Proc. of JSCE No. 433/V-15, pp. 231~238, August 1991

DEVELOPMENT OF MICROPLANE MODEL OF CONCRETE WITH PLURAL TYPES OF GRANULAR PARTICLES

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A fracture or damage model for monotonic behavior of plain concrete using microplane mechanism is developed. The microplane is defined as the contact surface between particles inside the material. Plain concrete is idealized to contain two types of particles which are aggregate and mortar particles in contrast to the Bazant's single particle model. It is assumed that the normal stress on the microplane of any orientation within the material is a function of the normal strain on the same microplane. This strain is assumed to be equal to the resolved component of the macroscopic strain tensor. In addition, shear strains are considered on the same microplanes. The model can represent experimentally observed macroscopic softening or damage behavior of concrete under the effect of different types of loading conditions.

Keywords : *concrete, fracture, softening, microplane, aggregate and mortar*

1. INTRODUCTION

In investigating the characteristics of concrete, which is composed of granulates in brittle cement paste matrix, a large research effort has been devoted to its constitutive modeling. The commonly used are the macroscopic models such as classical plastic models, deformation theory models, fracture models and continuum damage models. Despite of the significant initial success during the last 20 years, however, the macroscopic approach showed gradually its limitations. As pointed by Bazant *et al.*[3),4)], probably only relatively minor further improvement can be expected depending on the expanding experimental data. The main reason is the difficulty to deal in general with the microcracking and the actual failure of microstructure due to the fact that the microcracking make characteristics of concrete of a very complex nature with dilatancy, strain localization, softening and all other sorts of nonlinear phenomena. We can not expect to model all these aspects of the material by unified general theory, although some investigators recently tried to introduce microscopic consideration or damage parameters due to microcracking to the macroscopic models (Mazar[11)], Wu and Tanabe[12)]). Analyzing the material through the micromechanism seems to be rational and powerful because it enables us to look into far insight of the true damage or failing mechanisms of concrete under the effect of different types of monotonic and repeated loading. For these reasons, it appears preferable to describe the behavior of the material not globally but individually for the contacts between the particles of various orientation within the material and then superimpose the contributions from all the contact planes.

Micromechanical models were also analyzed for different kinds of materials such as metals, concrete and granular materials. Routhenburg *et al.*[1),2)], Chang *et al.*[5),6)] and Christoffersen *et al.*[7)] investigated the micromechanical behavior of granular materials. In their studies, only one kind of particle and one kind of contact were considered. Bazant *et al.*[3),4)] used the microplane model to obtain tensile strain softening curves of concrete. In his model, concrete has been assumed to have only one kind of microplane, which exists at the contact surfaces between aggregates. This assumption is not practically true because the main source of the damage or softening is the spreading of the microcracks into the mortar through the contact surface between aggregate and mortar. Moreover, in his analyses, only the normal microstrain was considered and an additional volumetric elastic strain was introduced to adjust the value of Poisson's ratio.

In the present study, as a continuation of modeling concrete through the microstructure, the development of the microplane model is investigated. To represent the different kinds of microplanes, concrete is idealized to have two types of particles, which are aggregate and mortar particles (refer to **Fig.1**). This idealization helps us to investigate the micromechanical behavior not only on the contact between aggregates but also on the contact between aggregate and mortar. In addition, to explain the behavior more reasonably, shear

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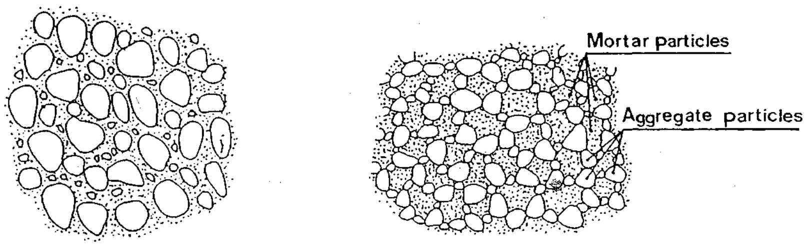
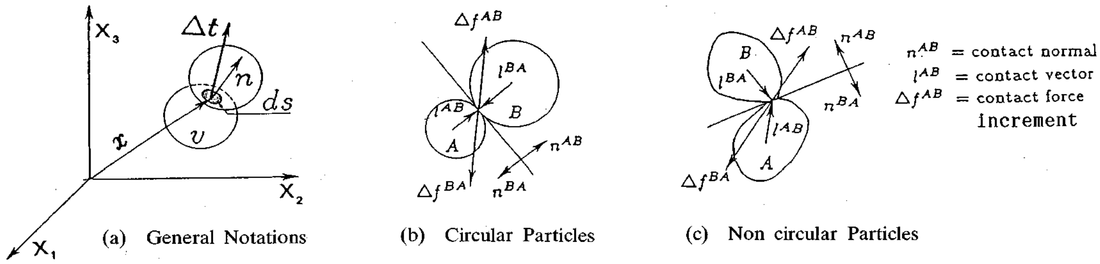


(a) Actual Distribution of Particles (b) Idealized Distribution of Particles

Fig.1 Distribution of Particles in Concrete.



(a) General Notations (b) Circular Particles (c) Non circular Particles

Fig.2 Contact Force, Contact Normal and Contact Vector.

strain on the microplane is considered. The current formulation has been shown to be capable of modeling the available macroscopic data of concrete.

2. THEORETICAL CONSIDERATION

(1) Generalization and idealization

The overall macroscopic stress in concrete is examined by considering the contact forces that transmitted by the contacting particles at the microscale. Concrete is idealized to be composed of two types of particles which are aggregate and mortar particles. In addition, every mortar particle is assumed to be surrounded by a specified number of aggregate particles as illustrated in **Fig.1**(b).

(2) Average stress tensor from averages of contact forces

If a region v is subjected to an incremental stress of $\Delta\sigma_{ij}$ and the stress state of this region is in equilibrium, the average stress increment $\Delta\bar{\sigma}_{ij}$ is defined[1),2),5)~7)] as :

$$\Delta\bar{\sigma}_{ij}=\frac{1}{v}\int_v \Delta\sigma_{ij}dv \quad\cdots\cdots(1)$$

Using this definition, the average stress increment for aggregate and mortar particles is written as :

$$\Delta\bar{\sigma}^a_{ij}=\frac{1}{v^a}\int_v \Delta\sigma_{ij}dv \quad\cdots\cdots(2)$$

$$\Delta\bar{\sigma}^m_{ij}=\frac{1}{v^m}\int_v \Delta\sigma_{ij}dv \quad\cdots\cdots(3)$$

where v^a and v^m are the volume of an aggregate particle and a mortar particle respectively. Using the equilibrium condition (i.e. $\Delta\sigma_{ij,j}=0$) and the divergence theory, the volume average integral in Eq.(1) can be converted into surface integral. The average incremental stress tensor $\Delta\bar{\sigma}_{ij}$ is reduced as follows :

$$\Delta\bar{\sigma}_{ij}=\frac{1}{v}\int_v \Delta\sigma_{ij}dv=\frac{1}{v}\int_v \Delta\sigma_{ik}x_{j,k}dv$$
$$=\frac{1}{v}\int_s \Delta\sigma_{ik}x_j n_k ds=\frac{1}{v}\int_s \Delta t_i x_j ds \quad\cdots\cdots(4)$$

where v is the volume of the particle, s is the boundary surface of the particle, x_j is the coordinates of a point on s, n_k is the normal on s at x_j and Δt_i is the traction increment on s at x_j as shown in **Fig.2**(a). For a finite number of contact points, the surface integral of the boundary contact forces increments can be replaced by the summation of these contact forces increments. Eq.(4) can then be written in the form :

$$\Delta\bar{\sigma}_{ij}=\frac{1}{v}\sum_{m=1}^{n}\Delta f_i^m x_j^m \quad\cdots\cdots(5)$$

where n is the number of contacts per particle, x_j is the contact vector (position vector) for the m contact and Δf_i is the contact force increment at the m contact. The contact vector can be measured from the centroid of the particle to the contact point[1),2),5),6)]. The average stress increment within the aggregate and mortar particles can be represented as :

$$\Delta\bar{\sigma}^a_{ij}=\frac{1}{v^a}\sum_{\alpha_1}\Delta f_i^{\alpha_1} l_j^{\alpha_1} \quad\cdots\cdots(6)$$

$$\Delta\bar{\sigma}^m_{ij}=\frac{1}{v^m}\sum_{\alpha_2}\Delta f_i^{\alpha_2} l_j^{\alpha_2} \quad\cdots\cdots(7)$$

where α_1 and α_2 are the notation to identify each contact point along the boundary of the aggregate and mortar particles respectively. Terms Δf_i^α and l_j^α refer to the scalar components of the contact forces increments Δf^α and contact vectors l^α at the contact locations (refer to **Fig.2**(b) and(c)).

Furthermore, we will use volume average to define the incremental mean stress for any representative volume by summing the stresses increments for all particles within this volume as follows :

$$\Delta\bar{\sigma}_{ij}=\frac{1}{V}\sum_{\alpha}\Delta\bar{\sigma}_{ij}^{a}v^{a}+\frac{1}{V}\sum_{m}\Delta\bar{\sigma}_{ij}^{m}v^{m} \quad\cdots\cdots(8)$$

Eq.(8) can be simplified to :

$$\Delta\bar{\sigma}_{ij}=\frac{1}{V}\left[\sum_{c1}\Delta f_i^{c1}l_j^{c1}+\sum_{c2}\Delta f_i^{c2}l_j^{c1}+\sum_{c2}\Delta f_i^{c2}l_j^{c2}\right] \quad\cdots\cdots(9)$$

where c_1 is the total number of contacts between aggregate particles, c_2 is the total number of contacts between aggregate and mortar particles, and V is the total volume. This equation is for large but finite number of particles. In Eq.(9), the first part, which represents the contribution of the contacts due to aggregates has two components. That is because of the assumption that every aggregate particle have two different kinds of contact points, which are the contacts between two aggregates and the contact between an aggregate and mortar particles. On the other hand, the second part of that equation which represents the contribution of the mortar has only one component because we have assumed that only one kind of contact exists around every mortar particle. This single contact is between aggregate and mortar particles.

The total number of contacts do not take into account the orientation of the contact normals. Because the case of two dimensional system is considered in the current study, a function $E(\theta)$ which considers the relative frequency of contacts with different orientations of normals will be defined. The number of contacts with normals between θ and $\theta+\Delta\theta$ will be $cE(\theta)\Delta(\theta)$. This function satisfies the next expression :

$$\int_0^{2\pi}E(\theta)d\theta=1 \quad\cdots\cdots(10)$$

For isotropic granular assemblies, Rothenburg[1),2)] defined the orientational distribution function $E(\theta)$ as :

$$E(\theta)=\frac{1}{2\pi} \quad\cdots\cdots(11)$$

Since the current study is restricted to isotropic material, the definition proposed by Rothenburg[1),2)] will be used.

Unfortunately, the calculation of the average incremental stress tensor using Eq.(9) requires exact knowledge of contact forces increments and contact vector terms for all particles. Equivalent simpler expressions can be developed by considering certain averages of grouped information. If the contacts are grouped within a finite number of orientational intervals, then, the grouped averages $\Delta\bar{f}\bar{l}(\theta)$ in Eq.(9) can be calculated.

Assuming that the number of contacts is very large, while the orientational interval $\Delta(\theta)$ is very small, Eq.(9) can be expressed in the next form

$$\Delta\bar{\sigma}_{ij}=\frac{1}{V}\Big[\sum_{\theta_g}\Delta\bar{f}_i^{c1}\bar{l}_j^{c1}(\theta)(c_1E(\theta)\Delta\theta)+\sum_{\theta_g}\Delta\bar{f}_i^{c2}\bar{l}_j^{c1}(\theta)(c_2E(\theta)\Delta\theta)+\sum_{\theta_g}\Delta\bar{f}_i^{c2}\bar{l}_j^{c2}(\theta)(c_2E(\theta)\Delta\theta)\Big] \quad\cdots\cdots(12)$$

For finite but large number of particles, the average incremental stress tensor from the discrete information is an accurate analogue to the incremental stress tensor of continuum mechanics and we will assume that $\Delta\bar{\sigma}_{ij}=\Delta\sigma_{ij}$. In the case of isotropic analysis (i.e. $\bar{l}_j(\theta)=\bar{l}n_j(\theta)$ and $E(\theta)=1/2\pi$), Eq.(12) can be written in the form :

$$\Delta\sigma_{ij}=\frac{c_1}{V}\int_0^{2\pi}\Delta\bar{f}_i^{c2}\bar{l}_j^{c1}(\theta)E(\theta)d\theta+\frac{c_2}{V}\int_0^{2\pi}\Delta\bar{f}_i^{c2}\bar{l}_j^{c1}(\theta)E(\theta)d\theta+\frac{c_2}{V^A}\int_0^{2\pi}\Delta\bar{f}_i^{c2}\bar{l}_j^{c2}(\theta)E(\theta)d\theta \quad\cdots\cdots(13)$$

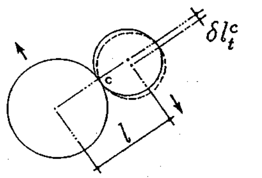
where $\bar{l}_1$ is the average radius of aggregates and $\bar{l}_2$ is the average radius of mortar particles, which can be obtained by assuming that every mortar particles is surrounded by n particles of aggregates (refer to **Fig.1**).

(3) Relation between average contact forces and strain tensor

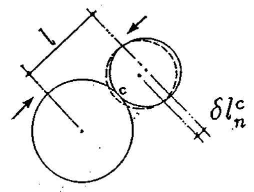
Neglecting the possible rotation between particles, the average contact force is linked with the contact displacement using the linear contact law as follow :

$$\Delta f_n^c=K_n\Delta\left(\frac{\delta l_n^c}{l}\right),\qquad \Delta f_s^c=K_s\Delta\left(\frac{\delta l_t^c}{l}\right) \quad\cdots\cdots(14)$$

where $(\delta l_n^c/l)$ represents the relative normal displacement between particle centers; $(\delta l_t^c/l)$ is the relative tangent displacement at the contact; l is the distance between particle centers in contact. These terms are illustrated in **Fig.3**, where K_n and K_s refer to the normal and shear stiffnesses of the contact, while, Δf_n and Δf_s are the components of the contact force increments. Eq.(13) for the average stress tensor contains only the averages of forces of the same orientation. Therefore, it is



(a) Shear Contact Displacement



(b) Normal Contact Displacement

Fig.3 Contact Displacement Components.

necessary to equate these averages of forces to quantities describing average displacement components for similarly oriented contacts.

If $\bar{\delta}_n^c(\theta)$ and $\bar{\delta}_t^c(\theta)$ are the average normal and shear displacements for a groups of contacts having the same orientation, thus, Eq.(14) can take the form :

$$\left.\begin{aligned}\Delta\bar{f}_n^c(\theta)&=K_n\Delta\bar{\delta}_n^c(\theta)\\ \Delta\bar{f}_s^c(\theta)&=K_s\Delta\bar{\delta}_t^c(\theta)\end{aligned}\right\}\cdots\cdots(15)$$

where

$$\Delta\bar{\delta}_n^c(\theta)=\Delta\left(\frac{\delta\bar{l}_n(\theta)}{\bar{l}}\right),\qquad \Delta\bar{\delta}_t^c(\theta)=\Delta\left(\frac{\delta\bar{l}_t(\theta)}{\bar{l}}\right)\cdots\cdots(16)$$

Here, we will assume that the normal and shear microstrains (ε_n, ε_t), which govern progressive cracking and failure of microplane are equal to the resolved components of the macroscopic strain tensor on the same microplane, i.e.

$$\varepsilon_n=\frac{\delta l_n}{l}=\varepsilon_{ij}n_in_j,\qquad \varepsilon_t=\frac{\delta l_t}{l}=\varepsilon_{ij}t_in_j\cdots\cdots(17)$$

In which n are the direction consines of the unit normal of the microplane and t are the direction consines of the unit tangent of the microplane i.e., $n=(\cos(\theta),\ \sin(\theta))$ and $t=(-\sin(\theta),\ \cos(\theta))$. It can be expected that Eq.(17) will hold true when exprcsscd as averages taken over the microplanes of similar orientations. It is reasonable to expect that :

$$\left.\begin{aligned}\Delta\bar{\delta}_n^c(\theta)&=\Delta\left(\frac{\delta\bar{l}_n(\theta)}{\bar{l}}\right)=\phi(\Delta\varepsilon_{ij}n_in_j)\\ \Delta\bar{\delta}_t^c(\theta)&=\Delta\left(\frac{\delta\bar{l}_t(\theta)}{\bar{l}}\right)=\phi(\Delta\varepsilon_{ij}t_in_j)\end{aligned}\right\}\cdots\cdots(18)$$

where ϕ is a function of ε_{ij} and the direction cosines.

(4) Stress strain relation

If the average normal and tangential contact forces in Eq.(15) are combined with Eq.(18) and the resulting value is introduced into Eq.(13), we can obtain the following incremental stress-strain relation :

$$\Delta\sigma_{ij}=D_{ijkl}\Delta\varepsilon_{kl}\cdots\cdots(19)$$

where

$$\begin{aligned}D_{ijkl}=&\frac{c_1\bar{l}_1\phi\bar{a}_1}{2\pi V}\int_0^{2\pi}(k_nn_in_jn_kn_l+k_st_in_jt_kn_l)^{c1}d\theta\\ &+\frac{c_2\bar{l}_1\phi\bar{a}_2}{2\pi V}\int_0^{2\pi}(k_nn_in_jn_kn_l+k_st_in_jt_kn_l)^{c2}d\theta\\ &+\frac{c_2\bar{l}_2\phi\bar{a}_2}{2\pi V}\int_0^{2\pi}(k_nn_in_jn_kn_l+k_st_in_jt_kn_l)^{c2}d\theta\end{aligned}\cdots\cdots(20)$$

where $\bar{a}_1$ and $\bar{a}_2$ are the averages of contact areas between aggregates alone and between aggregate and mortar particles, respectively. k_n and k_s are the normal and shear stiffnesses which can be obtained through the stress strain relationship of the microplane. Because of the lack of experimental data which can distinguish the behavior of different contacts, all contacts will be assumed to have the same properties in the current study. Now, Eq.(20) can be simplified to :

$$D_{ijkl}=\eta\int_0^{2\pi}(k_na_{ijkl}+k_sb_{ijkl})d\theta\cdots\cdots(21)$$

where

$$\eta=\frac{\phi}{2\pi}\left[\frac{c_1\bar{l}_1\bar{a}_1}{V}+\frac{c_2\bar{l}_1\bar{a}_2}{V}+\frac{c_2\bar{l}_2\bar{a}_2}{V}\right]$$

$$a_{ijkl}=n_in_jn_kn_l,\qquad b_{ijkl}=t_in_jt_kn_l$$

(5) Normal and shear stiffnesses of the microplane (k_n, k_s)

Normal stiffness of the microplane is taken as the ratio between the incremental microplane normal stress and normal strain, while, shear stiffness is defined as the ratio between the incremental microplane shear stress and shear strain as follows :

$$k_n=\frac{d\sigma_n}{d\varepsilon_n},\qquad k_s=\frac{d\tau_{nt}}{d\varepsilon_{nt}}\cdots\cdots(22)$$

where σ_n and ε_n are the normal stress and the normal strain on the microplane, while, τ_{nt} and ε_{nt} are the shear stress and the shear strain on the same microplane. Here, we made an assumption that k_s is related to k_n in the form :

$$k_s=\lambda k_n\cdots\cdots(23)$$

where λ is constant. This assumption will be valid in both tension and compression.

(6) Normal stress and normal strain relation on the microplane

a) Microplanes in tension

Since the normal microstress and the normal microstrain relation governs the progressive development of cracking on the microplane of any orientation, and since our aim is to describe softening, damage and the real failure of the microstructure, at which σ_n reduces to zero, σ_n as a function of ε_n must first rise up to the maximum limit, then, must decrease gradually up to zero. Hence, the following expression is assumed if $\varepsilon_n \geqq 0$:

$$\sigma_n = E_n \varepsilon_n e^{-k_t \varepsilon_n^p} \qquad (24)$$

Where E_n, k_t and p are positive constants.

b) Microplanes in compression

In the same manner, the next expression for microplane in compression (i.e. if $\varepsilon_n < 0$) is assumed :

$$\sigma_n = E_n \varepsilon_n e^{k_c \varepsilon_n^{p_1}} \qquad (25)$$

where E_n, k_c and p_1 are also positive constants.

3. THEORETICAL DERIVATION OF POISSON'S RATIO AND THE RELATION OF INITIAL MACROSTIFFNESS AND MICROSTIFFNESS

To check the value of Poisson's ratio, the case of uniaxial strain is considered (i.e. $\varepsilon_x = 0$ and $\varepsilon_y \neq 0$). Assuming that for small strains $\sigma_n = E_n \varepsilon_n$, $k_n = E_n$ and $k_s = \lambda k_n$, the following results can be obtained by using Eqs.(19) and (21);

$$\sigma_x = \eta E_n \int_0^{2\pi} (\sin^2(\theta)\cos^2(\theta) - \lambda \sin^2(\theta)\cos^2(\theta))\, d\theta \varepsilon_y = \eta E_n (1-\lambda)\frac{\pi}{4}\varepsilon_y \qquad (26)$$

$$\sigma_y = \eta E_n \int_0^{2\pi} (\sin^4(\theta) + \lambda \cos^2(\theta)\sin^2(\theta))\, d\theta \varepsilon_y = \eta E_n (3+\lambda)\frac{\pi}{4}\varepsilon_y \qquad (27)$$

Thus, $\sigma_x/\sigma_y = (1-\lambda)/(3+\lambda)$ and, in view of Hook's law of $\sigma_x/\sigma_y = \nu/1-\nu$, the relation between ν and λ is obtained as follow :

$$\nu = \frac{1-\lambda}{4} \qquad (28)$$

Since the elastic Poisson ratio of concrete is around 0.20, λ is suggested to be 0.20. In addition, from Eq.(27), the relation between the initial macroscopic and microscopic elastic stiffness E and E_n can be observed as follow :

$$E = \left[\frac{\pi}{4}(3+\lambda)\eta\right] E_n \qquad (29)$$

where η is a material parameter. This parameter, as defined in Eq.(21), depends on the size of particles, contact areas, number of contacts, and total volume of both aggregate and mortar. The value of this parameter will be selected to obtain a consistent values between the initial stiffnesses of the macroscopic experimental data and that of the microplanes.

4. NUMERICAL IMPLEMENTATION AND COMPARISON WITH THE TEST DATA

To insure the capability of the current model and to calibrate its results, different types of plane problems are examined, in which, these plane problems are subjected to several types of loading conditions. The results are compared with the available macroscopic test data as shown below.

(1) Uniaxial tension

In the beginning the parameter E_n is obtained using Eq.(29). In Eq.(29), the values of $\lambda = 0.20$ and $\eta = 1.0$ are used. Then, the values of the parameters k_t and p of the model which can describe the tensile behavior of the microplanes have been found to fit the experimental data of Evans and Marathe[8]. A careful attention has been made to obtain identical initial stiffnesses, peak stresses, and a reasonable hardening and softening tendencies. The other parameters for compressive microplanes k_c and p_1 are kept constant. The fits illustrated in **Figs.4**(a) ~ (f). In these figures, concrete has different characteristics. According to the experimental data, the peak stresses of concretes shown in **Fig.4**(a) ~ (f) are 16.14, 15.60, 21.10, 21.40, 26.70 and 31.70 kgf/cm^2, respectively. The values of $10^{-4}E$ (E is the initial stiffness) of these concrete are 15.50, 16.50, 17.50, 20.0, 15.25, and 13.5 kgf/cm^2, respectively. In **Fig.4**, the current model is shown by the solid lines, while, the dashed lines represent the experimental data[8]. For all cases constant values of $p = 1.5$, $p_1 = 2.0$ and $k_c = 20 \times 10^4$ are used. The values of $10^{-4}E_n$ for concretes shown in from (a) to (f) of **Fig.4**, are 6.17, 6.565, 6.963, 7.96, 6.068 and 5.37 kgf/cm^2, respectively, and those of $10^{-4}\ k_t$ are 29.5, 34.7, 24.0, 28.4, 13.3 and 8.70. As seen, a reasonable agreement with the test data can be noticed to be achieved. It should be noted that, we have only one parameter, k_t, to determine a good fitting with the test data. Through this small number of data, we try to obtain the relation between k_t values and macroscopic compressive strength and Young's modulus. Finally the obtained relation is shown in **Fig.5**. From **Fig.5**, it can be noticed that, as the compressive strength of concrete increases, the value of k_t decreases. This may be due to the fact that the stress strain relationship for high strength

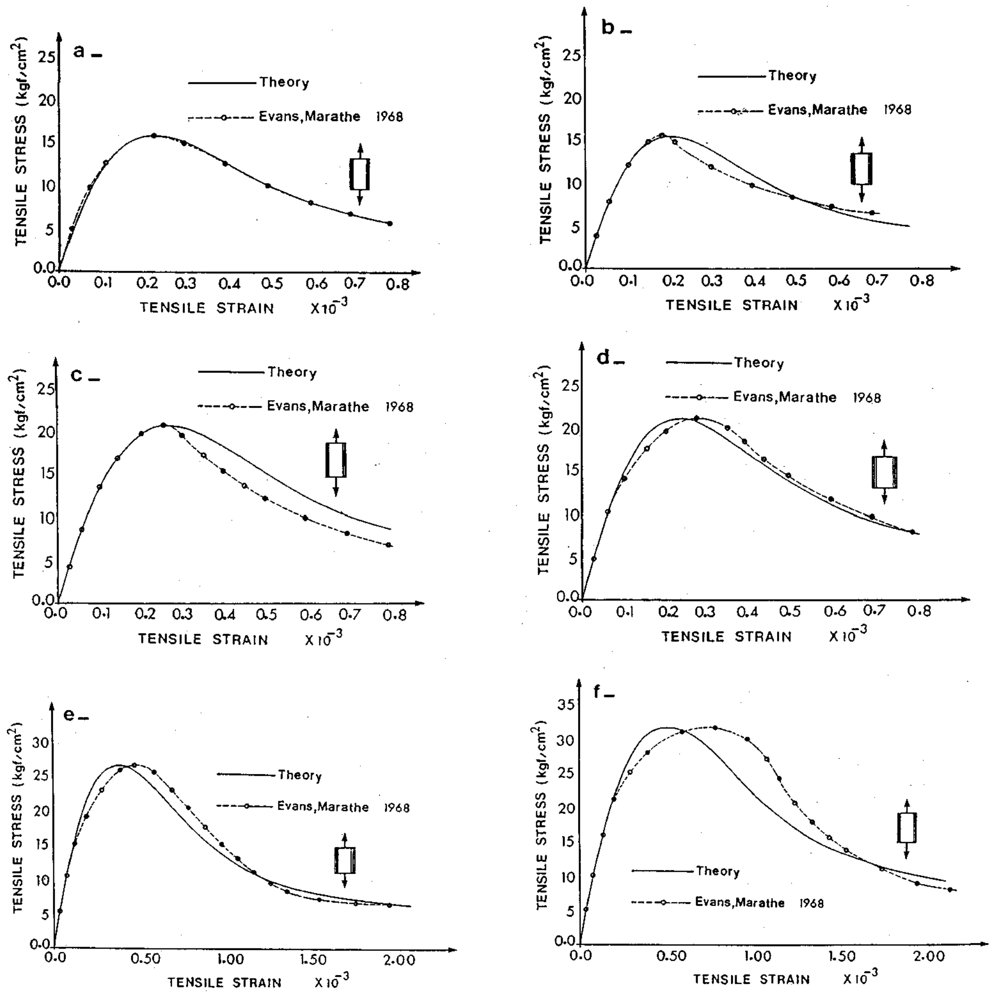
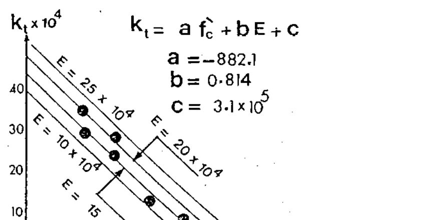


Fig.4 Fits of Current Study to Test Data of Evans *et al.*[8].



f_c'=compressive strength of concrete, E=Young's modulus

Fig.5 The Relationship between k_t, and f_c' for Different Values of E.

concrete, which is almost linear up to the fracture limit, is followed by a sudden drop of stress. This sudden drop occurs due to the highly softening phenomena which is controlled by the k_t parameter.

(2) Uniaxial compression

In the same manner, in the beginning E_n is obtained using Eq.(29). In this quation, the values of $\lambda=0.20$ and $\eta=1.0$ are used. The parameters p_1 and k_c have been obtained to get a good fits of the data of Desayi *et al.*[9] and Kupfer *et al.*[10], as shown in **Figs.6**(a)~(d). For all cases, $p=1.5$, $k_t=26\times10^4$ and $p_1=2.0$ are used. The values of $10^{-4}E_n$ for concretes a,b, c and d are 8.22, 13.08, 20.0 and 10.98 kgf/cm², and those of $10^{-4}k_c$ are 18.8, 22.2, 19.6, and 13.8. As can be seen, a good agreement with the experimental data is obtained, although, all parameters are fixed constant except k_c. Here again, the relation between k_c values and macroscopic compressive strength of concrete and Young's modulus were obtained and shown in **Fig.7**. From that figure, it is noticed that the variation of k_c due to the increase of the compressive strength of concrete is not too much. This may be due to the fact that a sudden failure

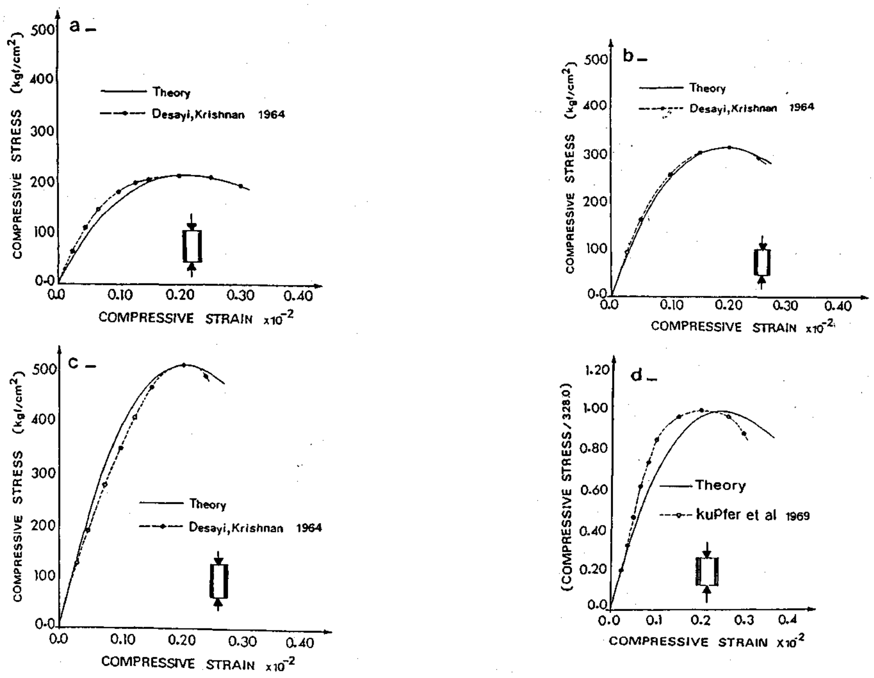


Fig.6 Fits of Present Study to Test Data of References 9), 10).

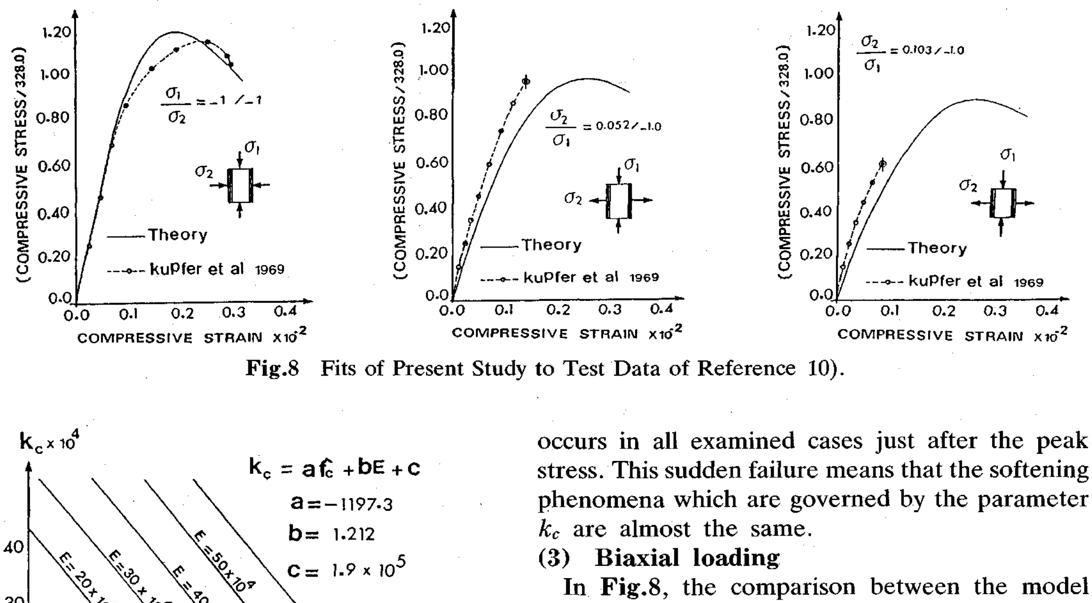
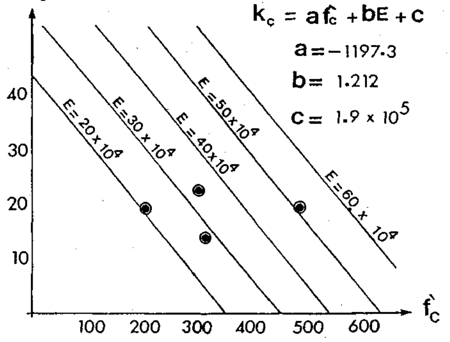


Fig.8 Fits of Present Study to Test Data of Reference 10).



f_c'=compressive strength of concrete, E=Young's modulus

Fig.7 The Relation between k_c, and f_c' for Different Values of E.

occurs in all examined cases just after the peak stress. This sudden failure means that the softening phenomena which are governed by the parameter k_c are almost the same.

(3) Biaxial loading

In **Fig.8**, the comparison between the model results and test data of Kupfer *et al.*[10] are presented. A reasonable agreement of fitting the test data is obtained. In **Fig.8**, the following values of the parameters are used, $\eta=1.0$, $\lambda=0.20$, $p=1.5$, $p_1=2.0$, $k_t=24.5\times10^4$, $k_c=13.8\times10^4$ and $E_n=10.80\times10^4$ kgf/cm². These values are obtained to satisfy the uniaxial behavior (tension and compression) and obtained from the proposed relations of k_t and k_c.

5. CONCLUSIONS

The monotonic behavior of plain concrete is investigated. The microplane model is analyzed to recognize the actual damage or failure of the microstructure under the effect of different types of loading conditions. The analysis is based on the micromechanical behavior of granular materials. Furthermore, to represent the different types of contacts in the material, plain concrete is idealized to have two types of particles (aggregates and mortar). This idealization enables us to study the actual micromechanism not only between the aggregates as investigated before in the previous research work but also on the contact between mortar and aggregates. It can be concluded that :

(1) Progressive fracture or damage of concrete due to microcracking can be described using the microplane model.

(2) Through the proposed model, the relation between the initial macroscopic and microscopic stiffnesses can be obtained.

(3) The macroscopic elastic Poisson's ratio is found to give the ratio between the shear stiffness and the normal stiffness of microplane.

(4) The proposed model, considering only one kind of microplane, is found to be consistent with the experimental data. However, this model can also consider two kinds of microplanes, but, experimental data are necessary to distinguish the behavior of the different microplanes. Also, a more comprehensive research must be conducted to study the density distribution, the nonuniformity of strain distribution, and the cyclic behavior of microplane to have a better understanding of the characteristics of the microstructure.

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(Received December 19, 1990)

異種粒子からなるマイクロプレーン集合としてのコンクリートのモデル化

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マイクロプレイン機構による単調載荷を受けるプレーンコンクリートの破壊構成モデルの開発を行った．マイクロプレインは材料粒子間の接触表面として定義されるが，Bazant らの単一粒子モデルを用いず，骨材間および骨材とモルタル間の２種マイクロプレインを考慮した．定式化においては，ある集合体に対して divergence 理論および仮想仕事原理によって平均化された応力-ひずみ関係が導かれた．また，マイクロプレインの垂直およびせん断剛性を導入し，数値シミュレーションならびに実験との同定によって本モデルの有用性，妥当性を示した．