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SUPPLY MODEL FOR COMMON BUS ROUTES UNDER DETERMINISTIC CONDITIONS

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Transit systems are characterized by networks with extensively overlapping routes, and frequently operating at or close to its capacity. This research addresses the problems of scheduling the buses from terminals and allocating a fleet of buses between routes in the system. An attempt is given to develop a model which recognizes the information on the time dependence of passenger arrival rate, running time between stops, bus capacity, O-D trip pattern and some parameters related to the relationship between dwell time and boarding and/or alighting passengers known under deterministic conditions. The solution developed is based on the decomposition of bus routes into single bus routes and proposed to be the central element of a short-range bus service planning process.

Keywords : *common routes, dwell time, running time, bus availability*

1. INTRODUCTION

On large bus networks operating important service, the problem of *common bus routes* where a passenger must select an appropriate bus which he will use to move within the common section must be taken into consideration[1]. These common routes are often observed in the bus networks in many busy cities in the developing countries. The illustration of common bus routes is shown in **Fig.1**. The common bus routes are represented by solid line while the other bus routes that do not deal with common sections are drawn with dashed line. The passengers who are waiting for the bus at bus stops, S_l and S_m, may have choice to take any bus from any route to take them traverse within the common routes.

There are many research works on the scheduling of public transit, such as minimum waiting time on a single O-D[2], the case where bus makes more than one trip[3),4)], effect of boarding passengers on pairing of buses and the scheduling[5),6)], and multiple O-D pattern with constant arrival[7]. However, the situation of common routes was ignored. In this paper, we solve the problems of determination of scheduling policy for public transit dealing with common routes for multiple O-D pattern, as well as providing the number of buses required to operate on each route in which arrival rate of passengers is regarded non-constant. In the scheduling policy, the problem faced in the common routes is quite difficult to be handled simultaneously. However, our approach allows the consideration of route decomposition for common bus routes which are treated independently as single routes.

2. GENERAL ASSUMPTIONS

Our goal is to minimize total wait time and travel time as well for all passengers rather than to minimize average wait time. Here, we do not consider the *transferring process* of passengers who intend to use more than one bus line in this system, since their arrivals at the next bus stops are included in the given arrival pattern at those bus stops. The operation of supply model developed does not allow passengers to board the buses at the terminals, which is merely aimed at the convenience of formulation. However, the situation in which passengers are allowed to board the bus at terminal can be involved easily by creating a dummy link connecting terminal with bus stop with zero travel time. Furthermore the following assumptions are adopted.

(1) Time dependence of passenger arrival

In this scheduling problem, deterministic but time dependent passenger arrivals are considered, which implies that we are treating average arrival pattern remain unchanged. For the situation where the bus frequency is high, the arrival pattern will not be affected by scheduling policy. Although this condition may not always hold, it is sufficient to assume this condition for the beginning stage of this sort of research. Arrival of passenger is given as some smooth function of time within a specific time period of interest, such as morning peak hour, day off-peak hour, afternoon peak hour or night off-peak hour. This kind of arrival can be represented as the cumulative number of passengers who arrive at a bus stop at an instant t.

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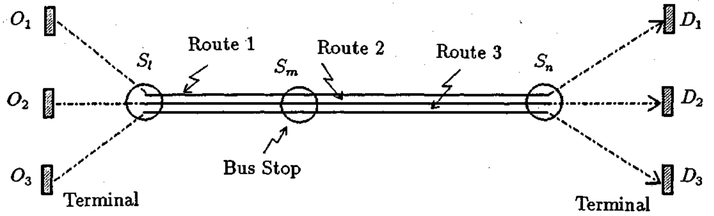

Fig.1 Simple Common Bus Routes.

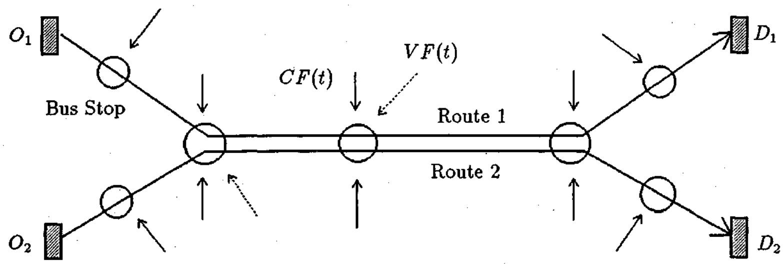

Fig.2 Captive and Variable Arrivals.

(2) Captive arrival and variable arrival

As mentioned above, in this model we deal with the situation where there are some stops partially served by common bus routes. Because of the duplicate services at stops on common routes, we therefore, separate the arriving passengers at those bus stops into two portions. Those who can only take the buses from a single route, as captive arrivals. These captive arrivals will also include some passengers in the common routes who can only take buses from a single route by some reason such for their commuting tickets. And those who can take any bus from any route as variable arrivals.

The condition of these two different arrivals is illustrated in **Fig.2**. All stops have captive arrival, $CF_{ir}(t)$, which is the cumulative arrival of passengers at stop i of route r at an instant t. At any stop which does not deal with common routes, there is a single captive arrival. But when the stop deals with common routes, there are multiple captive arrivals corresponding to each route. In addition, these common routes have also a single variable arrival, $VF_i(t)$, which is the cumulative passenger arrivals at stop i at an instant t who can take buses from any route, since they will travel within the common routes.

(3) O-D transition probability

Since passengers may board and alight at any bus stop, we need to know where each of them goes in order to obtain the optimal dispatch schedule as well as the total number of buses required. We assume that the number of passengers on board from stop i to stop l is proportional to the number of passengers boarding at stop i. Since the passengers' arrival rate at each stop is time dependent, the number of passengers traveling from stop i to stop l changes over time. The ratio mentioned above is called '*O-D Transition Probability*' from i to l. Furthermore R_{ilr} denotes the O-D transition probability from stop i to stop l of route r for the captive arrival, and has the following characteristic,

$$\sum_{l=i+1}^{D_r} R_{ilr}=1 \ \forall\, i;r \qquad (1)$$

where D_r is the destination of all buses of route r. While R_{il} denotes the O-D transition probability for the variable arrival, and for each of the common route considered has the following characteristic,

$$\sum_{l=i+1}^{E} R_{il}=1 \ \forall\, i;l \text{ within the common routes} \qquad (2)$$

where E is the last stop on the common routes considered. Since the R_{ilr}'s and R_{il}'s are constant over a given time period, the number of passengers carried along some links (adjacent stops) can be obtained easily, by multiplying the number of passengers boarding the bus at all stops before those links by their R_{ilr}'s and R_{il}'s and summing them all up.

(4) Vehicle movement

a) Running time

Travel time of transit system comes out to be crucial, because incorrect estimation may result in the delay in the schedule or bunching of buses. However, most of the available research works on the scheduling problem under deterministic condi-

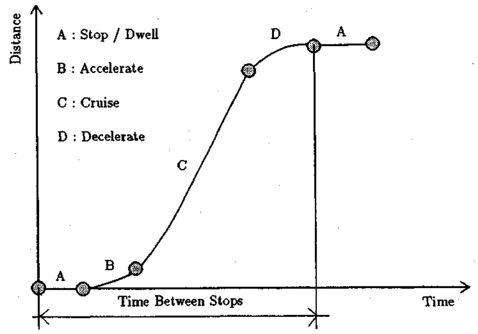

Fig.3 Components of Transit Travel Time.

tions assumed that this travel time is constant. In fact this assumption is not realistic. The components of transit travel time is illustrated in **Fig.3**. The four components are : Dwell (A), the time used by each bus to load and unload at a bus stop ; Accelerate (B), the time used by a bus in effort to attain its normal speed ; Cruise (C), the running time of a bus between two consecutive stops at normal speed and ; Decelerate (D), the time used to halt the bus[8]. However, in this model we separate these components into two major components ; (i) running time which includes accelerate, cruise, decelerate ; and (ii) dwell time.

Running time of road transit system is highly influenced by the traffic condition of that road which is characterized by the level of flow of the vehicles[9),10)]. Moreover the flow level of vehicles on the road varies within a specific time of period, ie. morning peak hour, day off-peak hour and so forth. Therefore in this model we assume that running time is time dependent and given as some function of time. The presentation of running time of a bus between two consecutive stops $(i,\ i+1)$ along route r is given by,

$$\Delta t_{ir}=f_{ir}(t)\ \forall\ i;r \qquad (3)$$

b) Dwell time

In the operation of public transit, dwell time is affected by either boarding or alighting passengers. Consequently in this model we adopt that the dwell time of a bus at each stop is a function of number of boarding and alighting passengers[11]. The dwell time of the jth bus at stop i of route r is represented as,

$$DW_{ijr}=\max\ (\alpha\times \text{boarding passengers.},\ \beta\times \text{alighting passengers.})\ \forall\ i;j;r \qquad (4)$$

where, α and β are parameters related to boarding and alighting time per passenger.

3. MODEL STRUCTURE

(1) Route decomposition

The principal difficulty in solving the optimal bus departures for bus routes dealing with common routes is that there are variable arrivals, $VF_i(t)$'s, which may take any first incoming bus to traverse within common routes (see **Fig.2**). The model developed here will treat them as separate routes, and the coincidence will be accommodated by appropriately modifying the variable arrivals in the system. This modification can be done by following *'Base Frequency'* and *'Surplus Frequency'* procedures explained as follows.

a) Base frequency procedure

This procedure is defined as a tool to find the lower bound frequency, $q_r(0)\ \forall\ r$, of buses operated on route r that does not violate the capacity constraint. The explanation is as followings.

First let us denote, $CF_{ilr}(T)$, as the total number of captive passengers traveling from stop i to stop l of route r within a given time period T, where i and l are consecutive stops. Also, let $VF_{ilr}(T)$ be the variable ones which have been decomposed for route r, so the total number of passengers traveling from stop i to stop l of route r is given by,

$$DF_{ilr}(T)=CF_{ilr}(T)+\delta^r_{il}VF_{ilr}(T)\ \forall\ il\in L_r \qquad (5)$$

where, L_r : set of stop pairs of route r

$$\delta^r_{il}=\begin{cases}1 & \text{if } il \text{ of route } r \text{ deals with common routes}\\ 0 & \text{otherwise}\end{cases}$$

In this base frequency procedure passengers are served nearly to the total bus capacity. So VF_{ilr} can be considered proportional to the frequency of buses on route r on common routes, so $VF_{il}(T)$ (total variable passengers on common link il within time period T) can be decomposed into each route in proportion to their *'Frequency Shares'* as $VF_{ilr}(T)$. Therefore equation (5) can be rewritten as,

$$DF_{ilr}(T)=CF_{ilr}(T)+\delta^r_{il}\frac{q_r}{\sum_k q_k}VF_{il}(T)\ \forall\ il;k\in X_{il} \qquad (6)$$

where, X_{il} : a set of routes between stop i and l that coincide.

q_k : bus frequency operated on route k

The following algorithm is used to generate lower bound, $q_r(0)$ for each q_r, which satisfies the feasibility requirement and gives minimum number of buses required in a specific time period, in the sense that no excess of bus capacity (C) is allowed on any link of any route[12),13)].

Base Frequency Algorithm

(1) For given routes, determine $CF_{ilr}(T)$, $VF_{il}(T)$ and X_{il} for every stop pair il and bus route r.

(2) Using only the captive passenger flow, determine the frequency from,

$$C\times q_r(0)=\max_{il}[CF_{ilr}(T)]\ \forall\, r;il\in L_r \cdots (7)$$

where, C : bus capacity

L_r : set of stop pairs of route r

set $n=0$.

(3) Compute $DF_{ilr}(T)$, using equation (6), where $q_r=q_r(n)$.

(4) Find the peak load link passenger flow on each route. If the peak load link of each route is the same as that identified in the previous iteration, STOP and set $q_r(0)=q_r(n)$. Otherwise set $n=n+1$

(5) And redefine $q_r(n)$ as the solution of the following system equations,

$$C\times q_r(n)=\max_{il}[DF_{ilr}(T)]\ \forall\, r;il\in L_r \cdots (8)$$

Then go to (3).

It is clear that there is a finite number of different combinations of peak links, one for each route, that can be found in step 3 of the algorithm. Accordingly the algorithm converges within a finite number of iterations.

b) Surplus frequency procedure

In general, supply of public transit is designed to minimize the total cost comprising of user and operating costs, which are functions of total wait/travel time and number of buses, respectively. In the base frequency procedure we can obtain the lower bound of feasible bus frequency which implies that the supply is given at, or close to, capacity. So in order to improve service quality, surplus frequency procedure is designed to provide the feasible value of q_r as,

$$q_r\geqq q_r(0)\ \forall\, r \cdots\cdots (9)$$

Therefore by this procedure we can increase the number of buses from the number given in base frequency procedure and obtain their minimized total wait/travel time as well as their scheduling policy.

(2) Model formulation

The problem is to find the optimal scheduling policy which minimizes the *Total Wait/Travel Time* of all passengers subject to *Bus Capacity Constraints*. Following is the formulation for problem based on the concept of route decomposition.

a) Constraints

To derive the constraints of this problem, let us find first the number of passengers boarding the jth bus of route r at stop i, from

$$[CF_{ir}(t_{ijr})-CF_{ir}(t_{ij-1r})] + \delta_{ir}[VF_{ir}(t_{ijr})-VF_{ir}(t_{ij-1r})]\ \forall\, i;j;r \cdots (10)$$

where, t_{ijr} : jth bus arrival at stop i of route r

$$t_{ijr}=t_{jr}+\sum_{k=0}^{i-1}\Delta t_{kr}+\sum_{k=1}^{i-1}DW_{kjr} \cdots\cdots (11)$$

t_{jr} : departure time of the jth bus of route r

Δt_{kr} : bus running time between stop k and $k+1$, see equation (3)

DW_{kjr} : the jth bus dwell time at stop k of route r, see equation (4)

$$\delta_{ir}=\begin{cases}1 & \text{if stop } i \text{ of route } r \text{ deals with common routes}\\ 0 & \text{otherwise}\end{cases}$$

Next is to find the number of passengers getting off the jth bus of route r at stop k, from

$$E_k(t_{jr})=\sum_{i=1}^{k-1}R_{ik}\{[CF_{ir}(t_{ijr})-CF_{ir}(t_{ij-1r})] + \delta_{ir}[VF_{ir}(t_{ijr})-VF_{ir}(t_{ij-1r})]\}\ \forall\, k;j;r \cdots\cdots (12)$$

R_{ik} is defined in equation (1) and (2). Knowing the number of boarding and alighting passengers at each stop, we can determine the number of passengers in the jth bus of route r on link $(i, i+1)$, $D_i(t_{jr})$, from the following equation,

$$D_i(t_{jr})=D_{i-1}(t_{jr})+[CF_{ir}(t_{ijr})-CF_{ir}(t_{ij-1r})] + \delta_{ir}[VF_{ir}(t_{ijr})-VF_{ir}(t_{ij-1r})] - E_i(t_{jr})\ \forall\, i;j;r \cdots\cdots (13)$$

So the constraints of this problem, that is the number of passengers in the bus on each link should be less or equal to bus capacity (C), are

$$D_i(t_{jr})\leqq C\ \forall\, i;j;r \cdots\cdots (14)$$

where $D_i(t_{jr})$'s are determined in equation (13).

b) Total wait/travel time

The objective of this scheduling problem is to minimize the total wait/travel time. This implies to minimize total wait time for all passengers at bus stops as possible and their travel time consisting of *In-Moving Vehicle Time* and *In-Stopping Vehicle Time*, for the varying running time and dwell time. This situation should hold since we would like to model an improved supply for bus services. In the following discussions we will determine the formulations of this objective function.

i) Wait time

In order to minimize total wait time for all passengers in this problem we adopt the necessary condition for optimum, that is, *every passenger should be able to take the first incoming bus after his/her arrival*[2]. This condition is to hold for both captive and variable arrivals. Furthermore the total wait time for all passengers on route r, W_r, is given as follows.

$$W_r=\sum_{j=1}^{N_r}\sum_{i=1}^{M_r}\int_{t_{ij-1r}}^{t_{ijr}}\{[CF_{ir}(t_{ijr})-CF_{ir}(t_{ij-1r})] + \delta_{ir}[VF_{ir}(t_{ijr})-VF_{ir}(t_{ij-1r})]\}dt\ \forall\, r$$

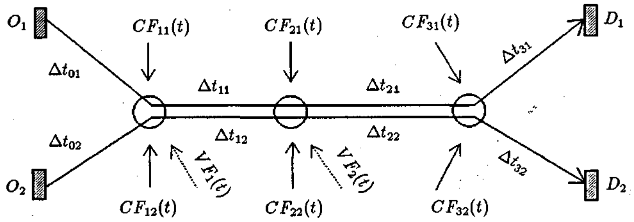

Fig.4 Two Bus Routes With Three Bus Stops.

Table 1 O-D Transition Probability For Captive Arrivals.

Origin/Destination	2	3	D1	D2
1	0.1/0.2	0.2/0.3	0.7	0.5
2	-	*0.3/0.2	0.7	0.8
3	-	-	1.0	1.0

*Note */* : route 1 / route 2*

Table 2 O-D Transition Probability For Variable Arrivals.

Origin/Destination	2	3
1	0.4	0.6
2	-	1.0

$$\cdots\cdots \quad (15)$$

where Mr and Nr are total number of stops and buses on route r respectively.

ii) In-moving vehicle time

This term can be defined as the time spent by all passengers to move from their origin to their destination not including the delay at bus stops. Since we know the number of passengers in the j th bus on link $(i, i+1)$ of route r, we can obtain the in-moving vehicle time of those passengers, VM_{ijr}, as follows

$$VM_{ijr}=D_i(t_{jr})\times \Delta t_{ir}\ \forall\ i;j;r \quad (16)$$

Hence for the total in-moving vehicle time of all passengers along route r, VM_r, is obtained as

$$VM_r=\sum_{j=1}^{Nr}\sum_{i=1}^{Mr} VM_{ijr}\ \forall\ r \quad (17)$$

iii) In-stopping vehicle time

At stop i of route r, for example, there is a certain number of passengers in the j th bus who have to be delayed for loading and unloading of other passengers for DW_{ijr} (see equation 4). This certain number of passengers is given by,

$$D_{i-1}(t_{jr})-E_i(t_{jr})\ \forall\ i;j;r \quad (18)$$

and the in-stopping vehicle time of those passengers at that stop is

$$VS_{ijr}=[D_{i-1}(t_{jr})-E_i(t_{jr})]\times DW_{ijr}\ \forall\ i;j;r \quad (19)$$

so that, the total in-stopping vehicle time of all passengers along route r, VS_r, is given by

$$VS_r=\sum_{j=1}^{Nr}\sum_{i=2}^{Mr} VS_{ijr}\ \forall\ r \quad (20)$$

Accordingly this scheduling problem can be represented in the following *Mathematical Programming*,

Minimize Total Wait/Travel Time

$$=W_r+VM_r+VS_r$$

subject to,

$$D_i(t_{jr})\leqq C\ \forall\ i;j;r$$

This kind of optimization problem can be solved recursively by using *Dynamic Programming*. The structure of dynamic programming algorithm for this optimization is chosen by deciding the *headway* between the $(j-1)$ th and the j th buses' departure time denoted by s_j as decision variables, and *cumulative headways* or the time interval between the beginning of time period of interest and the j th departure on route r, Q_{jr}, as the stage corresponding to each dispatched bus and state variables. Furthermore we have recursive relationship of this problem as,

$$r_j(Q_{jr})=\mathrm{Min}_{s_j}[r_{j-1}(Q_{j-1r})+f_j(s_j)] \quad (21)$$

where $r_j(Q_{jr})$ is the minimum travel time for the first j buses, and $f_j(s_j)$ is the travel time for the j th bus. The state of dynamics is given by

$$Q_{jr}=Q_{j-1r}+s_j \quad (22)$$

In order to show how this model works, we will try to solve the scheduling problem on a simple bus network dealing with common routes, so as to minimize total wait/travel time for all passengers in the following section.

4. APPLICATION OF THE MODEL

Suppose we have a simple bus network consisting of two bus routes and three bus stops on their common section. The network structure and O-D transition probability are given in **Fig.4**, **Table 1** and **Table 2**, respectively.

The running time of buses on each link are assumed to be time dependent, and we have

$\Delta t_{01}=5+t/24$; $\Delta t_{02}=6+t/20$

$\Delta t_{11}=\Delta t_{12}=10+t/12$; $\Delta t_{21}=\Delta t_{22}=8+t/15$

$\Delta t_{31}=6+t/20$; $\Delta t_{32}=5+t/24$

Table 3 Optimal Bus Departure Times.

Bus No/Route No	1	2
1	22'	28'
2	35'	43'
3	45'	57'
4	58'	-
Total Wait Time	432'	495'
Total Travel Time	5172'	5028'

Table 4 Constant Headway Bus Departure Times.

Bus No/Route No	1	2	Remarks
1	15'	20'	
2	30'	40'*	* Overcrowding on link (2,3)
3	45'	60'	
4	60'*	-	* Overcrowding on link (2,3)
Total Wait Time	1486'	1791'	
Total Travel Time	5300'	5149'	

The passenger arrivals at every bus stop is given by,

$$CF_{11}(t)=\begin{cases} 6(t-5)^2/360 & 5\leqq t\leqq 65 \\ 60 & t\geqq 65 \end{cases}$$

$$CF_{12}(t)=\begin{cases} 7(t-6)^2/360 & 6\leqq t\leqq 66 \\ 70 & t\geqq 66 \end{cases}$$

$$CF_{21}(t)=\begin{cases} 4(t-15)^2/360 & 15\leqq t\leqq 75 \\ 40 & t\geqq 75 \end{cases}$$

$$CF_{22}(t)=\begin{cases} 3(t-16)^2/360 & 16\leqq t\leqq 76 \\ 30 & t\geqq 76 \end{cases}$$

$$CF_{31}(t)=\begin{cases} (t-23)^2/360 & 23\leqq t\leqq 83 \\ 10 & t\geqq 83 \end{cases}$$

$$CF_{32}(t)=\begin{cases} 2(t-24)^2/360 & 24\leqq t\leqq 84 \\ 20 & t\geqq 84 \end{cases}$$

$$VF_1(t)=\begin{cases} 4(t-5)^2/360 & 5\leqq t\leqq 65 \\ 40 & t\geqq 65 \end{cases}$$

$$VF_2(t)=\begin{cases} 2(t-15)^2/360 & 15\leqq t\leqq 75 \\ 20 & t\geqq 75 \end{cases}$$

bus capacity $(C)=40$ passengers/bus, $\alpha=0.1$ and $\beta=0.08$ minute/passenger and $T=60$ minutes.

The solution of this problem can be simplified by separating the bus networks into single bus routes through the base and surplus frequency procedures, which results in the required number of buses to operate on each route and modified variable arrivals. Knowing the number of buses and modified variable arrivals, we can minimize total wait/travel time for all passengers by using dynamic programming. Solving the problem, the time line is divided into discrete parts that would ignore the discrete nature of passengers who use a bus. However, this discrete nature is masked by the use of continuous approximation of the arrival patterns. The solution of this problem is summarized in **Table 3**.

In addition, as a comparison, suppose we have other scheduling policy to dispatch the buses by constant headways within the time period (60 minutes). Using the same number of buses as in the optimal policy for each route, this scheduling policy gives results shown in **Table 4**. These results show that the policy requires more time for both wait and travel times. Comparison with the ones we obtained from the optimal departures, this total wait time is significantly reduced by nearly 72%, from 3 277' to 927', which of course gives convenience to passengers, and total travel time is reduced by nearly 2.5%, from 10 449' to 10 200'. Moreover this constant headway policy results in some overcrowding effect on link (2, 3) on the second bus of route 2 and the fourth bus of route 1 that gives inconvenience to passengers.

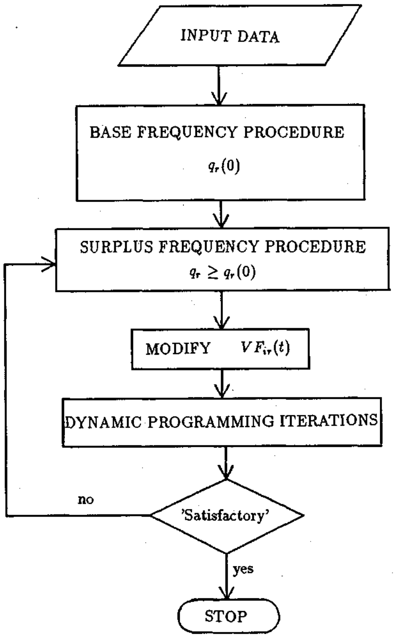

Fig.5 Computational Flowchart.

However, this optimum solution cannot guarantee that all passengers can take a bus within the wait time which they can tolerate. So in practice, we can improve the service quality by putting an upper limit to the waiting time, by adding the headway constraints to this model. It should be noted, however, that these additional constraints may result in no solution for some number of buses. If so we should increase the number of buses as explained in the surplus frequency procedure and modify all variable arrivals and capacity constraints accordingly. The computation time for this example, using small personal computer is less than 5 minutes. This computation time is mainly influenced by the number of buses and bus capacity, so in the real problem with larger number and capacity of buses the computation time can still be expected to be reasonable. Furthermore the general computational flowchart of this model is illustrated in **Fig.5**. In this figure The 'Satisfactory' term may include some judgement factor of bus operator regarding level of service and operating cost, however in this example we do not consider that situation and just run the model for 1 iteration.

5. BUS AVAILABILITY

In the model developed above we just dispatch bus without thinking whether it can be re-dispatched or not. Accordingly, the operator may be required to have less number of buses by re-dispatching buses that have been used. This section discusses about the bus availability that bridges this model to the practical application which is always limited by fleetsize. Our objective here is to fulfill the fleetsize by providing the fewest number of buses required to operate on the given bus networks.

In general the operation of bus service or any other public transit can be viewed as a two-way service, as shown in **Fig.6**, wherein buses only traverse along the given route, and each bus that has served one way may, of course, be used to serve the opposite way. However, we state here that a bus arriving first at a terminal will be the first to leave. The previous discussions enable us to determine the optimal schedule policy which results in optimal bus departure times. Also, by tracing the computation for the optimal departure times we can obtain the travel time of those buses to reach the opposite terminal. So based on the prescribed departure and travel time we want to determine heuristically the minimal fleetsize required to keep up a two-way bus route.

Suppose there is a given route of two-lane service. Let t_{jzr} denote departure time of the j th bus $\forall j$ ($j=1, 2, \ldots Nzr$, where Nzr is the total number of buses required to serve all demand along the z th direction of route r, $z=1, 2$) and y_{jzr} be the travel time of this departure until it reaches the opposite terminal.

The heuristic solution proposed here is based upon the observation that a new bus is required in the (j, z, r) th departure, namely the j th departure in z th direction of route r, whenever no bus, that has already been used on the route, is available. We obtain the required number of buses on a route by summing over all departures (j, z, r) where a

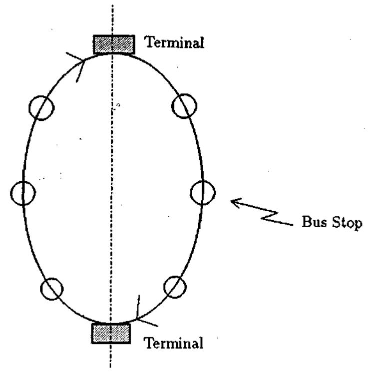

Fig.6 Two Way Lane Bus Service.

new bus must be assigned.

Consider the (j, z, r) th departure, in order to determine whether a new bus is needed to carry it out, it suffices to review all arrivals from the opposite terminal, namely $3-z$, and to check if one of them can provide a bus for this departure. This can be done by counting the arrivals in each j time interval between departures that precede the j th one, and discarding the buses that are needed for previous departures. If at least one bus remains, then it may serve the j th departure ; if not, a new bus is required. In order to make the computation of this observation possible, let us first form vector h_{zr} of route r, of Nzr components whose the j th component, h_{jzr}, means the number of buses arriving at the z th terminal between the $(j-1)$ th and j th departure times from the $(3-z)$ th terminal. This component can be defined as

$$h_{jzr}=\sum_{j'=1}^{N3-zr}\zeta_{jj'z}=\sum_{j'=tj'3-zr\leq tjzr}\zeta_{jj'z} \cdots\cdots\cdots (23)$$

where

$$\zeta_{jj'z}=\begin{cases}1 & \text{if } t_{j'3-zr}+y_{j'3-zr}\in(t_{j-1zr},t_{jzr})\\ 0 & \text{otherwise}\end{cases}$$

Now we define the number of buses remaining at the z th terminal of route r after the j th departure, $P_{jzr}\,\forall j;z;r$, these P_{jzr}'s can be defined in the light of the fact that,

$$P_{jzr}=\max(P_{j-1zr}+h_{jzr}-1,0)$$
$$\forall j=1,2,\ldots(Nzr-1);z;r \cdots\cdots\cdots (24)$$

where $P_{0zr}=0$. Like h_{jzr} values, we can arrange P_{jzr}'s in Nzr component vector $\boldsymbol{P}_{zr}$. If we define the following marker variables

$$\xi_{jzr}=\begin{cases}1 & \text{if } P_{j-1zr}+h_{jzr}=0\\ 0 & \text{otherwise}\end{cases}$$

it is obvious that the minimal number of buses needed to run the route r is

$$n_r=\sum_{z=1}^{2}\sum_{j=1}^{Nzr}\xi_{jzr}\,\forall r \cdots\cdots\cdots (25)$$

Since these ξ_{jzr}'s keep track on all the departures that require a new bus. Actually n_r is the number of zeros in the two vectors $\boldsymbol{h}_{zr}+\boldsymbol{P}_{zr}, z=1, 2$. Furthermore we can obtain the total number of buses required to operate on some routes of the given bus networks such as,

$$n=\sum_{r}\sum_{z=1}^{2}\sum_{j=1}^{Nzr}\xi_{jzr} \cdots\cdots (26)$$

Having known in advance the departure time and travel time of all buses, it is easy to determine the fleetsize of bus operation by using the abovementioned heuristic solution.

6. CONCLUSIONS

In this research an optimum supply model for public transit dealing with common routes is developed. It is proposed that the coincidence is separated by appropriately modifying the variable arrival patterns in proportional to their frequency shares. It may be difficult to obtain reliable data needed to fit this model. However it will be better to have optimized scheduling policy based on this information rather than dispatching buses without any optimization. Furthermore it is concluded that.

(1) In the case where arrival rate is high, continuous approximation of the arrival pattern gives fairly accurate results. Departure times can be chosen as control variables, otherwise the dynamic programming can be formulated in terms of loads rather than departure times.

(2) The improvement of service quality can be anticipated easily by introducing headway constraints to the model developed here. This improvement may, in general, give other solution that minimizes wait time but not necessarily total wait and travel time.

(3) This proposed model can be applied repeatedly to the given routes so that a planner may trace the so-called trade-off curve between number of buses and total wait/travel time in which over a reasonable range of frequencies the demand can be assumed to be inelastic to wait/travel time. This trade-off curve can thus be used for policy analysis and setting of service standards.

(4) It is possible to determine the fleetsize that is required to maintain two-way service by using heuristic solution proposed in this model. Furthermore this solution can be utilized as part of dynamic programming procedure to establish the optimal departure times of buses without violating the constraints of having a given number of them.

(5) The supply model developed in this research is assumed under deterministic conditions. So the reliability issues might be an interesting topic in the future research.

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非弾性需要下での重複区間をもつバスルートの供給モデル

島崎敏一・Sutanto SOEHODHO

大都市におけるバスのルートは，多くの場合，重複部分があり，また，容量に近い状態で運行されていることが多い．本論文は，そうした状況におけるバスのスケジューリング問題を解くものである．提案したモデルは，乗客の停留所への到着パターン，バスの容量，OD 表，停車時間などを含めたバスの停留所間運行時間などを入力として，乗客の総所用時間を最小化するようなバスのスケジュールを求めるものである．