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OPTIMAL CONTROL OF FLOOD USING FINITE ELEMENT METHOD

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This paper presents a method for optimal control of flood propagating through a reservior of river channel using the combination of the finite element method and optimal control theory. The numerical model of the flood propagation can be expressed by the linear two dimensional shallow water equation and the equation can be solved by the two step explicit schemes. For the control theory, the method presented by Sakawa and Shindo is effectively used. It is shown that the water elevation can be controlled as flat as possible by adjusting the discharge of the dam gate. This method is also adaptable to the control constraining the control outflow within the limit of inflow into a reservoir.

Keywords : *finite element method, optimal control, flood control, shallow water equation, Sakawa-Shindo method*

1. INTRODUCTION

Recently large scale dams are constructed along a river to protect human properties, which have often been damaged by a flood. There arises a new problem, i.e., how to control the dam gate. For instance, consider a flood flows into a reservior built up with a dam. If the water gate equiped to the dam is suddenly closed the reflective wave will be generated and propagated toward the upstream area of the reservoir. In case that capacity of the reservoir is not sufficient, the wave will cause an unexpected damage to the upstream area. The unexpected wave propagation will also happen in the downstream area. Recently, there happend some accidents in which the human properties of the upstream area around the reservior of dam was severely damaged by the reflective wave generated by the unsutable operation of the dam gate. Therefore, it is necessary to introduce the control of flood, in which the hydrodynamic behavior of flow through the whole field including dam reservoir, upstream and downstream rivers is considered. In the conventional flood analysis, only the analysis[1)~8)]itself or the control analysis without hydrodynamic model[9)~11)]was carried out. The flood control analysis with hydrodynamic model was obtained by Muskatirovic and Kapor[12)] and Kawahara and Kawasaki[13)]. But, those are limited in one dimensional model. To cope with the behavior of the wave propagation through the reservior, the planar propagation is essential to be clarified. Thus, the two dimensional analysis should be introduced. For this purpose, the finite element method seems to be one of the most powerful method.

Assume that the flood behavior is known in advance during the whole duration time and flood behavior can be expressed by the linear two dimensional shallow water equation. The control problem can be defined to minimize the water elevation and the control water discharge for the whole flow field, in which the control function is the water discharge through the dam. For the minimization technique, the conjugate gradient method and the Sakawa-Shindo method[15)] are used and compared about their efficieney. The optimal control system can be established introducing the discharge of the dam gate as the control function. To solve the control problem in this paper, the control discharge is included in the performance function not only because the cost of the control should be proportional to the control discharge but also because several constraints should be imposed on the control discharge. The quadratic functional of the water elevation and the control discharge is chosen as the performance function. The hydrograph of the flood is assumed to be given at the upstream of the reservior as a time function over the interval to be analyzed. The control problem is the fixed terminal time quadratic control problem. To solve the time dependent equations, both forward and backward integrations should be introduced. To do this, a two step explicit scheme has been used effectively, which was presented in the authors' previous papers[4),7)].

Several numerical studies are carried out to

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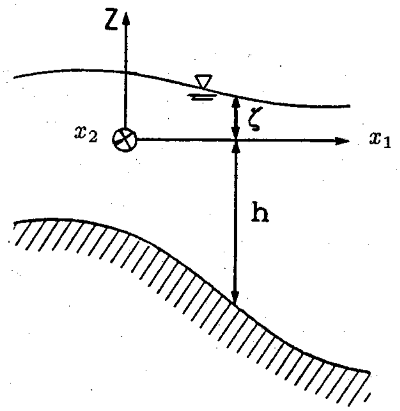

Fig.1

express the adaptability of the method presented in this paper. This paper presents a numerical procedure which can deal with the control problem in which the hydrodynamic model is included in the state equation. This paper also shows a possibility that the water elevation of the whole flow field can be controlled as flat as possible by adjusting the discharge at the gate equiped to the dam, knowing the hydrograph of the flood in advance. Moreover, it is detected out that the control can be performed constraning the outflow control discharge within the limit of inflow to the reservoir. But, in this case, it can be shown that the efficiency of the control is not as much reasonable as that of the control without constraint.

2. BASIC EQUATIONS

The wave propagation through the surface of a reservoir and/or river can be expressed by the linear two dimensional shallow water equation. Let the cartesian coordinate system $x_i (i=1,2)$ be introduced as shown in **Fig.1**. The time is represented by t. Denoting mean discharge and water elevation as q_i and ζ, the equations of motion and continuity can be written in the following forms.

$$\dot{q}_i + gh\zeta_{,i} \quad \cdots\cdots (1)$$

$$\dot{\zeta} + q_{i,i} = 0 \quad \cdots\cdots (2)$$

where g, h are gravity acceleration and water depth respectively and superscripted dot and subscripted comma mean partial differentiations with respect to time and coordinate respectively and the usual summation convention with repeated indices is employed. In this paper, linear equation system is used for the convenience of computational time and storage.

The wave is given at the upstream boundary S_u as the boundary conditon for mean discharge :

$$q_i = \hat{q}_i \quad \text{on } S_u \quad \cdots\cdots (3)$$

where superscripted caret represents a function given on the boundary. The wave control is assumed to be carried out by the outflow decided by the strategy following the optimal control theory for the operation of the water gate equiped to the dam. This can be expressed as :

$$q_i = \bar{q}_i \quad \text{on } S_c \quad \cdots\cdots (4)$$

where S_c is the boundary in which the water gate is equiped and superscripted bar denotes a function determined by the optimal control analysis. This bundary is referred to as the control boundary.

The initial conditions are given as :

$$\zeta = \hat{\zeta}^0 \quad \text{at } t = t_0 \quad \cdots\cdots (5)$$

$$q_i = \hat{q}_i^0 \quad \text{at } t = t_0 \quad \cdots\cdots (6)$$

where $\hat{\zeta}^0$ and $\hat{q}_i^0$ are water elevation and mean discharge at the initial stage defined on the whole domain.

3. OPTIMAL CONTROL THEORY

The optimal control theory employed in this paper is the quadratic control theory. Before application of the theory, the basic equations should be converted to the discritized forms. Based on the Galerkin formulation, equations (1) and (2) can be transformed into the weighted residual equations. Using the linear interpolation function based on the three node triangular element, the semi-descritized form of the finite element equation can be derived. The usual finite element procedure leads to the following equation.

$$[M]\{\dot{x}\} + [H]\{x\} + [A]\{f\} + [B]\{u\} = \{0\} \quad \cdots\cdots (7)$$

where

$$\{x\} = \begin{Bmatrix} q_i \\ \zeta \end{Bmatrix} \quad \cdots\cdots (8)$$

in which q_i and ζ mean dischrge and water elevation at all nodal points of the flow domain to be analyzed. The boundary condition (3) is transformed to the term $[A]\{f\}$ where

$$\{f\} = \begin{Bmatrix} \hat{q}_i \\ 0 \end{Bmatrix} \quad \cdots\cdots (9)$$

in which $\hat{q}_i$ denotes the discharge of the input wave at all nodal points on the upstream boundary S_u. The control term $[B]\{u\}$ is derived from equation (4) and

$$\{u\} = \begin{Bmatrix} \bar{q}_i \\ 0 \end{Bmatrix} \quad \cdots\cdots (10)$$

where $\bar{q}_i$ represents control outflow decided at the nodal points on the boundary S_c. The initial condition is expressed as

$$\{x_0\}=\begin{Bmatrix}\widehat{q}_i^0\\ \widehat{\zeta}^0\end{Bmatrix}\cdots\cdots(11)$$

where $\widehat{q}_i^0$ and $\widehat{\zeta}^0$ denote mean discharge and water elevation at all nodal points at the initial stage.

The wave control problem is formulated as to determine an optimal function $\{u\}$ that minimizes the performance function :

$$J=\frac{1}{2}\int_{t_0}^{t_f}(\{\zeta\}^T[S]\{\zeta\}+\{u\}^T[R]\{u\})dt\cdots(12)$$

under the state equation

$$\{\dot{x}\}=[C]\{x\}+[D]\{u\}+[F]\cdots\cdots(13)$$

with the initial condition $\{x_0\}$ where $[S]$, $[R]$ are weighting matrices and t_0, t_f are initial and final times of the time domain and

$$[C]=-[M]^{-1}[H]\cdots\cdots(14)$$

$$[D]=-[M]^{-1}[B]\cdots\cdots(15)$$

$$[F]=-[M]^{-1}[A]\{f\}\cdots\cdots(16)$$

The optimal function of equation (12) denotes the gross weight which means both of water elevation in control domain and outflow discharge as optimal function$\{u\}$. The final time t_f is fixed and assumed to correspond to the duration time of the whole flood. Namely, the control is continued until the flood is disappeared. The final state $\{x(t_f)\}$ is chosen as free, which means final water elevation should be coincide with the still water level.

For the optimization technique to seek the minimum value of J in equation (12) with the constraints of equation (13), both the conjugate gradient method and the Sakawa-Shindo method have been employed and compared each other about their efficiency of the computation. The conjugate gradient method searches for the minimum value of J along the conjugate direction of the gradient of J. To obtain the absolute value of the gradient, the line search technique should be introduced because the problem is itself nonlinear. It is well-known that the minimizing function u for J is coincident with the function that minimizes the Hamiltonian H defined below. To secure the stability, the Sakawa-Shindo method employes the modified Hamiltonian using the known constants. The efficiency of the method is wholly dependent on these constants.

4. THE CONJUGATE GRADIENT METHOD

To apply the optimal control theory, it is necessary to introduce the Hamiltonian function as :

$$H=\frac{1}{2}\{\zeta\}^T[S]\{\zeta\}+\frac{1}{2}\{u\}^T[R]\{u\}$$
$$+\{p\}^T([C]\{x\}+(D)\{u\}+[F])\cdots\cdots(17)$$

where $\{p\}$ denotes the Lagrange multiplier. The Euler-Lagrange equation and the transversality condition can be described as follows:

$$\{\dot{p}\}=-\frac{\partial H}{\partial\{x\}}=-[S]\{\zeta\}-[A]^T\{p\}\cdots\cdots(18)$$

$$\{p(t_f)\}=\{0\}\cdots\cdots(19)$$

The algorithm of the conjugate gradient method investigates the minimum of J among the direction of the conjugate direction. The gradient of J in this problem is

$$J_u=-\frac{\partial H}{\partial u}=-[R]\{u\}-[D]^T\{p\}\cdots\cdots(20)$$

The optimal control function u can be determined as the final value of the sequence :

$$\{u^{(i+1)}\}=\{u^{(i)}\}+\alpha^{(i)}\{s^{(i)}\}\quad i=1,2,\cdots,N\cdots(21)$$

where $\alpha^{(i)}$ is to minimize

$$J(u^{(i)}+\alpha^{(i)}s^{(i)})\cdots\cdots(22)$$

and

$$\{s^{(i)}\}=-\{J_u^{(i)}\}+\beta\{s^{(i-1)}\}\cdots\cdots(23)$$

in which β is the determined as if $\{s^{(i)}\}$ is conjugate with the gradient $\{J_u^{(i)}\}$.

To obtain the amplitude $\alpha^{(i)}$, the line search algorithm is introdued. The precise algorithm will be described in section 7.

5. THE SAKAWA-SHINDO METHOD

To secure the stability of the computation, the Hamiltonian function is modified in the following form.

$$K^{(i)}=H^{(i)}+(\{u^{(i)}\}-\{u^{(i-1)}\})^T[W^{(i)}](\{u^{(i)}\}$$
$$-\{u^{(i-1)}\})\cdots\cdots(24)$$

where superscripted (i) means i th iteration cycle and $[W^{(i)}]$ is a constant weighting matrix for the i th iteration ;

$$[W^{(i)}]=\begin{bmatrix}w_{(1)}&&&\\&w_{(2)}&&\\&&\ddots&\\&&&w_{(n)}\end{bmatrix}\cdots\cdots(25)$$

in which $w_{(1)}, w_{(2)},\cdots, w_{(n)}\geq 0$ are given constants. Because $K^{(i)}$ is not restrained with respect to $\{u^{(i)}\}$, the optimality condition can be written :

$$\frac{\partial K^{(i)}}{\partial\{u^{(i)}\}}=[R]\{u^{(i)}\}+[D]^T\{p^{(i-1)}\}$$
$$+2[W^{(i)}](\{u^{(i)}\}-\{u^{(i-1)}\})\equiv 0\cdots(26)$$

which leads to the optimam control as:

$$\{u^{(i)}\}=-([R]+2[W^{(i)}])^{-1}([D]^T\{p^{(i-1)}\}$$
$$-2[W^{(i)}]\{u^{(i-1)}\})\cdots\cdots(27)$$

The optimal control of the Sakawa-Shindo method can be summariged as follows. Assuming the appropriate stability constants $w_{(1)},w_{(2)},\cdots,w_{(n)}$, the minimum value of J in equation (12) can be obtained by equation (27) based on $\{p^{(i)}\}$ in equation (18) with (19) and $\{x^{(i)}\}$ in equation (13) with (11). Thus, the minimum value of J can be found by equation (12) solving equation system with respect to $\{p^{(i)}\}$ and $\{x^{(i)}\}$ respectively. The precise algorithm will be described in section 7.

6. NUMERICAL INTEGRATION IN TIME

To obtain the optimal control solution, time dependent differential equation (13) with (11) must be solved in the forward direction, from t_0 to t_f and equation (18) with (19) in the backward direction, from t_f to t_0, because the initial condition is given at the final time t_f. To solve these equations, the time marching numerical integration scheme is used. The total time interval to be analyzed is divided into a plenty of short time intervals, one of which is denoted by Δt. Representing time point by n, the forward two step explicit method can be applied to equation (13) : for the first step :

$$\{x^{n+\frac{1}{2}}\}=[\bar{M}]^{-1}[\tilde{M}]\{x^n\}-\frac{\Delta t}{2}[\bar{M}]^{-1}[H]\{x^n\} \qquad (28)$$

and for the second step :

$$\{x^{n+1}\}=[\bar{M}]^{-1}[\tilde{M}]\{x^n\}-\Delta t[\bar{M}]^{-1}[H]\{x^{n+\frac{1}{2}}\} \qquad (29)$$

starting from the initial condition equation (11). The backward two step explicit method is used for equation (18) :
for the first step :

$$\{p^{n+\frac{1}{2}}\}=([\bar{M}]^{-1}[\tilde{M}])^T\{p^n\}+\frac{\Delta t}{2}(([\bar{M}]^{-1}[H])^T\{p^n\}+[S]\{x^n\}) \qquad (30)$$

and for the second step :

$$\{p^{n+1}\}=([\bar{M}]^{-1}[\tilde{M}])^T\{p^n\}+\Delta t(([\bar{M}]^{-1}[H])^T\{p^{n+\frac{1}{2}}\}+[S]\{x^{n+\frac{1}{2}}\}) \qquad (31)$$

starting from the initial condition equation (19). In equations (28) ~ (31), the lumped coefficient matrix $[\bar{M}]$ is introduced to obtain the full explicit scheme. To secure the stability, the mixed coefficient matrix $[\tilde{M}]$ is used as follows

$$[\tilde{M}]=e[\bar{M}]+(1-e)[M] \qquad (32)$$

where e is referred to as the lumping parameter.

To compute equations (30) and (31), it is necessary to use the value of $\{x^n\}$, which is computed in the computation process in equations (28) and (29). Therefore, the values of $\{x^n\}$ should be stored inside the computer for the retrieval of the later computation. But a tremendous number of core storages are required to store all the values of $\{x^n\}$. However, the behaviors of these values do not show rapid changes. Thus, the values at every T time point pitch are stored for the use of the later computation,

$$T=M\Delta t \qquad (33)$$

where $M=10\sim100$ was used in the practical computation.

7. COMPUTATIONAL ALGORITHM

The computational algorithm employed in this paper is summarized in this section. To express the procedure of equations (28) and (29) with (11), the following abbreviated form is introduced.

$$\{x^{(i)}\}=\{x(u^{(i)})\} \qquad (34)$$

where superscripted (i) means the function is evaluated in the i th iteration cycle and $u^{(i)}$ denotes the optimal control function assumed at the i th iteration. Thus, equation (34) represents to solve equation (13) with (11) by the procedure of equations (28) and (29) assuming the control function as $u^{(i)}$. Similarly, the abbreviated form :

$$\{p^{(i)}\}=\{p(u^{(i)},x^{(i)})\} \qquad (35)$$

means to express the procedure to solve equation (18) with (19) by equations (30) and (31) assuming the control function as $u^{(i)}$.

(1) THE CONJUGATE GRADIENT METHOD

The computational algorithm of the conjugate gradient method[16] can be described as follows.

1. Assume initial control function $u^{(0)}(t), t\in[t_0,t_f]$
2. Solve $\{x^{(0)}\}=\{x(u^{(0)})\}$
3. Solve $\{p^{(0)}\}=\{p(u^{(0)},x^{(0)})\}$
4. Compute $\{s^{(0)}\}=-\{J_u^{(0)}\}=-([R]\{u^{(0)}\}-[D]^T\{p^{(0)}\})$
5. Determine amplitude $\alpha^{(i)}$ by minimizing $J(u^{(i)}+\alpha^{(i)}s^{(i)})$.
6. Compute $\{u^{(i+1)}\}=\{u^{(i)}\}+\alpha^{(i)}\{s^{(i)}\}$
7. Solve $\{x^{(i+1)}\}=\{x(u^{(i+1)})\}$
8. Solve $\{p^{(i+1)}\}=\{p(u^{(i+1)},x^{(i+1)})\}$
9. Compute $\{J_u^{(i+1)}\}=-([R]\{u^{(i+1)}\}-[D]^T\{p^{(i+1)}\})$
10. IF$\{J_u^{(i+1)}\}<\varepsilon$ THEN STOP ELSE $i=i+1$
11. Compute $\beta^{(i)}=\dfrac{\{J_u^{(i)}\}^T\{J_u^{(i)}\}}{\{J_u^{(i-1)}\}^T\{J_u^{(i-1)}\}}$
12. Compute $\{s^{(i)}\}=-\{J_u^{(i)}\}+\beta^{(i)}\{s^{(i-1)}\}$and GOTO 5

The parameter ε is a small number which expresses the convergence allowance. The flow chart of the computation is shown in **Fig.2**.

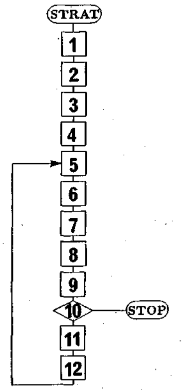

Fig.2 Flow chart

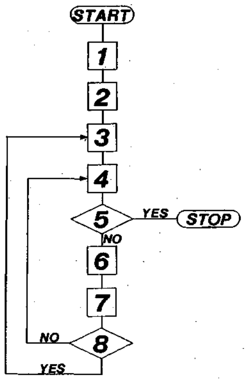

Fig.3 Flow chart

The amplitude α can be determined by minimizing $J(u+\alpha s)$ where the present position u and the search direction s are both given. Determine the amplitude α that minimizes $J(u)$ by means of $u+\alpha s$ on a quadratic line. Put $g(\alpha)=J(u+\alpha s)$, then it is converted to the problem of searching the minimum point of function $g(\alpha)$. This algorithm is called as the line search algorithm[13].

Three points $u_{(1)}$, $u_{(2)}$, $u_{(3)}$ are called as the u-shape three points where $u_{(1)}<u_{(2)}<u_{(3)}$, $J(u_{(1)})>J(u_{(2)})<J(u_{(3)})$. If the u-shape three points are found, the minimum point of $J(u)$ can get in between section $[u_{(1)},u_{(3)}]$. The value $J(u_{(0)})$ is conputed by the initial point $u_{(0)}$. The value $J(u_{(1)})$ is computed by $u_{(1)}=u_{(0)}+\alpha s$. If $J(u_{(0)})>J(u_{(1)})$, the direction is right. And the amplitude α doubles the step size again and determine $u_{(3)}$, then continue the same procedure. If $J(u_{(1)})<J(u_{(2)})$, the u-shape three points are found. If the u-shpae three points are found, $u_{(n+1)}$ divides the section $[u_{(n+2)},u_{(n)}]$ into 2:1 (or 1:2). $J_{(u)}$ is solved by the middle point between $u_{(n+2)}$ and $u_{(n+1)}$. Comparing both sides, the even intervals of u-shape three points are obtained. If the iteration of the three points approach is complete, $J(u)$ can be obtained by the parabolic interpolation of the three points. the minimum point of the parabola through three points is given as follows ;

$$u_{(4)} = \frac{1}{2}\frac{(u_{(1)}^2-u_{(2)}^2)J(u_{(3)})+(u_{(2)}^2-u_{(3)}^2)J(u_{(1)})+(u_{(3)}^2-u_{(1)}^2)J(u_{(2)})}{(u_{(1)}-u_{(2)})J(u_{(3)})+(u_{(2)}-u_{(3)})J(u_{(1)})+(u_{(3)}-u_{(1)})J(u_{(2)})} \cdots\cdots\cdots\cdots\cdots (36)$$

Assumming this point as the initial point, the next iteration can be carried out. The amplitude α is replaced with $\alpha/10$. The u-shape three points are found again. If the amplitude α is obtained as small enough as less than the preassigned allowance value, the final amplitude α is obtained.

(2) THE SAKAWA-SHINDO METHOD

The Sakawa-Shindo method[15] employed in this paper is described in this section. The computation of $J(x^{(i)},u^{(i)})$ is performed as :

$$J(x^{(i)},u^{(i)})=\frac{1}{2}\int_{t_0}^{t_f}(\{\zeta^{(i)}\}^T[S]\{\zeta^{(i)}\}+\{u^{(i)}\}^T[R]\{u^{(i)}\})dt \cdots\cdots\cdots (37)$$

Using the equations (34) and (35), the computational algorithm can be described as follows.

1. Assume initial control function $u^{(0)}(t), t\in[t_0,t_f]$
   Solve $\{x^{(0)}\}=\{x(u^{(0)})\}$ and set $i=1$
2. Compute $J(x^{(0)},u^{(0)})$
3. Solve $\{p^{(i-1)}\}=\{p(u^{(i-1)},x^{(i-1)})\}$
4. Solve control function $u^{(i)}$ by equation (27)
5. Compute $e=\sum_{k=1}^{n}\|u^{(i)}(k)-u^{(i-1)}(k)\|$

IF $e<\varepsilon$ THEN STOP
ELSE

6. Solve $\{x^{(i)}\}=\{x(u^{(i)})\}$
7. Compute $J(x^{(i)},u^{(i)})$
8. Compute $JJ=J(x^{(i)},u^{(i)})-J(x^{(i-1)},u^{(i-1)})$
   IF $JJ<0$ THEN $i=i+1$ and GOTO 3
   ELSE choose larger $[W^{(i)}]$ and GOTO 4

The parameter ε is a small number which expresses the convergence allowance. For the stability of the computation at the first stage, the matrix $[W^{(i)}]$ should be chosen rather large value, for instance, comparable to the order of 10-1 000 was chosen in the following numerical examples. But, according as computation converges and $[W^{(i)}]$ tends smaller, ther variation of the control function computed takes large values. Therefore, the initial value of $[W^{(0)}]$ should be chosen as small as possible. The flow chart of the computation is shown in **Fig.3**.

8. CONPARISON BETWEEN THE CONJUGATE GRADIENT AND THE SAKAWA-SHINDO METHODS

To validate the adaptability of the present control method, a simple one dimensional channel problem has been solved. **Fig.4** shows the finite element idealization and boundary conditions. Total numbers of nodal points and finite elements are 123 and 160 respectively. On the boundary S_u, the input flow is specified as a time function shown in **Fig. 5**. The normal velocity component on the boundary S_1 is given to be zero. On the boundary S_c, the outflow discharge is controlled. Without any control, the discharge shown by the dotted line in **Fig. 6** is obtained. This and input flow in **Fig.5** are mutually congruent. This fact shows that the exact computation can be carried out using the present computer program.

In **Fig.6**, the solid line and the cross symbol are controls of outflow discharge, which are computed

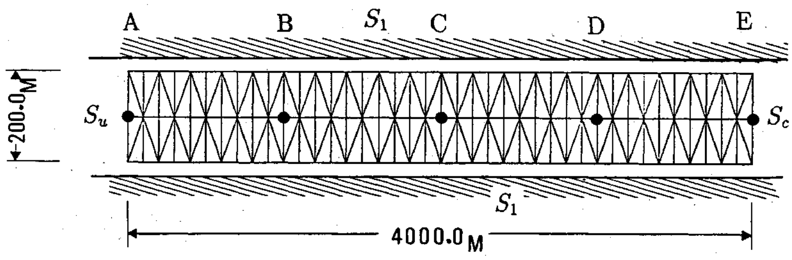

Fig.4 Finite element idealization

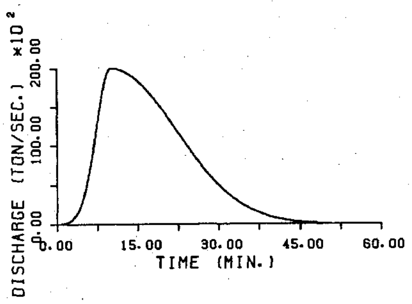

Fig.5 Input flow at boundary S_u

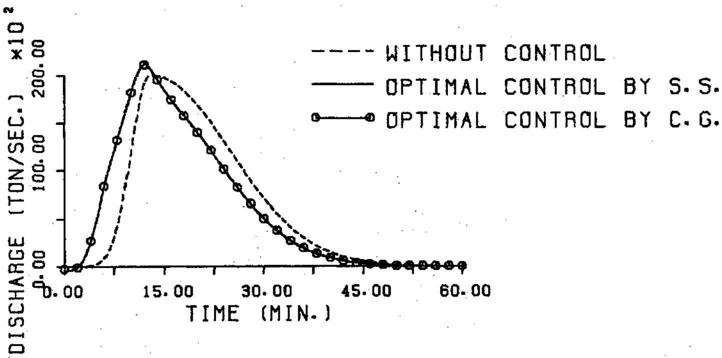

Fig.6 Control discharge at boundary S_c

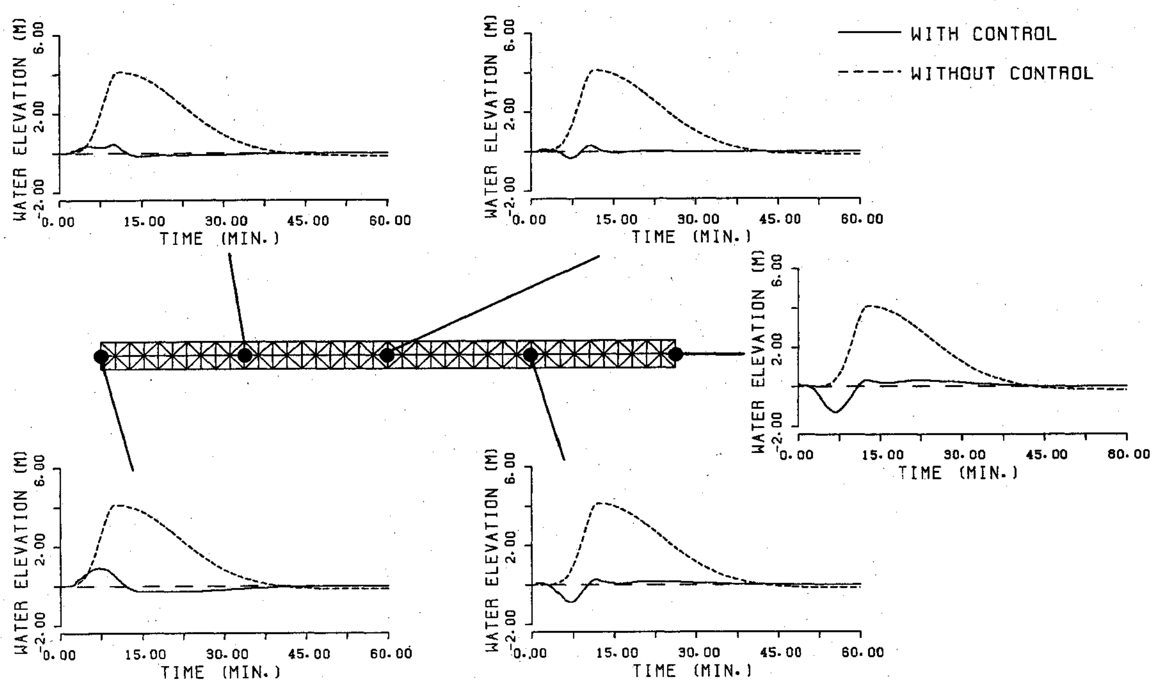

Fig.7 Computed water elevation at various observa tional points

by the Sakawa-Shindo method and the conjugate gradient method repectively. In this figure, both results of the computed control of outflows can be shown to be completely coincident. **Fig.7** represents the computed water elevation at the various observation points. In the figure, the dotted line shows the computed water elevation without control and the solid line is the controlled water elevation. It is observed from the computed results that the water elevation can be controlled to be almost coincident with the still water level by controlling the water outflow on boundary S_c

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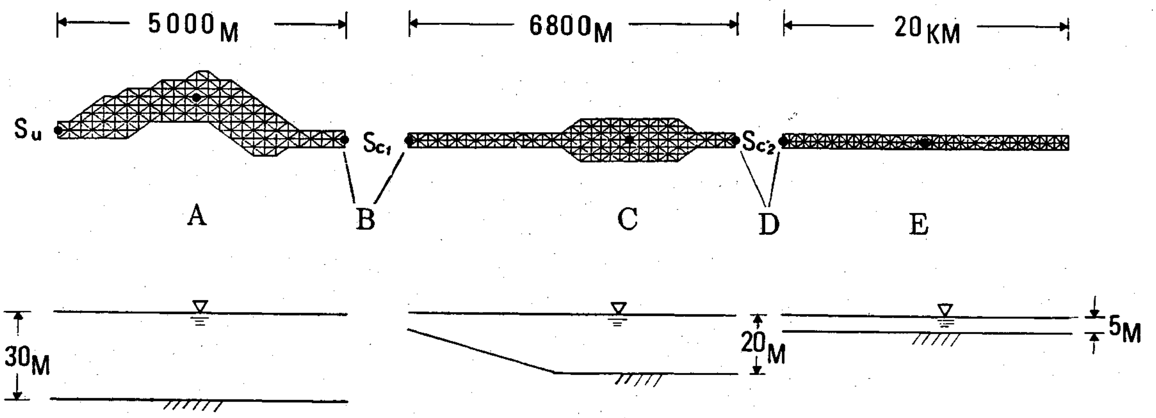

Fig.8 Finite element idealization with water depth

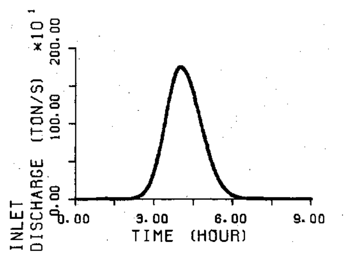

Fig.9 Input flow at bonudary S_u

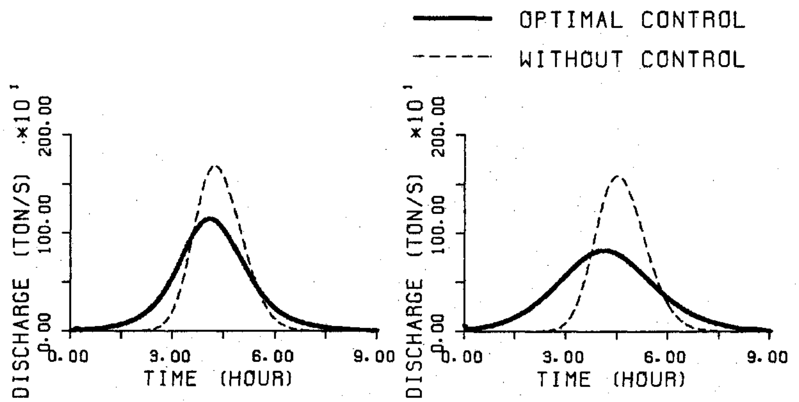

Fig.10 Control discharge at boundary S_{c1} and S_{c2}

following the way represented in **Fig. 6.** The outflow on boundary S_c should be larger than the normal outflow at the time before the peak value. For the parameters $[S]=1.0$ and $[R]=0.0001$ are used. It can be stated that the water elevation can be controlled as almost flat as possible if the parameters will be chosen suitable. It has been shown that the way how to control the discharge can be obtained by the method presented in this paper.

In this computation, both numerical results obtained by the conjugate gradient and the Sakawa-Shindo methods are comletely coincident. The core storage requirements of the conjugate gradient method and the Sakawa-Shindo method were approximately 5Mbites and 7Mbites respectively. The computation time of the conjugate gradient method is twice as long as that of the Sakawa-Shindo method. Considering this fact, the Sakawa-Shindo method is employed in the computation shown in the following section.

9. DAM CONTROL ANALYSIS

(1) TWO DAMS CONTROL

In case of the flood control problem by a dam, both upstream and downstream conditions have to be considered at the same time. In the present analysis, the control problem in which two dams are located along the river is carried out. The river basin used is shown in **Fig.8** with water depth and width. The total numbers of nodal points and finite elements of upstream are 140 and 224, of middlestream are 153 and 232 and of downstream are 123 and 160 respectively. The lumping parameter $e=0.9$ and time increment $\Delta t=4.5$ (sec.) are used. **Fig.9** shows the input flood discharge on boundary S_u.

The computed results are shown in **Figs.9~11.** Assume that the flood which comes to the boundary S_u is expressed by a time function shown in **Fig.9**. There are two dams at the control points denoted by B and D in **Fig.8**. Controlling the discharges of the gates as shown in **Fig.10**, the controlled water elevations at points A, C, E can be represented in **Fig.11**. In **Fig.10** the solid line

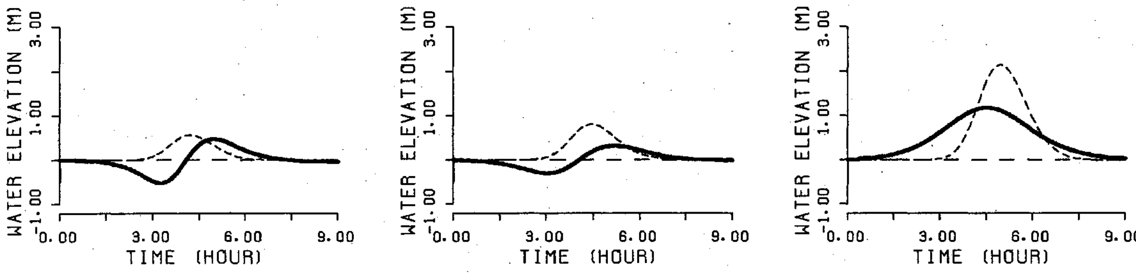

Fig.11 Computed water elevation at various observational points

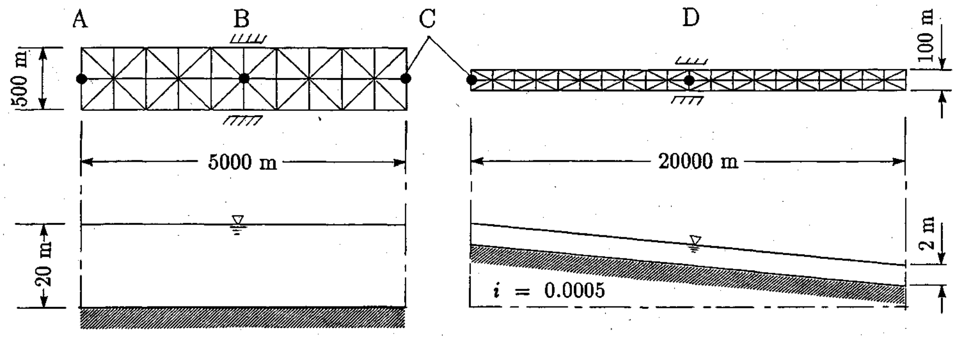

Fig.12 Finite element idealization with water depth

shows the control obtained by the present theory and the dotted line is the discharge at the gates without any control. In **Fig.11**, the solid line shows the water elevation obtained by the present theory. The water elevations without any control are also represented by the dotted line. It is shown in **Fig.11** that the controlled water elevations are all computed as smaller values than those obtained without control. Referring to the discharge controlled at gates B and D, the discharges at the gates have to flow out larger amounts than those of the inflow. Namely, it is necessary to flow out the water discharge at the gates in advance and to keep the water elevation lower than the still water level. Thus, it can be concluded that the discharges at the dam gates should be flowed out in advance before the peak value of the flood arrives. It is interesting to see in the computed result that the downstream control flow should start earlier than the starting time of the upstream control. For the parameters, $[S]=1.0$ in area A to C, $[R]=0.001$ on S_{c1}, S_{c2} are used. Using the different parameters $[R]$'s, the weighted controls for each area can be performed.

(2) CONTROL WITH CONSTRAINT ON CONTROL FLOW

To protect the downstream area, it is sometimes required that the outflow at a dam should not exceed the inflow at the same time. Therefore, in the present analysis, the flood control problem with the constraint on the control flow is carried out. This constraint can be expressed as follows.

$$\{u(t)\} \leq \{f(t)\} \qquad (38)$$

where $f(t)$ means the inflow flowed into the reservoir of the dam. To express the river flow, the equation of motion is modified in the following form to consider the inclination of the river bed.

$$\dot{q}_i + g(h+\zeta)(h+\zeta+z)_{,i} + \frac{gn^2}{(h+\zeta)^{\frac{4}{3}}} u q_i = 0 \qquad (39)$$

where z is altitude, u is velocity and n is Manning coeffcient of roughness. The river basin used is shown in **Fig.12** with water depth and width. The total numbers of nodal points and finite elements of upstream are 33 and 40 and of downstream are 63 and 80 respectively. Lumping parameter $e=0.9$, time increment $\Delta t = 5.0$ (sec.) and Manning coefficient of roughness $n=0.04$ are used. **Fig.13** shows the input flood discharge on boundary S_u.

The computed results are shown in **Fig.13～15**. Assume that the flood which comes to the boundary S_u is expressed by a time function shown in **Fig.13**. There is a dam at the control point denoted by C in **Fig.12**. Controlling the discharge of the gate as shown in **Fig.14**, the controlled water elevations at points B, D can be represented in **Fig.15**. In **Fig.14** the thin solid line shows the control without any constraint, the thick solid line shows the control with constraint and the dotted line in the discharge at the gates without control. In

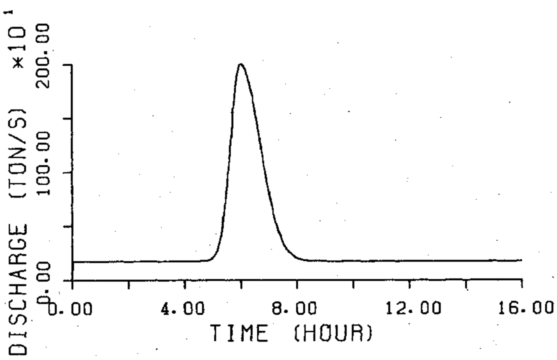

Fig.13 Input flow at boundary S_u

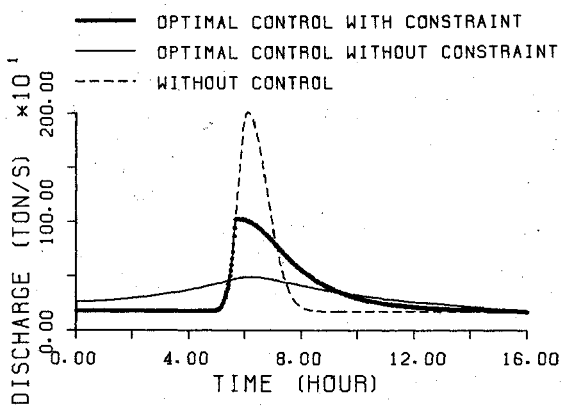

Fig.14 Control discharge at boundary S_c

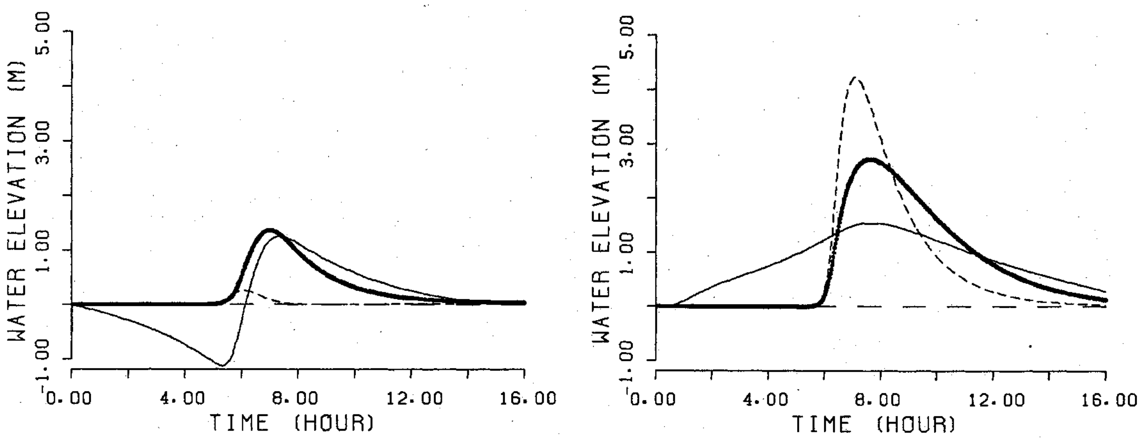

Fig.15 Computed water elevation at point B and D

Fig.15, the thin solid line shows the controlled water elevetion without constraint and the thick solid line shows the controlled water elevation with constraint. The water elevation without control are also represented by the dotted line. It is shown that the water elevation with constraint should be controlled more repidly using larger amount of volume than the one without constrant. Thus, the water elevation with constraint is less efficient than the one without constraint at the downstream area. This fact shows that the efficiency of flood control is rested on the point that the dam gate should be flowed out in advance before the peak value of the flood arrives. For the parameters, [S]=1.0 and [R]=0.01 are used.

10. CONCLUSION

The outcomes of the present paper are summarized as follows.

1. The optimal control theory for the flood propagating through a reservior and river channel has been formulated based on the quadratic control theory combined with the finite element method assuming the flood is known in advance.

2. For the control technique, Sakawa and Shindo method is effectively used. Because this method is one of the most effective techniques on the point of computational time and core storage requirement. The effeciency depends on the selection of the known constants.

3. Two step explicit schemes with the lumped coefficient matrix have shown to be efficient for the numerical integration in time not only in the forward direction but also in the backward direction. The lumping parameter $e=0.9$ is used in the computation in this paper.

4. It has been shown that the water elevation in the whole flow field can be controlled as flat as possible by adjusting the discharge of the dam gates following the strategy obtained in this paper. It is necessary to discharge through the dam gate in advance before the peak value of the flood arrives.

5. It is sometimes necessary that the control outflow is limited by several constraints. The method presented in this paper can be adaptable to the control analysis with constraint. It is natural that the control without constraint is much more efficient than that with constraint with respect to the outflow.

In this paper, the linear shallow water equation and the quadratic optimal control theory have been used, but the extensions to the nonlinear theory is straightfoward. A part of this research has been carried out with the help of Mr. Tomoyuki Kawasaki, graduate student of Chuo university. The computations in this paper have been carried out using FACOM VP-30 of Chuo University. A part of this research has been supported by the Grant in Aid of Science and Engineering, Ministry of Education, No. 01613001.

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有限要素法を用いた洪水の最適制御

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本論文は，洪水の際の効果的なダムゲートの操作方法に関して，有限要素法による流体解析法と最適化手法を連成した制御解析手法について研究したものである．最適化手法としては，共役勾配法，Sakawa-Shindo 法の 2 法について検討し，1 ダムの流入出問題によって，収束性，計算時間，記憶容量等について調べている．また応用として，複数の領域への適応を行い，従来の放流法との比較を行っている．