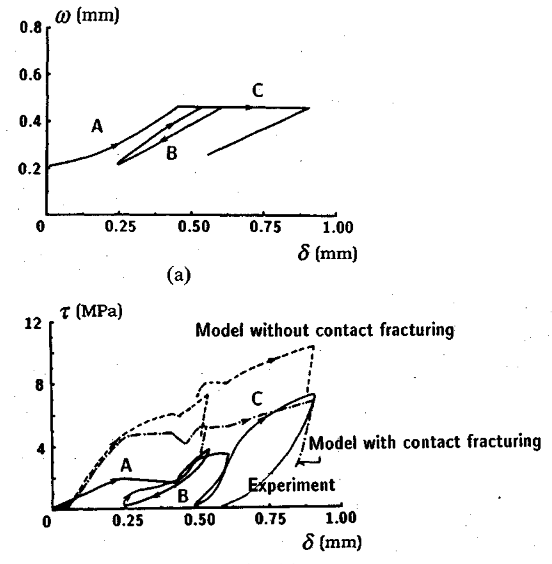


Fig.13 Proposed model relating anisotropic plasticity parameter K_r and the local "displacement ratio $|\gamma|$".



—— Experiment
------ Model without contact fracturing
——Model with contact fracturing

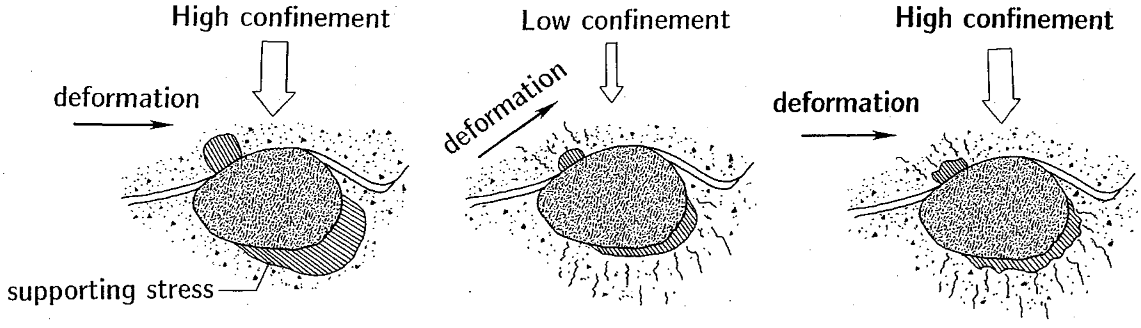
Fig.14 Experimental observations and comparative analytical results illustrating the effect of contact fracturing caused by the specialized paths *A* and *B*. (a) Deformational path. (b) Transferred shear stress response.

This anisotropic parameter K_r will affect stiffness of contact stress which in turn will change yielding level because elastic range, $\omega'_{\theta\max}-\omega'_{\theta p}$ [**Fig.3**], must be kept constant according to Maekawa's elasto-plastic and fracture model[7]. The parameter K_r is assumed to be a function of the displacement ratio $|\gamma|$ [Eq.12] which serves as a suitable parameter relating global displacement direction to a local one of a contact unit. The simple trial model for K_r is shown in **Fig.13** and it should be noted that K_r can reversibly fluctuate between the lower value and the upper one depending on the parameter $|\gamma|$ which in turn is changing according to global loading direction at a point of deformational history. This fluctuation causes the reversible altering of yielding level f'_y which is a characteristic of this anisotropic property of concrete stress transfer[8].

Incorporating the simple model of anisotropic plasticity into the basic contact density model, one can obtain a trial stress transfer model tailored for the anisotropic property of which improved analytical result is also shown in **Fig.11**. The shifting of stiffness at the transition point can well be taken into account by the trial model.

(3) The Assumption of Contact Fracturing

This assumption of contact fracturing is analogous to the previous assumption of anisotropic plasticity in the sense that it is also related to changing of plastic yielding level in contact stress model. Both contact fracturing and anisotropic plasticity play an important role in the so-called specialized paths as evidenced by the discrepancy



(a) no fracturing (b) progressive fracturing (c) damaged state

Fig.15 Scenario at a contact unit subjected to fracturing of plastic zone and of supporting stress distribution.

of the basic contact density model which does not consider both factors as shown in **Fig.7**.

Fig.14 shows a selected case sensitive to the contact fracturing factor. Crack deformation starts with specialized path *A* and *B* and finally crack width constant path *C*. The analytical result of contact density model starts to depart from the experimental one when the crack plane deforms along path *A*, *B* which is natural because the model does not consider effect of anisotropy along specialized path. However, as the deformational path changes to constant crack width path *C* which is the path under high confinement around contact units in which the original model was proved to be successful[4], the experimental response is still lower than that of the model. This suggests that there must be some irrecoverable damage inflicted on the concrete crack plane during the deformation along the specialized path *A* and *B*.

The irrecoverable damage may be physically interpreted at the level of a contact unit as shown in **Fig.15**. The contact unit undergoes deformation along constant crack width path which is the path with high confinement inducing sizable plastic zone and supporting stress distribution without any fracturing [**Fig.15(a)**]. Then the unit deforms under a specialized path in **Fig.15(b)** of which unconfined nature inducing smaller plastic zone and supporting stress, i.e. the anisotropic property explained in previous section. Moreover, the unconfined state also brings about fracturing in surrounding mortar which is irrecoverable and causes lasting damage of the plastic zone and the supporting stress distribution even when the unit is under high confinement path again [**Fig.15(c)**].

The elasto-plastic model for contact stress in Eq.5 is modified for the effect of contact fracturing by introducing a contact fracture parameter K_c $(0 \leq K_c \leq 1)$ as follows.

$$\sigma'_c(\theta)=\begin{cases} K_c R_s(\omega'_\theta-\omega'_{\theta p}) & \text{for } \omega'_\theta \geq \omega'_{\theta p} \\ 0 & \text{for } \omega'_\theta < \omega'_{\theta p} \end{cases} \cdots\cdots (19)$$

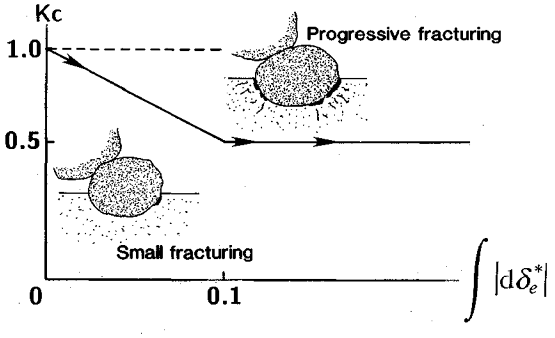


Fig.16 Proposed model for contact fracture parameter K_c.

The parameter on trial is a function of "accumulated effective frictional slip" $\int |d\delta_e^*|$ as

$$K_c = g\left(\int_{\text{Path}} |d\delta_e^*|\right) \cdots\cdots\cdots\cdots (20)$$

the term $d\delta_e^*$ is a variant of the effective frictional slip $d\delta_e$ [Eq.15] and is defined as

$$d\delta_e^* = \begin{cases} d\delta_e & \text{when } d\omega'_\theta > 0 \\ 0 & \text{when } d\omega'_\theta \leq 0 \end{cases} \cdots\cdots\cdots\cdots (21)$$

in which $d\omega'_\theta$ is the infinitesimal local displacement in normal direction computed from Eq.13.

Fig.16 shows the proposed model for contact fracture parameter K_c which is irrecoverable and thus causes a permanent reduction of contact yielding stress level f'_y.

A trial model exclusively modified for this effect of contact fracturing is obtained by including the above simple model for fracturing into the basic contact density model. The trial model can better anticipate reduction of the transferred shear stress along path *C* after deterioration under the specialized paths as can be seen by comparing analytical results shown in **Fig.14(b)**. It should be noted that the trial model with consideration of contact fracturing alone still yields higher stress responses along path *A* and *B* since the path involves, besides the effect of contact fracturing, the effect of anisotropic plasticity which is not taken into account by the model.

5. CONCLUDING REMARKS

Using the original contact density model as the appropriate analytical tool, basic characteristics of concrete stress transfer along cracks were able to be comprehended and identified as the geometry of crack surface, the anisotropy of contact plasticity, the local fracturing and the contact friction. These were separately introduced as basic assumptions into the framework of contact density model in forms of constituent microscopic models having clear physical meaning in connection to the observed characteristic. As a result, the basic contact density model is customized into respective trial ones according to the basic characteristics. The models were then successfully applied on selected experimental responses related to the respective mechanisms. Applicability of the separate trial models on the various aspects of stress transfer assured that the contact density framework provided suitable parameters in concert with natures of concrete stress transfer.

The study thus confirmed that the contact density concept can serve as a very good foundation on which a wider applicable model can be based towards a development of the universal model for concrete stress transfer as the ultimate goal.

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コンクリートひびわれ面における応力伝達機構の定性分析

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接触面密度の概念に基づき，コンクリートひびわれ面における応力伝達機構について，解析と実験による定性分析を行い，構造物中の変形履歴を再現した経路と，高強度コンクリートについて感度解析を実施した．ひびわれ開口とせん断すべりが混成する場合，摩擦・破壊・塑性の異方性が接触面近傍で顕在化すること，高強度コンクリートのひびわれ面形状の平滑化が，応力伝達特性に大きな影響を及ぼすことが判明した．