投稿論文(英文)
PAPER

Proc. of JSCE No. 433/V-15, pp. 215~222, August 1991

DISPLACEMENTS AT SHEAR CRACK IN BEAMS WITH SHEAR REINFORCEMENT UNDER STATIC AND FATIGUE LOADINGS

Hussein Mokhtar HASSAN*, Sabry A. FARGHALY** and Tamon UEDA***

Reinforced concrete beams with vertical or inclined, and plain and/or deformed stirrups of various reinforcement ratios and diameters are tested under static and repeated loadings. Opening and shearing displacements along shear cracks as well as stirrup strains and concrete deformations along stirrups are measured. Effects of bond characteristics, ratio and diameter of stirrup on stirrup slip and crack displacement in the direction of stirrup are revealed. Based on the test results, a method to predict opening displacement of shear crack under static and fatigue loadings is presented. It is found that the predicted values agree with the test results.

Keywords : *shear crack, shear reinforcement, fatigue loading, beam*

1. INTRODUCTION

In practice reinforced concrete structures subjected to repeated loading may suffer from not only excessive flexural cracking but also excessive shear cracking. Relatively large crack opening displacement (crack width) induced by excessive shear cracking would be a major cause of corrosion of shear reinforcement like in case of flexural cracking. Furthermore, along a shear crack shearing displacement, which causes significantly early fatigue fracture of shear reinforcement at its intersection with shear crack[1], is found. Prediction of displacements along shear crack under repeated loading, therefore, is necessary to introduce a rational way for controlling the crack displacements.

Mechanism of crack spacing which affects magnitude of crack width has been found quite different in shear crack from that in flexural crack[2]. Along a flexural crack there is opening displacement mainly, but there is shearing displacement as well along a shear crack. A shear crack generally crosses shear and tension reinforcement diagonally, while a flexural crack intersects perpendicularly tension reinforcement. Those facts on shear cracking tell the necessity of study on shear cracking mechanism besides study on flexural cracking. There have been, therefore, some studies[3)~8)] on shear cracking, mainly on shear crack width. In those studies factors to affect shear crack width were disclosed which were shear reinforcement characteristics, such as its strain, bond characteristics, angle with member axis, spacing and size, ratio of shear span to effective depth, web width, and concrete strength[1),9)]. As effect of repeated loading, it was reported that shear crack width increased significantly with loading cycles [5),6)] and that increment of shear crack width was almost in proportion to logarithm of loading cycles[1)]. Although some prediction methods of shear crack width have been proposed in the previous studies[3),10)], it cannot be said that mechanism of shear cracking is well understood like flexural cracking. Especially prediction of shearing displacement as well as effect of repeated loading on crack displacements, which are not considered in case of flexural cracking, require further study.

In this study experimental works were conducted on eight rectangular beams with vertical or inclined stirrups with various bond characteristics, reinforcement ratios and diameters. The beams were subjected to repeated loading. The opening displacement (crack width) and shearing displacement along shear cracks, concrete strains along stirrups as well as stirrup strains were measured carefully. As the results, the formulae for crack displacements along a shear crack were proposed. A part of the test results was reported already in the previous paper[11)].

2. OUTLINE OF EXPERIMENT

Eight reinforced concrete beams of rectangular cross section were tested. All the beams had the same size of 3 600×200×500 mm (length×width ×height) and effective depth of 440 mm. Also they

* Former Graduate Student, Dr.Eng., Department of Civil Engineering, University of Tokyo (Aba-Al-Wakf Al-Balad, Maghagha, Menia, Egypt)
** Associate Professor, Dr.Eng., Department of Civil Engineering, Assuit University
*** Member of JSCE, Dr.Eng., Associate Professor, Department of Civil Engineering, Hokkaido University

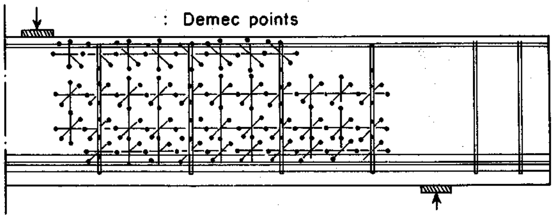

Fig.1 Specimen.

Table 1 Details of Specimens.

specimen	spacing of stirrups (mm)		shear reinforcement ratio (%)		concrete strength (MPa)
	a/d		a/d		
	2.0	4.0	2.0	4.0	
VP (13)	160	430	0.83	0.31	27.60
VD (13)	155	410	0.82	0.31	27.27
IP (13)	230	610	0.82	0.31	21.53
ID (13)	215	570	0.83	0.31	24.10
OVD (13)	85	230	0.84	0.31	34.90
VP (10) *	70	190	0.91	0.33	28.38
VD (10) *	85	230	0.84	0.31	29.90
IP (10) *	120	320	0.90	0.33	34.42
ID (10) *	100	270	0.84	0.32	34.49
	a/d = 3.0		a/d = 3.0		
	plain	def.	plain	def.	
VDP (13)	300	300	0.44	0.42	33.10
IDP (13)	300	300	0.63	0.60	30.39
VP(13)D(10)	150	80	0.89	0.89	24.31

V : vertical, I : inclined, D : deformed, P : plain,
( ) : stirrup diameter, * : specimens in Ref. [1]

had the same tension and compression reinforcement, five of D25 and two of D13 respectively, except specimen VP (13)D(10) with six of D25 for tension reinforcement and specimen OVD (13) with two of D32 and two of D25 for tension reinforcement and two of D10 for compression reinforcement. Shear reinforcement was plain or/and deformed stirrups with U-shape. **Fig.1** shows configuration of specimen VDP(13). The details of all the specimens are given in **Table 1**. Concrete cylinder strength for each specimen is also given in **Table 1**. The properties of steel used are given in **Table 2**.

Before carrying out the test, the Demec points, which are points for contact strain gage with accuracy of 0.001 mm, were cemented on the concrete surface at a level of mid-depth for specimens VP(13), VD(13), IP(13) and ID(13) in order to measure concrete deformations in shear cracking region. For specimens VDP(13), IDP(13), OVD(13) and VP(13)D(10), the Demec points were cemented at various levels (see **Fig.1**). Wire strain gages were attached to measure strain in each stirrup at a distance of 5 cm above the centroid of the tension reinforcement for specimens VP(13), VD(13), IP(13), ID(13) and OVD(13), and at 8 cm for specimen VP(13)D(10). For specimens VDP(13) and IDP(13), strain gages were attached at an interval of 5 cm for each

Table 2 Properties of Steel.

type of bar	bar	diameter (mm)	area (mm^2)	fy (MPa)	Es x 10^3 (MPa)
deformed	D10	9.53	71.3	379.4	166.7
	D13	12.70	126.5	376	173
	D25	25.40	506.7	405	190
	D32	30.70	794	336	190
plain	P10	9	63.6	362	183.9
	P13	13	133	340	190

Es : Young's modulus, fy : yield strength

stirrup. Loads for specimens IDP(13) and VP(13)D(10) were increased statically up to 294 kN (shear force of 147 kN) and then released to zero and increased to failure of the specimens. Loads at failure were 580 kN and 666 kN respectively (shear force of 290 kN and 333 kN). The other specimens were loaded statically during the first 100 cycles and then dynamically with 210 cycles per minute. For specimens VP(13), VD(13), IP(13), ID(13), VP(10), VD(10), IP(10) and ID(10), maximum and minimum loads were 368 kN and 147 kN for the first stage which was one million cycles, and 417 kN and 98 kN for the second stage which was continued to failure of the specimen. For specimen OVD(13), maximum and minimum loads were kept constant, 417 kN and 98 kN. For specimen VDP(13), maximum and minimum loads were 343 kN and 137 kN up to 10^5 cycles and then loaded statically until failure at 588 kN. The dynamic loading was stopped at logarithmic interval to take the measurement.

3. TEST RESULTS AND DISCUSSIONS

(1) Crack opening and shearing displacements

Opening and shearing displacements of shear cracks were calculated from the Demec points measurements by the same way as in Ref.1). Variations of the opening and shearing displacements under static and repeated loading for one of the measured locations in specimen VDP(13) are shown in **Fig.2**. It can be seen from this figure that the opening and shearing displacements along shear cracks increase significantly under effect of repeated loading. There are sizable residual crack opening and shearing displacements at complete unloading.

(2) Stirrup strain distribution

It was observed that strain of each stirrup differed greatly according to stirrup and shear crack locations as reported in the previous study[8]. Variation of stirrup strain was also observed within a stirrup leg. **Fig.3** shows distribution of measured strains of plain stirrup No.2 and deformed stirrup No.2 in specimen VDP(13) at maximum applied repeated load of the first and 10 000th cycles. It is clear that at a crack-stirrup intersection, the strain is greater and decreases toward stirrup hook. Rate

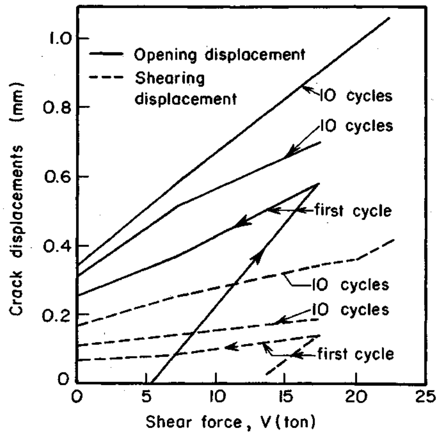

Fig.2 Relationships between Crack Displacements and Shear Force under Fatigue Loading.

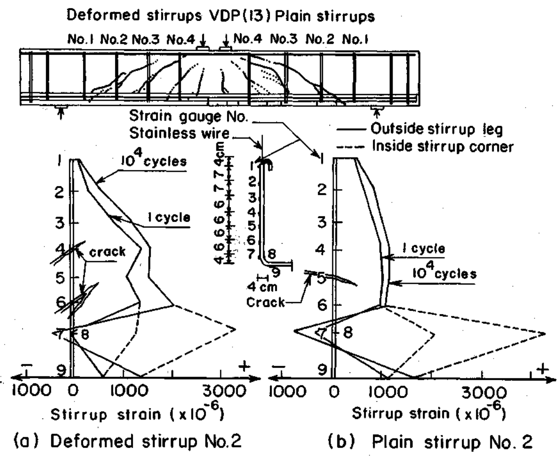

Fig.3 Distribution of Stirrup Strain[11].

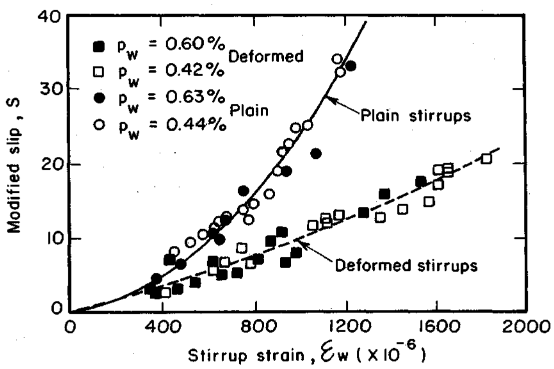

Fig.4 Relationship between Modified Slip and Stirrup Strain at Crack-Stirrup Intersection.

of decreasing is greater for the deformed stirrup in comparison with the plain stirrup. This difference is considered to be due to difference in bond characteristics between the plain and deformed stirrups[11].

(3) Stirrup slip-strain relationship at crack-stirrup intersection

Slip of stirrup at an intersection with a shear crack can be determined by integration of the stirrup strain from a fixed end of the stirrup to the intersection considered[12]. The slip of stirrup is considered to be a major component of crack displacements at a shear crack. In case of the deformed stirrup the strain at the fixed end, which is at the starting point of hook, was zero or rather small, and the measured slips there were mostly negligible. In case of the plain stirrup, however, significant slips were observed because of large strain at hook and its poor bond characteristics. For the case with the hook slip, the slip was obtained by adding the hook slip to integration of the stirrup strain from the starting point of hook to the first crack-stirrup intersection. **Fig.4** shows slip-strain relationships for the plain and deformed stirrups with different web reinforcement ratios. The measured slips are modified by considering effects of bar diameter and concrete strength according to the study in Ref.12), although the bar diameters in this study (10 and 13 mm) were less than those in Ref.12) (19, 25 and 32 mm)

$$S=\frac{1\,000s}{\phi}K_{fc} \qquad (1)$$

where S : modified slip, s : slip (mm), ϕ : bar diameter (mm), K_{fc} : coefficient for effect of concrete strength $(=f_c'/19.6)^{2/3})$, f_c' : concrete cylinder strength (MPa). It seems that the web reinforcement ratio has little effect on the relationships. The experimental results can be expressed as follows.

$$S=4\times10^3\varepsilon_w+20\times10^6\varepsilon_w^2 \quad \text{for plain stirrup} \qquad (2\text{-a})$$

$$=8\times10^3\varepsilon_w+2\times10^6\varepsilon_w^2 \quad \text{for deformed stirrup} \qquad (2\text{-b})$$

where ε_w : stirrup strain at crack-stirrup intersection. For the same stirrup strain, the slip of plain stirrup is much greater than that of deformed stirrup. The modified slip of deformed stirrup with bar diameter of 13 mm and cover of around 15 mm in this study was found to be slightly greater than that of deformed bar with bar diameter of 19, 25 or 32 mm embedded in massive concrete, which is reported in the previous study[12].

(4) Crack displacement in direction of stirrup and effect of web reinforcement ratio

Crack displacement in the direction of stirrup at a crack-stirrup intersection is considered to be caused by difference between elongation of stirrup and concrete. In **Fig.5** the crack displacements

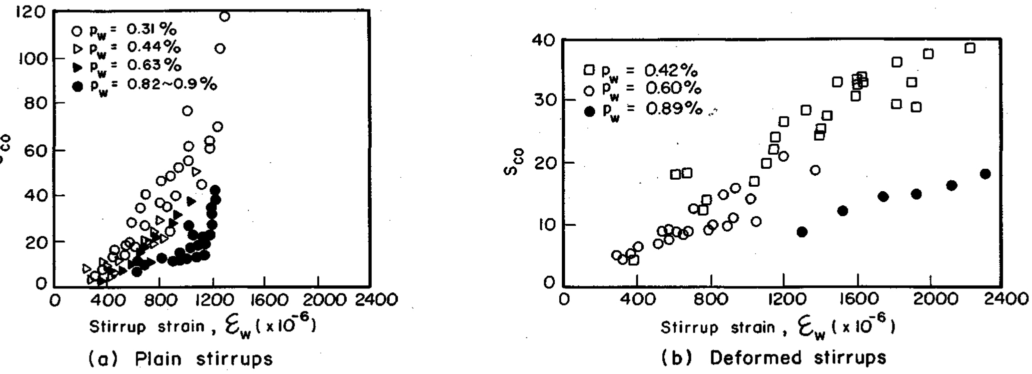

Fig.5 Relationship between Modified Crack Displacement and Stirrup Strain at Crack-Stirrup Intersection.

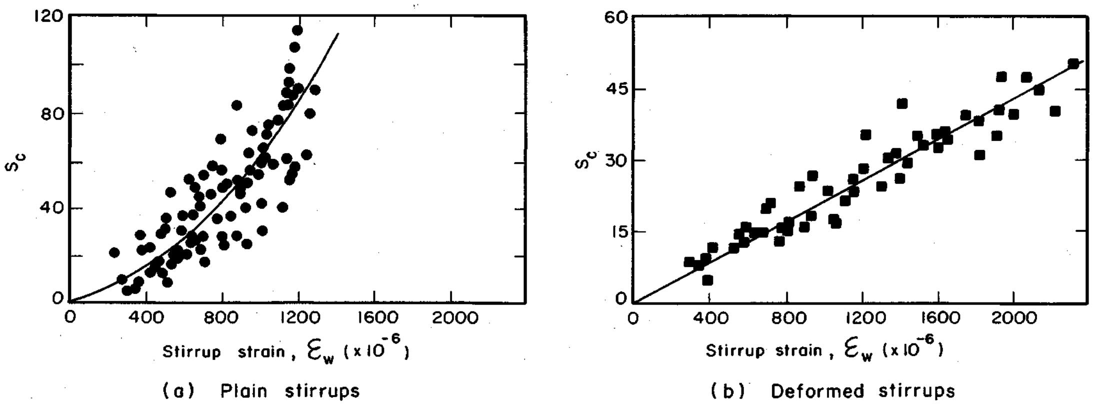

Fig.6 Relationship between Remodified Crack Displacement and Stirrup Strain at Crack-Stirrup Intersection.

measured by the Demec points are shown with corresponding stirrup strains measured at the crack-stirrup intersections. Now it is assumed that the elongation of concrete is proportional to that of stirrup, which is determined to be "slip"in Sec.3(3). Since the slip depends on bar diameter and concrete strength, the measured crack displacements are modified as follows (see Eq.(1)).

$$S_{co}=\frac{1\,000s_c}{\phi}K_{fc} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where S_{co} : modified crack displacement in direction of stirrup, s_c : crack displacement in direction of stirrup (mm). It is observed in **Fig.5** that the modified crack displacements in the direction of stirrup for the same stirrup strain are different among cases of different web reinforcement ratios in this study. The smaller web reinforcement ratio is, the greater is the crack displacement. The reason for this difference may be considered in such a way that a greater web reinforcement causes a greater concrete elongation for the same stirrup strain, which means a smaller crack displacement. Another possible factor causing difference in crack displacement is crack spacing. Since a shear span with a higher web reinforcement ratio was subjected to a higher shear stress level, observed spacings were smaller with a higher web reinforcement ratio. It was reported, however, in the previous study[11] that effect of shear crack spacing on crack displacement was little. Further study on the effect of web reinforcement ratio on the crack displacement is considered to be necessary. In this study this effect was simply taken into account by taking another modification factor, K_p as shown in Eq.(4). The factor, K_p, which is a function of web reinforcement ratio, was chosen to obtain a better fitting curve for the experimental results in this study.

$$S_c=K_pS_{co}=\frac{1\,000s_c}{\phi}K_{fc}K_p \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

where S_c : remodified crack displacement in direction of stirrup, $K_p=(p_w/0.004)^{1.3}$, $p_w=A_w(\sin\alpha+\cos\alpha)/b_ws_p$: web reinforcement ratio, A_w : area of web reinforcement within spacing, s_p, b_w : web width, α : angle between web reinforcement and member axis. The remodified crack displacements

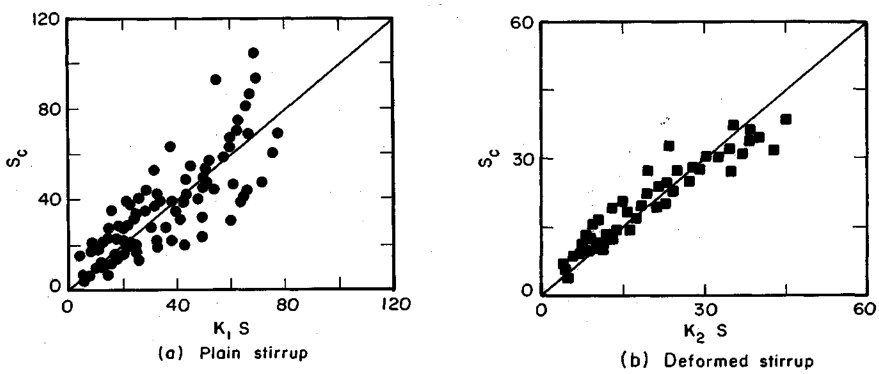

Fig.7 Relationship between Remodified Crack Displacement and Modified Slip at Crack-Stirrup Intersection.

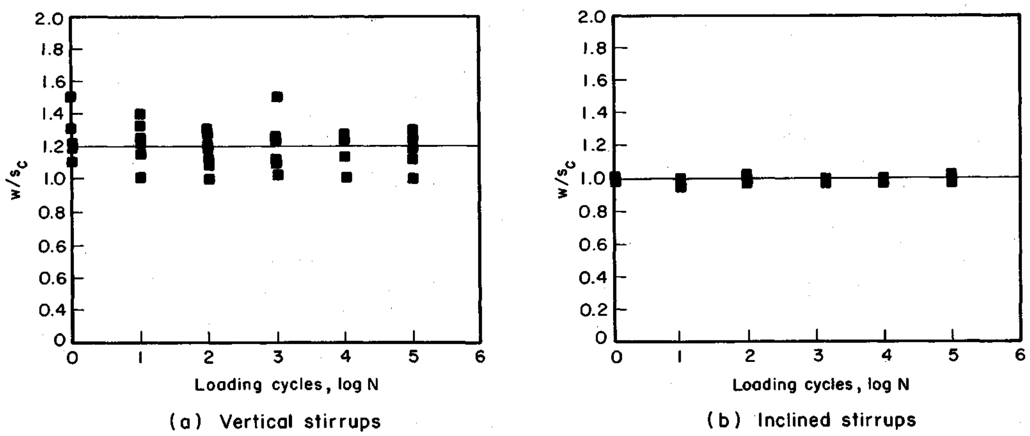

Fig.8 Ratio of Shear Crack Width to Crack Displacement in Stirrup Direction.

for both plain and deformed stirrups are given in **Fig.6**.

(5) Rclationship between crack displacement in stirrup direction and stirrup slip

Since the crack displacement in the stirrup direction is caused mainly by elongation of the stirrup (or slip of the stirrup), relationship between the crack displacement given by Eq.(4) and the stirrup slip given by Eq.(1) were carefully observed. Rather proportional relationships can be seen between them as shown in **Fig.7**, being expressed by the following equation.

$$S_c = K_1 S \quad \cdots\cdots (5)$$

where $K_1 = 2.4$ for plain stirrup and 2.0 for deformed stirrup. The crack displacement is caused by the slip of stirrup from both sides of the crack. The slip given by Eq.(1), however, is the one occurring at the side above the crack. The value, 2.4 of K_1 for plain stirrup, therefore, means that there is more slip at the side below the crack, which is 1.4 times of the slip at the side above the crack. For the case of deformed stirrup it is considered that equal amount of slip takes place at each side of the crack. The slip below the crack is caused by the elongation of the stirrup of not only vertical part but also horizontal part beyond the lower bend of the stirrup[11].

(6) Crack displacement normal to crack (crack opening displacement)

Since a shear crack crosses stirrups diagonally, the crack displacement in the direction of stirrup is not a crack width which is a crack displacement normal to crack. The crack displacement in the stirrup direction can be expressed by crack displacements normal and parallel to crack, w and δ as follows.

$$s_c = w \sin(\theta + \alpha) - \delta \cos(\theta + \alpha) \quad \cdots\cdots (6)$$

where θ : angle of shear crack to member axis. **Fig.8** shows that the ratio of the crack displacement normal to crack to that in the stirrup direction which is given by Eq.(6). It can be seen that the ratio for vertical stirrup is decreasing slightly as loading cycle increases but that for inclined stirrup the ratio is almost constant under cyclic loading. The decreasing ratio indicates, as seen from Eq.(6), that the ratio of crack displacement normal

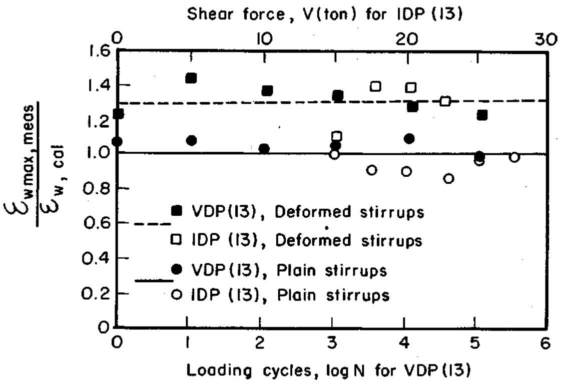

Fig.9 Ratio of Maximum Measured Stirrup Strain to Calculated One.

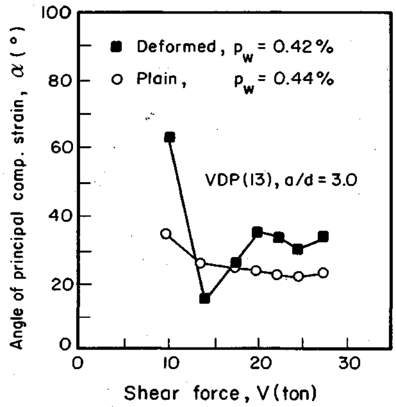

Fig.10 Angle of Principal Compressive Concrete Strain.

to crack to that parallel to crack (the ratio of crack opening displacement to shearing displacement) is decreasing. This fact corresponds with the experimental fact in the previous study[1]. The ratio for inclined stirrup is always around one because inclined stirrups were intersected by shear cracks almost perpendicularly.

(7) Calculation of crack displacement normal to crack

If the ratio of crack displacement normal to crack, w to displacement parallel to crack, δ is known, the relationship between crack displacement normal to crack (crack width) and stirrup strain, ε_w can be derived by using Eqs.(2) to (6). Although the ratio of w/s_c is decreasing slightly for vertical stirrup, it can be said from **Fig.8** that generally this ratio is more or less constant, which means that the ratio of δ/w is also constant. The following equation is then introduced.

$$w = K_2 s_c \quad (7)$$

where $K_2 = 1.2$ for vertical stirrup and 1.0 for inclined stirrup.

On the other hand, stirrup strain under fatigue loading can be calculated by using the following equation[8].

$$\varepsilon_w = \frac{V_{max} - V_{co} 10^{-0.0326(1-r^2)\log N}}{A_w E_w (z/s_p)(\cos\alpha + \sin\alpha)} \quad (8)$$

where ε_w : stirrup strain at applied maximum shear force, V_{max}, N : loading cycles, $r = V_{min}/V_{max}$, E_w : elastic modulus of stirrup. V_{min} : applied minimum shear force, V_{co} : shear force carried by concrete under static loading, expressed by Eq.(9)[13].

$$V_{co} = 0.2 f_c'^{1/3}(1 + \beta_p + \beta_d) b_w d \quad (9)$$

where $\beta_p = p^{1/2} - 1$, $p = 100 A_s / b_w d$, A_s : area of tension reinforcement, d : effective depth (mm), $\beta_d = (1\,000/d)^{1/4} - 1$, $z = d/1.15$. **Fig.9** indicates the ratio of the maximum measured stirrup strain at intersection with shear crack to strain calculated by Eq.(8). The measured stirrup strain for deformed stirrup is greater than the calculated one, but they are more or less equal for plain stirrup. From **Fig.9**, the following equation is introduced for simplicity.

$$\varepsilon_{wmax} = K_3 \varepsilon_w \quad (10)$$

where ε_{wmax} : maximum stirrup strain at intersection with shear crack, $K_3 = 1.3$ for deformed stirrup and 1.0 for plain stirrup. This difference in the ratio could be explained by difference in inclination of diagonal concrete stress flow existing in a shear cracking zone. As shown in **Fig.10**, the observed inclination of the principal compressive strain in concrete between shear cracks was greater in case of deformed stirrup than in case of plain stirrup in several specimens. Once truss mechanism is assumed, a greater inclination gives a greater stirrup stress. At present, however, the reason why a higher inclination in case of deformed stirrup is not clarified. Further investigation will be needed.

Fig.11 and **Fig.12** show crack displacement normal to shear crack (shear crack width) predicted by the proposed equations and the measured one. It can be said that the predicted shear crack width is in good agreement with the measured one generally. In **Fig.12(b)** the shear crack widths measured in the study of Leonhardt and Walther[3] are compared with the predicted ones by the equations proposed in this study. Although the predicted crack widths of plain stirrup rather overestimate the measured ones, it can be said that there is agreement between the measured and predicted ones for both plain and deformed stirrups.

The shear crack widths predicted by the proposed formulae are compared with those by another prediction formula proposed by Placas and Regan[4] in **Fig.12**. The Placas' and Regan's formula

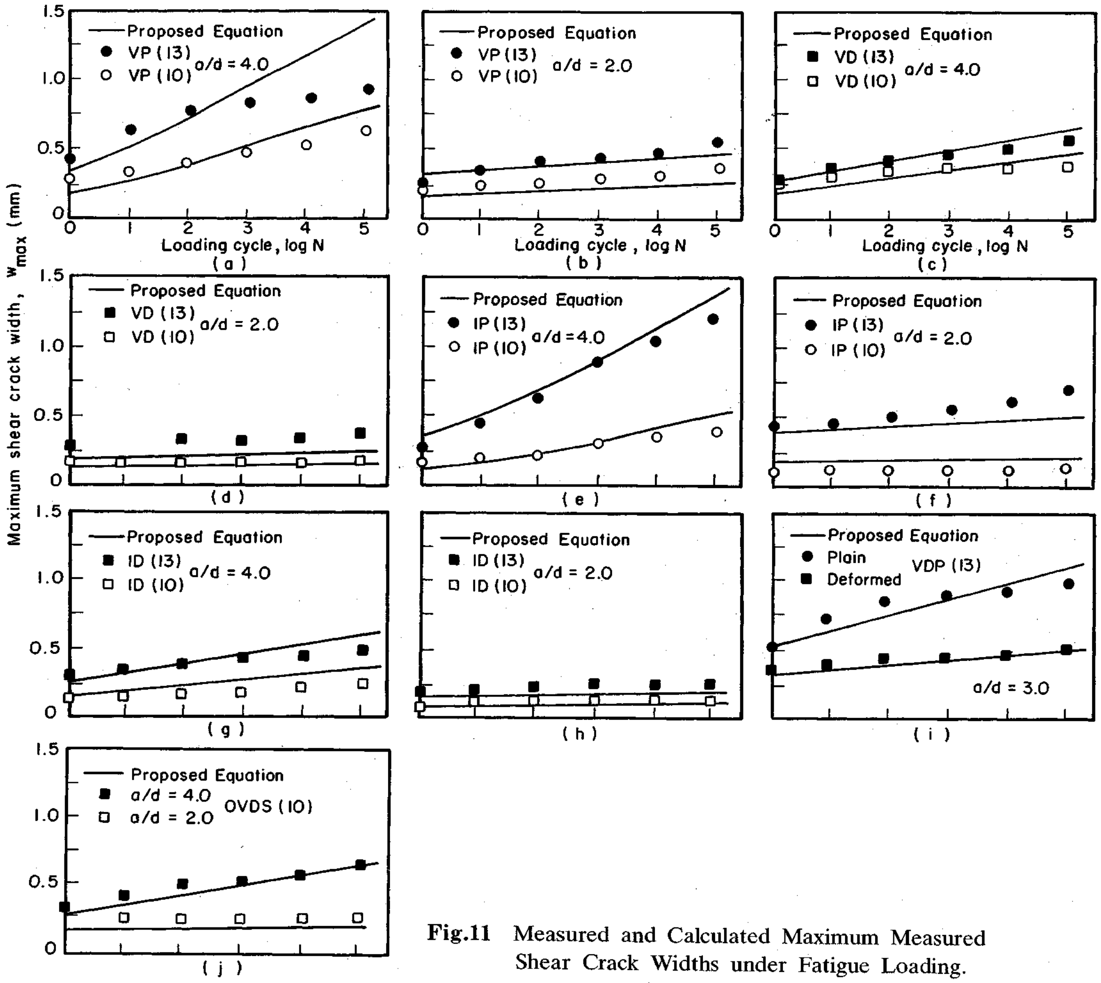

Fig.11 Measured and Calculated Maximum Measured Shear Crack Widths under Fatigue Loading.

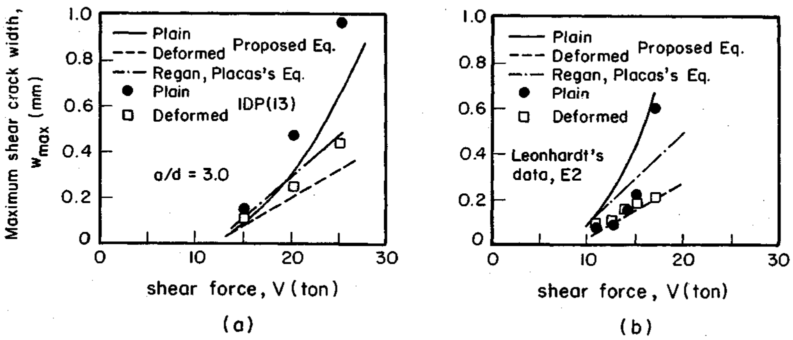

Fig.12 Measured and Calculated Maximum Shear Crack Widths.

is only for static loading and does not distinguish cases between plain and deformed stirrups. Shear crack width for deformed stirrup can be predicted reasonably by both the Placas' and Regan's and the authors' formulae, but the authors' formulae can give better prediction for plain stirrup.

4. CONCLUSIONS

An experimental investigation was conducted with eight rectangular beams with stirrups subjected to repeated loadings. From the careful observation of crack displacements along shear cracks and stirrup strains, the following conclusions were derived.

(1) It was observed that the relationship between the stirrup slip from the side above a shear crack and the stirrup strain at an intersection of the stirrup with the shear crack was affected little by web reinforcement ratio or hook slip.

(2) The relationship between the crack displacement in the direction of stirrup and the stirrup strain at the crack-stirrup intersection was observed, however, to be changed by the web reinforcement ratio. Namely the less web reinforcement ratio was, the greater was the crack displacement. It is probably because a greater elongation of concrete was caused by a greater web reinforcement ratio.

(3) The crack displacement in the direction of stirrup was observed to be greater than twice as much as the stirrup slip from the side above the shear crack, which means that there was more slip from the side below the shear crack.

(4) A method to predict the crack displacement normal to shear crack (shear crack width) was proposed for the cases of both plain and deformed stirrups with any angle with member axis under both monotonic and cyclic loadings. The predicted crack displacements agreed reasonably with the measured ones in the authors' and the previous studies.

5. ACKNOWLEDGEMENT

This study was supported in part by the Grant-in-Aid for Scientific Research (Project No.60750442) from the Ministry of Education, Science and Culture, Japanese Government. The authors would like to express their gratitude to Prof. Hajime Okamura, the University of Tokyo, for his valuable guidance to this study and Mr.Takuya Koshiji, a former graduate student of the University of Tokyo, for his assistance in the experiment.

REFERENCES

1) Hassan, H.M., Ueda, T., Tamai, S. and Okamura, H. : Fatigue Test of Reinforced Concrete Beams with Various Types of Shear Reinforcement, Transactions of JCI, Vol.7, pp.277-284, 1985.

2) Koshiji, T. : Shear Cracking Behavior in Reinforced Concrete Beams, Bachelor's Thesis submitted to University of Tokyo, March 1987 (in Japanese).

3) Leonhardt, F. and Walther, R. : The Stuttgart Shear Tests, 1961, Translation No.III, Cement and Concrete Association, London, England, 1964.

4) Placas, A. and Regan, P.E. : Shear Failure of Reinforced Concrete Beams, ACI Journal, Vol.68, pp.763-773, October 1971.

5) Price, K.M. and Edwards, A.D. : Fatigue Strength in Shear of Prestressed Concrete I-Beams, ACI Journal, Vol.68, pp.282-292, April 1971.

6) Hanson, J.M., Hulsbos, C.L. and Van Horn, D.A : Fatigue Tests of Prestressed Concrete I-Beams, Journal of the Structural Division, ASCE, Vol.96, No.ST 11, pp.2443-2464, November 1970.

7) Haddadin, M.J., Hong, S. and Mattock, A.H. : Stirrup Effectiveness in Reinforced Concrete Beams with Axial Force, Journal of the Structural Division, ASCE, Vol.97, No.ST9, pp.2277-2297, September 1971.

8) Ueda, T. and Okamura, H. : Behavior in Shear of Reinforced Concrete Beams under Fatigue Loading, Concrete Library International, JSCE, No.2 pp.37-69, December 1983.

9) The Joint ASCE-ACI Task Committee 426 on Shear and Diagonal Tension of the Committee on Masonry and Reinforced Concrete of the Structural Division : The Shear Strength of Reinforced Concrete Members, Journal of the Structural Division, ASCE, Vol.99, No.ST6, pp.1091-1187, June 1973.

10) CEB-FIP : Model Code for Concrete Structures, CEB-FIP International Recommendations, Third Edition, Comite Euro-International du Beton, 1978.

11) Hassan, H.M. and Ueda, T. : Relative Displacements along Shear Crack of Reinforced Concrete Beam, Proceedings of JCI 9th Conference, Vol.9-2, pp.699-704.

12) Niwa, J., Chou, L.L., Shima, H. and Okumura, H. : Nonlinear Spring Element for Strain-Slip Relationship of a Deformed Bar, Seminar on Finite Element Analysis of Reinforced Concrete Structures, Vol.2, JCI, pp.227-235, May 1985.

13) Okamura, H. and Higai, T. : Proposed Equation for Strength of Reinforced Concrete Beams without Web Reinforcement, Proceedings of JSCE, No.300, pp.131-141, August 1980.

(Received August 16, 1990)

静的および疲労荷重下のせん断補強されたはりのせん断ひびわれでの変位

Hussein Mokhtar HASSAN・Sabry A. FARGHALY・上田多門

異形鉄筋もしくは丸鋼をスターラップとして鉛直もしくは斜めに配置したはりの疲労試験を行い，せん断ひびわれでの変位を測定している．スターラップの定着部および曲げ引張領域での折曲げ部でのすべりがせん断ひびわれ変位に与える影響を明らかにするとともに，スターラップの角度，付着性状，径，コンクリート強度，荷重の大きさと繰返し回数の影響を考慮したせん断ひびわれ変位の推定方法を提示している．