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MODELING OF CONCRETE IN EARLY AGE AS ELASTO-PLASTIC POROUS CONTINUUM

Denzil LOKULIYANA*, Takeshi INOUE**, and Tada-aki TANABE***

In the analysis of early age concrete structures with initial thermal stress fields caused by hydration heat of cement, the consideration of time dependent constituive relation is very important. In this study, early age concrete is modeled as a two phase material consisting of aggregates having elastic behaviour and cement paste which fills the voids between aggregates together with water having elasto-plastic behaviour. Plastic formulation was performed with Drucker-Prager failure criterion with the consideration of the effect of maturity of concrete on the cohesion and angle of internal friction of cement paste. An experimental study was carried out to investigate the time dependent characteristics of these two material parameters. Finally, a numerical simulation was carried out using Finite Element Method for this two phase non-homogeneous material and results were compared with experimental data.

Keywords : *early age concrete, two phase material, elasto-plastic*

1. INTRODUCTION

In the analysis of massive concrete structures, the thermal behaviour due to hydration heat of cement at a very early age is very important. To predict the actual behaviour of early age concrete structures, a clear information on material properties which are time dependent, is necessary. These material properties which govern the constitutive relation are cohesion, angle of internal friction, Young's modulus, Poisson ratio, porosity, etc. Many research works have been done to model the constitutive relation for hardened concrete with the effect of these material properties. But it is foreign to early age concrete and hence, in this study, a constitutive model for carly age concrete is proposed and the effect of the material properties on the model was discussed.

In this analysis, early age concrete is assumed to contain aggregates and cement paste with voids between aggregates. Hence, concrete can be modeled as two phase non-homogeneous porous material having elastic behaviour for aggregates and elasto-plastic behaviour for cment paste with water diffusion in void system. Although the well known Mohr-Coulomb failure criterion is simple and has clear physical meaning, since corners of the irregular hexagon of this failure surface can cause difficulties in the real computation, the Drucker-Prager failure criterion was employed with the consideration of the effect of maturity of concrete on the cohesion and angle of internal friction of cement paste to determine the plastic behaviour of early age concrete. Moreover, the effect of degradation of cohesion and angle of internal friction of concrete with damage parameter proposed by Wu and Tanabe[1] is also incorporated. The force equilibrium and flow continuity equations proposed by Contri, Majorana and Schrefler[2] and Lewis and Schrefler[3] is modified to consider the time effect. The variation of cohesion and angle of internal friction with age was determined experimentally by triaxial compressive test. The sensitivity of other material parameters on the model obtained from three dimensional analysis was numerically discussed. Finally, a numerical simulation was carried out using Finite Element Method for this two phase non-homogeneous material and an uniaxial compressive test was performed at different ages to compare with the numerical results.

2. MATHEMATICAL MODELING OF EARLY AGE CONCRETE BY SATURATED POROUS TWO PHASE MATERIAL

In this formulation early age concrete is treated as non-homogeneous, two phase porous material of aggregates and cement paste where voids are saturated with water (**Fig.1**). Hence, the behaviour of aggregates can be considered as perfectly elastic material while cement paste is assumed as elasto-plastic permeable material.

The force equilibrium equation and continuity equation of pore water proposed by Contri, Majorana and Schrefler[2] and Lewis and Schrefler[3] are developed to incorporate the time effect and the combined equation is solved with the consideration of plastic behaviour of cement paste which

* Graduate Student, Nagoya University (Nagoya 464, Japan)
** Engineer, Japan Highway Public Corporation
*** Member of JSCE, Dr. Eng., Professor, Nagoya Uuiversisty

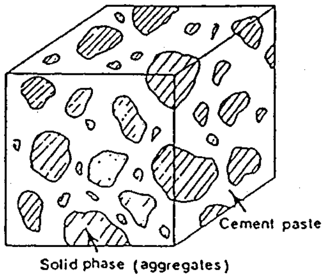

Fig.1 Two Phase Concrete Model

is mainly governed by degradation of cohesion and angle of internal friction with damage parameter proposed by Wu and Tanabe[1)].

(1) Plastic Formulation for Cement Paste

a) Plastic strain of Cement Paste

If the assumed failure function F is considered to be a function of stress state and plastic strain history, from the classical plasticity theory, plastic deformation takes place when $F \geq 0$ and if $F<0$ the behaviour of cement paste is still in elastic region. Then, from the yield condition

$$F=F(\{\sigma'\},\{\varepsilon_c^p\})=0 \quad \cdots\cdots (1)$$

Where, $\{\sigma'\}$ = effective stress and $\{\varepsilon_c^p\}$ = plastic strain. The consistency condition of yield surface, i.e., $dF=0$ is

$$\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T d\{\sigma'\}+\left\{\frac{\partial F}{\partial\{\varepsilon_c^p\}}\right\}^T d\{\varepsilon_c^p\}=0 \quad \cdots\cdots (2)$$

Where

$$d\{\sigma'\}=[D_C^e]d\{\varepsilon_c^e\} \quad \cdots\cdots (3)$$

Here, $\{D_C^e\}$ is the elastic stiffness matrix for cement paste.

According to the associated flow rule, plastic strain takes the value of

$$d\{\varepsilon_c^p\}=d\lambda\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\} \quad \cdots\cdots (4)$$

But, the incremental from of the total strain $d\{\varepsilon_c^T\}$ of cement paste is

$$d\{\varepsilon_c^T\}=d\{\varepsilon_c^e\}+d\{\varepsilon_c^p\}+d\{\varepsilon_c^{pr}\}+d\{\varepsilon_c^t\},$$

$$d\{\varepsilon_c'\}=d\{\varepsilon_c^e\}+d\{\varepsilon_c^p\} \quad \cdots\cdots (5)$$

Where, $\{\varepsilon_c^e\}$, $\{\varepsilon_c^p\}$, $\{\varepsilon_c^{pr}\}$, $\{\varepsilon_c^t\}$ and $\{\varepsilon_c'\}$ are the elastic strain, plastic strain, strain caused by pore water pressure, thermal strain due to hydration heat of cement and effective strain, respectively. Here, the values of $d\{\varepsilon_c^{pr}\}$ and $d\{\varepsilon_c^t\}$ are

$$d\{\varepsilon_c^{pr}\}=-\{m\}\left(\frac{1}{3k_s}\right)dp,\ d\{\varepsilon_c^t\}=\{m\}\alpha dT \quad \cdots (6)$$

Where $\{m\}^T=\{111000\}$, k_s=average bulk modulus of solid phase, dp=incremental pore water pressure, α=linear thermal expansion coefficent and dT=temperature increment.

Substituing eqns. (3), (4) and (5) into eqn. (2), the value of $d\lambda$ can be obtained as

$$d\lambda=\frac{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]d\{\varepsilon_c'\}}{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}+h} \quad \cdots\cdots (7)$$

Here, h is called as hardening parameter defined as

$$h=-\left\{\frac{\partial F}{\partial\{\varepsilon_c^p\}}\right\}^T\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\} \quad \cdots\cdots (8)$$

Then, solving for plastic strain using eqns. (4) and (7), it can rewritten as

$$d\{\varepsilon_c^p\}=\frac{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]d\{\varepsilon_c'\}}{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}+h} \quad \cdots\cdots (9)$$

b) Plastic stress-strain relation for cement paste

The effective stress-strain relation for cement paste

$$d\{\sigma'\}=[D_C^e][d\{\varepsilon_c'\}-d\{\varepsilon_c^p\}] \quad \cdots\cdots (10)$$

Using eqn. (9) eqn. (10) can be written as,

$$d\{\sigma'\}=\left\{[D_C^e]-\frac{[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]}{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}+h}\right\}d\{\varepsilon_c'\}$$

$$=[D_C^{ep}]d\{\varepsilon_c'\} \quad \cdots\cdots (11)$$

Where the elasto-plastic stiffness matrix $[D_C^{ep}]$ for cement paste is

$$[D_C^{ep}]=[D_C^e]-\frac{[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]}{\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}^T[D_C^e]\left\{\frac{\partial F}{\partial\{\sigma'\}}\right\}+h} \quad \cdots\cdots (12)$$

(2) Elasto-Plastic Stiffness Matrix of Concrete

Since in the analysis, concrete is modeled as a two phase material of aggregates and cement paste, the stiffness matrix for concrete is formulated by assuming that the total strain components take the summation of the weighted strains of individual materials. When the volume and incremental form of total strain for concrete, aggregates and cement paste is V and $d\{\varepsilon^T\}$, V_A and $d\{\varepsilon_A^T\}$, V_C and $d\{\varepsilon_C^T\}$ respectively, the total strain

$$d\{\varepsilon^T\}=\frac{V_A}{V}d\{\varepsilon_A^T\}+\frac{V_C}{V}d\{\varepsilon_C^T\} \quad \cdots\cdots (13)$$

can be written with $V=V_A+V_C$.

where

$$d\{\varepsilon_A^T\}=d\{\varepsilon_A^e\}+d\{\varepsilon_A^{pr}\}+d\{\varepsilon_A^t\},$$

$$d\{\varepsilon_C^T\}=d\{\varepsilon_C^e\}+d\{\varepsilon_C^p\}+d\{\varepsilon_C^{pr}\}+d\{\varepsilon_C^t\} \quad \cdots\cdots (14)$$

Here, superscripts '*e*', '*p*', '*pr*' and '*t*' refers to elastic, plastic, pore pressure and temperature components of strain while the subscripts '*A*' and

'C' refer to aggregates and cement paste respectively.

Now, the effective stress $d\{\sigma'\}$, for concrete takes the following relations.

$$d\{\sigma'\}=[D^e_A]d\{\varepsilon^e_A\}=[D^e_C]d\{\varepsilon^e_C\} \cdots\cdots (15)$$

using eqn. (11) it can be rewritten as,

$$d\{\sigma'\}=[D^e_A]d\{\varepsilon^e_A\}=[D^{ep}_C]d\{\varepsilon'_C\} \cdots\cdots (16)$$

Here, $[D^e_A]$ is the elastic stiffness matrix for solid phase. Then,

$$d\{\varepsilon^e_A\}=[D^e_A]^{-1}d\{\sigma'\},\ d\{\varepsilon'_C\}=[D^{ep}_C]^{-1}d\{\sigma'\} \cdots\cdots (17)$$

As in eqn. (13), the incremental form of effective strain for concrete can be written as

$$d\{\varepsilon'\}=\frac{V_A}{V}d\{\varepsilon^e_A\}+\frac{V_C}{V}d\{\varepsilon'_C\} \cdots\cdots (18)$$

Hence, the following relation can be obtained using eqns. (13), (14) and (18).

$$d\{\varepsilon'\}=d\{\varepsilon^T\}-d\{\varepsilon^{pr}\}-d\{\varepsilon^t\} \cdots\cdots (19)$$

where

$$d\{\varepsilon^{pr}\}=\frac{V_A}{V}d\{\varepsilon^{pr}_A\}+\frac{V_C}{V}d\{\varepsilon^{pr}_C\},$$

$$d\{\varepsilon^t\}=\frac{V_A}{V}d\{\varepsilon^t_A\}+\frac{V_C}{V}d\{\varepsilon^t_C\}$$

Then, from eqn. (17) and (18),

$$d\{\varepsilon'\}=\left[\frac{V_A}{V}[D^e_A]^{-1}+\frac{V_C}{V}[D^{ep}_C]^{-1}\right]d\{\sigma'\} \cdots\cdots (20)$$

Hence, the elasto-plastic stiffness matrix $[D^{ep}_T]$, for concrete with the effect of time becomes

$$[D^{ep}_T]=\left[\frac{V_A}{V}[D^e_A]^{-1}+\frac{V_C}{V}[D^{ep}_C]^{-1}\right]^{-1} \cdots\cdots (21)$$

(3) Application of Drucker-Prager type of Failure Criterion

a) Drucker-Prager Failure Criterion

Although it is known that the Mohr-Coulomb criterion which was also employed in the analysis by Contri, Majorana and Schrefler[2], is simple and it has clear physical meaning, since the corners of the irregular hexagon of this failure surface can cause considerable difficulties and complications in obtaining numerical solutions, the following Drucker-Prager failure criterion is employed. As in Mohr-Coulomb criterion, this also gives lesser value for tensile strength over compressive strength.

$$F=\alpha I_1+\sqrt{J_2}-k \cdots\cdots (22)$$

where $I_1=\sigma_{kk}$ and $J_2=\frac{1}{2}s_{ij}s_{ij}$ are the first invariant of the stress tensor σ_{ij} and second invariant of deviatoric stress tensor s_{ij}, respectively, and α and k are material constants which vary with degree of hydration and age of concrete.

On the basis of the above consideration, there are several ways proposed by Chen[4] to approximate the Mohr-Coulomb hexagonal surface by Drucker Prager cone. As for example if it is made to agree along the compressive meridian where $\theta=60°$, the two material constants take the values of

$$\alpha=\frac{2\sin\phi^*}{\sqrt{3}(3-\sin\phi^*)},\quad k=\frac{6c^*\cos\phi^*}{\sqrt{3}(3-\sin\phi^*)} \cdots\cdots (23)$$

and similarly when it is made to agree along the tensile meridian where $\theta=0°$, then the values of the material constants become

$$\alpha=\frac{2\sin\phi^*}{\sqrt{3}(3+\sin\phi^*)},\quad k=\frac{6c^*\cos\phi^*}{\sqrt{3}(3+\sin\phi^*)} \cdots\cdots (24)$$

where c^* and ϕ^* are defined as the mobilized cohesion and friction of cement paste.

But in this study compressive failure is considered and hcnce the matching was done only along the compressive meridian while considering effect of degree of hydrantion and age of concrete on these material constants.

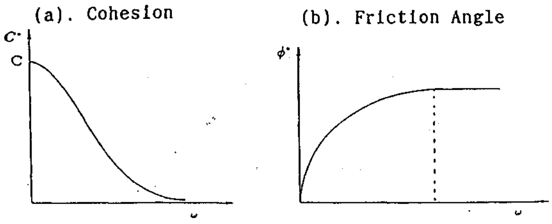

Fig.2 Damage vs. Cohesion and Friction Angle

b) Definition of mobilized friction angle and cohesion

From the above definition, the two material constants for cement paste can be determined from the so-called mobilized cohesion c^* and angle of internal friction ϕ^* which are not constants, but depend on the plastic strain history through the damage parameter ω. Since in general, the value of ϕ^* should be an increasing function of ω, while c^* may be expected to be a decreasing function of ω, the relation proposed by Wu and Tanabe[1] is used and defined as follows (**Fig.2**(a), 2(b)).

$$c^*=c\exp[-(a\omega)^2] \cdots\cdots (25)$$

$$\phi^*=\begin{cases}\phi\sqrt{2\omega-\omega^2}, & \omega\leq 1\\ \phi, & \omega>1\end{cases} \cdots\cdots (26)$$

in which c and ϕ are the cohesion and friction angle which are depend on the rate of hydration and age of cement paste and 'a' is a material constant. Damage parameter ω becomes unity when compressive stress reaches to its strength f'_c.

c) Definition of damage parameter

In this study, the failure surface is assumed to be varying with damage parameter in the stress space, and then, from the concept[4] of effective stress σ_e and effective plastic strain ε_p, the incremental from of plastic work dw^p can be related as

$$dW^p=\{\sigma\}^T d\{\varepsilon^p\}=\sigma_e d\varepsilon_p \cdots\cdots (27)$$

Damage parameter is defined as the damage of the material accumulated due to the progressive growth of microcracks and it can be expressed as[1]

$$\omega=\frac{\beta}{\sigma_e\varepsilon_0}\int dW^p \cdots\cdots (28)$$

where β is a material constant, and

$$\varepsilon_0=\frac{f_c'}{E_c} \cdots\cdots (29)$$

Here, E_c denotes the modulus of elasticity of concrete, and f_c' is the uniaxial compressive strength. And then, substituting eqn. (27) into eqn. (28) we also have

$$\omega=\beta\int\frac{1}{\varepsilon_0}d\varepsilon_p \cdots\cdots (30)$$

(4) Formulation of Equilibrium equation from Principle of Virtual Work

With the presence of pore water pressure p, the relation of effective strcss σ' and total stress σ will become

$$\{\sigma'\}=\{\sigma\}+\{m\}p \cdots\cdots (31)$$

where the sign of tensile stress is taken as positive.

Form the concept of this effective stress which governs the total linear and non-linear behaviour can be related to the strain components which were described in the previous section from the eqn. (19), as in the form of

$$d\{\sigma'\}=[D_T^{ep}]\left(d\{\varepsilon^T\}+\{m\}\left(\frac{1}{3k_s}\right)dp-\{m\}\alpha dT\right) \cdots\cdots (32)$$

Then the equilibrium equation using the principle of virtual work becomes

$$\int_\Omega\delta\{\varepsilon^T\}^T\{\sigma\}d\Omega-\int_\Omega\delta\{u\}^T\{b\}d\Omega-\int_\Gamma\delta\{u\}^T\{t\}d\Gamma=0 \cdots\cdots (33)$$

here, vector $\{b\}$, $\{t\}$ shows the body force and surface traction vectors and, Ω, Γ denotes the volume and surface area of concrete body. Now, using the appropriate shape functions, this equilibrium equation can be rewritten in differential form as

$$K_T\frac{d\{\bar{u}\}}{dt}-L\frac{d\{\bar{p}\}}{dt}-A\frac{d\{\bar{T}\}}{dt}-\frac{d\{f\}}{dt}=0 \cdots\cdots (34)$$

where matrices K_T, L, A are the tangent stiffness, effect of pore water pressure and volume change of solid phase, and effect of temperature, respectively. The vector $\{f\}$ denotes the effect of external force on displacement.These can be defined as

$$K_T=\int_\Omega B^T[D_T^{ep}]Bd\Omega,$$

$$L=\int_\Omega B^T\{m\}\bar{N}d\Omega-\int_\Omega B^T[D_T^{ep}]\{m\}\left(\frac{1}{3k_s}\right)\bar{N}d\Omega,$$

$$A=\int_\Omega B^T[D_T^{ep}]\{m\}\alpha\bar{N}d\Omega,$$

$$f=\int_\Omega N^T\{b\}d\Omega+\int_\Gamma N^T\{t\}d\Gamma \cdots\cdots (35)$$

here, N, $\bar{N}$ and B is the shape function for displacement, pore pressure and temperature, and strain displacement matrix respectively.

(5) Formulation of the Flow Continuity Equation

In this study, the flow of pore water is considered to follow Darcy's law and then, if the fluid having coefficient of permeability of k flows with a velocity of v under head h, it can be written as

$$v=-k\nabla h=-k\nabla\left[\frac{\gamma z+p}{\gamma}\right] \cdots\cdots (36)$$

where γ is the specific gravity of the fluid, z is the vertical coordinate of the point considered and $\nabla^T=\{\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z\}$.

Now, from the mass conservation law, the amount of fluid accumulation in a controlled unit volume is equal to the difference of the amount of inflow to and out flow from this volume. The following contributions to the amount of fluid accumulation is adopted in the formulations.

(1) Due to total strain change

$$\frac{\partial\varepsilon_v}{\partial t}=\{m\}^T\frac{\partial\{\varepsilon\}}{\partial t}$$

(2) Due to the change of volume of grains caused by changes of hydrostatic pressure

$$(1-\zeta)\left(\frac{1}{k_s}\right)\frac{\partial p}{\partial t}$$

where ζ is the porosity which depends on the rate of hydration of cement.

(3) Due to change of fluid volume

$$\zeta\left(\frac{1}{k_f}\right)\frac{\partial p}{\partial t}$$

where k_f is the bulk modulus of fluid.

(4) Due to the change of fluid volume caused by temperature

$$-3\zeta\beta\frac{\partial T}{\partial t}$$

where β is the thermal expansion coefficient of water.

(5) Due to the change of grain size by effective stresses

The change of effective stress gives rise to the mean hydrostatic pressure of

$$-\frac{1}{3}\{m\}^T\frac{\partial\{\sigma'\}}{\partial t}\left(\frac{1}{1-\zeta}\right)$$

which leads to fluid accumulation and since it acts on the total volum $(1-\zeta)$ of solid, the amount of fluid accumulation for this reasoning is

$$-\frac{1}{3k_s}\{m\}^T\frac{\partial\{\sigma'\}}{\partial t}$$

Substituting the value of effective stress from eqn. (32), the above term becomes

$$-\frac{1}{3k_s}\{m\}^T[D_T^{ep}]\left(\frac{\partial\{\varepsilon\}}{\partial t}+\{m\}\left(\frac{1}{3k_s}\right)\frac{\partial p}{\partial t}-\{m\}\alpha\frac{\partial T}{\partial t}\right)$$

Hence, now the continuity equation for the fluid with the consideration of mass conservation law can be obtained as

$$-\nabla^T k'\nabla[(\gamma z+p)]+\left(\{m\}^T-\left(\frac{1}{3k_s}\right)\{m\}^T[D_T^{ep}]\right)\frac{\partial\{\varepsilon\}}{\partial t}+\left[\frac{(1-\zeta)}{k_s}+\zeta\left(\frac{1}{k_f}\right)-\left(\frac{1}{3k_s}\right)^2\{m\}^T[D_T^{ep}]\{m\}\right]\frac{\partial p}{\partial t}+\frac{1}{3k_s}\{m\}^T[D_T^{ep}]\{m\}\alpha\frac{\partial T}{\partial t}-3\zeta\beta\frac{\partial T}{\partial t}=q \quad \cdots\cdots(37)$$

and can be reduced to the form

$$f(p)-q=0 \quad \cdots\cdots(38)$$

where q is the amount of flow to the controlled volume from an external source and $k'=k/\gamma$.

Then applying the Method of Weighted Residual (MWR), eqn. (38) becomes

$$\int_\Omega \delta p^T\{f(p)-q\}d\Omega=0,$$

$$H\{\bar{p}\}+S\frac{d\{\bar{u}\}}{dt}+L^T\frac{d\{\bar{u}\}}{dt}+W\frac{d\{\bar{T}\}}{dt}-f_p=0 \quad \cdots\cdots(39)$$

where

$$H=\int_\Omega(\nabla\bar{N})^T k'\nabla\bar{N}d\Omega,\quad S=\int_\Omega\bar{N}^T s\bar{N}d\Omega,$$

$$s=\left[\frac{(1-\zeta)}{k_s}+\zeta\left(\frac{1}{k_f}\right)-\left(\frac{1}{3k_s}\right)^2\{m\}^T[D_T^{ep}]\{m\}\right],$$

$$L^T=\int_\Omega\bar{N}^T\left(\{m\}^T-\left(\frac{1}{3k_s}\right)\{m\}^T[D_T^{ep}]\right)Bd\Omega,$$

$$W=\int_\Omega\bar{N}^T\left(\frac{1}{3k_s}\right)\{m\}^T[D_T^{ep}]\{m\}\alpha\bar{N}d\Omega-\int_\Omega 3\zeta\bar{N}^T\beta\bar{N}d\Omega,$$

$$f_p=+\int_\Omega\bar{N}^T q d\Omega-\int_\Omega(\nabla\bar{N})^T k'\nabla\gamma z d\Omega \quad \cdots\cdots(40)$$

(6) Linkage of Equilibrium Equation and Continuity Equation

Finally, from the eqns. (34) and (39) the matrix representation of the linked equilibrium and flow continuity equations can be written as

$$\begin{bmatrix}[0] & [0]\\ [0] & -[H]\end{bmatrix}\begin{Bmatrix}\{\bar{u}\}\\ \{\bar{p}\}\end{Bmatrix}+\begin{bmatrix}[K_T] & -[L]\\ -[L^T] & -[S]\end{bmatrix}\begin{Bmatrix}d\{\bar{u}\}/dt\\ d\{\bar{p}\}/dt\end{Bmatrix}=\begin{Bmatrix}d\{f\}/dt+Ad\{\bar{T}\}/dt\\ -f_p+Wd\{\bar{T}\}/dt\end{Bmatrix} \quad \cdots\cdots(41)$$

In this equation, all of the matrices do not violate the symmetricity and hence if the initial conditions are known it can be solved for the displacement. The same equation can be transformed to the following form for the time increment Δt.

$$\begin{bmatrix}[K_T] & -[L]\\ -[L^T] & \frac{\Delta t}{2}[H]-[S]\end{bmatrix}\begin{Bmatrix}\{\bar{u}\}\\ \{\bar{p}\}\end{Bmatrix}_{(t+\Delta t)}-\begin{bmatrix}[K_T] & -[L]\\ -[L^T] & \frac{\Delta t}{2}[H]-[S]\end{bmatrix}\begin{Bmatrix}\{\bar{u}\}\\ \{\bar{p}\}\end{Bmatrix}_{(t)}=\Delta t\begin{Bmatrix}d\{f\}/dt+Ad\{\bar{T}\}/dt\\ -f_p+Wd\{\bar{T}\}/dt\end{Bmatrix} \quad \cdots\cdots(42)$$

3. DETERMINATION OF MATERIAL PARAMETERS

(1) Cohesion and Angle of Internal Friction of Early Age Concrete

Triaxial compressive test was performed to determine the cohesion and angle of internal friction of cement paste at the age of 12, 24, 36, and 48 hrs. using a cylindrical test specimen having the diameter of 5.0 cm and height of 10.0 cm. The water cement ratio used was 0.35 and the test was carried out under 25°C of room temperature.

The experimental results obtained for cohesion of early age cement paste from triaxial compressive test is shown in **Fig.3**. Since when hydration takes place cement paste becomes hardened, the obtained results for cohesion which increases with maturity are reasonable. Moreover, it can be seen that the development of cohesion during first 12 hrs. after casting is comparatively less than that of the development during other time intervals. The casting temperature used is 25°C with a room temperature of 20°C and it can be said that if higher temperature is used as casting temperature, higher values for initial strength can be obtained. The value of maturity, t_e, is expressed as follows[5].

$$t_e=\int_0^t\exp\left\{\frac{U_h}{R}\left(\frac{1}{293}-\frac{1}{K}\right)\right\}dt \quad \cdots\cdots(43)$$

Here, K= the temperature of test specimen in ^0K, U_h= activating energy of hydration (J/mol) and R = general gas constant (J/mol.K).

The variation of angle of internal friction with time is shown in **Fig.4**. It can be seen that the angle of internal friction almost does not vary with time and has the values in the range of 25°C to 35°C since it is a property of individual materials. Since less experimental studies have been done on this

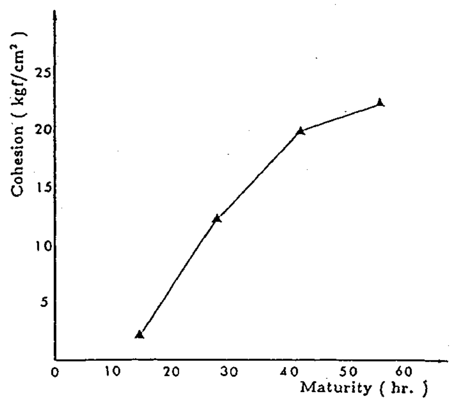

Fig.3 Cohesion with Maturity

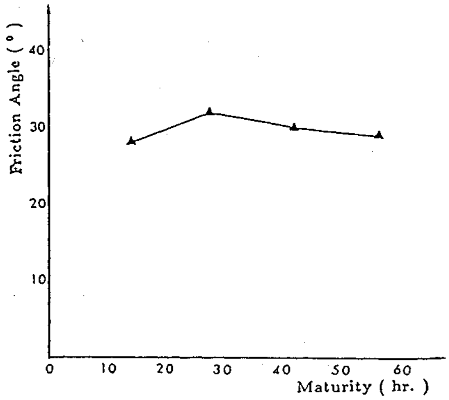

Fig.4 Friction Angle with Maturity

field, the obtained results can be used in thermal stress analysis of early age concrete.

(2) Consideration on Porosity and Permeability

Though it was not able to perform the experiments to determine the variation of porosity and permeability of early age concrete with time, and since there is no much information on these, it was considered in the following way.

Porosity may be obtained from the mix proportions in which the total amount of water is considered to be filling the porous voids. Initial value of 0.23 is considered for porosity with a considerable reduction with hardening of concrete. With the water cement ratio of 0.35, the calculated value for porosity of hardened concrete became 0.09. Hence, the values of porosity for 12, 24, 36 and 48 hrs. were taken as 0.15, 0.14, 0.12 and 0.10, respectively.

For the cofficient of permeability, it is approximated to the value of soil at early age and assumed to be reduced gradually until it reaches to the value of hardened concrete, when hydration has taken place. Hence, the coefficent of permeabilty of early age concrete can be assumed to take the values between 1.0×10^{-6} and 1.0×10^{-12} cm/s which are the values of soil and hardened concrete, respectively. The values of 5.0×10^{-7} cm/min, 5.0×10^{-8} cm/min, 5.0×10^{-9} cm/min and 5.0×10^{-9} cm/min for 12, 24, 36 and 48 hrs. were selected and employed in the calculation.

4. VERIFICATION OF THE PROPOSED MODEL

(1) Sensitivity of Material Parameters on the Proposed Model

The effect of each material parameters of early age concrete on the stress-strain relation and pore water pressure-strain relation are demonstrated in **Figs.5** to **18**. Young's modulus of 4.0×10^{5} kg/cm² for solid phase, and 5.0×10^{4} kg/cm² for cement paste, value of cohesion as 12.0 kgf/cm² and friction angle as 30° for cement paste, 0.20 of Poisson ratio, 0.15 of porosity and 5.0×10^{-7} cm/min of permeability for concrete were used in the calculations.The displacement rate equal to 1.0×10^{-3} cm/min, bulk modulus of water as 2.2×10^{-4} kg/cm² and ratio of solid : cement paste equal to 7 : 3 were also employed.

Fig.5 and **6** show the effect of Young's modulus of cement paste and it can be seen that the yield strain is influenced by this factor. For the larger values of Young's modulus yield strain becomes smaller. This implies that , with the increase of Young's modulus of cement paste, material reaches to its yield point sooner without changing its yield strength. The increase of Young's modulus decreases pore water pressure.

The effect of Poisson ratio of cement paste is shown in **Figs.7** and **8.** It can be noticed that there is no much effect on the stress-strain relation but, the reduction of pore pressure gives a slight increment to the stiffness of transverse direction.

Figs.9 and **10** show the influence of volume ratio of aggregates and cement paste on the proposed model. Since larger values of this ratio gives higher stiffnesses because it contains more aggregates compared to cement paste, a similar explanation can be given as in the case where Young's modulus is varied.

The effect of porosity is illustrated in **Figs.11** and **12.** The term porosity is defined as the ratio of volume of the solid phase to volume of water contained. Since for the same value of material ratio, the variation of porosity dose not have much effect on the stiffness, there is no influence on yield stress, strain and pore pressure.

The variation of cohesion and angle of internal friction are shown in **Figs.13** to **16.** The yield condition is influenced by these factors since, the

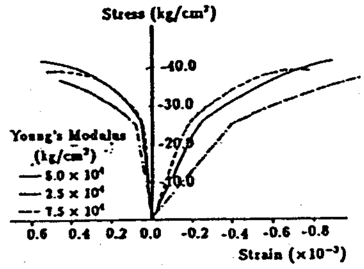

Fig.5 Stress-strain Relation

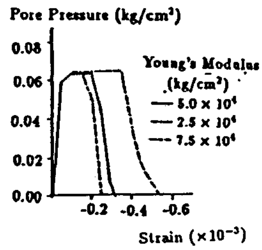

Fig.6 Pore Pressure-strain Relation

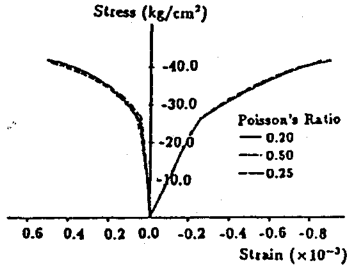

Fig.7 Stress-strain Relation

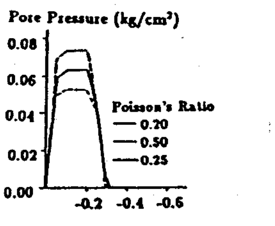

Fig.8 Pore Pressure-strain Relation

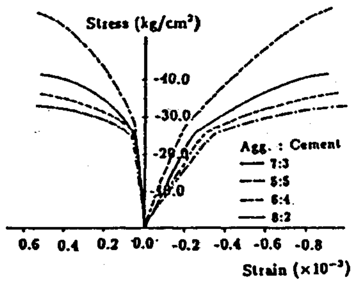

Fig.9 Stress-strain Relation

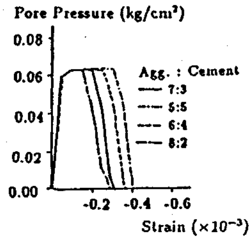

Fig.10 Pore Pressure-strain Relation

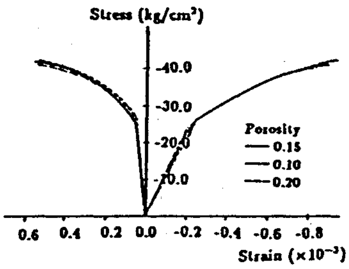

Fig.11 Stress-strain Relation

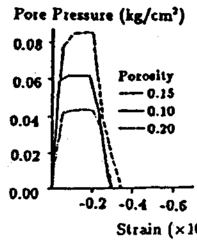

Fig.12 Pore Pressure-strain Relation

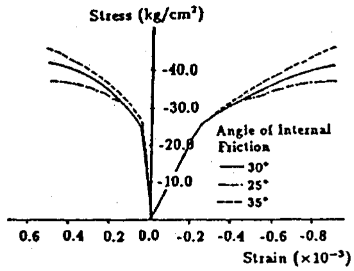

Fig.13 Stress-strain Relation

material constants k and α used in the assumed yield function are dependent on cohesion and angle of internal friction.Therefore, it can be concluded that the cohesion and angle of internal friction of early age concrete have much effect on the yield stress, yield strain and pore water pressure. As shown in **Figs.13** and **14**, uniaxial compressive strength and pore water pressure increases with the increase of the angle of internal friction. Same variation is observed for cohesion also but, not that much as for angle of internal friction.

The effect of loading rate is show in **Fig.17**. Since there is no time to drain for the pore water for rapid loading, pore water pressure increases with the decrease of effective compressive stress. On the other hand, for slow loading rate, this stress component becomes higher since water is allowed to drain.

The variation of permeability is shown in **Fig.18**, but it gave considerable effect only on pore water pressure and not for stress-strain relation.

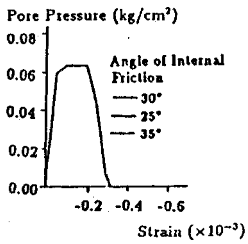

Fig.14 Pore Pressure-strain Relation

(2) Uniaxial Compressive Test

Uniaxial Compressive Test was performed to verify the validity of the proposed model which predicts the deformational characteristic of early age concrete at different age of 12, 24, 36 and 48 hrs. The mix proportions of the test specimen used in the test is shown in **Table 1**. The obtained

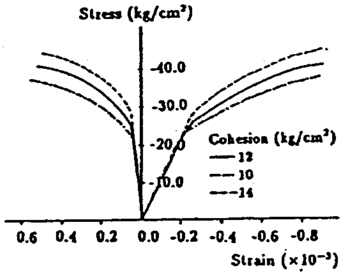

Fig.15 Stress-strain Relation

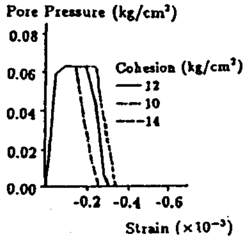

Fig.16 Pore Pressure-strain Relation

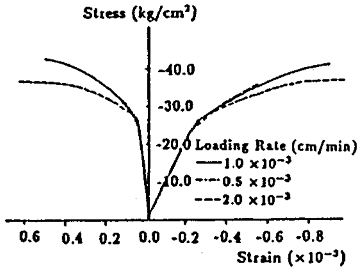

Fig.17 Stress-strain Relation

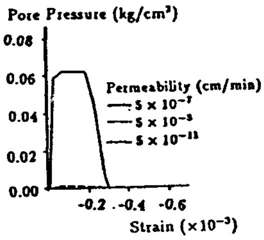

Fig.18 Pore Pressure-strain Relation

Table 1 Mix Proportions of the Test Specimen

Max. agg. size (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Units (kg/m^3)			
					W	C	S	G
25	7	2	55.0	40.9	189	344	724	1049

compressive strength of concrete at 12, 24, 36 and 48 hrs. is 10 kgf/cm^2, 51 kgf/cm^2, 73 kgf/cm^2, and 99 kgf/cm^2, respectively. From this test, it is discovered that the plastic behaviour occurs at about one half of the uniaxial compressive strength. However, no plastic strains appears in the first one third region of the uniaxial compressive strength. Hence, it can be considered that elastic behaviour occurs in this region. This implies that yielding has taken place within the region of 1/3 to 1/2 of the uniaxial compressive strength. The value of plastic strain became one tenth of the total strain at the point of two third of the strength. When the strain takes the value of 1.1×10^{-3}, stress became constant.

(3) Comparison of Numerical results

The results obtained are shown in **Figs.19** and **20** and show a good agreement with calculations. In this calculation, the values of the constants 'a' and β at each time were selected as 1.0 and 0.65, respectively. From these, it can be seen that the results are identical in the elastic region for any time and hence this implies that the assumed elastic deformation for aggregates is reasonable. But it can be suggested that a proper evaluation on the real value of the material ratio and the Young's modulus is necessary.

Although the angle of internal friction and cohesion has much influence on the yield point, in the elastic region since the value of ϕ is made to zero the material constants of Drucker-Prager failure criterion α became zero and only the other material constant k influences on the failure criterion. Hence, failure criterion became a function of stress state, and the value of k and the yield point varied according to the value of cohesion. However, the results agree in the vicinity of the yield point also and hence, the assumed variations of cohesion and angle of internal friction were reasonable.

The behaviour after yielding is largely affected by the values of parameters β and 'a'. The results agreed well until the point of uniaxial compressive strength. Since the variation of cohesion and angle of internal friction have much influence on plastic strain rather than damage parameter, in the plastic region the results show a good agreement.

Even if the effect of temperature caused by hydration of cement has influence on the stress analysis of early age concrete, it was neglected for simplicity.

Due to the lack of experimental information on the effect of pore water pressure, a good comparison cannot be given, but it was found that the value of pore water pressure is almost zero in the elastic region and developed in the plastic region. But, this value was less than the one-tenth of applied compressive stress.

5. CONCLUSION

In this study a constitutive relation for early age concrete which is modeled as a two phase elasto-plastic material with the consideration of the effect of rapid hydration of cement is modeled and the evaluation of the various model parameters and the accuracy are verified experimentally. Triaxial compressive test was performed to determine the various parameters of cement paste at different ages.

From the numerical results, it is understood that the constitutive relation of early age concrete can be obtaned by considering the effect of cohesion and angle of internal friction of cement paste which

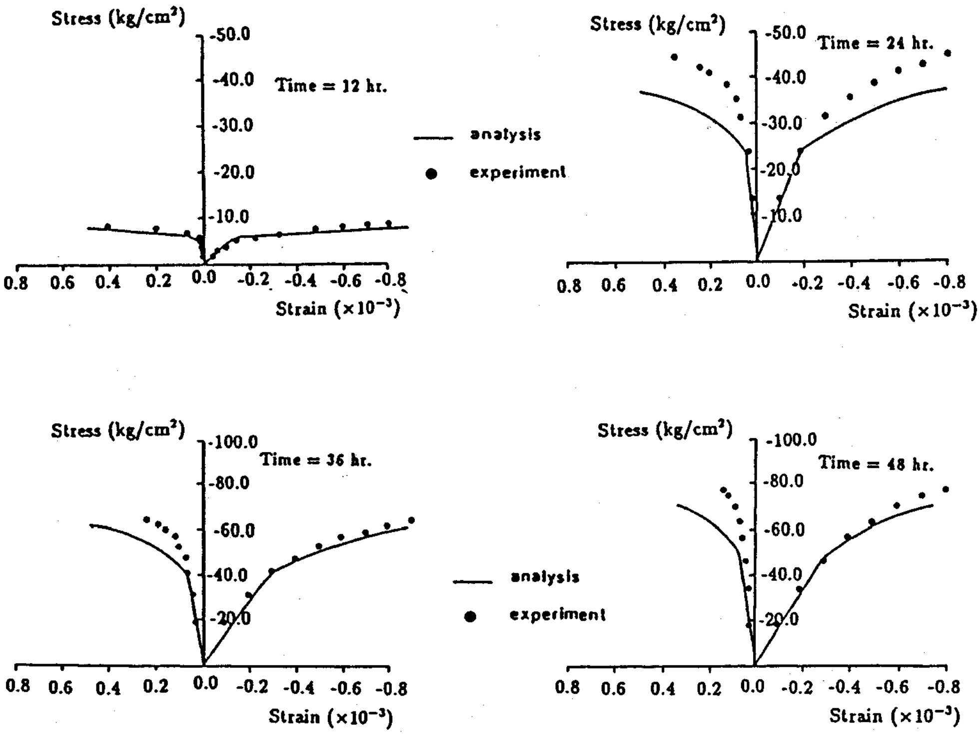

Fig.19 Stress-strain Relation

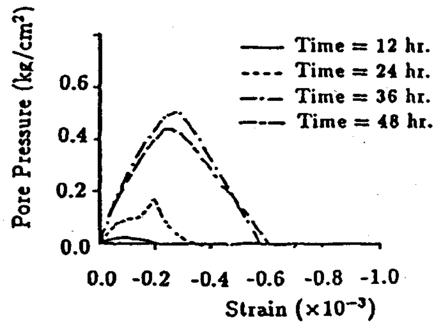

Fig.20 Pore Pressure-strain Relation

with time. The failure of early age concrete is mainly influenced by cohesion of cement paste which increases with time while the angle of internal friction remained almost unchanged. The pore water pressure is almost zero in the elastic region and is developed in the plastic region having the value which is less than one-tenth of the applied compressive stress.

Since the real effect of age on some of the material parameters were not included in the present analysis, a detailed experimental investigation is necessary. However, this present model can be used to predict the behaviour of early age concrete when a full investigation on material parameters is completed.

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REFERENCES

1) Wu, Z. S. and Tanabe, T : A Hardening-softening Model of Concrete Subjected to compressive Loading, Journal of Structural Engineering, Architectural Institute of Japan, Vol.36B, pp.153~162, 1990.
2) Contri, L., Majorana, C. E. and Schrefler, B. A. : Proceeding of International Conference on Concrete of Early Ages, Vol.1, 1982, pp.193~198. (Ecole Nationale des Ponts et chaussees, Paris 6-7-8 April, 1982)
3) Lewis, R. W. and Schrefler, B. A. : A Fully Coupled Consolidation Model of the Subsidence of Venice, Water Resources Research, Vol.14, pp.223~230, 1978.
4) Chen, W, F, : Plasticity in Reinforced Concrete, McGraw-Hill Book Company, 1982.
5) Mats, E. : Thermal Stresses in Concrete Structures at Early Ages, Thesis Presented to the Lela University of Technology, Requirement for the Degree of Doctor of Engineering in 1989.

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粘弾性多孔連続体としての若材令コンクリートのモデル化

Denzil LOKULIYANA・井上　健・田辺忠顕

本研究では，若材令コンクリートを弾性体である骨材と水で飽和された空隙を有する弾塑性透水体としてのセメントペーストからなる２相材料としてモデル化し，その変形挙動の定式化を行った．塑性の定式化は，時間の経過に伴う水和の程度に応じて変化する Drucker-Prager タイプの降伏基準を導入して行った．最終的には，提案したモデルの有限要素解析を試み，実験値との比較検討を行うことでその妥当性を確かめた．