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ON QUADRATIC APPROXIMATION METHOD FOR OPTIMUM DESIGN OF TRUSS STRUCTURES

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Structural optimization method using the quadratic approximation functions for the structural responses are studied. There are two kinds of the design variables in the shape optimization of truss structures. For the sectional variables, the linear approximation functions are used for the calculations of the structural responses. For the geometric variables, the quadratic approximation functions are proposed. The functions are approximately generated by using the elements on the main diagonal and ignoring the off-diagonal elements of the Hessian. Several numerical examples show the reliability and the efficiency of the method proposed

Keywords : *quadratic approximation function, structural optimization, move-limits, geometric variables, structural optimization system*

1. INTRODUCTION

It has been a long time since the approximation concepts for the structural optimization began to study. The approximation concepts, in a wide meaning, involve the linking of the design variables, the screening of the constraints and the approximation of the functions. This paper studies the approximation of the functions.

Generally speaking, there are two kinds of the approximation methods. One is the method presented by Schmit and Farshi[1] and Schmit and Miura[2),3)]. In this method, the sub-problem approximated by the linear functions with respect to the reciprocal variables is solved by the method of the inscribed hyperspheres or the method of extended penalty function method. The new approximation method presented by Vanderplaats and Salajegheh[4] can be included in this category. These methods lay emphasis rather on the quality of the approximation function.

Other methods were presented by Schmit and Fleury. In these methods, objective and constraints are approximated by the simple separable function and the sub-problem is solved applying the dual theorem. The methods are called as dual method[5),6)] and new dual method[7]. The newly presented MMA[8] (method of moving asymptotes) can be included in this category. These methods lay emphasis on the separability of the approximation function and make it possible to apply the dual theorem effectively to solving the sub-problem.

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In structural optimization, most of the computing time is occupied by the structural analyses. Many structural analyses are necessary during the optimization process. For the sake of this reason, approximation methods have been proposed to replace an exact analysis by the approximation calculations. In approximation techniques, the exact analysis is carried out only for the generation of the approximation formulation. It is expected, in this case, that the approximation functions are simple and the quality of the functions are high. The high quality means, in this paper, that the structural responses calculated by the approximation functions are close to the values calculated by the exact structural analysis. Now, in different structural system design, how to make a high quality approximation function has become the one of the big themes in the application of structural optimization.

As written above, based on the concept of approximation, dual method and new dual method have been produced. New dual method is more applicable than dual method and be said as more general. But for the sake of the generality, the method has some unsatisfactory points, such as unsatisfactory agreement to original function, and too conservative approximation, which lead to the convergency of optimization getting poorer.

In the optimum design of structures, the design variables usually involve the variables dealing with the sectional sizes (abbreviated as sectional variables after) and the geometric variables dealing with the coordinates of joints. As reported already, the displacements and the stresses of the truss structures can be regarded as linear functions with respect to the reciprocal variables of the sectional variables. The partially approximating methods[9] using these approximation functions gave the good results in the truss optimization where the design variables are the sectional variables only.

On the other hand, various studies on the approximation functions with respect to the geometric variables have been done[10]. It was concluded that the stress and the displacement were nonlinear functions of the geometric variables, and the high quality approximation formulations were not readily available by using only the first order information. Therefore a method using the second order information is presented here. This study is based on the principal that the more terms of the Taylor series expansion give the better agreement with the original function. The second order information is provided by the Hessian where only the elements on the main diagonal are used. The reason for using only the elements on the main diagonal is to consider the computer storage space and the expensive calculating time. This paper, laying emphasis on the quality of the approximation function, studies the method using the quadratic approximation functions (abbreviated as the quadratic approximation method after) to improve the reliability of the convergence of the structural optimization.

2. LINEAR APPROXIMATION FUNCTIONS

Before giving an explanation of the new approximation method, the linear approximation functions using in dual method and new dual method are investigated.

As an example, following cubic function is considered. The function is represented in **Fig.1** by a thin curved line.

$$g(x)=\frac{1}{8}(x-4)(x^2-8x+4) \cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

In **Fig.1**, the curved line of equation (1) is divided into four parts, A to D, by their curvatures and the gradients.

In dual method, the approximation function of $g(x)$ is formulated as follows,

$$\bar{g}(x)=g(x^0)+\left(\frac{dg}{dx}\right)^0 x^0\left(1-\frac{x^0}{x}\right) \cdots\cdots\cdots\cdots \quad (2)$$

Also, in new dual method, the approximation function of $g(x)$ is formulated as follows,

$$\bar{g}(x)=\begin{cases} g(x^0)+\left(\frac{dg}{dx}\right)^0(x-x^0) : \left(\frac{dg}{dx}\right)^0 \geqq 0 & \cdots\cdots\cdots \text{(3-a)} \\ g(x^0)+\left(\frac{dg}{dx}\right)^0 x^0\left(1-\frac{x^0}{x}\right) : \left(\frac{dg}{dx}\right)^0 < 0 & \cdots\cdots\cdots \text{(3-b)} \end{cases}$$

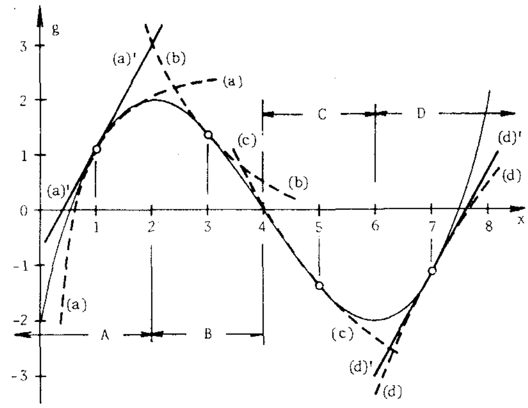

Fig.1 A cubic function and its linear approximation functions.

where, $\left(\frac{dg}{dx}\right)^0$ is the gradient of $g(x)$ corresponding to $x=x^0$.

These approximation functions are generated at four approximation points, $x^0=1$, 3, 5, and 7. Equation (2) to equation (3-b) are calculated around these four approximation points and shown in **Fig.1** also. In this figure, equation (2) and equation (3-b) are represented by broken lines and equation (3-a) is represented by thick lines. This figure shows the relation of the approximation functions to the original function [equation (1)]. As equation (2) is for dual method, and equation (3-a) and equation (3-b) are for new dual method, it can be considered that the original function is approximated by lines (a), (b), (c) and (d) around each approximation point in dual method, and by lines (a)′, (b), (c) and (d)′ in new dual method.

Following three matters are given from this figure.

① The approximation functions of dual method almost agree with the original function in region C and A. ② The approximation functions of dual method and new dual method disagree with the original function in region B and D, and in region A, B and D respectively. ③ The approximation functions of new dual method are convex in all regions and more conservative than those of dual method.

When dual method was published, it was applied successfully to the minimum weight design of truss structures where only the sectional variables were the design variables. It is well known that the relation of the stresses or the displacements of truss structures to the sectional variables are similar to the curved line in region C of **Fig.1**. So, it can be said, as one of the reasons of the initital success of dual method, that it owed to the numerical examples taken.

As shown in **Fig.1**, the regions where the linear approximation functions almost agree with the original function are limited to the neighbourhood of each approximation point. So, in the optimization methods based on the linear approximation, SLP (sequential linear programming), dual method and new dual method, the move-limits play an important part. Generally speaking, the move-limits are the useful and so important parameters, but, have some difficulties from the standpoint of the numerical calculations. For example, ill-suited initial values of the move-limits lead to the very slow convergence or unusable design. It appears that the optimization method without the move-limits or the method where the move-limits don't play so important part is preferable.

3. SHAPE OPTIMIZATION OF TRUSS STRUCTURES AND PARTIALLY APPROXIMATING METHOD

The shape optimization problem of truss structures of this paper is defined as follows.

○ Objective : $O(X, Y)=\sum_{i=1}^{N} L_i(Y)x_i \rightarrow \min$ ……… (4)

○ Constraints :

stresses $g^s=\sigma_{ik}(X, Y)-\sigma_{ai}\leqq 0$ $(i=1\sim NM, k=1\sim NLC)$ ……… (5)

displacements $g^d=\delta_{jk}(X, Y)-\delta_a\leqq 0$ $(j\in P, k=1\sim NLC)$ ……… (6)

slenderness ratio $g^r=r_i(x_i, l_i)-r_a\leqq 0$ $(i=1\sim NM)$ ……… (7)

upper and lower $x_i^L\leqq x_i\leqq x_i^U$ $(i=1\sim N)$, $y_j^L\leqq y_j\leqq y_j^U$ $(j=1\sim M)$ ……… (8)

○ Design variables :

$X=\{x_1, x_2,\cdots, x_N\}$, $Y=\{y_1, y_2,\cdots, y_M\}$ ……… (9)

where, O is the objective, g^s are the stress constraints, g^d are the displacement constraints, g^r are the constraints on slenderness ratios. x_i is the sectional variable and is sectional area in this paper, N is the number of the sectional variables, y_j is the geometric variable, M is the number of the geometric variables. L_i is the sum of the lengths of the members which are linked to the i-th sectional variables. σ_{ik} is the stress of the i-th member in k-th loading condition, δ_{jk} is the displacement of the j-th freedom in k-th loading condition, r_i is the slenderness ratio of the i-th member. σ_{ai} is the allowable stress of the i-th member, and when considering the buckling, is the function of the sectional area and the member length. δ_a is the allowable value of the displacement, r_a is the upper value of the slenderness ratio, NM is the number of the members, NLC is the number of the loading conditions, P is the set of the freedoms those displacements are constrained. x_i^L, x_i^U, y_j^L, y_j^U are the lower and the upper limits of x_i and y_j respectively.

The optimization problem formulated by equation (4) to equation (9) is called as primal problem in this paper. In dual method or new dual method, all of the functions in equation (4) to equation (7) are approximated by the linear functions with respect to the reciprocal or/and direct variables. On the other hand, in the partially approximating method[9] of this paper, only the terms those are related to the structural analysis, σ_{ik} of equation (5) and δ_{jk} of equation (6), are replaced by the approximation functions. Equation (5) and equation (6) of the primal problem are replaced by the following equations in the sub-problem of r-th iteration.

$$\bar{g}^{s(r)}=\bar{\sigma}_{ik}(X, Y, X^{(r-1)}, Y^{(r-1)})-\sigma_{ai}\leqq 0 \quad (i=1\sim NM, k=1\sim NLC) \cdots\cdots (10)$$

$$\bar{g}^{d(r)}=\bar{\delta}_{jk}(X, Y, X^{(r-1)}, Y^{(r-1)})-\delta_a\leqq 0 \quad (j\in P, k=1\sim NLC) \cdots\cdots (11)$$

where, $\bar{\sigma}_{ik}$ is the approximation function of the stress and $\bar{\delta}_{jk}$ is the approximation function of the displacement. $X^{(r-1)}$, $Y^{(r-1)}$ are the optimum design of the previous iteration and are the approximation points for this iteration. In this formulation, the allowable stresses are considered as constant values.

In this approximation method, as shown in the above, only the terms those are related to the structural analysis are approximated and the other terms are left as they were in the primal problem. So, if the stresses and the displacements are approximated by the functions those are simple and agree with the true structural responses well, it can be expected that the efficiency of the optimization procedure is improved.

4. APPROXIMATION FUNCTIONS OF STRESSES AND DISPLACEMENTS

The stress approximation functions of equation (10) and the displacement approximation functions of equation (11) are not necessary to be separable in the approximation method of this paper. It is to be desired that the quality of these functions are high in their agreement with the true structural responses and the functions are easy to calculate.

The approximation functions which satisfy those requirments are thought to be different according to the kinds of their variables. The approximation functions with respect to the sectional and geometric variables are presented in the following sections respectively.

(1) Approximation Functions with Respect to Sectional Variables

The stresses and the displacements of truss structures were generally approximated by the linear functions with respect to the reciprocal variables. But, from the numerical and the theoretical studies[11),12)], it was concluded that the force approximations with respect to the reciprocal variables were superior to the others for the stress approximations. So, in this paper, following approximation functions are used in the r-th iteration.

Approximation functions for stresses with respect to the sectional variables ;

$$\bar{\sigma}_{ik}(X,\ X^{(r-1)})=\frac{1}{A_i}\left\{(F_{ik})^{(r-1)}+\sum_{p=1}^{N}\left(\frac{\partial F_{ik}}{\partial x_p}\right)^{(r-1)}x_p^{(r-1)}\left(1-\frac{x_p^{(r-1)}}{x_p}\right)\right\}$$
$$(i=1\sim NM,\ k=1\sim NLC)\cdots\cdots\cdots(12)$$

Approximation functions for displacements with respect to the sectional variables ;

$$\bar{\delta}_{jk}(X,\ X^{(r-1)})=(\delta_{jk})^{(r-1)}+\sum_{p=1}^{N}\left(\frac{\partial \delta_{jk}}{\partial x_p}\right)^{(r-1)}x_p^{(r-1)}\left(1-\frac{x_p^{(r-1)}}{x_p}\right)$$
$$(j\in P,\ k=1\sim NLC)\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

where, A_i is the sectional area of the i-th member, F_{ik} is the axial force of the i-th member in k-th loading condition. $(\cdot)^{(r-1)}$ means the value of the function in the brackets corresponding to $X^{(r-1)}$.

Equation (12) and equation (13) will be used as the approximation functions with respect to the sectional variables in this paper.

(2) Approximation Functions with Respect to Geometric Variables[13)]

The relations of the stresses and the displacements to the geometric variables are not so simple as to the sectional variables. Every four types of the region A to region D of **Fig.1** are to be appeared. It seems clear, from the study on **Fig.1**, that the approximation function based on the first derivatives only has its limit. These will be explained numerically in chapter 5.

For the geometric variables, accordingly, quadratic approximation function is studied and proposed in this paper.

The quadratic approximation function of a multivariable function $f(X)$ corresponding to X^0 is defined as follows,

$$f(X)\fallingdotseq f(X^0)+\sum_{p=1}^{N}\left(\frac{\partial f}{\partial x_p}\right)^0(x_p-x_p^0)+\frac{1}{2}(X-X^0)^T[H]^0(X-X^0)\ \cdots\cdots\cdots(14)$$

where, $[H]^0$ is the Hessian consisting of the second derivatives of $f(X)$. An element of the Hessian is as follows,

$$H_{pq}=\left(\frac{\partial^2 f}{\partial x_p \partial x_q}\right)^0\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

Although the quadratic approximation functions of this paper is formulated according to equation (14), the every terms of the second derivatives of equation (15) are difficult in their numerical calculations and require much computer storage space. So, it is proposed to use only the elements on the main diagonal of the Hessian. In this way, equation (14) becomes :

$$f(X)\fallingdotseq f(X^0)+\sum_{p=1}^{N}\left(\frac{\partial f}{\partial x_p}\right)^0(x_p-x_p^0)+\frac{1}{2}\sum_{p=1}^{N}\left(\frac{\partial^2 f}{\partial x_p^2}\right)^0(x_p-x_p^0)^2\cdots\cdots\cdots\cdots(16)$$

The effect of ignoring the off-diagonal elements will be investigated in the numerical examples.

According to equation (16), the approximation functions for stresses and displacements with respect to the geometric variables are formulated as follows,

Approximation functions for stresses with respect to the geometric variables ;

$$\bar{\sigma}_{ik}(Y,\ Y^{(r-1)})=\frac{1}{A_i}\left\{(F_{ik})^{(r-1)}+\sum_{q=1}^{M}\left(\frac{\partial F_{ik}}{\partial y_q}\right)^{(r-1)}(y_q-y_q^{(r-1)})+\frac{1}{2}\sum_{q=1}^{M}\left(\frac{\partial^2 F_{ik}}{\partial y_q^2}\right)^{(r-1)}(y_q-y_q^{(r-1)})^2\right\}$$
$$(i=1\sim NM,\ k=1\sim NLC)\cdots\cdots\cdots(17)$$

Approximation functions for displacements with respect to the geometric variables ;

$$\bar{\delta}_{jk}(Y,\ Y^{(r-1)})=(\delta_{jk})^{(r-1)}+\sum_{q=1}^{M}\left(\frac{\partial \delta_{jk}}{\partial y_q}\right)^{(r-1)}(y_q-y_q^{(r-1)})+\frac{1}{2}\sum_{q=1}^{M}\left(\frac{\partial^2 \delta_{jk}}{\partial y_q^2}\right)^{(r-1)}(y_q-y_q^{(r-1)})^2$$
$$(j\in P,\ k=1\sim NLC)\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

These functions naturally contain the second derivatives, so, when the derivatives are calculated by the finite difference method, one more additional structural analysis is needed comparing with the calculation of the first derivatives. This means that, if there are m variables, m additional structural analyses are needed for one iteration. Suppose the number of iteration is k, $(m\times k)$ additional analyses are necessary to compare with the linear approximation method. However, on the other hand, high quality approximations generated by this method will improve the reliability and the

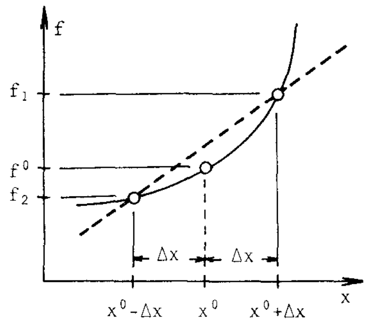

Fig.2 Finite difference calculation

efficiency of the optimization.

(3) Summary of Approximation Functions and Second Derivatives

In the previous sections, the approximation functions with respect to the sectional variables and the geometric variables are presented separately. In this section, these functions are combined and presented. They are as follows.

Approximation functions for stresses ;

$$\bar{\sigma}_{ik}(X,\ Y,\ X^{(r-1)},\ Y^{(r-1)}) = \frac{1}{A_i}\Bigg[(F_{ik})^{(r-1)} + \sum_{p=1}^{N}\left(\frac{\partial F_{ik}}{\partial x_p}\right)^{(r-1)} x_p^{(r-1)}\left(1-\frac{x_p^{(r-1)}}{x_p}\right) + \sum_{q=1}^{M}\left\{\left(\frac{\partial F_{ik}}{\partial y_q}\right)^{(r-1)}(y_q-y_q^{(r-1)}) + \frac{1}{2}\left(\frac{\partial^2 F_{ik}}{\partial y_q^2}\right)^{(r-1)}(y_q-y_q^{(r-1)})^2\right\}\Bigg]$$

$$(i=1\sim NM,\ k=1\sim NLC) \quad \cdots\cdots\cdots (19)$$

Approximation functions for displacements ;

$$\bar{\delta}_{jk}(X,\ Y,\ X^{(r-1)},\ Y^{(r-1)}) = (\delta_{jk})^{(r-1)} + \sum_{p=1}^{N}\left(\frac{\partial \delta_{jk}}{\partial x_p}\right)^{(r-1)} x_p^{(r-1)}\left(1-\frac{x_p^{(r-1)}}{x_p}\right) + \sum_{q=1}^{M}\left\{\left(\frac{\partial \delta_{jk}}{\partial y_q}\right)^{(r-1)}(y_q-y_q^{(r-1)}) + \frac{1}{2}\left(\frac{\partial^2 \delta_{jk}}{\partial y_q^2}\right)^{(r-1)}(y_q-y_q^{(r-1)})^2\right\}$$

$$(j\in P,\ k=1\sim NLC) \quad \cdots\cdots\cdots (20)$$

In equation (19) and equation (20), the first and second derivativese are contained. Even though they can be obtained analytically in some cases of truss structures, they are calculated in this paper by the finite difference method as shown below.

The curved line shown in **Fig.2** represents a relation of a function $f(x)$ to the variable x. f^0 is the value of the function corresponding to x^0. Δx is an increment of x, f_1 corresponds to $x^0+\Delta x$, f_2 corresponds to $x^0-\Delta x$. The derivatives are calculated by the following equations respectively.

The first derivatives with respect to the sectional variables :

$$\frac{\partial f}{\partial x}=\frac{f_1-f^0}{\Delta x} \quad \cdots\cdots\cdots (21)$$

The first and second derivatives with respect to the geometric variables :

$$\frac{\partial f}{\partial x}=\frac{f_1-f_2}{2\Delta x},\quad \frac{\partial^2 f}{\partial x^2}=\frac{f_1-2f^0+f_2}{(\Delta x)^2} \quad \cdots\cdots\cdots (22)$$

The reason why the first derivatives with respect to the geomeric variables are calculated by the central finite difference method is that the three values are calculated for the second derivatives.

Table 1 Loading condition of 25-members truss

nodal point	loding condition 1			loding condition 2		
	Px	Py	Pz	Px	Py	Pz
1	0	22.7	90.6	-4.53	22.7	45.3
2	0	22.7	-90.6	0	22.7	45.3
3	0	0	0	-2.27	0	0
4	0	0	0	-2.27	0	0

(t f)

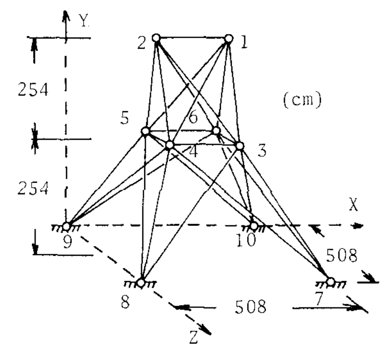

Fig.3 25-members truss

5. NUMERICAL EXAMPLES

Several numerical examples will be shown here to demonstrate the effect of the quadratic approximation method by comparing with the linear approximation method.

(1) Shape Optimization Problem of Truss Structures

Shape optimization problem of truss structures calculated in this paper are explained here.

One is the 25-members truss shown in **Fig.3**. Loading conditions are represented in **Table 1**. Number of design variables is 9 including 3 geometric variables. The geometric variables are x- and z- coordinates of nodal points 3 to 6 and x-coordinates of nodal points 7 to 10. They are linked each other to ensure the structural symmetry.

Table 2 Loading condition of 50-members truss

nodal	loding condition 1		
point	Px	Py	Pz
13	0	50.0	0
14	0	50.0	0

(tf)

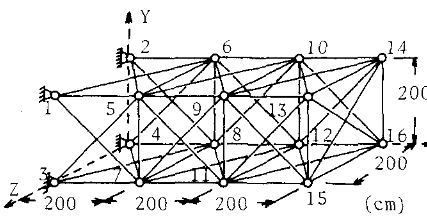

Fig.4 50-members truss

Table 3 Initial values and optimum geometric values of 25-members truss

design variables		case (1)	(2)	(3)	(4)	optimum
nodal point and direction	3,4,5,6-X	10	10	50	50	32
	3,4,5,6-Z	10	10	50	50	85
(cm)	7,8,9,10-X	30	30	50	50	113
sectional area (cm²)		50	100	50	100	——

Table 4 Initial values and optimum geometric values of 50-members truss

design variables		case (1)	(2)	(3)	(4)	optimum
nodal point and direction	7,8-Y	20	20	50	50	3
	11,12-Y	30	30	50	50	15
(cm)	15,16-Y	40	40	50	50	90
sectional area (cm²)		50	100	50	100	——

Sectional areas of 25 members are linked to 6 sectional variables. Four couples of the initial values are given to this problem. They are shown in **Table 3**.

Another one is the 50-members truss shown in **Fig.4**. Loading conditions are represented in **Table 2**. Number of design variables is 7 including 3 geometric variables. The geometric variables are y-coordinates of nodal points 7, 8, y-coordinates of nodal points 11, 12 and y-coordinates of nodal points 15, 16. Sectional areas of 50 members are linked to 4 sectional variables. Four couples of the initial values are given to this problem. They are shown in **Table 4**.

(2) Structural Responses by Quadratic Approximation Functions

In this paper, as an approximation function with respect to the geometric variables, the quadratic functions of equation (17) and equation (18) were proposed. Those functions are generated here in the case of 25 members truss of **Fig.3**, and the values of the functions are compared with the true structural responses and the values calculated by the linear approximation functions.

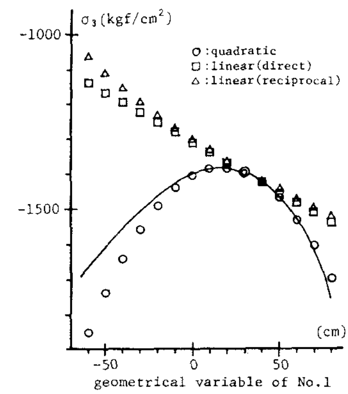

Fig.5 a σ_3-y_1

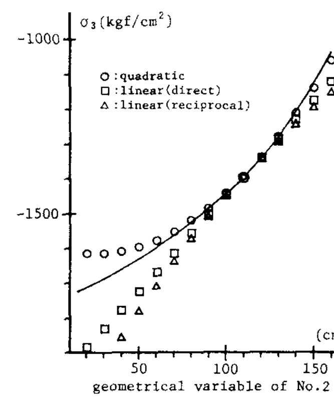

Fig.5 b σ_3-y_2

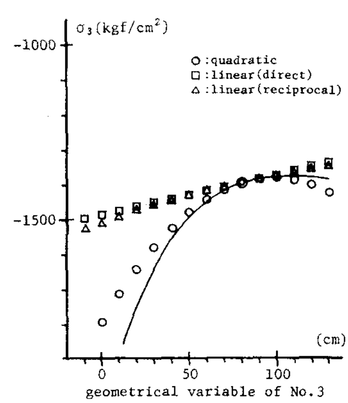

Fig.5 c σ_3-y_3

Fig.5 a to **Fig.5 c** show the relations of the stresses of member-3 (connecting nodal point 1 to 4) to three geometric variables respectively. Rigid line corresponds to the true structural responses, ◎

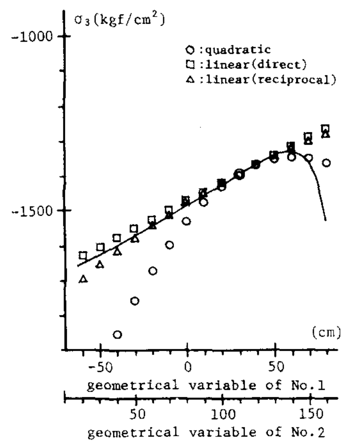

Fig.6 a σ_3-y_1, y_2

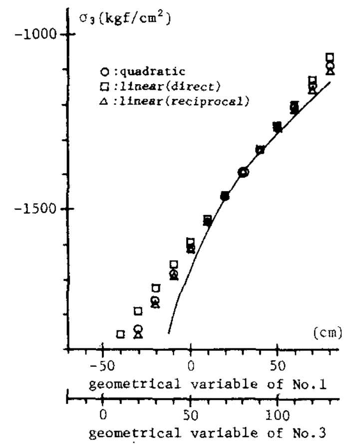

Fig.6 b σ_3-y_1, y_3

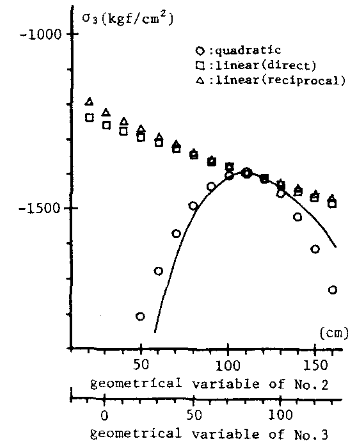

Fig.6 c σ_3-y_2, y_3

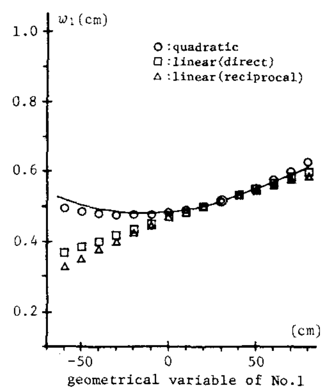

Fig.7 a w_1-y_1

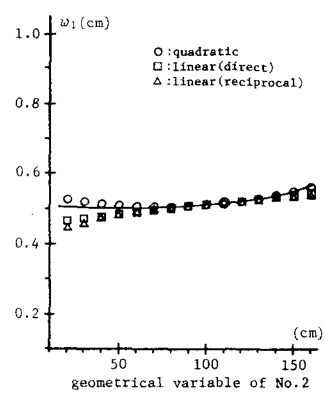

Fig.7 b w_1-y_2

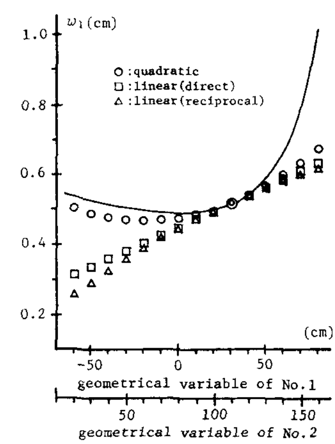

Fig.8 w_1-y_1, y_2

represent the approximation points. The values calculated by each approximation function generated at these approximation points are shown also in this figure. ○ corresponds to the quadratic approximation, □ corresponds to the linear approximation with repect to direct variable, and △ corresponds to the linear approximation with respect to reciprocal variable.

From these figures, it can be said that the quadratic approximation function shows the better agreement with the true structural responses than the linear approximation.

In these figures, however, the responses with respect to only one variable were shown. The better agreement mentioned above is, in a sense, a natural result, because only the main diagram of Hessian is considered and the other second derivatives are neglected in the quadratic approximation function. So, next, two variables are changed simultaneously and the stresses with respect to the variables are calculated. **Fig.6 a** to **Fig.6 c** show the results. **Fig.6 a** shows the stresses with respect to the geometric variable 1 and 2, **Fig.6 b** and **Fig.6 c** show the relations coresponding to the variable 1 and 3, and 2 and 3 respectively. Generally speaking, even if the two variables are changed simultaneously, the quadratic approximation function show good agreement.

In the same manner as the above, the displacements of a nodal point 1 in z-direction is calculated and the results are shown in **Fig.7 a**., **Fig.7 b** and **Fig.8**. **Fig.7 a** and **Fig.7 b** show the results with respect to one variable, and **Fig.8** shows the results with respect to two variables.

These are the part of the results calculated in this study. It can be said that the quadratic approximation functions of this paper show the good agreement and are better than the linear approximation functions.

(3) Results of Optimization

In the quadratic approximation method of this paper, the second derivatives with respect to the

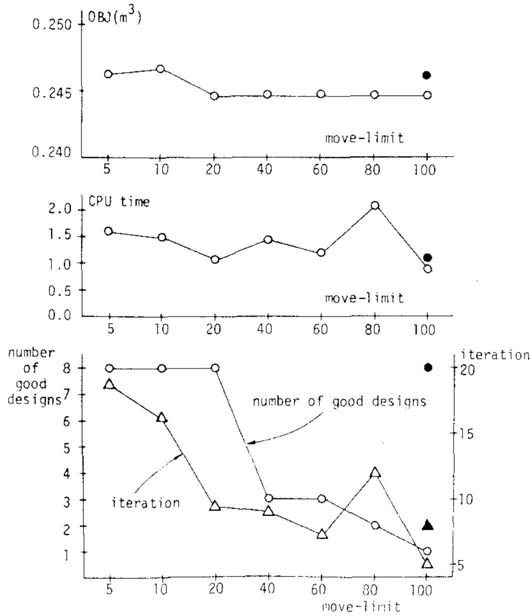

Fig.9 a Optimization results of 25-members truss.

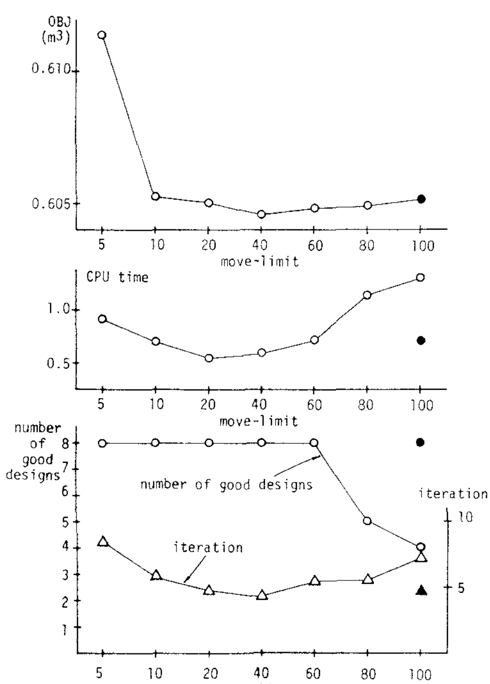

Fig.9 b Optimization results of 50-members truss.

geometric variables must be calculated. On the other hand, the quality of the approximation function is improved by introducing the second order information, when applying them to the structural optimization, the reduction of the number of the iteration and the improvement of the convergence reliability can be expected. Also, the move-limits are expected to be unnecessary in this approximation method.

In this section, quadratic and linear approximation method are applied to the shape optimization problem of the truss structures described above. Both of the scaled problem[14] and nonscaled problem are solved for each initial value. The results of the optimization of 8 cases are arranged and compared. For the linear approximation method, optimum design is calculated under the several values of the move-limits on the geometric variables, they are 5 cm, 10 cm, 20 cm, 40 cm, 60 cm, 80 cm, and 100 cm. For the quadratic approximation method, the value of the move-limit is fixed to 100 cm. This optimization problem with the move-limit being 100 cm is almost equal to the problem with no move-limit. The results of optimum design of 25-members truss and 50-members truss are shown in **Fig.9 a** and **Fig.9 b** respectively. ○ and △ represent the results by linear approximation method, and ● and ▲ represent the results by quadratic approximation method. The axis of abscissas corresponds to the values of move-limits.

Lower figure shows the two kinds of the results. One is the number of the cases, where the feasible designs are obtained, among 8 cases described before, and the other is the average number of the iterations in which the feasible designs are obtained. The former corresponds to the left axis of ordinates and the latter corresponds to the right. Middle figure shows the average CPU time (sec) and upper figure shows the average values of the objective. In these figures, only the cases in which the feasible designs were obtained are considered. Among these four values presented in these figures, the number of the cases of the lower figure and average value of the objective will be the parameters to estimate the reliability of the convergence, and the average number of the iterations and the average CPU time will be the paremeters to estimate the efficiency.

From these figures it is concluded that the linear approximation method is a good method in so far as the value of the move-limit is chosen reasonably. However it is also pointed out that the reliability and the efficiency of this method highly depend on the move-limit. On the other hand, the results by the quadratic approximation method are almost same with the best one among the results by the linear approximation method for the various values of the move-limits. Taking into consideration that the move-limit is basically unnecessary for this quadratic approximation method, it can be concluded that the method is in no way inferior to the

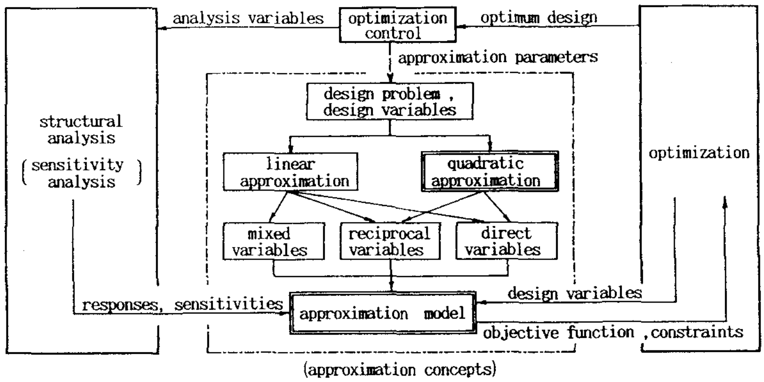

Fig.10 Structural optimization system using approximation method.

linear approximation method with move-limits.

The optimum values of the geometric variables are shown in the right columns of **Table 3** and 4 respectively.

6. STRUCTURAL OPTIMIZATION SYSTEM USING APPROXIMATION METHOD

In this chapter, the structural optimization system including the approximation method of this paper is proposed.

In structural optimization, the approximation method in which the sub-problems consisting of some approximation functions are solved iteratively has better efficiency and reliability than the method in which the primal problem is solved directly by some mathematical programming, SLP, GRG, etc[10),13)]. Especially, for the optimum design of the large scaled structures, the approximation method is considered to be the indispensable techniques.

Design variables can be classified into two parts from the standpoint of the approximation. One is the variable like the sectional variable of framed structure. The behaviour of the structure in regard to these variables are simple and well known, and so the linear approximation function with respect to these variables shows the better agreement with the true structural responses.

And the other is the variable like the geometric variable studied in this paper. The behaviour of the structure in regard to these variables are not so simple, and the linear approximation function with respect to these variables does not always show the better agreement with the true responses. For these variables, quadratic approximation function is recommended.

For the optimum design of the structures with several kinds of the design variables, the general purpose structural optimization system shown in **Fig.10** is proposed.

Although the basic structure of this system is same as the other system of approximation concepts, but, in this system, the most reasonable approximation function can be selected depending on the relation of the structural responses to the design variables.

7. CONCLUDING REMARKS

In structural optimization, the use of the quadratic approximation function partially was proposed. The method was applied to the shape optimization of truss structures and the results were compared with the results by the linear approximation method.

The conclusions are as follows :

(1) In structural optimization, the method using the quadratic approximation functions for the structural responses with respect to the geometric variables were proposed.

(2) The quadratic approximation function of this paper was generated by using only the elements on the main diagonal of the Hessian to reduce the required computer storage space and the computing time.

(3) Numerical calculation of 25-members truss showed that the quadratic approximation function represented the better agreement with the true structural responses than the linear approximation function.

(4) Both of the linear approximation function and the quadratic approximation function were applied to the optimum design of 25-member truss and 50-member truss, and the results were compared. As a result, it was given that, in so far as the reasonable values were chosen for the move-limits, the linear approximation method was good method, but the reliability and the efficiency of the method highly depend on the move-limits. It was also given that the quadratic approximation method showed the

almost best results among the results by the linear approximation method.

(5) By taking into consideration that the move-limits were basically unnecessary for this quadratic approximation method, the quadratic function seemed to be superior as the approximation function with respect to the variables like the geometric variables to the linear approximation function.

(6) This method can be applied not only to the truss structures but also to the framed structures and the shape optimization of the continuum. For those purposes, the general optimization system using approximation method was proposed also.

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2次近似法によるトラス構造物の最適設計に関する研究

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構造物の最適設計において，近似法を用いることは常識になってきている．トラス構造物において，設計変数が断面積であれば，構造応答は1次近似式によりある程度近似可能であるが，幾何変数に対しては十分な近似度は得られない．その結果最適化の過程がムービリミットに大きく依存することになる．そこで，幾何変数に関しては2次近似関数を用いることを提案し，数値計算によりその信頼性，有効性を示している．