Structural Eng./Earthquake Eng. Vol. 8, No. 2, 65s-74s, July 1991
Japan Society of Civil Engineers (Proc. of JSCE No. 432/I-16)

OPTIMUM DESIGN ANALYSIS OF STRUCTURAL CABLE NETWORKS

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A method of optimum design calculation for structural cable networks is presented. In the analysis, the solution satisfies not only statical and design conditions regarding the equilibrium shape and member forces under stationary loadings, but also conditions under superimposed loadings as to member forces and deformation as well as cross-sectional areas of members. Optimization technique is used, in which the objective function is formulated in terms of "Desirability" with regard to shape, deformation and member forces. Numerical examples show that a variety of solutions are possible in accordance with the design philosophy by adjusting the value of a weight attached to each term of the objective function.
Keywords : *cable net/truss, optimum design, minimizing technique, shape determination*

1. INTRODUCTION

In structural cable networks, such as cable net and cable truss, the equilibrium shape of the structures, in many cases, does not conform to the preset target shape because of the geometrical nonlinearity of the cable members. In such a case attempts are made to bring the equilibrium shape as close to the required shape as possible by controlling member forces and lengths.

The numerical process used for this purpose was named shape determination analysis first by the authors[1),2)] and later several practical methods were presented[3)~5)]. Among them, Refs.4) and 5) show the method utilizing the optimization technique in which the objective function is formulated in terms of "Desirability" as to shape and member forces and solutions are obtained as the stationary point of the objective function.

All the above mentioned methods[1)~5)], however, discuss the shape determination problem under fixed loads only (such as dead weight), which sets limit to the practical application of the theory. As the fixed loads include the dead weight of members, sectional areas of members must be assumed prior to shape determination. On the other hand, member sections should be so designed that stress and deformation satisfy design conditions (allowable stress, etc.) under superimposed (additional) loads. The shape determination, therefore, should be finished before these additional loads are applied. Hence, in practical design, it is not rational to make shape determination and section calculation separately and, from the viewpoint of optimum design, to establish the method which enables us to perform both of these analyses in an automatic process becomes indispensable.

Until recently we could find out only a few works of this category. Jendo[8)] discusses the minimum weight design of a catenary cable under fixed loads and shows the method for equistressed cable design, in which, however, stresses and deformation under additional load are not considered.

Nishino, Duggal and Loganathan[9)] recently presented a new method of cable design analysis. In it they claim that the method copes with the analysis under multiple loading conditions which have never been discussed in the foregoing shape determination analyses and that a variety of solutions are possible owing to setting appropriate design criteria (Five criteria are mentioned regarding shape, stresses, dead weight etc.). Perhaps, at present, their method of analysis can be the most refined and sophisticated one as an optimization technique of cable assemblies.

The basic idea of coping with such multiple loading conditions, however, was already presented, though orally, by the authors more than a decade ago[6)] and, later, one of the authors[7)], though in a limited publication, discussed in detail the optimum design method of cable networks.

This paper gives a unification of the above-mentioned authors' previous works[6),7)] together with a newly tried numerical example to show a method of optimum design analysis of cable networks which covers totally the shape determination analyses by the authors[4),5)] in the past.

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2. STATICAL AND DESIGN CONDITIONS

In the following analysis, while the solution satisfies both statical and design conditions preset as to the equilibrium shape and member forces under stationary loadings ("Completed state" under fixed loads), it satisfies the design or restraint conditions under superimposed loadings ("Deformed state" under additional loads) as to member forces and deformation as well as cross-sectional area of members.

Followings are the statical and design conditions prescribed in this analysis:

Statical condition S-1 : Internal and external forces shall be in stable equilibrium:

Statical condition S-2 : No compressive force shall act on any members:

Design condition D-1 : The shape in the equilibrium state is to be as close to the preset target shape as possible:

Design condition D-2 : Tension of preset value shall act on a specially assigned member:

Design condition D-3 : Tension of an arbitrarily selected member is to be as close to the preset objective value as possible:

Design condition D-4 : Member forces shall not exceed the prescribed allowable values both in the completed state under fixed loads and in the deformed state under additional loads:

Design condition D-5 : Nodal displacements due to additional loads shall be as small as possible:

Design condition D-6 : Total weight of structural members shall be as small as possible.

D-1 is the design requirement of shape prescribed from the structural scheme or functional viewpoint. D-2 is to assume a situation in which the design of a cable anchorage can be made easy by, for example, setting the tensions of boundary members beforehand. These member forces, therefore, are no longer design variables but ones specified as constants. D-3 is the condition imposed on the member forces that become design variables. This is to cope with, for example, the design requirements for achieving a nearly uniform distribution of the member forces over the whole structure by preventing the member forces from becoming too large or too small locally, or for keeping the tensions of prestressing members at prescribed values wherever possible. D-4 is an ordinary design condition for calculating a member cross-section. D-5 controls the extent of structural deformation when additional loads are applied to a completed cable network, so that the function of the structure shall not be spoiled. D-6 is self-evident with regard to the cross-sectional design of members.

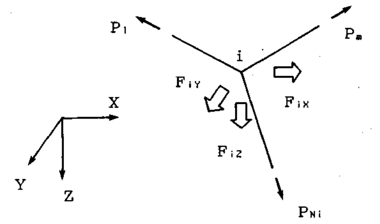

Fig.1 Equilibrium at a joint.

3. FORMULATION AS AN OPTIMIZATION PROBLEM

(1) Conditon S-1

First, we consider the equilibrium equations of completed state under fixed loads. At an arbitrary nodal point i of a cable assembly **(Fig.1)**

$$\left.\begin{aligned}&\sum_{m=1}^{N_i}(X_i-X_j)\varphi_m=F_{iX}\quad \sum_{m=1}^{N_i}(Y_i-Y_j)\varphi_m=F_{iY}\\&\sum_{m=1}^{N_i}(Z_i-Z_j)\varphi_m=F_{iZ}\end{aligned}\right\}\quad\cdots\cdots(1)$$

where m is number of members connected to point i; N_i is total number of members joined at point i; j is number of the other end of member ij; X_i, X_j etc. are nodal coordinates of member m; $\varphi_m=P_m/L_m$ is tension coefficient (P_m and L_m are tension and length of member m, respectively); F_{iX} etc. are applied forces to point i. Similar equations are obtained at every nodal point and their total number is equal to f, the degrees of freedom of the structure. Eq. (1) becomes in matrix form

$$\boldsymbol{\Phi}_T\boldsymbol{X}_T=\boldsymbol{F}\cdots\cdots(2)$$

where $\boldsymbol{\Phi}_T$ is an $f\times 3n$ matrix consisting of φ_m(n: total number of joints); $\boldsymbol{X}_T$ and $\boldsymbol{F}$ are $3n\times 1$ nodal coordinates vector and $f\times 1$ external force vector, respectively. Here, we classify the nodal points of a cable network into two kinds, boundary and non-boundary (free) points. Writing the nodal coordinates of the former $\boldsymbol{X}_C$ and the latter $\boldsymbol{X}$, we rewrite Eq. (2) in the form

$$[\boldsymbol{\Phi}:\boldsymbol{\Phi}_C]\begin{bmatrix}\boldsymbol{X}\\ \cdots\\ \boldsymbol{X}_C\end{bmatrix}=\boldsymbol{F}\cdots\cdots(3)$$

where $\boldsymbol{\Phi}$ and $\boldsymbol{\Phi}_C$ are $f\times f$ and $f\times(3n-f)$ matrices, respectively. Therefore,

$$\boldsymbol{X}=\boldsymbol{\Phi}^{-1}(\boldsymbol{F}-\boldsymbol{\Phi}_C\boldsymbol{X}_C)\cdots\cdots(4)$$

The fixed load $\boldsymbol{F}$ is rewritten in the form

$$\boldsymbol{F}=\boldsymbol{F}_C+\boldsymbol{F}_R\cdots\cdots(5)$$

where $\boldsymbol{F}_R$ is a vector of member dead weight and $\boldsymbol{F}_C$ is a vector of fixed loads except member dead weight. We rewrite $\boldsymbol{F}_R$ in the following form

$$\boldsymbol{F}_R=\boldsymbol{C}_T[L_m]\boldsymbol{A} \quad \text{or} \quad \boldsymbol{F}_R=\boldsymbol{C}_T[A_m]\boldsymbol{L} \cdots\cdots (6)$$

where $[L_m]$ and $[A_m]$ are diagonal matrices of M_T-order consisting of member lengths and cross-sectional areas, respectively (M_T : total number of members). $\boldsymbol{L}$ and $\boldsymbol{A}$ are vectors of M_T-order having member lengths and sectional areas, respectively, as their elements. $[L_m]\boldsymbol{A}$ and $[A_m]\boldsymbol{L}$, therefore, mean member volume vectors. $\boldsymbol{C}_T$ is an $f\times M_T$ matrix which connects the member volume vectors with fixed load vector $\boldsymbol{F}_R$.

Assuming that the weight of a member is distributed equally to both ends of the member, we can write the m-th column of $\boldsymbol{C}_T$ in the form

$$\boldsymbol{C}_{Tm}=[0, \ldots, 0, \frac{1}{2}\gamma_m C_X, \frac{1}{2}\gamma_m C_Y, \frac{1}{2}\gamma_m C_Z, \quad 0, \ldots, 0, \frac{1}{2}\gamma_m C_X, \frac{1}{2}\gamma_m C_Y, \frac{1}{2}\gamma_m C_Z, \quad 0, \ldots, , 0]^T \cdots\cdots (a)$$

where γ_m is weight per unit volume of the m-th member and C_X, C_Y and C_Z are direction cosines of gravitational force with regard to X, Y and Z-axes, respectively.

Meanwhile, a cable network is composed of many structural ropes,each of which is erected between two anchoring points and is considered to be divided into several structural members spanning two adjacent nodal points. This means that members within a single rope have certain uniform sectional value. Hence, the sectional area vector $\boldsymbol{A}$ is rewritten in the form

$$\boldsymbol{A}=\boldsymbol{C}_P\boldsymbol{A}_P \cdots\cdots (7)$$

where $\boldsymbol{A}_P$ is a vector of N_P-order with uniform sectional areas and $\boldsymbol{C}_P$ is an $M_T\times N_P$ matrix, of which m-th row is

$$\left.\begin{matrix} 1 \quad\quad k \quad\quad N_P \\ [0, \ldots, 0, 1, 0, \ldots, 0] \end{matrix}\right\} \cdots\cdots (b)$$

when the cross-sectional area of the m-th member is in common with that of the k-th member. (N_P means the total number of uniform sectional values.) Hence, Eq. (6) becomes

$$\boldsymbol{F}_R=\boldsymbol{C}_T[L_m]\boldsymbol{C}_P\boldsymbol{A}_P \cdots\cdots (8)$$

From Eqs. (4) and (5), nodal coordinates in the completed state are

$$\boldsymbol{X}=\boldsymbol{\Phi}^{-1}(\boldsymbol{F}_C+\boldsymbol{F}_R-\boldsymbol{\Phi}_C\boldsymbol{X}_C) \cdots\cdots (9)$$

and in the deformed state under additional loads, the equilibrium equations are written in the form

$$\boldsymbol{X}'=\boldsymbol{\Phi}_L^{-1}(\boldsymbol{F}_C+\boldsymbol{F}_R+\boldsymbol{F}_L-\boldsymbol{\Phi}_{LC}\boldsymbol{X}_C) \cdots\cdots (10)$$

where $\boldsymbol{X}'$ and $\boldsymbol{F}_L$ are joint coordinates vector after deformation and an additional load vector of $f\times 1$, respectively. $\boldsymbol{\Phi}_L$ and $\boldsymbol{\Phi}_{LC}$ are matrices consisting of tension coefficients in the deformed state, i. e.

$$\boldsymbol{\Phi}_L=\boldsymbol{\Phi}|_{\varphi_T=\varphi_L}, \quad \boldsymbol{\Phi}_{LC}=\boldsymbol{\Phi}_C|_{\varphi_T=\varphi_L} \cdots\cdots (c)$$

where φ_T and φ_L mean tension coefficient vectors in the completed and the deformed state, respectively. The elements of φ_L are

$$\varphi_{Lm}=\frac{P_m+P'_m}{L'_m} \qquad m=1, 2, \ldots, M_T \cdots\cdots (d)$$

where P_m is tension in the m-th member in completed state and P'_m means its increment in deformed state. L'_m is member length in deformed state, i.e.,

$$P'_m=\frac{E_mA_m}{L_m}e_m-\alpha E_mA_m\Delta T, \quad L'_m=L_m+e_m \qquad m=1, 2, \ldots, M_T \cdots\cdots (e)$$

where L_m is length of the m-th member in completed state and E_m, A_m and e_m are Young's modulus, sectional area and elastic elongation, respectively ; α and ΔT are coefficient of linear expansion and temperature change, respectively. In the following analysis, we use the approximation

$$\varphi_{Lm}=\frac{P_m+P'_m}{L_m}=\varphi_m+\frac{E_mA_m}{L_m^2}e_m-\alpha\frac{E_mA_m}{L_m}\Delta T \qquad m=1, 2, \ldots, M_T \cdots\cdots (11)$$

by considering the small difference between member lengths in the completed and the deformed state.

(2) Condition S-2

The conditions of incompressibility of members are

$$\left.\begin{matrix} \varphi_k>0 & k=1, 2, \ldots, M \\ \varphi_{Lm}\geqq 0 & m=1, 2, \ldots, M_T \end{matrix}\right\} \cdots\cdots (12)$$

for completed and deformed state, respectively, where M is the total number of members excluding those which are under the control of the following design condition D-2.

(3) Condition D-2

We rewrite the tension coefficient vector φ_T (in completed state) in the form

$$\varphi_T=\begin{bmatrix} \varphi \\ \varphi_C \end{bmatrix} \cdots\cdots (f)$$

where φ is an $M\times 1$ unknown tension coefficient vector and φ_C is an $(M_T-M)\times 1$ vector of tension coefficients in members which are under control of Condition D-2. The elements of φ_C are no longer design variables, but prescribed constants.

(4) Condition D-4

We express this condition in the form

$$P_m\leqq\frac{P_{cm}}{n_c}, \quad P_m+\frac{E_mA_m}{L_m}e_m-\alpha E_mA_m\Delta T\leqq\frac{P_{cm}}{n_s} \qquad m=1, 2, \ldots, M_T \cdots\cdots (13)$$

for completed and deformed state, respectively, where P_{cm} is ultimate strength of the m-th member, and n_c and n_s are safety factors for completed and deformed state. In the following analysis we assume P_{cm} is proportional to sectional area A_m, i.e.,

$$P_{cm}=kA_m \quad (k: \text{proportional constant})$$
$$m=1,2,\ldots,M_T \quad \cdots\cdots\cdots\cdots (g)$$

Eqs. (13) become, therefore,

$$L_m\varphi_m \leqq C_cA_m,$$
$$L_m\varphi_m+\frac{E_mA_m}{L_m}e_m-\alpha E_mA_m\Delta T\leqq C_sA_m$$
$$m=1,2,\ldots,M_T \quad \cdots\cdots (14)$$

where $C_c=k/n_c$ and $C_s=k/n_s$.

(5) Conditions D-1, D-3, D-5 and D-6

We define the quantity

$$W=\sum_{i=1}^{f}\left(\frac{R_i}{q_{Ri}}\right)^2+\sum_{k=1}^{M}\left(\frac{\varphi_k-\varphi_{0k}}{q_{\varphi k}}\right)^2+\sum_{i=1}^{f}\left(\frac{x_i}{q_{xi}}\right)^2+\sum_{m=1}^{M}\left(\frac{L_mA_m}{q_{Am}}\right)^2$$
$$=\|Q_RR\|^2+\|Q_\varphi(\varphi-\varphi_0)\|^2+\|Q_Xx\|^2+\|Q_A[L_m]A\|^2 \quad \cdots\cdots\cdots\cdots (15)$$

where R_i is difference between joint coordinates of target shape and those of completed equilibrium shape, φ_k is an unknown tension coefficient, φ_{0k} is a target tension coefficient (target tension divided by member length in target shape), x_i is joint displacement and R, φ, φ_0 and x are vector expression of the above quantities.

In Eq. (15), the reciprocals of $q_{Ri}, q_{\varphi k}, q_{xi}$ and q_{Am} mean the weights given to design variables R_i, $\varphi_k-\varphi_{0k}$, x_i and L_mA_m in compliance with designer's requirement ; Q_R, Q_φ, Q_x and Q_A are diagonal matrices of which elements are $(1/q)$-values. The mathematical meaning of these weights is discussed in detail in Refs. 4) and 5).

Vector R in Eq. (15) is rewritten in the form

$$R=\Phi^{-1}(F_C+F_R-\Phi_CX_C)-X_0 \quad \cdots\cdots\cdots\cdots (16)$$

by using Eq. (9), where X_0 is the coordinates vector of target shape. Joint displacement vector x has the form

$$x=\Phi_L^{-1}(F_C+F_R+F_L-\Phi_{LC}X_C)-X \quad \cdots\cdots (17)$$

We call the reciprocal of W of Eq. (15) "Desirability"[4),5)] in cable network design and consider that we attain to the design optimum when we enhance the desirability at a maximum.

Consequently, the prime subject of the analysis boils down to the solution of the optimization problem : "Obtain the values of φ and A_P which minimize the objective function W of Eq. (15) under the conditions (12) and (14), where m unknown tension coefficients and N_P uniform sectional areas of cable members are taken as independent variables".

4. NUMERICAL PROCESS OF SOLUTION

We use Gauss's method to get correction vectors. Let $\varphi^{(i)}$ and $A_P^{(i)}$ be approximate solutions at the i-th iteration, and $\Delta\varphi^{(i)}$ and $\Delta A_P^{(i)}$ be correction vectors for each of them. In order to express the vectors R, x and $[L_m]A$ in Eq. (15) in terms of correction vectors, we expand them in Taylor series with respect to $\Delta\varphi^{(i)}$ and $\Delta A_P^{(i)}$, and rewrite them R_L, x_L and $\{[L_m]A\}_L$, respectively, i.e.,

$$\left.\begin{aligned}R_L&=R^{(i)}+M_1^{(i)}\Delta\varphi^{(i)}+M_2^{(i)}\Delta A_P^{(i)}\\x_L&=x^{(i)}+M_3^{(i)}\Delta\varphi^{(i)}+M_4^{(i)}\Delta A_P^{(i)}\\\{[L_m]A\}_L&=\{[L_m]A\}^{(i)}+M_5^{(i)}\Delta\varphi^{(i)}+M_6^{(i)}\Delta A_P^{(i)}\end{aligned}\right\} \quad \cdots\cdots (18)$$

where terms of higher order are neglected. Superscript i means the value at the i-th iteration and

$$\left.\begin{aligned}&M_1^{(i)}=\left[\frac{\partial R_j}{\partial\varphi_k}\right]^{(i)},\quad M_2^{(i)}=\left[\frac{\partial R_j}{\partial A_{P,n}}\right]^{(i)},\\&M_3^{(i)}=\left[\frac{\partial x_j}{\partial\varphi_k}\right]^{(i)},\quad M_4^{(i)}=\left[\frac{\partial x_j}{\partial A_{P,n}}\right]^{(i)},\\&M_5^{(i)}=\left[\frac{\partial(L_mA_m)}{\partial\varphi_k}\right]^{(i)},\quad M_6^{(i)}=\left[\frac{\partial(L_mA_m)}{\partial A_{P,n}}\right]^{(i)}\\&\begin{pmatrix}j=1,2,\cdots,f\ ; & m=1,2,\cdots,M_T\\k=1,2,\cdots,M\ ; & n=1,2,\cdots,N_P\end{pmatrix}\end{aligned}\right\} \quad \cdots\cdots (19)$$

We will now formulate $M_1^{(i)}$ through $M_6^{(i)}$ in Eqs. (19) omitting superscript i in the following.

M_1 is easily formulated by using Eqs. (6) and (16) and through some calculative process in the form

$$M_1=-(E-\Phi^{-1}C_TN_4)^{-1}\Phi^{-1}[\Delta X_k] \quad \cdots\cdots (20)$$

where E is a unit matrix, and

$$\left.\begin{aligned}N_4&=\left[\frac{\partial(L_mA_m)}{\partial X_j}\right]=[A_m]\left[\frac{\partial L_m}{\partial X_j}\right]\\&j=1,2,\ldots,f\\&m=1,2,\ldots,M_T\end{aligned}\right\} \cdots\cdots (21)$$

ΔX_k is an $f\times 1$ vector of which elements are the coordinates differences of both ends of the k-th member and is written in the form

$$\Delta X_k=[0,\ldots,0,X_A-X_B,Y_A-Y_B,Z_A-Z_B,\ 0,\ldots,0,X_B-X_A,Y_B-Y_A,Z_B-Z_A,\ 0,\ldots,0]^T \quad \cdots\cdots (h)$$

M_2 is derived directly from Eqs. (6), (7) and (16) and takes the form

$$M_2=(E-\Phi^{-1}C_TN_4)^{-1}\Phi^{-1}C_T[L_m]C_P \quad \cdots\cdots (22)$$

Next, from Eqs. (17) and (h)

$$\frac{\partial x}{\partial\varphi_k}=-\Phi_L^{-1}\left\{\frac{\partial\Phi_L}{\partial\varphi_k}(X+x)+\frac{\partial\Phi_{LC}}{\partial\varphi_k}X_C\right\}$$

$$+\boldsymbol{\Phi}_L^{-1}\frac{\partial \boldsymbol{F}_R}{\partial \varphi_k}-\frac{\partial \boldsymbol{X}}{\partial \varphi_k} \quad \cdots\cdots (23)$$

Here, we write the tension coefficient vector in deformed state φ_L in the form

$$\varphi_L=\begin{bmatrix}\varphi\\ \hline \varphi_C\end{bmatrix}+\varphi' \quad \cdots\cdots (24)$$

where φ and φ_C mean the unknown and the known tension coefficient vector in completed state and φ' is an incremental tension coefficient vector due to additional loadings. The element of φ' are

$$\varphi'_m=\frac{E_mA_m}{L_m^2}e_m-\alpha\frac{E_mA_m}{L_m}\Delta T \qquad m=1,2,\ldots,M_T \quad \cdots\cdots (\mathrm{i})$$

$\boldsymbol{\Phi}_L$ and $\boldsymbol{\Phi}_{LC}$ in Eq. (23) are also rewritten in the form

$$\boldsymbol{\Phi}_L=\boldsymbol{\Phi}+\boldsymbol{\Phi}', \quad \boldsymbol{\Phi}_{LC}=\boldsymbol{\Phi}_C+\boldsymbol{\Phi}'_C \quad \cdots\cdots (25)$$

where the first terms consist of φ and φ_C, and the second terms consist of φ' only.

Differentiating Eqs. (25) and substituting in Eq. (23) we find

$$\frac{\partial \boldsymbol{X}}{\partial \varphi_k}=-\boldsymbol{\Phi}_L^{-1}\left\{\Delta \boldsymbol{X}_k+\Delta \boldsymbol{x}_k+\boldsymbol{N}_5\frac{\partial \varphi'}{\partial \varphi_k}\right\}$$
$$+\boldsymbol{\Phi}_L^{-1}\frac{\partial \boldsymbol{F}_R}{\partial \varphi_k}-\frac{\partial \boldsymbol{X}}{\partial \varphi_k} \quad \cdots\cdots (26)$$

where $\Delta \boldsymbol{x}_k$ is a similar vector to $\Delta \boldsymbol{X}_k$ (Eq. (h)) and consists of joint displacement differences and $\boldsymbol{N}_5$ is an $f\times M_T$ matrix having a column vector $\Delta \boldsymbol{X}_m+\Delta \boldsymbol{x}_m$ for the m-th member.

Eq. (26) is further rewritten through certain calculative process in the form

$$\frac{\partial \boldsymbol{x}}{\partial \varphi_k}=-\boldsymbol{\Phi}_L^{-1}\left\{\Delta \boldsymbol{X}_k+\Delta \boldsymbol{x}_k+\boldsymbol{N}_5\left(\boldsymbol{N}_1\frac{\partial \boldsymbol{R}}{\partial \varphi_k}\right.\right.$$
$$\left.\left.+\boldsymbol{N}_2\frac{\partial \boldsymbol{x}}{\partial \varphi_k}\right)\right\}+\boldsymbol{\Phi}_L^{-1}\boldsymbol{C}_7\boldsymbol{N}_4\frac{\partial \boldsymbol{R}}{\partial \varphi_k}-\frac{\partial \boldsymbol{R}}{\partial \varphi_k} \quad \cdots\cdots (\mathrm{j})$$

where

$$\left.\begin{aligned}\boldsymbol{N}_1&=\left[\frac{\partial \varphi'}{\partial \boldsymbol{X}}\right]=\left[\frac{\partial\left(\frac{E_mA_m}{L_m^2}e_m-\alpha\frac{E_mA_m}{L_m}\Delta T\right)}{\partial X_j}\right],\\ \boldsymbol{N}_2&=\left[\frac{\partial \varphi'}{\partial \boldsymbol{x}}\right]=\left[\frac{\partial\left(\frac{E_mA_m}{L_m^2}e_m\right)}{\partial x_j}\right]\\ &\qquad m=1,2,\ldots,M_T\\ &\qquad j=1,2,\ldots,f\end{aligned}\right\} \quad \cdots\cdots (27)$$

Then, by the definition of $\boldsymbol{M}_3$ in Eq. (19) we obtain

$$\boldsymbol{M}_3=-(\boldsymbol{E}+\boldsymbol{\Phi}_L^{-1}\boldsymbol{N}_5\boldsymbol{N}_2)^{-1}\{\boldsymbol{\Phi}_L^{-1}([\Delta \boldsymbol{X}_k+\Delta \boldsymbol{x}_k]$$
$$+\boldsymbol{N}_5\boldsymbol{N}_1\boldsymbol{M}_1-\boldsymbol{C}_7\boldsymbol{N}_4\boldsymbol{M}_1)+\boldsymbol{M}_1\} \quad \cdots\cdots (28)$$

where $[\Delta \boldsymbol{X}_k+\Delta \boldsymbol{x}_k]$ is an $f\times M$ matrix consisting of a column vector $\Delta \boldsymbol{X}_k+\Delta \boldsymbol{x}_k$ $(k=1,2,\ldots,M)$.

$\boldsymbol{M}_4$ is formulated in the similar way to $\boldsymbol{M}_3$. Differentiating Eq. (17) with respect to $A_{P,n}$ and considering Eqs. (25) and (26), we get

$$\frac{\partial \boldsymbol{x}}{\partial A_{P,n}}=-\boldsymbol{\Phi}_L^{-1}\boldsymbol{N}_5\frac{\partial \varphi'}{\partial A_{P,n}}+\boldsymbol{\Phi}_L^{-1}\frac{\partial \boldsymbol{F}_R}{\partial A_{P,n}}-\frac{\partial \boldsymbol{X}}{\partial A_{P,n}} \quad \cdots\cdots (29)$$

in which we can write

$$\frac{\partial \varphi'}{\partial A_{P,n}}=\sum_{j=1}^{f}\frac{\partial \varphi'}{\partial X_j}\frac{\partial X_j}{\partial A_{P,n}}+\sum_{j=1}^{f}\frac{\partial \varphi'}{\partial x_j}\frac{\partial x_j}{\partial A_{P,n}}$$
$$+\sum_{m=1}^{M_T}\frac{\partial \varphi'}{\partial A_m}\frac{\partial A_m}{\partial A_{P,n}}$$
$$=\boldsymbol{N}_1\frac{\partial \boldsymbol{R}}{\partial A_{P,n}}+\boldsymbol{N}_2\frac{\partial \boldsymbol{x}}{\partial A_{P,n}}+\boldsymbol{N}_3\boldsymbol{e}_n \quad \cdots\cdots (30)$$

where $\boldsymbol{e}_n$ is an $N_P\times 1$ vector having unity for the n-th element and zero elements elsewhere and

$$\boldsymbol{N}_3=\left[\frac{E_m}{L_m}\left(\frac{e_m}{L_m}-\alpha\Delta T\right)\right]\boldsymbol{C}_P \qquad (m=1,2,\ldots,M_T) \quad \cdots\cdots (\mathrm{k})$$

The bracket [ ] means a diagonal matrix of $M_T\times M_T$. From Eqs. (29) and (30) we get through several calculations

$$\boldsymbol{M}_4=-(\boldsymbol{E}+\boldsymbol{\Phi}_L^{-1}\boldsymbol{N}_5\boldsymbol{N}_2)^{-1}\{\boldsymbol{\Phi}_L^{-1}\boldsymbol{N}_5(\boldsymbol{N}_1\boldsymbol{M}_2+\boldsymbol{N}_3)$$
$$+(\boldsymbol{\Phi}_L^{-1}-\boldsymbol{\Phi}^{-1})\boldsymbol{C}_7(\boldsymbol{N}_4\boldsymbol{M}_2+[L_m]\boldsymbol{C}_P)\} \quad \cdots\cdots (31)$$

We get $\boldsymbol{M}_5$ and $\boldsymbol{M}_6$ directly by using Eqs. (20), (21) and (22) in the form

$$\boldsymbol{M}_5=\boldsymbol{N}_4\boldsymbol{M}_1, \quad \boldsymbol{M}_6=\boldsymbol{N}_4\boldsymbol{M}_2+[L_m]\boldsymbol{C}_P \quad \cdots\cdots (32)$$

Thus, the value of objective function at the $(i+1)$-st iteration becomes from Eqs. (15) and (18)

$$W^{(i+1)}=\|\boldsymbol{Q}_R\boldsymbol{R}_L\|^2+\|\boldsymbol{Q}_\varphi(\varphi^{(i)}+\Delta\varphi^{(i)}-\varphi_0)\|^2$$
$$+\|\boldsymbol{Q}_X\boldsymbol{x}_L\|^2+\|\boldsymbol{Q}_A\{[L_m]\boldsymbol{A}\}_L\|^2$$
$$=\|\boldsymbol{Q}_R(\boldsymbol{R}^{(i)}+\boldsymbol{M}_1^{(i)}\Delta\varphi^{(i)}+\boldsymbol{M}_2^{(i)}\Delta\boldsymbol{A}_P^{(i)})\|^2$$
$$+\|\boldsymbol{Q}_\varphi(\varphi^{(i)}+\Delta\varphi^{(i)}-\varphi_0)\|^2+\|\boldsymbol{Q}_X(\boldsymbol{x}^{(i)}$$
$$+\boldsymbol{M}_3^{(i)}\Delta\varphi^{(i)}+\boldsymbol{M}_4^{(i)}\Delta\boldsymbol{A}_P^{(i)})\|^2+\|\boldsymbol{Q}_A(\{[L_m]\boldsymbol{A}\}^{(i)}$$
$$+\boldsymbol{M}_5^{(i)}\Delta\varphi^{(i)}+\boldsymbol{M}_6^{(i)}\Delta\boldsymbol{A}_P^{(i)}\|^2 \quad \cdots\cdots (33)$$

The correction vectors $\Delta\varphi^{(i)}$ and $\Delta\boldsymbol{A}_P^{(i)}$ which are to minimize $W^{(i+1)}$ are determined by

$$\frac{\partial W^{(i+1)}}{\partial \Delta\varphi_k^{(i)}}=0, \quad \frac{\partial W^{(i+1)}}{\partial \Delta A_{P,n}^{(i)}}=0$$
$$k=1,2,\ldots,M, \quad n=1,2,\ldots,N_P \quad \cdots\cdots (34)$$

From Eqs. (33) and (34) we obtain

$$\begin{bmatrix}\boldsymbol{H}_{11} & \boldsymbol{H}_{12}\\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22}\end{bmatrix}\begin{bmatrix}\Delta\varphi^{(i)}\\ \Delta\boldsymbol{A}_P^{(i)}\end{bmatrix}=-\begin{bmatrix}\boldsymbol{G}_1\\ \boldsymbol{G}_2\end{bmatrix} \quad \text{or}$$

$$\boldsymbol{H}\begin{bmatrix}\Delta\varphi^{(i)}\\ \Delta\boldsymbol{A}_P^{(i)}\end{bmatrix}=-\boldsymbol{G} \quad \cdots\cdots (35)$$

where

$$\left.\begin{aligned}
&\boldsymbol{H}_{11}=\boldsymbol{M}_1^{(i)T}\boldsymbol{Q}_R^2\boldsymbol{M}_1^{(i)}+\boldsymbol{Q}_\varphi^2+\boldsymbol{M}_3^{(i)T}\boldsymbol{Q}_X^2\boldsymbol{M}_3^{(i)}\\
&\qquad+\boldsymbol{M}_5^{(i)T}\boldsymbol{Q}_A^2\boldsymbol{M}_5^{(i)},\\
&\boldsymbol{H}_{12}=\boldsymbol{M}_1^{(i)T}\boldsymbol{Q}_R^2\boldsymbol{M}_2^{(i)}+\boldsymbol{M}_3^{(i)T}\boldsymbol{Q}_X^2\boldsymbol{M}_4^{(i)}\\
&\qquad+\boldsymbol{M}_5^{(i)T}\boldsymbol{Q}_A^2\boldsymbol{M}_6^{(i)},\\
&\boldsymbol{H}_{21}=\boldsymbol{H}_{12}^T,\\
&\boldsymbol{H}_{22}=\boldsymbol{M}_2^{(i)T}\boldsymbol{Q}_R^2\boldsymbol{M}_2^{(i)}+\boldsymbol{M}_4^{(i)T}\boldsymbol{Q}_X^2\boldsymbol{M}_4^{(i)}\\
&\qquad+\boldsymbol{M}_6^{(i)T}\boldsymbol{Q}_A^2\boldsymbol{M}_6^{(i)},\\
&\boldsymbol{G}_1=\boldsymbol{M}_1^{(i)T}\boldsymbol{Q}_R^2\boldsymbol{R}^{(i)}+\boldsymbol{Q}_\varphi^2(\varphi^{(i)}-\varphi_0)\\
&\qquad+\boldsymbol{M}_3^{(i)T}\boldsymbol{Q}_X^2\boldsymbol{x}^{(i)}+\boldsymbol{M}_5^{(i)T}\boldsymbol{Q}_A^2\{[L_m]A\}^{(i)}\\
&\boldsymbol{G}_2=\boldsymbol{M}_2^{(i)T}\boldsymbol{Q}_R^2\boldsymbol{R}^{(i)}+\boldsymbol{M}_4^{(i)T}\boldsymbol{Q}_X^2\boldsymbol{x}^{(i)}\\
&\qquad+\boldsymbol{M}_6^{(i)T}\boldsymbol{Q}_A^2\{[L_m]A\}^{(i)}\\
&\boldsymbol{Q}_R^2=\boldsymbol{Q}_R^T\boldsymbol{Q}_R=\boldsymbol{Q}_R\boldsymbol{Q}_R,\quad \boldsymbol{Q}_\varphi^2=\boldsymbol{Q}_\varphi^T\boldsymbol{Q}_\varphi=\boldsymbol{Q}_\varphi\boldsymbol{Q}_\varphi,\\
&\boldsymbol{Q}_x^2=\boldsymbol{Q}_X^T\boldsymbol{Q}_X=\boldsymbol{Q}_X\boldsymbol{Q}_X,\quad \boldsymbol{Q}_A^2=\boldsymbol{Q}_A\boldsymbol{Q}_A
\end{aligned}\right\} \cdots\cdots (36)$$

In order to get the correction vectors in Eq. (35) with rapid convergence, we use the maximum neighborhood method proposed by Marquardt[10]. Following the method we construct an expression

$$\{[h_{jj}]^{-1}\boldsymbol{H}[h_{jj}]^{-1}+\lambda\boldsymbol{E}\}\begin{bmatrix}\Delta\varphi_*^{(i)}\\ \Delta A_{P*}^{(i)}\end{bmatrix}=-[h_{jj}]^{-1}\boldsymbol{G} \cdots\cdots (37)$$

in relation to Eq. (35), where

$$\begin{bmatrix}\Delta\varphi^{(i)}\\ \Delta A_P^{(i)}\end{bmatrix}=[h_{jj}]^{-1}\begin{bmatrix}\Delta\varphi_*^{(i)}\\ \Delta A_{P*}^{(i)}\end{bmatrix}\cdots\cdots (1)$$

and $[h_{jj}]$ is a diagonal matrix where $h_{jj}(j=1, 2, ..., M+N_P)$ is square root of a diagonal element of matrix $\boldsymbol{H}$. λ is a prescribed positive number which changes the direction of a correction vector. In Refs. 4) and 5) discussion is made as to the numerically experimental characteristics of the maximum neighborhood method as well as how to preset the values of λ. The solution of Eq. (37) gives a set of correction vectors $\Delta\varphi^{(i)}$ and $\Delta A_P^{(i)}$ with regard to each value of λ.

Further, we improve the correction vectors by a numerical method developed by the authors[4),5)]. When we preset n_B values for λ in Eq. (37), we obtain n_B sets of correction vectors. Then, we write n_B sets of solutions in the $(i+1)$st iteration in the form

$$\begin{bmatrix}\varphi_n^{(i+1)}\\ A_{P,n}^{(i+1)}\end{bmatrix}=\begin{bmatrix}\varphi^{(i)}\\ A_P^{(i)}\end{bmatrix}+S_n\begin{bmatrix}\Delta\varphi_n^{(i)}\\ \Delta A_{P,n}^{(i)}\end{bmatrix}\cdots\cdots (38)$$

where subscript n means the n-th value of λ ($n=1, 2, ..., n_B$) and S_n is a step-size to improve correction vectors given by Eq. (37). We determine the value of S_n (Optimum step-size) so as to minimize

$$W_n^{(i+1)}=W(\varphi_n^{(i+1)},\ A_{P,n}^{(i+1)})=W(S_n)\cdots\cdots (39)$$

$$n=1, 2, ..., n_B$$

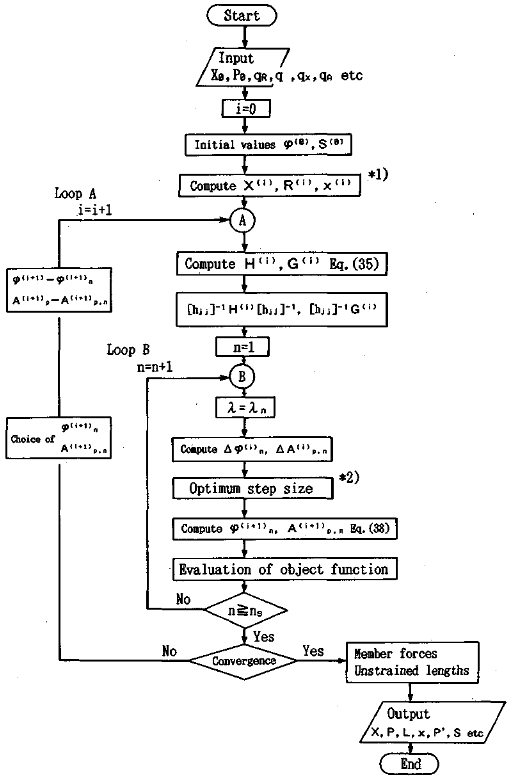

Fig.2 Flow of computation.

In Refs. 4) and 5) the detailed discussion is made as to how to determine the optimum step-size and we refrain from repeating the discussion here.

Fig.2 shows the flow of computation. Loop A minimizes the object function and Loop B determines correction vectors and optimum step-size. At places marked $*1)$ and $*2)$ finite deformation analysis is done for finding out the equilibrium state at each stage. The conditions (12) and (14) are taken into account at $*2)$. Judgment of convergence is made by

$$\frac{W^{(i)}-W^{(i+1)}}{W^{(i)}}<\varepsilon_C\cdots\cdots (40)$$

where ε_C is an arbitrary small number.

5. NUMERICAL EXAMPLES

Following assumptions are made for two numerical examples shown in this chapter : ① Young's modulus $E=2.0\times10^7$ t/m^2 ; ② Unit weight of member $\gamma=8.32$ t/m^3 ; ③ Proportional constant between member area and breaking strength $k=1.32\times10^5$ t/m^2 ; ④ Safety factor (Breaking strength/Allowable tension) = 3.0 (for completed state) and 2.7 (for deformed state) ; ⑤ Convergence condition (Eq. (40)) $\varepsilon_C=10^{-3}$; ⑥

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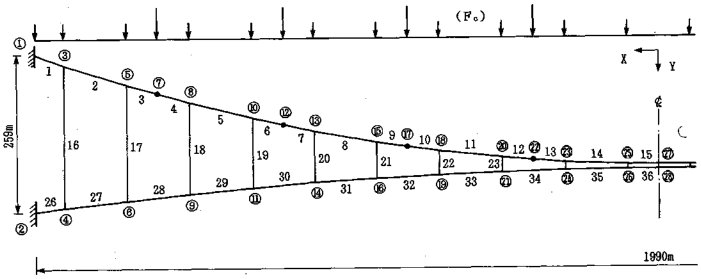

Fig.3 Example 1 : Cable girder.

Table 1 Load, shape and displacement (Example 1).

Node No.	Target shape X	Target shape Y	Vertical load F_C	Vertical load F_L	Completed shape X	Completed shape Y	Displacement x	Displacement y
1	995.000	0	0	0	995.000	0	0	0
2	995.000	259.000	0	0	995.000	259.000	0	0
3	950.000	16.341	7.974	3.680	950.008	16.242	0.016	0.069
4	950.000	252.988	0	0	949.492	253.215	-0.006	0.067
5	850.000	49.847	8.203	3.786	850.026	49.737	0.038	0.193
6	850.000	240.625	0	0	849.872	240.694	-0.014	0.189
7	800.000	65.160	9.424	2.504	800.035	65.002	0.047	0.254
8	750.000	79.519	8.076	3.728	749.951	79.454	0.052	0.302
9	750.000	229.635	0	0	749.883	229.336	-0.020	0.299
10	650.000	105.395	8.039	3.711	650.016	105.309	0.060	0.402
11	650.000	220.019	0	0	649.874	219.615	-0.023	0.399
12	600.000	116.921	9.326	2.458	599.895	116.803	0.062	0.452
13	550.000	127.511	7.945	3.667	549.879	127.445	0.061	0.488
14	550.000	211.777	0	0	549.860	211.380	-0.024	0.487
15	450.000	145.896	7.919	3.655	449.922	145.823	0.057	0.562
16	450.000	204.909	0	0	449.877	204.554	-0.023	0.560
17	400.000	153.697	10.256	2.426	399.912	153.579	0.054	0.598
18	350.000	160.574	7.856	3.626	349.903	160.482	0.049	0.622
19	350.000	199.414	0	0	349.900	199.130	-0.020	0.621
20	250.000	171.566	7.840	3.619	249.956	171.417	0.038	0.671
21	250.000	195.293	0	0	249.939	195.171	-0.015	0.670
22	200.000	175.684	9.214	2.407	199.949	175.455	0.031	0.696
23	150.000	178.886	7.809	3.604	149.917	178.681	0.024	0.706
24	150.000	192.545	0	0	149.977	192.769	-0.010	0.706
25	50.000	182.543	7.804	3.602	50.020	182.233	0.008	0.733
26	50.000	191.172	0	0	49.983	192.159	-0.003	0.733
27	0	183.000	5.100	1.200	0	182.656	0	0.739
28	0	191.000	0	0	0	192.648	0	0.761
Unit		m	ton (1ton=9.8kN)					m

Table 2 Member force and tension coefficient (TC) (Example 1).

Member No.	Target value Tension	Target value TC	Completed state Tension	Completed state TC	Deformed st. Tension
1	1585.90	33.122	1585.846	33.125	1713.218
2	1571.90	14.905	1572.045	14.906	1698.110
3	1558.80	29.809	1558.944	29.812	1683.889
4	1550.70	29.809	1547.951	29.757	1671.862
5	1539.60	14.905	1540.405	14.913	1663.682
6	1526.50	29.750	1525.687	29.734	1647.737
7	1523.60	29.811	1522.884	29.797	1644.590
8	1515.50	14.905	1515.952	14.910	1637.063
9	1508.50	29.809	1508.052	29.801	1628.460
10	1504.50	29.809	1504.095	29.801	1624.142
11	1499.50	14.905	1500.102	14.911	1619.772
12	1495.50	29.809	1495.160	29.802	1614.398
13	1493.60	29.811	1492.440	29.788	1611.404
14	1491.50	14.905	1492.929	14.919	1611.901
15	1490.60	29.811	1489.804	29.795	1608.504
16	8.99	0.038	7.202	0.030	7.137
17	12.40	0.065	11.875	0.062	11.673
18	12.40	0.083	12.765	0.085	12.519
19	12.40	0.108	12.337	0.108	12.112
20	12.40	0.147	12.097	0.144	11.867
21	12.40	0.210	12.030	0.204	11.819
22	12.40	0.319	12.116	0.312	11.898
23	12.40	0.523	12.214	0.515	11.984
24	12.40	0.908	12.335	0.903	12.081
25	9.30	1.078	8.760	1.016	8.633
26	220.30	4.852	221.651	4.882	208.227
27	220.10	2.184	224.770	2.230	211.175
28	219.70	2.184	223.566	2.222	210.073
29	219.40	2.184	223.226	2.222	209.791
30	219.10	2.184	222.960	2.222	209.574
31	218.90	2.184	222.802	2.223	209.454
32	218.70	2.184	222.633	2.223	209.321
33	218.60	2.184	222.525	2.223	209.244
34	218.50	2.184	222.425	2.223	209.176
35	218.40	2.184	222.238	2.222	209.026
36	218.40	4.368	222.319	4.446	209.115
Unit	Tension in ton; TC in ton/m (1ton=9.8kN)				

Values for λ (Eq. (37)) $=10^{-3}$ and 10^{-1}.

(1) Example 1 (cable girder)

We show an example of design calculation for a two-dimensional cable girder in **Fig.3** to determine its completed shape and cross sectional area of each member. Target shape and joint loads are given in the left half of **Table 1** where F_C means fixed load, excluding the dead weight of members, and F_L means additional joint load due to uniformly distributed load of 0.0478 ton/m along the cable length and concentrated loads at points 7, 12, 17 and 22. The left half part of **Table 2** shows the target values of member forces and tension coefficients. (We preset these values referring to the approximate solution by membrane analysis of 2-dimensional cable girder.) Here, upper chord members (Nos.1～15), suspension members (Nos. 16～25) and lower chord members (Nos.26～36) shall have respective uniform cross-sections and it is assumed that dead weight of a member is divided into two equal portions applied to both ends of the member.

This cable girder is a model of footbridge for the purpose of erecting main cables of a long span suspension bridge and its design priority is given to the conformity of the target and the completed shape. We, therefore, set the q-value in Eq. (15), which controls the size of desirable domain for each design variable, as follows ; $q_{Ri}=0.01$ m, $q_{Xi}=100$ m ($i=1, ..., f$), $q_{\varphi k}=1\ 000$ ton/L_k ($k=1, ..., 15$), $q_{\varphi k}=10$ ton/L_k ($k=16, ..., 25$), $q_{\varphi k}=100$ ton/L_k ($k=26, ..., 36$) and $q_{Am}=1$ m^3 ($m=1, ..., M_T$), where $f=50$ and $M_T=36$.

Table 3 Member cross-sectional area (m^2).

Member	No.	Initial	Optimum
Upper chord	1~15	0.03	0.0360
Web	16~25	0.0005	0.000555
Lower chord	26~36	0.01	0.0105

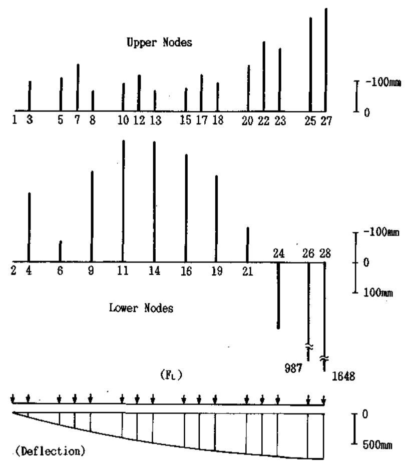

Fig.4 Difference between target and completed shapes, and deflection due to additional loads.

The background of setting these q-values is as follows ; upper chord members shall have shape-weighted solution rather than member force-weighted one and, on the other hand, web members shall have member force-weighted solution, while the solution for lower chord members shall have in-between characteristics.

The left half part of the structure is analyzed due to its symmetry. At the beginning of numerical process, we have to set the initial values for unknown tension coefficients and sectional areas and we use the values of target tension coefficients in **Table 2** and values in **Table 3** given by the results from membrane approximation. Numerical results are shown in **Tables 1** through **4.** In the right half of **Table 1** are shown the completed shape and displacement due to additional loads. Member forces in the completed and the deformed state are shown in the right half of **Table 2** and the optimum solution for member cross sections in **Table 3. Fig.4** shows the diagram of discrepancy of the completed shape in vertical direction from the target shape, together with deflection due to additional loads. This figure clearly shows the effect of setting q-values in Eq. (15) as to the completed shape. As is shown above, the weights for the shape ($1/q_{Ri}$) are uniform to all members and the weights for the web member forces ($1/q_{\varphi k}$) are hundred times as heavy as those of upper chord members. As a result, upper nodes in completed state show small differences from target shape, while lower nodes show rather big differences due to the member force-weighted solution for web members.

Table 4 Convergence of objective function (Example1).

i	$W^{(i)}$	$\lvert \varepsilon_c \rvert$
0	3.833×10^7	
1	7.289×10^5	0.384
2	5.306×10^4	0.272
3	5.295×10^4	2.073×10^{-3}
4	5.300×10^4	0.944×10^{-3}

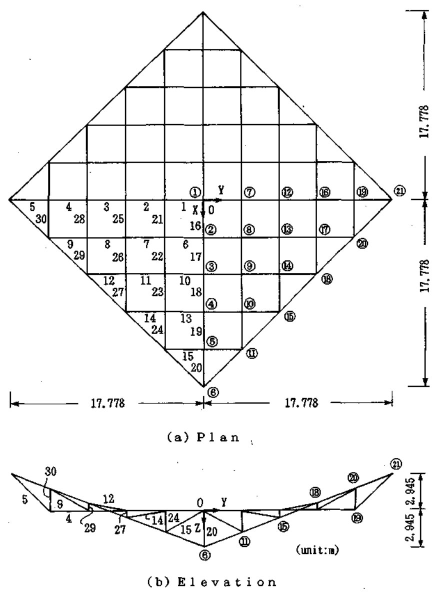

Fig.5 Example 2 : Diagonal cable net.

Table 4 shows how the objective function converges with the increasing number of iteration. The CPU-occupying time for this numerical process by scalar computation in FACOM VP-200 was eight seconds.

(2) Example 2 (3-dimensional diagonal cable net)

We analyze a diagonal-type cable net of which target shape is shown in **Fig.5** and in **Table 5**. This is the same structure as was already analyzed in Refs. 4) and 5). We will here determine the completed state and member sectional areas. In **Tables 5** and **6**, fixed loads F_C excluding dead weight, target shape and target tensions are given. Fixed load F_R due to dead weight of a member is equally divided into two joint loads. Additional loads F_L shall have half the magnitude of F_C. All the main suspension cables (Nos. 1~15) shall have a uniform cross section and all the secondary cables (Nos. 16~30) another uniform cross section. We specify the desirable domain of solution by quantities

Table 5 Load, target shape, completed shape and joint displacement (Example 2).

Joint No.	Load F_c	Target shape			Completed shape			Joint displacement		
		X	Y	Z	X	Y	Z	x	y	z
1	0.125	0	0	0	0	0	-0.058	0	0	0.011
2	0.252	3.536	0	0	3.511	0	0.213	-0.003	0	0.040
3	0.254	7.071	0	0	7.015	0	0.600	-0.004	0	0.035
4	0.256	10.707	0	0	10.592	0	1.156	-0.003	0	0.023
5	0.259	14.142	0	0	13.972	0	1.886	-0.003	0	0.015
6	0	17.778	0	2.945	17.778	0	2.946	0	0	0
7	0.251	0	3.536	0	0	3.443	-0.172	0	0.001	0.018
8	0.503	3.536	3.536	0	3.514	3.468	0.093	-0.003	0.002	0.038
9	0.508	7.071	3.536	0	7.021	3.505	0.470	-0.003	0.002	0.033
10	0.515	10.707	3.536	0	10.593	3.515	1.010	-0.003	0.001	0.019
11	0	14.142	3.536	1.768	14.142	3.536	1.768	0	0	0
12	0.252	0	7.071	0	0	6.874	-0.510	0	0.002	0.020
13	0.506	3.536	7.071	0	3.520	6.928	-0.266	-0.002	0.003	0.033
14	0.510	7.071	7.071	0	7.044	7.021	0.081	-0.002	0.002	0.022
15	0	10.707	7.071	0.589	10.707	7.071	0.589	0	0	0
16	0.255	0	10.707	0	0	10.374	-1.079	0	0.003	0.019
17	0.510	3.536	10.707	0	3.523	10.465	-0.876	-0.001	0.003	0.019
18	0	7.071	10.707	-0.589	7.071	10.707	-0.589	0	0	0
19	0.259	0	14.142	0	0	13.635	-1.806	0	0.003	0.015
20	0	3.536	14.142	-1.768	3.536	14.142	-1.768	0	0	0
21	0	0	17.778	-2.945	0	17.778	-2.946	0	0	0
unit	(ton)									(m)
N. B.	Joints Nos. 6, 11, 15, 18, 20 and 21 are anchoring points (See Fig. 5).									

(1ton=9.8kN)

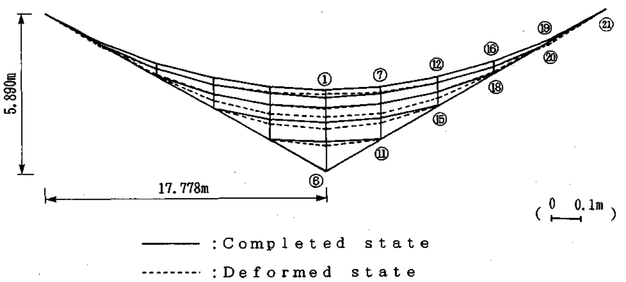

Fig.6 Completed shape and deformation.

Table 6 Tension coefficient and member force.

Member No.	Target value		Completed state		Deformed state (Member force)
	φ_{om}	P_{om}	φ_m	P_m	
1	2.828	10.0	2.826	9.732	11.033
2	2.829	10.0	2.827	9.744	11.063
3	2.750	10.0	2.748	9.751	11.079
4	2.911	10.0	2.910	9.734	11.107
5	2.137	10.0	2.136	9.185	10.630
6	2.828	10.0	2.826	9.832	12.534
7	2.829	10.0	2.828	9.852	12.573
8	2.750	10.0	2.750	9.884	12.645
9	2.588	10.0	2.589	9.801	12.637
10	2.828	10.0	2.827	9.935	12.473
11	2.829	10.0	2.829	10.030	12.578
12	2.715	10.0	2.716	10.194	12.768
13	2.828	10.0	2.828	9.971	11.807
14	2.790	10.0	2.791	10.021	11.881
15	2.529	10.0	2.530	8.984	9.936
16	0.707	2.5	0.709	2.454	1.745
17	0.707	2.5	0.708	2.471	1.760
18	0.688	2.5	0.689	2.452	1.727
19	0.728	2.5	0.729	2.484	1.752
20	0.534	2.5	0.534	2.046	1.246
21	1.414	5.0	1.415	4.958	4.201
22	1.414	5.0	1.415	4.944	4.174
23	1.375	5.0	1.376	4.931	4.144
24	1.294	5.0	1.295	4.643	3.775
25	1.414	5.0	1.415	4.973	4.307
26	1.414	5.0	1.416	4.989	4.320
27	1.357	5.0	1.359	4.993	4.313
28	1.414	5.0	1.415	4.977	4.433
29	1.395	5.0	1.397	4.968	4.412
30	1.265	5.0	1.266	4.540	4.194
	(t/m)	(t)	(t/m)	(t)	(t)

(1ton=9.8kN)

Table 7 Cross-sectional area (m^2).

Member	Initial value	Solution
Nos. 1-15	2.0×10^{-4}	2.61×10^{-4}
Nos. 16-30	1.0×10^{-4}	1.14×10^{-4}

$$\left.\begin{array}{ll} q_{Rj}=1.0\text{ m}, & q_{\varphi k}=1.0\text{ t/m}, \\ q_{xj}=0.05\text{ m}, & q_{Am}=10^{-3}\text{m}^3 \end{array}\right\} \cdots\cdots\cdots\cdots \text{(m)}$$

where $j=1,2,\ldots,f$; $k=1,2,\ldots,M$; $m=1,2,\ldots,M_T$; $f=35$, $M=30$ and $M_T=30$.

It should be noticed that as to completed shape and member forces the design conditions of this model are unchanged from those of the model in Refs. 4) and 5) except that fixed loads due to member dead weight are not taken into account in Refs. 4) and 5). Solutions as to shape and member force are shown in the right half of **Tables 5** and **6**, and optimum sectional areas are obtained as shown in **Table 7**, where initial value means the initial assumption for starting numerical processing. **Fig.6** shows the sketch of the completed and the deformed shape of the cable net. The completed shape in **Table 5** is quite similar to those of Refs. 4) and 5). Refs. 9) and 11) also analyze the same model cited from Refs. 4) and 5) as a shape determination problem. Ref. 9) gives a slightly different, though almost similar, shape, which is mainly due to the difference of objective function. Ref. 11) shows quite the same result as the authors' where maximum difference of vertical coordinate is 0.076 m.

6. CONCLUSION

We presented a method of design analysis of cable network by utilizing the optimization technique, where not only the design conditions for shape and member forces in completed state but also the constraint conditions for stresses and displacements in deformed state and for member dead weights were taken into account, to determine the design shape, design member forces and sectional areas of members in an automatic process. Gauss iteration and maximum neighborhood method are used in minimizing the objective function, which has tension coefficients and member sectional areas as inde-pendent variables, together with authors' method for finding out optimum step-size.

By the present method, various kind of solutions are obtained in compliance with the design philosophy. By increasing the weight of constraint for deformation, for instance, we can obtain a cable network with high stiffness or we can reduce total dead weight by setting large weight to dead weight term of objective function.

The so-called shape determination analyses presented by the authors in the past are totally covered by the present method.

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(Received October 16, 1990)

ケーブル構造の最適設計計算法

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本文の手法によれば，固定荷重下でのケーブル構造の形状決定に関する諸条件と同時に，後載荷重の作用下で部材力，変形および部材断面に課せられる設計条件をすべて満たすような解が得られる．構造の形状，変形，応力に関する設計上の“望ましさ”を示す量によって目的関数が定式化され，その停留値として解が与えられる．設計変数に任意の重みを付加することにより，設計の目的に応じた種々の解を得ることができる．