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AN ANALYSIS FOR LONG-TERM BEHAVIOR OF STEEL-CONCRETE COMPOSITE GIRDERS BY TAKING INTO ACCOUNT OF CREEP RECOVERY

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A phenomenon of creep recovery in the concrete structures is relatively recent knowledge in the fields of RC- and PC-structures. Particularly, the effects of creep recovery on the long-term behavior in the steel-concrete composite girders are not yet clarified completely. Presented in this paper is an analytical method of creep in the composite girders by taking into account of the creep recovery. The parametric study based on this exactly analytical method of creep was conducted. A cross-sectional parameter of composite girders, creep coefficient of flow creep and initial loading age were selected in the parametric analyses. In a case where the creep recovery of concrete was considered in the creep analysis, it was found that change of stress resultants or stresses due to the creep were smaller than the results where the creep recovery was neglected.

Keywords : *creep analysis, creep recovery, delayed elastic creep, flow creep, long-term behavior, steel-concrete composite girder*

1. INTRODUCTION

Owing to the creep of concrete, redistribution of stresses, loss of prestressing forces and additional displacements of the girder are occurred. The creep analysis is, therefore, unavoidable for designing the steel-concrete composite girder.

Since the pioneering work done by Sattler[1], a lot of analytical method of creep have been proposed for designing the composite girder bridges[2)~8)]. Among them, Trost[4] showed a new analytical method of creep by using relaxation coefficient, where the delayed elastic strain component of creep (hereafter referred to as delayed elastic creep) was considered, and also Haensel[7] proposed a simplified treatment of delayed elastic creep in the composite girders. However, the analytical method of creep by considering the creep recovery (see **Fig.1**) in the composite girders is not yet clarified completely, because the creep recovery of concrete is relatively recent knowledge. Therefore, it is an important problem to examine the effects of delayed elastic creep and creep recovery in composite girders.

In the field of prestressed concrete structures, Wakasa and Izawa[9] have been investigated the effects of delayed elastic creep. Thereafter, the analytical methods and the effects of delayed elastic creep and creep recovery are also reported by Hoshino and Saeki[10] as well as Watanabe and Muguruma[11].

This paper reports a more accurate analytical method of creep by taking into account of all the creep strains and clarifies the effect of the creep recovery in the composite girders.

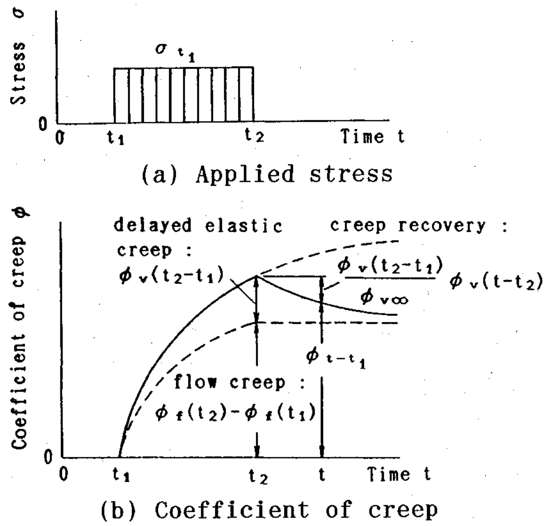

Fig.1 Creep due to loading at time t_1 and unloading at time t_2

Firstly, the concept of creep recovery and the treatments of delayed elastic creep, flow strain component of creep (hereafter referred to as flow creep) and creep recovery to analyze the creep are clearly predicated in this paper. Secondly, the fundamental relationship between stress and strain in the concrete at a time t is derived and discussed. The exact analytical solutions of creep by this method, where normal forces and bending moments in the concrete slab as well as the steel girder are analyzed separately, are presented under the typical loading conditions of the composite girder. Also presented is an approximate solution for the

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practical purposes. Finally, new creep coefficient η by considering the delayed elastic creep and creep recovery as well as the relaxation of stress in concrete is proposed herein.

2. RELATIONSHIP BETWEEN STRESS AND STRAIN IN CONCRETE TAKING INTO ACCOUNT OF DELAYED ELASTIC CREEP AND CREEP RECOVERY[11)~13)]

At present, it is a well known fact that the creep strain in the concrete structures consists of two components i.e., delayed elastic creep and flow creep strain. Delayed elastic and flow creep strain can easily be introduced to the creep analysis of composite girders. The analytical procedure of creep recovery is, however, generally complicated, because the creep recovery occurs not only when the applied stress removes instantaneously but also when concrete stress gradually decreases.

(1) The concept of creep recovery

In general, it is understood that the phenomenon of the creep recovery in the concrete structures occurs only in the case where the applied loads or stresses are removed instantaneously (see **Fig.1**). Removal of a part of dead load, release of the prestressing force of PC-bars or jack up or down of the intermediate supports of continous composite girders, etc. can be pointed out as examples.

However, the stresses in continually stressed concrete member are succeedingly relaxed and decreased their intensities. For the continually decreased stresses (see **Fig.3**) the effects of creep recovery should, therefore, be considered.

(2) Expression for coefficient of creep

When a concrete is subjected to constant load at a time t_1 and this load is removed at a time t_2 as illustrated in **Fig.1**, the coefficient of creep ϕ_{t-t_1} of concrete at a time $t>t_2$ can be expressed as follows :

$$\phi_{t-t_1}=\phi_v(t_2-t_1)+\{\phi_f(t_2)-\phi_f(t_1)\} -\frac{\phi_v(t_2-t_1)}{\phi_{v\infty}}\phi_v(t-t_2) \quad \cdots\cdots(1)$$

where $\phi_v(t_2-t_1)$: coefficient of creep for delayed strain (elastic) according to Davis-Glanville's law

$\{\phi_f(t_2)-\phi_f(t_1)\}$: coefficient of creep for flow strain (plastic) according to Whitney's law

and $\phi_{v\infty}$: value of $\phi_v(t-t_2)$ at time $t=\infty$

The coefficients of creep of the first and second terms in the right-hand side of Eq.(1) are applicable to the initial loading stress σ_{t_1} and the corresponding change of stress $\Delta\sigma_{t-t_1}$. The last one in Eq.(1) is applicable to the stress σ_{t_2} of after removement of the load and the corresponding

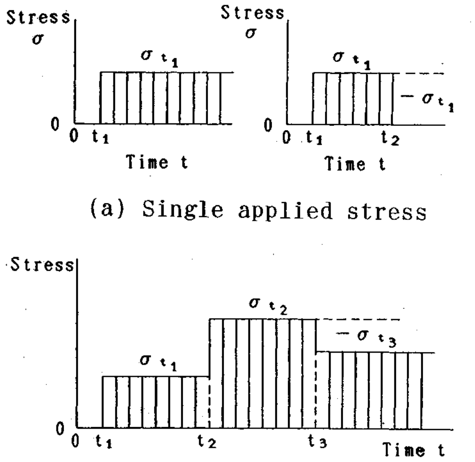

Fig.2 Various stresses applied to composite girder

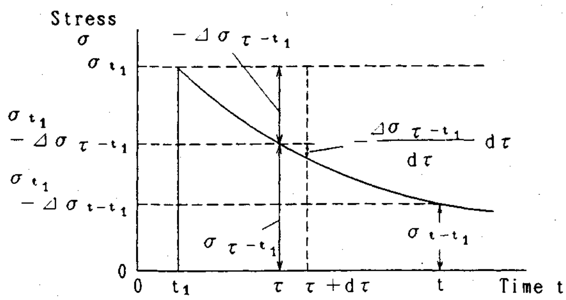

Fig.3 Stress in concrete with gradually decreasing

change of stress $\Delta\sigma_{t-t_2}$. Incidentally, for the coefficient of creep of the third term at the right-hand side of Eq.(1), a factor $\phi_v(t_2-t_1)/\phi_{v\infty}$ is multiplied. This is based on the assumption that the progress of creep recovery is analogous one to the delayed elastic creep.

(3) Relationship between stress and strain in concrete when stress gradually decreases

Generally, various sustained stresses, illustrated in **Fig.2** (b), apply to the composite girders. For analyzing such stresses, it is assumed that the principle of superposition can be used on the analogy to elastic problems, because Davis-Glanville's law is valid for the evaluation of coefficient of creep. Therefore, the following analytical method is only indicated for the case where a single sustained stress is applied.

Now, we consider a stress condition where stress σ_{t_1} applied at a time t_1 and hereafter the stress decreases gradually as shown in **Fig.3**. For a time t, the stress σ_{t-t_1} is expressed as follows :

$$\sigma_{t-t_1}=\sigma_{t_1}-\Delta\sigma_{t-t_1} \quad \cdots\cdots(2)$$

where $\Delta\sigma_{t-t_1}$ is the change of stress from time t_1 to t and the corresponding compressive stress σ_{t_1} is

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assumed as positive.

Then, the change of strain in concrete at a time t can be written as the following formula by referring **Fig.3** and using Eq.(1).

$$\Delta\varepsilon_{t-t_1}=\frac{\sigma_{t-t_1}}{E_b}[\phi_v(t-t_1)+\{\phi_f(t)-\phi_f(t_1)\}]+\frac{\Delta\sigma_{t-t_1}}{E_b}-\frac{1}{E_b}\int_{t_1}^{t}\frac{d\Delta\sigma_{\tau-t_1}}{d\tau}\{\phi_v(\tau-t_1)-\frac{\phi_v(\tau-t_1)}{\phi_{v\infty}}\phi_v(t-\tau)\}d\tau-\frac{1}{E_b}\int_{t_1}^{t}\frac{d\Delta\sigma_{\tau-t_1}}{d\tau}\{\phi_f(\tau)-\phi_f(t_1)\}d\tau \quad \cdots\cdots(3)$$

After integrating by parts the third and fourth terms of the right-hand side of the above equation, the basic equation for the relationship between stress and strain in concrete, which is subjected to gradually decreasing stress, can be obtained as follows :

$$\Delta\varepsilon_{t-t_1}=\frac{\sigma_{t_1}}{E_b}[\phi_v(t-t_1)+\{\phi_f(t)-\phi_f(t_1)\}]+\frac{\Delta\sigma_{t-t_1}}{E_b}+\frac{1}{E_b}\int_{t_1}^{t}\Delta\sigma_{\tau-t_1}\frac{d\phi_v(\tau-t_1)}{d\tau}d\tau+\frac{1}{E_b}\int_{t_1}^{t}\Delta\sigma_{\tau-t_1}\frac{d\phi_f(\tau)}{d\tau}d\tau-\frac{1}{E_b\phi_{v\infty}}\int_{t_1}^{t}\Delta\sigma_{\tau-t_1}\cdot\frac{d\{\phi_v(\tau-t_1)\phi_v(t-\tau)\}}{d\tau}d\tau \quad \cdots\cdots(4)$$

The fifth term in Eq.(4) means the effect of the creep recovery. Therefore, in the case where the creep recovery is ignored, the fifth term in Eq.(4) can be neglected. Furthermore, in the case where the delayed elastic creep is also ignored as is proposed by Dischinger's method, all the additional terms ϕ_v in Eq.(4) can be omitted.

3. SOLUTIONS OF CREEP FOR TYPICAL LOADING CONDITIONS

(1) Analytical assumptions

The analysis of creep is conducted on the basis of the following assumptions :

i) Cross-sectional plane of the girder still remains plane under various loading conditions.

ii) The girder behaves as a beam with full composite interaction between concrete slab and steel girder.

iii) Effective width of concrete slab is constant.

iv) Young's modulus of concrete is constant and used at age of 28 days.

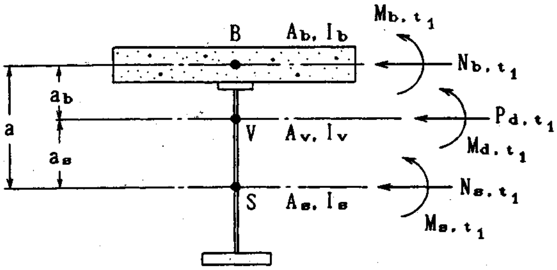

Fig.4 Stress resultants due to sustained bending moment M_{d,t_1} and normal force P_{d,t_1} at a time t_1

(2) Stress resultants in composite girder at a time t_1

Through the sustained applications of bending moment M_{d,t_1} and normal force P_{d,t_1} to the neutral axis of composite girder at a time t_1 shown in **Fig.4**, the following stress resultants of concrete slab and steel girder, of which positive directions are illustrated by the arrows, can be obtained from the equilibrium conditions of these forces and the compatibility condition of the deformation :

$$\left.\begin{aligned}N_{b,t_1}&=\frac{A_b}{nA_v}P_{d,t_1}+\frac{A_s a_s}{I_v}M_{d,t_1}\\M_{b,t_1}&=\frac{I_b}{nI_v}M_{d,t_1}\\N_{s,t_1}&=\frac{A_s}{A_v}P_{d,t_1}-\frac{A_s a_s}{I_v}M_{d,t_1}\\M_{s,t_1}&=\frac{I_s}{I_v}M_{d,t_1}\end{aligned}\right\}\cdots\cdots(5)_{1\sim4}$$

where

N_{b,t_1} : normal force in concrete slab at a time t_1

M_{b,t_1} : bending moment in concrete slab at a time t_1

N_{s,t_1} : normal force in steel girder at a time t_1

M_{s,t_1} : bending moment in steel girder at a time t_1

E_b : Young's modulus of concrete

E_s : Young's modulus of steel

$$n=E_s/E_b \quad \cdots\cdots(6)$$

: Young's modulus ratio between steel and concrete

A_b : cross-sectional area of concrete slab

A_s : cross-sectional area of steel girder

A_v : cross-sectional area of composite girder transformed into steel

I_b : geometric moment of inertia with respect to neutral axis of concrete slab

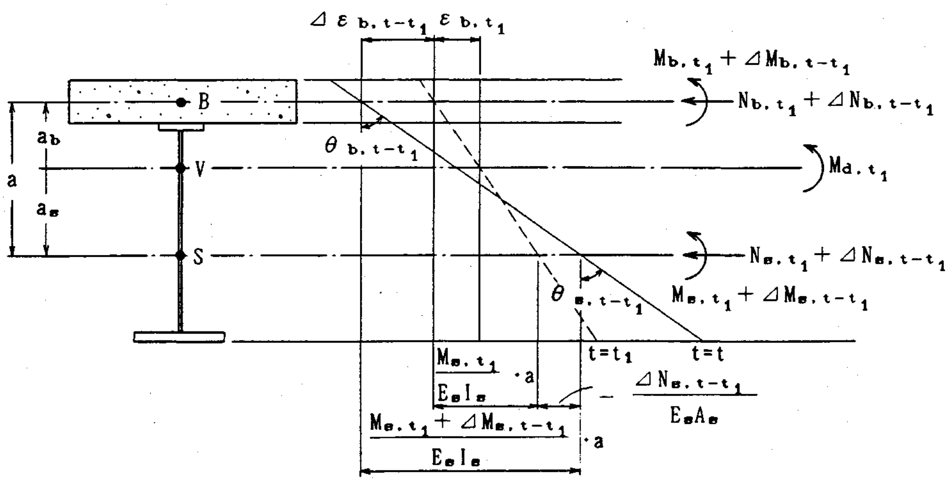

Fig.5 Distributions of strain in composite girder at time $t=t_1$ and $t=t$ due to M_{d,t_1}

I_s : geometric moment of inertia with respect to neutral axis of steel girder

I_v : geometric moment of inertia of composite girder transformed into steel

$a=a_s+a_b$ ……………………………………(7)

: distance between neutral axes of concrete slab and steel girder

a_b : distance between neutral axes of concrete slab and composite girder

a_s : distance between neutral axes of steel and composite girder

Therefore, the cross-sectional area and geometric moment of inertia of composite girder transformed into steel are given by the following equations.

$$\left.\begin{aligned} &A_v=A_b/n+A_s \\ &I_v=(A_b a_b^2+I_b)/n+A_s a_s^2+I_s \end{aligned}\right\} \cdots\cdots\cdots (8)_{1\sim2}$$

(3) Change of stress resultants at a time *t* due to sustained bending moment

a) Equilibrium condition of stress resultants

Fig.5 shows the distributions of strain in the composite girder at the initial loading time t_1 and at an arbitrary time t due to sustained bending moment M_{d,t_1}. From this figure, the equilibrium condition of stress resultants satisfies the following equations at time $t=t_1$:

$$\left.\begin{aligned} &N_{b,t_1}+N_{s,t_1}=0 \\ &M_{b,t_1}+N_{b,t_1}a+M_{s,t_1}=M_{d,t_1} \end{aligned}\right\} \cdots\cdots\cdots\cdots (9)_{1\sim2}$$

Similarly, the equations

$$\left.\begin{aligned} &\Delta N_{b,t-t_1}+\Delta N_{s,t-t_1}=0 \\ &\Delta M_{b,t-t_1}+\Delta N_{b,t-t_1}a+\Delta M_{s,t-t_1}=0 \end{aligned}\right\} \cdots (10)_{1\sim2}$$

fulfills at time $t=t$, where $\Delta N_{b,t-t_1}$ and $\Delta M_{b,t-t_1}$ are the changes of normal forces and bending moments of the concrete slab in a period of time $t-t_1$, respectively. While, $\Delta N_{s,t-t_1}$ and $\Delta M_{s,t-t_1}$ are also the changes of normal force and bending moment of steel girder in a period of time $t-t_1$, respectively.

b) Changes of strain and curvature in a period of time $t-t_1$

The change of normal force at the neutral axis of the concrete slab results in $\Delta N_{b,t-t_1}$. Then, $\Delta\sigma_{t-t_1}$ and σ_{t_1}/E_b in Eq.(4) can be expressed as $\Delta N_{b,t-t_1}/A_b$ and $M_{d,t_1}a_b/(E_sI_v)$, respectively. Therefore, the change of strain at the centroid B of concrete slab can be given by the following equation :

$$\begin{aligned} \Delta\varepsilon_{b,t-t_1}=&\frac{M_{d,t_1}}{E_sI_v}a_b[\phi_v(t-t_1) \\ &+\{\phi_f(t)-\phi_f(t_1)\}]+\frac{\Delta N_{b,t-t_1}}{E_bA_b} \\ &+\frac{1}{E_b}\int_{t_1}^{t}\frac{\Delta N_{b,\tau-t_1}}{A_b}\frac{d\phi_v(\tau-t_1)}{d\tau}d\tau \\ &+\frac{1}{E_b}\int_{t_1}^{t}\frac{\Delta N_{b,\tau-t_1}}{A_b}\frac{d\phi_f(\tau)}{d\tau}d\tau \\ &-\frac{1}{E_b\phi_{v\infty}}\int_{t_1}^{t}\frac{\Delta N_{b,\tau-t_1}}{A_b} \\ &\cdot\frac{d\{\phi_v(\tau-t_1)\phi_v(t-\tau)\}}{d\tau}d\tau \cdots\cdots\cdots (11) \end{aligned}$$

Whereas, the change of strain in the steel girder at the view point B shown in **Fig.5**, can be represented by the following equation :

$$\begin{aligned} \Delta\varepsilon_{s,t-t_1}=&\frac{M_{s,t_1}+\Delta M_{s,t-t_1}}{E_sI_s}a \\ &-\left(\frac{M_{s,t_1}}{E_sI_s}a-\frac{\Delta N_{s,t-t_1}}{E_sA_s}\right) \\ =&\frac{\Delta M_{s,t-t_1}}{E_sI_s}a+\frac{\Delta N_{s,t-t_1}}{E_sA_s} \cdots\cdots\cdots\cdots (12) \end{aligned}$$

In the similar manner, the changes of curvature of concrete slab and steel girder can, respectively, be set by the following equations :

$$\Delta\theta_{b,t-t_1}=\frac{M_{b,t_1}}{E_bI_b}[\phi_v(t-t_1)+\{\phi_f(t)-\phi_f(t_1)\}]+\frac{\Delta M_{b,t-t_1}}{E_bI_b}+\int_{t_1}^{t}\frac{\Delta M_{b,\tau-t_1}}{E_bI_b}\frac{d\phi_v(\tau-t_1)}{d\tau}d\tau+\int_{t_1}^{t}\frac{\Delta M_{b,\tau-t_1}}{E_bI_b}\frac{d\phi_f(\tau)}{d\tau}d\tau-\frac{1}{\phi_{v\infty}}\int_{t_1}^{t}\frac{\Delta M_{b,\tau-t_1}}{E_bI_b}\cdot\frac{d\{\phi_v(\tau-t_1)\phi_v(t-\tau)\}}{d\tau}d\tau \quad\cdots\cdots(13)$$

$$\Delta\theta_{s,t-t_1}=\frac{\Delta M_{s,t-t_1}}{E_sI_s}\quad\cdots\cdots(14)$$

c) Compatibility conditions of strain and curvature

The following equations can be given as the compatibility conditions at a time t :

$$\left.\begin{aligned}\Delta\varepsilon_{b,t-t_1}&=\Delta\varepsilon_{s,t-t_1}\\ \Delta\theta_{b,t-t_1}&=\Delta\theta_{s,t-t_1}\end{aligned}\right\}\quad\cdots\cdots(15)_{1\sim2}$$

d) Solutions for change of stress resultant due to creep

Substituting Eq.(11) through Eq.(14) into Eqs.(15), and solving the simultaneous equations of Eqs.(10) and (15), the changes of stress resultants can be obtained as follows :

$$\left.\begin{aligned}\Delta N_{b,t-t_1}&=-\Delta N_{s,t-t_1}\\&=-M_{d,t_1}\cdot\frac{D_2(1-\eta_M+D_M)-D_1D_V}{(1-\eta_N+D_N+D_1a)(1-\eta_M+D_M)-D_1D_Ma}\cdot\phi(t-t_1)\\ \Delta M_{b,t-t_1}&=-M_{d,t_1}\cdot\frac{D_V(1-\eta_N+D_N+D_1a)-D_2D_Ma}{(1-\eta_N+D_N+D_1a)(1-\eta_M+D_M)-D_1D_Ma}\cdot\phi(t-t_1)\\ \Delta M_{s,t-t_1}&=-(\Delta N_{b,t-t_1}a+\Delta M_{b,t-t_1})\end{aligned}\right\}\quad\cdots\cdots(16)_{1\sim3}$$

where

$$\left.\begin{aligned}&D_1=\frac{E_bA_b}{E_sI_s}a,\quad D_2=\frac{E_bA_b}{E_sI_v}a_b,\quad D_N=\frac{E_bA_b}{E_sA_s}\\&D_M=\frac{E_bI_b}{E_sI_s},\quad D_V=\frac{E_bI_b}{E_sI_v}\end{aligned}\right\}\quad\cdots\cdots(17)_{1\sim5}$$

and

$$\eta_N=\frac{1}{\Delta N_{b,t-t_1}}\left[-\int_{t_1}^{t}\Delta N_{b,\tau-t_1}\frac{d\phi_v(\tau-t_1)}{d\tau}d\tau-\int_{t_1}^{t}\Delta N_{b,\tau-t_1}\frac{d\phi_f(\tau)}{d\tau}d\tau+\frac{1}{\phi_{v\infty}}\int_{t_1}^{t}\Delta N_{b,\tau-t_1}\cdot\frac{d\{\phi_v(\tau-t_1)\phi_v(t-\tau)\}}{d\tau}d\tau\right]\quad\cdots\cdots(18)_1$$

$$\eta_M=\frac{1}{\Delta M_{b,t-t_1}}\left[-\int_{t_1}^{t}\Delta M_{b,\tau-t_1}\frac{d\phi_v(\tau-t_1)}{d\tau}d\tau-\int_{t_1}^{t}\Delta M_{b,\tau-t_1}\frac{d\phi_f(\tau)}{d\tau}d\tau+\frac{1}{\phi_{v\infty}}\int_{t_1}^{t}\Delta M_{b,\tau-t_1}\cdot\frac{d\{\phi_v(\tau-t_1)\phi_v(t-\tau)\}}{d\tau}d\tau\right]\quad\cdots\cdots(18)_2$$

as well as

$$\phi(t-t_1)=[\phi_v(t-t_1)+\{\phi_f(t)-\phi_f(t_1)\}]\quad\cdots\cdots(19)$$

e) Practical solution

In order to obtain a practical solution of η_N and η_M, the following two additional assumptions are introduced. These assumptions have generally been recognized by the test results[14].

i) The progress of coefficients of creep can be expressed approximately by the exponential function.

ii) The change of stress resultants is proportional to the progress of coefficients of creep.

From the above assumptions, the progress of coefficients of creep and the changes of stress resultant can be expressed by the following formulas.

$$\left.\begin{aligned}&\phi_v(t)=\phi_{v\infty}(1-e^{-k_1t})\\&\phi_f(t)=\phi_{f\infty}(1-e^{-k_2t})\\&\phi(t-t_1)=\phi_{v\infty}\{1-e^{-k_1(t-t_1)}\}\\&\qquad+\phi_{f\infty}\{e^{-k_2t_1}-e^{-k_2t}\}\end{aligned}\right\}\quad\cdots\cdots(20)_{1\sim3}$$

and

$$\left.\begin{aligned}\Delta M_{b,\tau-t_1}&=\Delta M_{b,t-t_1}\frac{\phi(\tau-t_1)}{\phi(t-t_1)}\\ \Delta N_{b,\tau-t_1}&=\Delta N_{b,t-t_1}\frac{\phi(\tau-t_1)}{\phi(t-t_1)}\end{aligned}\right\}\quad\cdots\cdots(21)_{1\sim2}$$

where the coefficients k_1 and k_2 represent the characteristic of the progress of creep and have no dimension.

Substitution of Eqs.(20) and (21) into Eqs.$(18)_{1\sim2}$, and their integration gives the following equation.

$$\eta=-\eta_N=-\eta_M=\frac{1}{2}\phi(t-t_1)+\frac{\phi_{v\infty}}{\phi(t-t_1)}\left[\frac{1}{2}\phi_{v\infty}\cdot\{1-e^{-2k_1(t-t_1)}-2k_1(t-t_1)e^{-k_1(t-t_1)}\}\right.$$

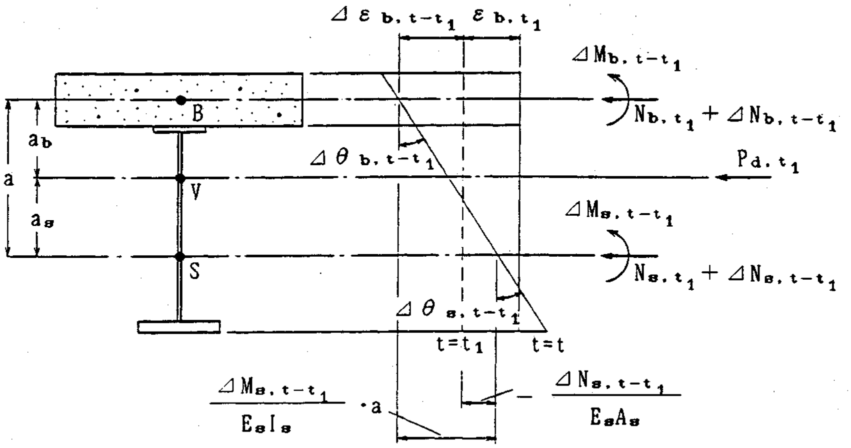

Fig.6 Distributions of strain in composite girder at time $t=t_1$ and $t=t$ due to P_{d,t_1}

$$+\phi_{f\infty}\left\{\frac{k_1}{k_1-k_2}\left(e^{-k_1(t-t_1)-k_2t_1}-e^{-k_2t}\right)\right.$$

$$\left.+\frac{k_1}{k_1+k_2}\left(e^{-k_2t_1}-e^{-k_1(t-t_1)-k_2t}\right)\right\}\Bigg] \cdots\cdots (22)$$

It is possible to define another kind of coefficient η' where the creep recovery is neglected. Then, the fifth term of the right-hand side in Eq.(4) is neglected and through the similar analysis, thus η' can be given by the following equation.

$$\eta'=-\eta_N'=-\eta_M'=\frac{1}{2}\phi(t-t_1) \cdots\cdots\cdots (23)_{1\sim3}$$

This result coincides with the basic equation of Dischinger's method[15], if the change of stress in concrete is assumed as the following equation in a form similar to Eq.(21).

$$\Delta\sigma_{\tau-t_1}=\Delta\sigma_{t-t_1}\frac{\phi(\tau-t_1)}{\phi(t-t_1)} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (24)$$

By integrating Eq.(4) with the adoptions of Eqs.$(20)_{1\sim3}$ and (24), the relationship between stress and strain in concrete proposed by Dischinger's method is given, which leads to :

$$\varepsilon_{t-t_1}=\frac{\sigma_{t_1}}{E_b}\{1+\phi(t-t_1)\}$$

$$+\frac{\Delta\sigma_{t-t_1}}{E_b}\left\{1+\frac{1}{2}\phi(t-t_1)\right\} \cdots\cdots\cdots\cdots (25)$$

It is noted that the delayed elastic creep is, naturally, neglected in Dischinger's method.

(4) Change of stress resultants at a time *t* due to sustained normal force

When a sustained normal force P_{d,t_1} applies to the V axis of composite girder at a time t_1 as shown in **Fig.6**, the changes of stress resultants in a period of time $t-t_1$ are given by performing the similar expansion of the above equations as well as preceding section. Thus :

$$\left.\begin{aligned}
\Delta N_{b,t-t_1}&=-\Delta N_{s,t-t_1}\\
&=-P_{d,t_1}\\
&\cdot\frac{(1+\eta+D_M)D_N'}{(1+\eta)(1+\eta+D_N+D_M+D_1a)+D_ND_M}\\
&\cdot\phi(t-t_1)\\
\Delta M_{b,t-t_1}&=P_{d,t_1}\\
&\cdot\frac{D_MD_N'a}{(1+\eta)(1+\eta+D_N+D_M+D_1a)+D_ND_M}\\
&\cdot\phi(t-t_1)\\
\Delta M_{s,t-t_1}&=-(\Delta N_{b,t-t_1}a+\Delta M_{b,t-t_1})
\end{aligned}\right\} \cdots\cdots (26)_{1\sim3}$$

where $D_N'=E_bA_b/(E_sA_v)$, D_1 and D_N as well as D_M are identical with the rigidity ratios given by Eqs.(17). The coefficient η is also given by Eq.(20).

(5) Change of statically indeterminate forces

The analysis concerning the change of statically indeterminate forces such as a continuous composite girders can be estimated by the following manner. Firstly, the total change of stress resultants are composed of two parts, i.e., statically determinate and statically indeterminate forces. Therefore, the total change of stress resultants at a time t are given by the following equations as already pointed out by references 4) and 5).

$$\left.\begin{aligned}
\Delta M^T_{s,t-t_1}&=\Delta M^0_{s,t-t_1}+\Delta M^1_{s,t-t_1}\\
\Delta N^T_{s,t-t_1}&=\Delta N^0_{s,t-t_1}+\Delta N^1_{s,t-t_1}\\
\Delta M^T_{b,t-t_1}&=\Delta M^0_{b,t-t_1}+\Delta M^1_{b,t-t_1}\\
\Delta N^T_{b,t-t_1}&=\Delta N^0_{b,t-t_1}+\Delta N^1_{b,t-t_1}
\end{aligned}\right\} \cdots\cdots (27)_{1\sim4}$$

where the superscripts are defined as follows :

T : total change of stress resultant
0 : change of stress resultants in statically determinate system

1 : change of stress resultants due to change of statically indeterminate forces

and the change of stress resultants in the statically determinate system is already known (for example, see Eqs.$(16)_{1\sim3}$).

Secondly, we can derive the statically indeterminate forces by focusing the deformation of steel girder alone. The elastic equation for statically indeterminate system with n-th orders can be written in the following form.

$$\overset{n\times n}{[\delta^{ij}]}\overset{n\times 1}{\{\Delta X^i\}}=\overset{n\times 1}{\{Q^i\}} \cdots\cdots (28)$$

where

$$\delta^{ij}=\int\frac{\overline{M}_s^i\overline{M}_s^j}{E_sI_s}dx \cdots\cdots (29)$$

$\overline{M}_s^i$, $\overline{M}_s^j$: bending moments of steel girder in statically determinate system when $X=1$ is applied to the points i and j, respectively

ΔX^i : change of statically indeterminate forces

$$Q^i=-\int\frac{\overline{M}^i\,\Delta M^0_{s,t-t_1}}{E_sI_s}dx \cdots\cdots (30)$$

$\Delta M^0_{s,t-t_1}$: change of bending moment in steel girder corresponds to statically determinate system

Thus, the change of statically indeterminate forces is given by

$$\{\Delta X^i\}=[\delta^{ij}]^{-1}\{Q^i\} \cdots\cdots (31)$$

and the change of bending moment $\Delta M^1_{s,t-t_1}$ in the steel girder is also given by the following equation.

$$\Delta M^1_{s,t-t_1}=\sum_{i=1}^{n}\Delta X^i\overline{M}_s^i \cdots\cdots (32)$$

The change of normal force is derived from the equilibrium condition of forces in composite section, thus :

$$\Delta N^1_{s,t-t_1}=-\Delta N^1_{b,t-t_1} \cdots\cdots (33)$$

The change of stress resultants in concrete slab can also be derived from the compatibility conditions of strain and curvature between concrete slab and steel girder by using Eqs.(15) and (33), namely

$$\left.\begin{aligned}\Delta M^1_{b,t-t_1}&=\frac{D_M}{1+\eta}\Delta M^1_{s,t-t_1}\\ \Delta N^1_{b,t-t_1}&=\frac{D_1}{1+\eta+D_N}\Delta M^1_{s,t-t_1}\end{aligned}\right\}\cdots\cdots (34)_{1\sim2}$$

Furthermore, the changes of applied forces to the whole cross section of composite girder are given from the equilibrium condition of forces as follows :

$$\left.\begin{aligned}\Delta N^1_{v,t-t_1}&=\Delta N^1_{b,t-t_1}+\Delta N^1_{s,t-t_1}=0\\ \Delta M^1_{v,t-t_1}&=\Delta M^1_{s,t-t_1}-\Delta N^1_{s,t-t_1}a+\Delta M^1_{b,t-t_1}\end{aligned}\right\}\cdots\cdots (35)_{1\sim2}$$

where $\Delta N^1_{v,t-t_1}$ and $\Delta M^1_{v,t-t_1}$ are the change of normal force and bending moment, respectively, of the cross section of the composite girder due to ΔX^i.

Finally, the total forces induced to cross section throughout the composite girder at a time t are given by the following equations :

$$\left.\begin{aligned}N_{v,t}&=N_{v,t_1}+\Delta N^0_{v,t-t_1}+\Delta N^1_{v,t-t_1}\\ M_{v,t}&=M_{v,t_1}+\Delta M^0_{v,t-t_1}+\Delta M^1_{v,t-t_1}\end{aligned}\right\}\cdots (36)_{1\sim2}$$

where N_{v,t_1} and M_{v,t_1} are initial normal force and bending moment applied to whole cross section of the composite girder at a time t, respectively, and

$$\left.\begin{aligned}\Delta N^0_{v,t-t_1}&=\Delta N^0_{b,t-t_1}+\Delta N^0_{s,t-t_1}=0\\ \Delta M^0_{v,t-t_1}&=\Delta M^0_{s,t-t_1}-\Delta N^0_{s,t-t_1}a+\Delta M^0_{b,t-t_1}\end{aligned}\right\}\cdots\cdots (37)_{1\sim2}$$

4. DERIVATION AND PHYSICAL MEANING OF COEFFICIENT η IN COMPUTATION.

The coefficient η derived by this analytical method can be related to the relaxation coefficient ρ proposed by Trost[4], so that Eq.(4) is expressed as :

$$\begin{aligned}\Delta\varepsilon_{t-t_1}&=\frac{\sigma_{t_1}}{E_b}[\phi_v(t-t_1)+\{\phi_f(t)-\phi_f(t_1)\}]\\ &\quad+\frac{\Delta\sigma_{t-t_1}}{E_b}(1+\rho[\phi_v(t-t_1)\\ &\quad+\{\phi_f(t)-\phi_f(t_1)\}])\\ &=\frac{\sigma_{t_1}}{E_b}\phi(t-t_1)+\frac{\Delta\sigma_{t-t_1}}{E_b}\{1+\rho\phi(t-t_1)\}\end{aligned}\cdots\cdots (38)$$

According to this method, the integration of Eq.(4) under the assumption of Eqs.(20) and (24) results in the following relationship between stress and strain in the concrete :

$$\Delta\varepsilon_{t-t_1}=\frac{\sigma_{t_1}}{E_b}\phi(t-t_1)+\frac{\Delta\sigma_{t-t_1}}{E_b}(1+\eta) \cdots\cdots (39)$$

By comparing this equation with Eq.(38), it is found that the coefficient η means the coefficient of creep by taking into account of the relaxation of concrete stress. In this method, if Eq.(39) is adopted as the basic equation, the coefficient of relaxation ρ_{NK} is expressed as follows :

$$\eta=\rho_{NK}\phi(t-t_1) \cdots\cdots (40)$$

then

$$\begin{aligned}\rho_{NK}&=\frac{1}{2}+\frac{\phi_{v\infty}}{\{\phi(t-t_1)\}^2}\Big[\frac{1}{2}\phi_{v\infty}\\ &\quad\cdot\{1-e^{-2k_1(t-t_1)}-2k_1(t-t_1)e^{-k_1(t-t_1)}\}\\ &\quad+\phi_{f\infty}\Big\{\frac{k_1}{k_1-k_2}(e^{-k_1(t-t_1)-k_2t_1}-e^{-k_2t})\end{aligned}$$

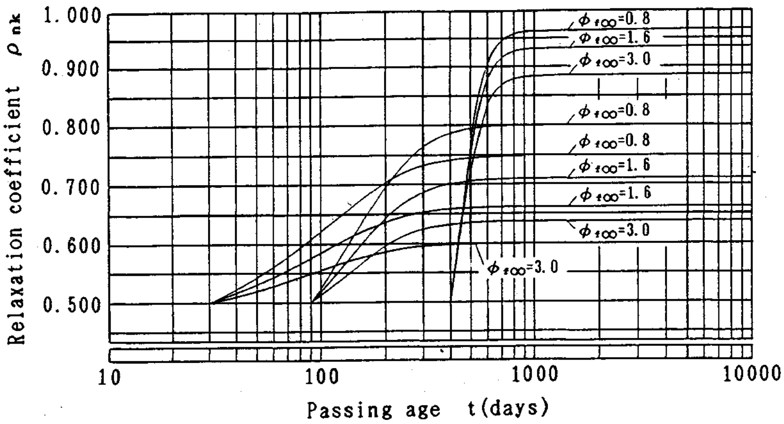

Fig.7 Numerical results of relaxation coefficient ρ_{NK}

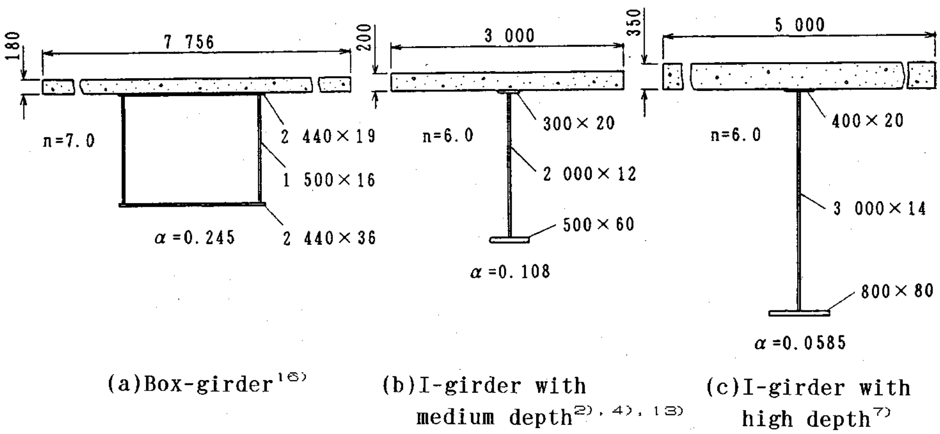

Fig.8 Three different cross sections of composite girders

$$+\frac{k_1}{k_1+k_2}\,(e^{-k_2t_1}-e^{-k_1(t-t_1)-k_2t})\Big\}\Big] \quad \cdots\ (41)$$

Consequently, the relaxation coefficient $\rho_{NK,\infty}$ at the time $t=\infty$ is given by :

$$\rho_{NK,\infty}=\frac{1}{2}+\frac{\phi_{v\infty}}{\phi(\infty)^2}\left(\frac{1}{2}\,\phi_{v\infty}+\phi_{f\infty}\,\frac{k_1}{k_1+k_2}\,e^{-k_2t_1}\right) \quad \cdots\cdots (42)$$

Fig.7 depicts the computed results of ρ_{NK} at an arbitrary time t under the conditions of the following parameters :

$\phi_{v\infty}=0.4$ (constant)
$\phi_{f\infty}=0.8$, 1.6 and 3.0
$k_1=0.02000$ (constant)[11),14),16)]
$k_2=0.00670$ (constant)[11),14),16)]
$t_1=30$, 90 and 400 days

It is found from **Fig.7** that the values of ρ_{NK} become large as the initial loading ages are set later. The values of ρ_{NK} also become large in accordance with decrease of the creep coefficients, if the initial loading ages are set as the same ages. In the case of ordinary design conditions, it follows that : $t_1=30$ days, $\phi_{v\infty}=0.4$ and $\phi_{f\infty}=1.6$, then $\rho_{NK}\cong 0.65$ may be applied to design as the final value.

5. NUMERICAL EXAMPLES

(1) Comparison with the results obtained by Dischinger and Trost

In order to examine the validity of this analytical method of creep, various numerical calculations were executed. As one of example, a typical loading condition was adopted to the composite girder with the section shown in **Fig.8**(b). Applying the sustained bending moment $M_{d,0}=1{,}105$ tf·m (bending moment due to the jack down of the support) to the composite section at time $t_1=0$, the change of stress resultants at time $t=\infty$ was calculated, where the following parameters are used :

$\phi_{v\infty}=0.4$, $\phi_{f\infty}=1.6$
$k_1=0.0200$[11),14),16)], $k_2=0.00670$[11),14),16)]
$E_s=2.1\times10^6$ (kgf/cm²)
$E_b=3.5\times10^5$ (kgf/cm²)
$D_1=0.04300$ (1/cm)

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Table 1 Change of stress resultants for cross section shown in Fig.8 (b)

Change of stress resultants	Consideration of creep recovery					Without consideration of creep recovery			
	① Trost's method	This method: ② Exact solution	This method: ③ Approximate solution	①/②	③/②	④ Dischinger's method	⑤ This method	⑤/④	⑤/②
$\Delta N_{b,\infty}$ tf	-94.2	-96.3	-97.1	0.978	1.008	-98.6	-98.7	1.001	1.025
$\Delta M_{b,\infty}$ tf·m	-1.67	-1.99	-1.99	0.839	1.000	-1.94	-2.24	1.155	1.126
$\Delta M_{s,\infty}$ tf·m	146.2	149.7	151.0	0.977	1.009	153.2	153.6	1.003	1.026

1 tf = 9.8 kN , 1 tf·m = 9.8 kN·m

Table 2 Parameters

Cross-sectional parameter of composite girder α	Initial loading age t_1 (days)	Coefficients of creep: Flow creep $\phi_{f\infty}$	Coefficients of creep: Delayed elastic creep $\phi_{v\infty}$
0.0585	28	0.8	0.4
0.108	90	1.6	
0.245	365	3.0	

$D_2 = 0.00463$ (1/cm)
$D_N = 1.66667$, $D_M = 0.00934$
$D_V = 0.00268$, $\eta = 1.27133$
$\eta' = 1.00000$

These results are shown in **Table 1** together with the results calculated by Dischinger's and Trost's method. In this Table, the additional approximate solutions were calculated by using Eqs.(44). Namely, the numerical values of D_1, D_2, D_N, D_M and D_V result in :

$$\left.\begin{array}{l} D_2(1+\eta+D_N) \gg D_1 D_V \\ (1+\eta+D_N+D_1 a)(1+\eta+D_M) \gg D_1 D_M a \end{array}\right\} \quad \cdots\cdots (43)_{1-2}$$

Therefore, according to these equations, the following equations can be put as the approximate solutions in this method.

$$\left.\begin{array}{l} \Delta N^a_{b,t-t_1} = -M_{d,t_1} \cdot \dfrac{D_2}{(1+\eta+D_N+D_1 a)} \cdot \phi(t-t_1) \\ \Delta M^a_{b,t-t_1} = -M_{d,t_1} \\ \quad \cdot \dfrac{D_V(1+\eta+D_N+D_1 a) - D_2 D_M a}{(1+\eta+D_N+D_1 a)(1+\eta+D_M)} \\ \quad \cdot \phi(t-t_1) \\ \Delta M^a_{s,t-t_1} = -(\Delta M^a_{b,t-t_1} + \Delta N^a_{b,t-t_1} a) \end{array}\right\} \quad \cdots\cdots (44)_{1-3}$$

It is recognized the following facts from **Table 1** :

i) The results of this method fall within the values between Dischinger's and Trost's method with the exception of $\Delta M_{b,\infty}$. Also the differences between this method and Trost's method are caused by the function for coefficient of creep and assumption for the change of stress resultants.

ii) The changes of stress resultant by taking into account of the creep recovery are smaller than the results where creep recovery is neglected.

iii) The approximate solutions give the safety and accurate results in comparison with the exact solutions.

Table 3 Effects of parameter α ($\times M_{d,0}$)

Stress resultant	Parameter	① Consideration of creep recovery	② Without consideration of creep recovery	②/①
Normal force of concrete slab	α	$\Delta N_{b,\infty}$ ($\times 10^{-1}$)	$\Delta N'_{b,\infty}$ ($\times 10^{-1}$)	
	0.245	-2.10812	-2.19922	1.043
	0.108	-0.87133	-0.89300	1.025
	0.0585	-0.33447	-0.33835	1.012
Bending moment of concrete slab	α	$\Delta M_{b,\infty}$ ($\times 10^{-3}$)	$\Delta M'_{b,\infty}$ ($\times 10^{-3}$)	
	0.245	-1.91536	-2.13970	1.117
	0.108	-1.80275	-2.03016	1.126
	0.0585	-3.17747	-3.59342	1.131
Bending moment of steel girder	α	$\Delta M_{s,\infty}$	$\Delta M'_{s,\infty}$	
	0.245	0.236970	0.247350	1.044
	0.108	0.135457	0.139023	1.026
	0.0585	0.086913	0.088307	1.016

(2) Parametric study

To evaluate the effects of creep recovery, three parameters, i.e., cross-sectional parameter of composite girder α, coefficient of flow creep $\phi_{f\infty}$ and initial loading age t_1 were selected as listed in **Table 2**, where α is defined[2] as follows :

$$\alpha = \frac{A_s I_s}{A_v I_v} \cdots\cdots (45)$$

These values listed in **Table 2** are corresponded to the cross sections illustrated in **Fig.8**.

The changes of stress resultants, in which the composite girders are subjected to the sustained

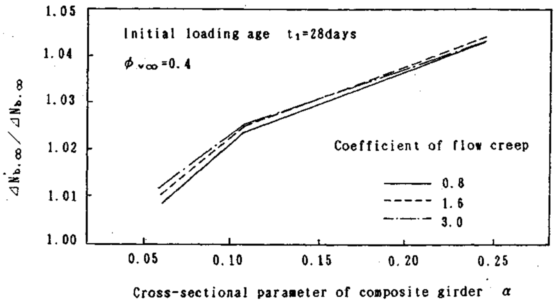

Fig.9 Effects of parameter $\phi_{f\infty}$

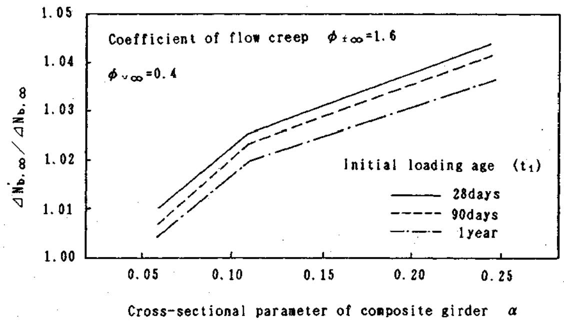

Fig.10 Effects of parameter t_1

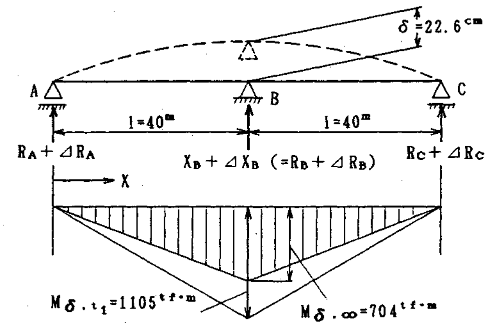

Fig.11 Two spans comtinuous composite girder

Table 4 Calculated results in two spans continuous composite girder

Change of stress resultants	① Consideration of creep recovery	② Without consideration of creep recovery	②/①
$\Delta N^T_{b,\infty}$ tf	-259.72	-278.83	1.074
$\Delta M^T_{b,\infty}$ tf·m	-2.60	-2.96	1.138
$\Delta M^T_{s,\infty}$ tf·m	0	0	

bending moment $M_{d,t_1}=1.0$ tf·m at time $t_1=28$ days, were calculated at time $t=\infty$. The results are summarized in **Table 3**. It is found from this Table that the effect of creep recovery becomes large as the value of α is larger with the exception of the bending moment in concrete slab. This may be caused by the fact that the stress resultants are decided according to the rigidity ratio between concrete slab and steel girder.

Besides, the effects of the parameters $\phi_{f\infty}$ and t_1 are plotted in **Figs.9** and **10**, respectively. **Fig.9** shows that the effect of the coefficient of flow creep is negligibly small. The absolute values of the change of stress resultants, naturally, become large as the $\phi_{f\infty}$ is larger. It reveals from **Fig.10** that the effect of initial loading ages is greater than the coefficients of flow creep. In this case, the effect of the creep recovery becomes small as the initial loading ages take later. Naturally, the absolute values of the change of stress resultants become also small in accordance with the late of the initial loading ages.

(3) Two spans continuous composite girder[2),4)]

Now, we consider a two spans continuous composite girder with constant cross section through span length as shown in **Fig.11**. The amount of jack down of the intermediate support δ is taken as 22.6 cm. The cross-sectional shape and dimensions of composite girder are same as shown in **Fig.8** (b).

Induced bending moment $M_{d,t_1}=1{,}105$ tf·m at the intermediate support of the girder is introduced by the jack down of the support. From the preceding calculated example, the change of bending moment at an arbitrary point of steel girder in the statically determinate system is $3.742x$ (unit : tf·m). This moment means $\Delta M^0_{S,\infty}$. From Eq.(31), the change of reaction in the intermediate support at time $t=\infty$ is given by :

$$\Delta X_{B,\infty}=7.4840 \text{ tf}$$

Thus, the change of bending moment in the steel girder is given by Eq.(32) as follows :

$$\Delta M^1_{S,\infty}=-3.742x$$

Therefore, it follows that :

$$\Delta M^T_{S,\infty}=\Delta M^0_{S,\infty}+\Delta M^1_{S,\infty}=0$$

Another changes of the stress resultants at a time t are given by Eqs.(33) and (34). Then, the change of the stress resultants, where the creep recovery is neglected, can be calculated. These results are listed in **Table 4**.

It reveals from comparisons with **Table 4** and **1** that the effect of creep recovery in the statically indeterminate system is greater than the statically determinate system.

6. CONCLUSION

In order to clarify the effects of the creep recovery on long-term behaviors of steel-concrete composite girders, the exact solutions of creep taking into account of delayed elastic and flow creep as well as creep recovery were derived under the typical loading conditions and various numerical examples were executed and discussed in this paper. The conclusions are summarized as follows :

(1) A new creep coefficient η considering all the creep phenomena and relaxation of concrete stress is the main parameter.

(2) The changes of stress resultant or stresses by taking into account of the creep recovery are smaller than the results where the creep recovery is neglected. In the case of statically indeterminate system, this tendency is stronger than the statically determinate system. Therefore, the conventional design method, in which the effect of creep recovery is neglected, more or less over estimates the creep stress from 3 to 14 % than the design by taking into account of creep recovery.

(3) The effects of the coefficient of flow creep and the initial loading age of concrete on the calculated results with or without consideration of the creep recovery are small enough to be ignored. However, the effect of the cross-sectional parameter of composite girder is remarkably recognized. Naturally, the absolute values of change of stress resultants become large as the flow creep coefficients are larger. Also the absolute values of change of stress resultants become smaller as the initial loading ages are set as later.

(4) The approximate solutions (Eq.$(44)_{1\sim3}$), which are also proposed in this method, give the safety and accurate results in comparison with the exact solutions.

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回復クリープを考慮した鋼・コンクリート合成桁の経時挙動の一解析

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コンクリートの回復クリープ現象は比較的最近の知見であるため，鋼・コンクリート合成桁のクリープ解析結果に及ぼす回復クリープの影響は，未だ明らかにされていない．そこで，著者らは，厳密に回復クリープを考慮したクリープ解を導き，かつ数値計算を実施して，その影響を種々の条件下で評価した．その結果，使用限界状態における設計法において回復クリープを考慮した場合は，無視した場合よりも設計上有利な結果が得られ，今後，種々なクリープ解析に利用できることを明らかにした．