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INTERACTION OF TUNED LIQUID DAMPER (TLD) AND STRUCTURE —THEORY, EXPERIMENTAL VERIFICATION AND APPLICATION—

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The interaction between rectangular Tuned Liquid Damper (TLD) and structure is investigated both experimentally and theoretically. A TLD-structure interaction model is developed where the dynamic interaction force is theoretically evaluated by applying the nonlinear shallow water wave theory. Good agreements are found between the experimental results and the theoretical simulation within the range where no breaking of wave occurs inside the TLD. Effectiveness of TLD is demonstrated for sinusoidal forced excitation. An example of TLD design procedure is also given using the TLD-structure interaction model.

Keywords : tuned liquid damper, horizontal motion, interaction, rectangular tank, liquid motion, nonlinear wave analysis, experiment, harmonic forced excitation

1. INTRODUCTION

The performance of Tuned Liquid Damper (TLD) was earlier investigated by the authors using free-oscillation experiments[1]. It was found that additional damping due to TLD is amplitude-dependent. Liquid motion inside TLD, which is dependent on amplitude of excitation, plays a central role in the vibration suppressing mechanism. Utilizing nonlinear shallow water wave theory, a model was recently proposed by the authors to predict the liquid motion inside the rectangular TLD[2]. In the model, the damping of liquid motion was semi-analytically evaluated. The model is found to be valid in the region of relatively small vibration amplitude where no breaking of wave in TLD occurs.

The present study concentrates on the interaction of a rectangular TLD and a structure which is subjected to an external harmonic force. Application of TLD to a particular structure vibrating horizontally is also a goal of this study. Hence, a commercially available, cheap and handy, rectangular plastic tank partially filled with water is used as TLD, and the tested structure has a horizontal natural frequency and a damping factor of a bridge girder where TLD is planned to be installed.

A TLD-structure interaction model is proposed and is verified through the forced excitation experiment. This interaction model is expected to be useful as an aid in TLD design.

In designing, those properties of TLD are preferred which help in suppressing the structure vibration to a low level for a wide range of excitation frequencies. Extensive numerical simulations using the proposed model can easily be conducted to select the optimal TLD properties. An example of designing of TLD is shown. The design example is made with regard to an actual constraint for applying TLD to a structure,

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e.g., space or cost constraint that leads to the limit number of TLD tanks.

2. TLD-STRUCTURE INTERACTION MODEL

In modeling the interaction between rectangular TLD and structure, the following analysis is used for liquid motion inside the TLD. The tank and the liquid parameters used in the liquid motion analysis are shown in **Fig. 1**. The basic equations which serve to define liquid motion inside rectangular tank are [for details, see Ref. 2)]

$$\frac{\partial \eta}{\partial t}+h\sigma\frac{\partial(\phi u(\eta))}{\partial x}=0, \quad (1)$$

$$\frac{\partial}{\partial t}u(\eta)+(1-T_H^2)u(\eta)\frac{\partial}{\partial x}u(\eta)+g\frac{\partial \eta}{\partial x}+gh\sigma\phi\frac{\partial^2\eta}{\partial x^2}\frac{\partial \eta}{\partial x}=-\lambda u(\eta)-\ddot{x}_s, \quad (2)$$

where η=the liquid surface elevation, u=velocity of liquid in x-direction relative to the tank, $\sigma=\tanh(kh)/(kh)$, $\phi=\tanh(k(h+\eta))/\tanh(kh)$, k=wave number, $T_H=\tanh(k(h+\eta))$, g=gravitational acceleration and h=liquid depth.

The following damping coefficient λ is used which includes the effects of the boundary layer along the wall as well as the bottom and the free-surface contamination [Ref. 2)]:

$$\lambda=\frac{1}{(\eta+h)}\frac{8}{3\pi}\sqrt{\omega\nu}(1+(2h/b)+S), \quad (3)$$

where ω=angular frequency of excitation, S=the width of the tank, ν=kinematic viscosity of liquid, S= the surface contamination factor which accounts for the damping due to the contamination of the liquid surface. The value of $S=1$, which corresponds to fully contaminated surface, is used in this study.

The present TLD-structure interaction model is a single-degree-of-freedom (SDOF) structure attached with TLD (**Fig. 2**). Two forces are exerted on the structure, namely an external sinusoidal force F_e and a TLD base shear force F. The equation of motion for the SDOF structure is, therefore, expressed as

$$m_s\ddot{x}_s+c_s\dot{x}_s+k_sx_s=F+F_e. \quad (4)$$

And hence,

$$\ddot{x}_s+2\omega_s\zeta_s\dot{x}_s+\omega_s^2x_s=\frac{1}{m_s}(F+F_e), \quad (5)$$

where "˙" stands for the derivative with respect to time t. $\omega_s=(k_s/m_s)^{1/2}$ is undamped angular frequency of the structure, while $\zeta_s=c_s/2m_s\omega_s$ is critical damping ratio of the structure.

TLD base shear force, F, is calculated and expressed as

$$F=\frac{1}{4}m_w\frac{gh}{a}(\Gamma'_n-\Gamma'_0), \quad (6)$$

where m_w=mass of liquid, Γ'=a nondimensional function of water surface elevation η at the end wall. The shear stress along the bottom of the container is neglected because the magnitude of the velocity of the structure is much smaller than that of the fluid velocity. Subscript "n" defines the position at $x=a$ and "0" at $x=-a$.

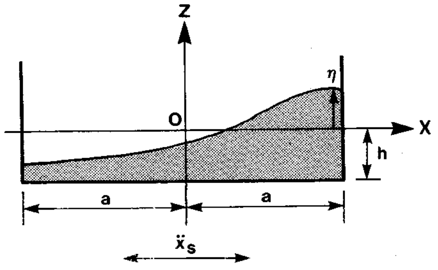


Fig. 1 Tank and parameters.

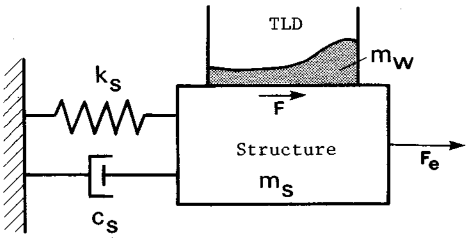


Fig. 2 TLD-structure interaction model.

$$\Gamma'=\Gamma'(\eta)=\frac{1}{h^2}\left((\eta+h)^2+\frac{2}{k^2}\frac{\partial^2\eta}{\partial x^2}(\eta+h-h\phi\sigma)\right). \quad\cdots\cdots (7)$$

Numerical simulation is conducted under the condition that the structure and the liquid are initially at rest. The basic equations of liquid motion [Eqs. (1), (2)] are discretized with respect to x into difference equations. Eqs. (1), (2) and (5), which are coupled, are then solved using the Runge-Kutta-Gill method. The time interval Δt is 1/60 of the period of the external force. Computation is continued until 220 cycles of external harmonic force.

3. FORCED EXCITATION EXPERIMENT OF TLD-STRUCTURE INTERACTION

Frequency sweep forced excitation experiment was performed in order to demonstrate the effectiveness of TLD and to verify the proposed TLD-structure interaction model.

(1) Experiment set-up and procedure

The structure model is a hanging single-degree-of-freedom steel platform vibrating in a shear-type horizontal motion (**Fig. 3** and **Photo 1**). The structural dynamic properties are set to simulate the bridge girder of concern. The natural frequency of the structure model, f_s, is 0.91 Hz (natural period $T_s=$ 1.10 sec) and the structural damping ratio ζ_s is 0.32 % (logarithmic decrement $\delta_s=0.02$). The total mass of structure m_s (including an excitor) is 168 kg. TLD with plain water was placed on the platform.

A sinusoidal inertia force of the oscillating part of an excitor, which was placed on the platform was used as the external force exerted on the structure. The amplitude of harmonic external force was, hence, kept constant by means of keeping constant the amplitude of acceleration of the oscillating part relative to the platform ; i. e., [absolute acceleration of the oscillating part] subtracted by [absolute acceleration of the platform] was kept constant in amplitude. Two levels of input force for the studied frequency range were

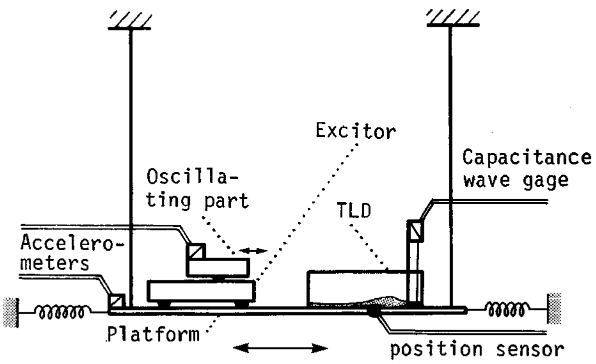


Fig. 3 Schematic view of experiment set-up.

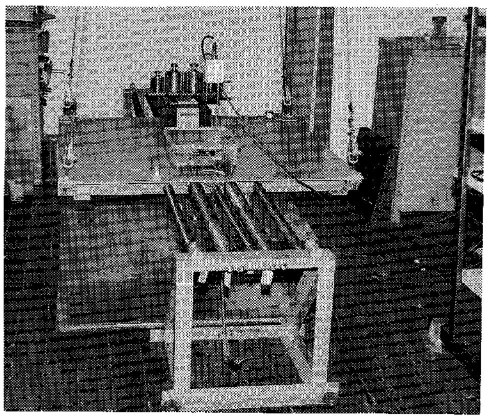


Photo 1 Experiment set-up.

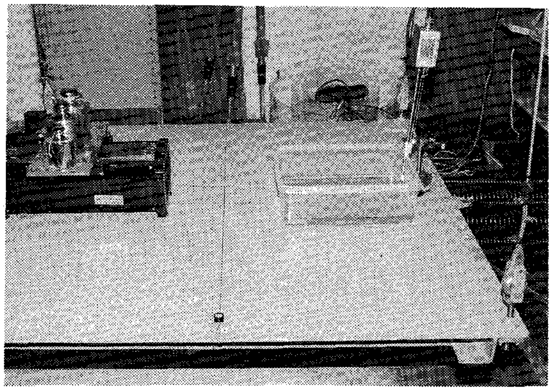


Photo 2 Commercially available plastic tank.

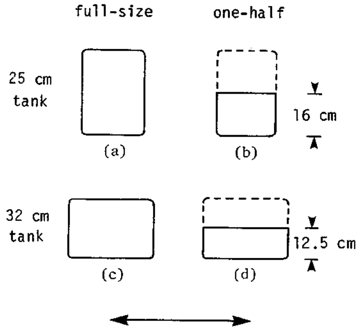


Fig. 4 Tanks used in the experiment.

exerted to the structure, causing the structure without TLD to vibrate at resonance ($f/f_S=1.00$) with the steady-state amplitude A_0 of 1.0 cm and 3.0 cm respectively.

The excitation frequency, f, was varied such that the frequency ratio f/f_S ranged from 0.85 to 1.15. The frequency was increased or decreased at an interval of 0.005 Hz after the steady-state has been reached at the preceding frequency. The digitized data of steady-state structural response x_S and liquid surface elevation at the end wall of the tank, η_0, were collected. The structural response was detected by a position sensor, while the liquid surface elevation was measured by a capacitance wave gage.

The prototype tank used as the TLD was a commercially available rectangular plastic tank as shown in Photo 2. However, the tank was not in a perfect rectangular shape, e.g., the walls were very slightly inclined, the corners and edges were rounded and the bottom was not perfectly flat. The tank was approximately 32 cm long and 25 cm wide and the height of the tank was 12 cm.

The tanks used in the study are schematically depicted in Fig. 4. The rectangular tank was used with either the 25 cm side (Fig. 4(a)) or 32 cm side (Fig. 4(c)) parallel to the direction of motion of the platform. In order to study the effect of mass ratio μ, experiments using a half tank, as shown in Figs. 4 (b) and (d), were also conducted (μ=mass of liquid m_w/mass of structure m_S). To make a one-half tank, laminated wooden partition was used to divide the original tank.

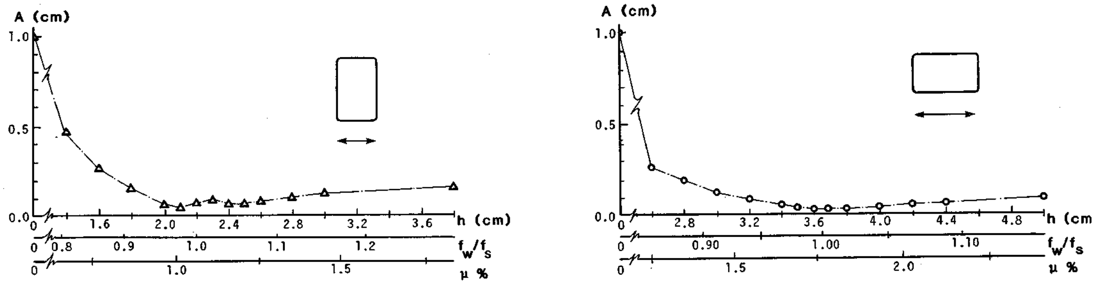
(2) Selection of water depth for the frequency sweep experiment

The proper water depth, h, to be used in the frequency sweep experiments is selected from the results of a preliminary experiment.

In selecting the proper water depth for TLD, it is to be realized that the major constraint for applying TLD to an actual structure is the number of TLD tanks that can be installed rather than an upper limit of mass ratio μ, since the cost of the tank is a major expense in TLD installation. The proper water depth is selected in order to make the TLD suppress the structural vibration most in a wide range of excitation frequencies, because the structure may be subject to an external force whose frequency changes from time to time.

However, in this experiment to select the proper water depth, the external force frequency (i.e. the excitor frequency), f, was retained at $f=0.91$ Hz ($=f_s$) to reduce the experimental time. The condition of $f/f_S=1.00$ is a resonant state for the SDOF structure without TLD, which may be regarded as the most critical condition. The experiment for selecting the water depth was performed using the set-up as described in the preceding section. The harmonic external force was maintained at the amplitude that would make the structure to have a steady-state response of 1.00 cm at resonance without TLD. Different water depths were tried.

The experimental result shown in Fig. 5(a) is the relation between the water depth in the TLD, h, and its corresponding structural response amplitude at steady-state, A, for full-size 25 cm tank; Fig. 5(b) shows the relation for full-size 32 cm tank. It can be seen that as liquid depth, h, increases from zero along with its corresponding mass ratio, μ, structural response A decreases. This decrease continues



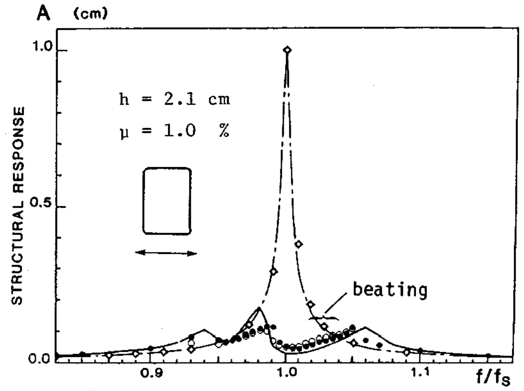
(a) full-size 25 cm tank

(b) full-size 32 cm tank

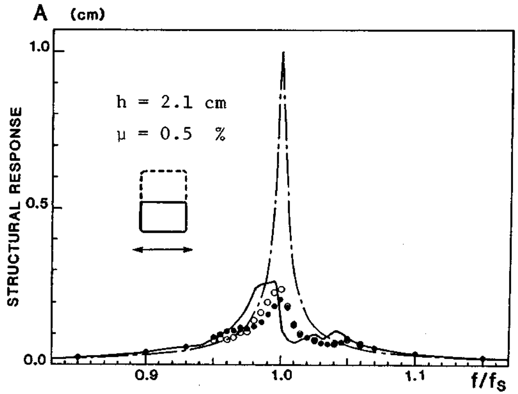
Fig. 5 Structural response A vs. water depth h, $(f/f_S)=1.00$.

until h reaches a certain level, about 2.1 cm for 25 cm tank and 3.6 cm for 32 cm tank, where further increase in h leads to an increase in structural response, even though mass ratio increases.

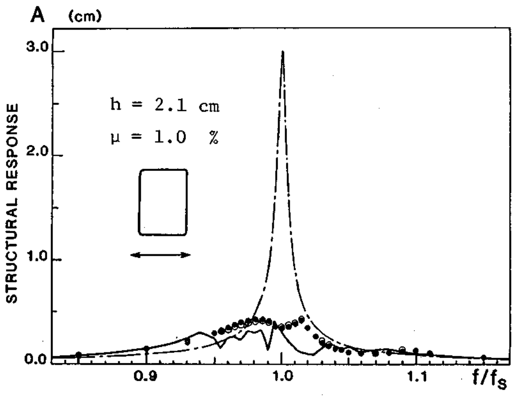
On this experimental basis, water depth of 2.1 cm for 25 cm tank was selected. Water depth lower than 2.1 cm, i. e. lower mass ratio, might suppress the vibration more if a higher mass ratio (e. g. 1.0 %) is



a) A_0= 1.0 cm, full-size 25 cm tank



b) A_0= 1.0 cm, one-half 25 cm tank

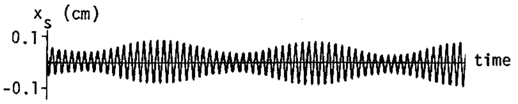


c) A_0= 3.0 cm, full-size 25 cm tank

- simulation ——— no TLD, ——— with TLD
- experiment ● sweep forward, ○ sweep backward,
◇ structure without TLD

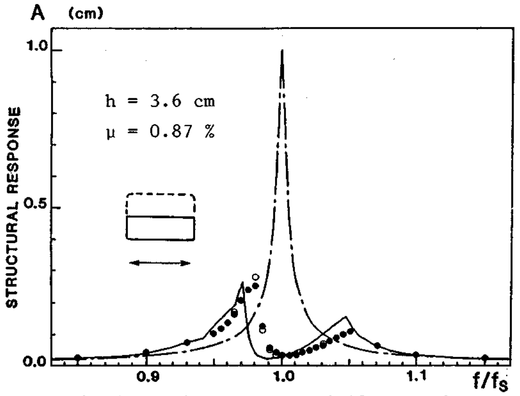
Fig. 6 Frequency response curves; 25 cm tank.

(a) Time history of water surface elevation

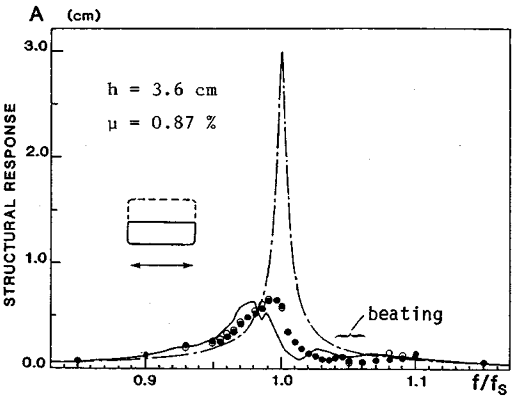


(b) Time history of structural response

Fig. 7 An example of beating; 25 cm tank, f/f_s=1.03, A_0=1.0 cm.



a) A_0= 1.0 cm, one-half 32 cm tank



b) A_0= 3.0 cm, one-half 32 cm tank

- simulation ——— no TLD, ——— with TLD
- experiment ● sweep forward, ○ sweep backward

Fig. 8 Frequency response curves; 32 cm tank.

attained. However, this would mean that more tanks of the given size are needed; it may not be practically appreciated. Similarly, water depth of 3.6 cm was chosen for 32 cm tank (**Fig.** 5(b)).

(3) Results and discussion on the frequency sweep experiment

a) Response of structure

The experimental results using TLD of full-size 25 cm tank (mass ratio $\mu=1.0\%$, $h=2.1$ cm), are shown in **Fig.** 6(a) as (●) and (○). Steady-state structural displacement amplitude is determined from the root-mean-square over 20 cycles and then multiplied by $\sqrt{2}$. This enables the structural response amplitude with beating, if any, to be compared with the response amplitude without beating. This steady-state amplitude is, then, plotted against frequency ratio; the plot is referred as "frequency response curve". The results of the frequency sweep test toward higher frequency (●) and backward (○) show no significant effect of initial condition on structural response and waves. It can be seen in **Fig.** 6(a) that at the region of resonance ($f/f_s\approx1.00$), the vibration amplitude is drastically reduced to about 5 %, while the local peaks at other frequencies are also much smaller than 1.0 cm. The results of the numerical simulation using the TLD-structure interaction model, shown as a solid line (——), agree qualitatively with the experimental results. Beating response was observed at the frequency range $f/f_s=1.015\sim1.04$ (e.g., **Fig.** 7). However, since the numerical simulation considers only two-dimensional motion of liquid, it cannot predict the effect of liquid motion perpendicular to the structural vibration, namely "cross-wave"$^{3)}$. This wave was experimentally observed in the cases where beating was noted. The averaged amplitude in the experiment is close to the simulated value. More discussion will be made in Sect. 3(3)b).

Fig. 6(b) shows the structural frequency response curves in the case with one-half tank. The water depth h ($h=2.1$ cm) is the same as in the full-size 25 cm tank and hence mass ratio μ reduces to half (i.e., $\mu=0.5\%$). The simulation also shows reasonable agreement with the experiment.

Results for a larger exciting force are presented next. The frequency response curves using a full-size tank ($\mu=1.0\%$) with the resonance amplitude $A_0=3.0$ cm is shown in **Fig.** 6(c). The response around the resonance frequency reduces to about 15 % when TLD is attached. The simulation, however, underestimates the response, i.e., it overestimates the TLD effectiveness. This can be attributed to the neglecting of wave breaking in the mathematical model. In the experiment, the liquid surface elevation was high and mild wave breaking was observed. In wave hydrodynamics, it is recognized that the breaking of wave on a very mild slope beach occurs when wave height (measured from wave crest to wave trough) is an order of liquid depth (h). The experimental result in Ref. 2) shows that when wave breaking occurs, the damping of TLD becomes large and may be over the optimal value [Ref.4)], hence ignoring the wave breaking can lead to overestimate of TLD effectiveness.

The results of the experiments using one-half 32 cm tank are depicted in **Fig.** 8(a) and (b) for $A_0=$ 1.0 cm and 3.0 cm, respectively. The water depth h is 3.6 cm and, therefore, mass ratio becomes 0.87 %.

Fig. 8(a) shows the results for $A_0=1.0$ cm. The experimental results of both sweeping forward and backward have reasonable agreement with the numerical simulation. It can be seen that the frequency response curve has two sharp peaks which are not as clearly seen in **Fig.** 6(a). Studying Eq. (3) indicates that the higher liquid depth, h, in the 32 cm tank TLD ($h=3.6$ cm) leads to a lower damping coefficient of liquid, λ, than that of the 25 cm tank TLD ($h=2.1$ cm). In linear or almost linear system, sharp peaks are associated with low damping.

The frequency response curve for one-half 32 cm tank with $A_0=3$ cm is shown in **Fig.** 8(b). Beating response was observed at the frequency range $f/f_s=1.04\sim1.06$. The structural response reduces to about 20 %. Unlike in the 25 cm tank, the force which yields the amplitude $A_0=3.0$ cm did not cause wave breaking in the 32 cm tank. Reasonable agreement is also observed between the experimental result and the simulation. Compared with **Fig.** 8(a), the absence of peak at $f/f_s>1$ appears to be due to the

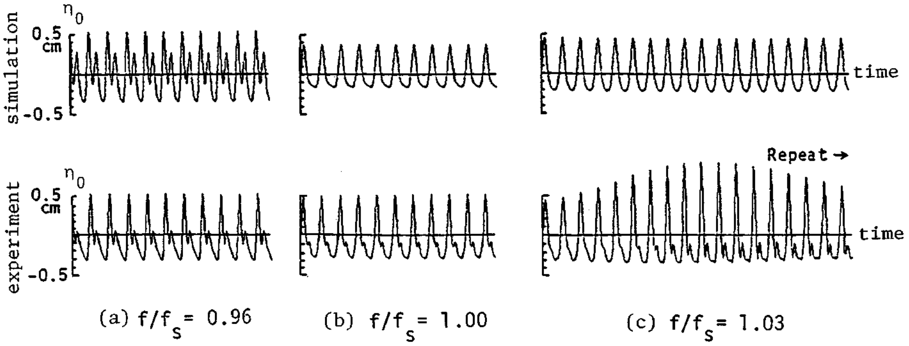


Fig. 9 Time histories of water surface elevation η_0; full-size 25 cm tank, $h=2.1$ cm, $\mu=1.00$ %, $A_0=1.0$ cm.

hardening-type nonlinearity of liquid motion[2].

b) Water surface elevation

Example of time history of water surface elevation η_0 obtained from experiments and simulation for selected frequency ratio f/f_s of 0.96, 1.00 and 1.03 in **Fig. 6**(a), are shown in **Figs. 9**(a) ~ (c). For $f/f_s=0.96$ (**Fig. 9**(a)), it can be seen that η_0 has two peaks per cycle. This shows the effect of nonlinearity of liquid motion as described in Ref. 2). The experimental result and the simulation agree well in both shape and peak value; but with some differences in the second peak. At $f/f_s=1.0$ (**Fig. 9**(b)), the numerical computation cannot simulate the water surface elevation equally well.

At $f/f_s=1.03$, beating was observed in the experiment. Beating occurred at a certain frequency range under a certain excitation amplitude. For 25 cm tank, beating was observed at the frequency range $f/f_s=1.015\sim1.04$ with $A_0=1.0$ cm (**Fig. 6**(a)), while for 32 cm tank it occurred at $f/f_s=1.04\sim1.06$ with $A_0=3.0$ cm (**Fig. 8**(b)). **Fig. 9**(c) is an example; **Fig. 7** shows the same case.

According to Ref. 5), beating of liquid motion in a rectangular tank may be caused by either the cross wave[3] or by nonlinear self-interaction. Naturally the cause of beating could also be their combination. Examples of beating obtained from experiment as well as numerical simulation are shown in Ref. 5). The simulation by the present model, however, did not predict the beating as observed in the experiment. Cross wave was always observed during the beating structural response, but this was also observed during non-beating stationary response. The specific cause of beating in the present experiment cannot be clearly identified.

The simulated structural response amplitudes in the frequency range where beating occurred in the experiment are not significantly different from the averaged response amplitudes obtained from the experiment, as shown in **Fig. 6**(a) at $f/f_s=1.015\sim1.04$ and **Fig. 8**(b) at $f/f_s=1.04\sim1.06$.

4. EXAMPLE OF TLD DESIGN

In applying TLD to an actual structure, suitable TLD properties have to be selected with regard to real constraints and external load conditions. Granting that an economical rectangular tank has been selected, the major constraint is the number of tanks that can be installed; the number of tanks is limited due to either the available space or the overall cost of the tanks. Considering either orientation of the tank, water depth has to be selected properly. In the preliminary experiment as discussed in Sect. 3(2), the liquid depth was selected that made the TLD suppress the vibration most at the frequency ratio $f/f_s=1.00$. However, in applying TLD to an actual structure which is subject to various excitation frequencies, one may have to pay attention to a wide range of frequency ratio (f/f_s) rather than only at one excitation frequency.

As an example, a hypothetical situation is considered herein. Suppose that the structure with the aforementioned dynamic properties (f_s=0.91 Hz, δ_s=0.02) has a horizontal vibration problem due to harmonic excitation whose frequency may change from time to time ranging from 0.85 Hz to 1.15 Hz. Maximum amplitude of structural vibration reaches 1.0 cm at resonance. The effective mass of the bridge vibration mode of concern is 215 000 kg. To suppress the vibration, only 600 rectangular plastic tanks shown in **Fig. 4** (a) or (c) are decided to be installed inside the bridge girder. The tanks may be oriented such that the girder motion is parallel to either the 25-cm side or the 32-cm side.

The proposed TLD-structure interaction model has been verified through the experiments as discussed

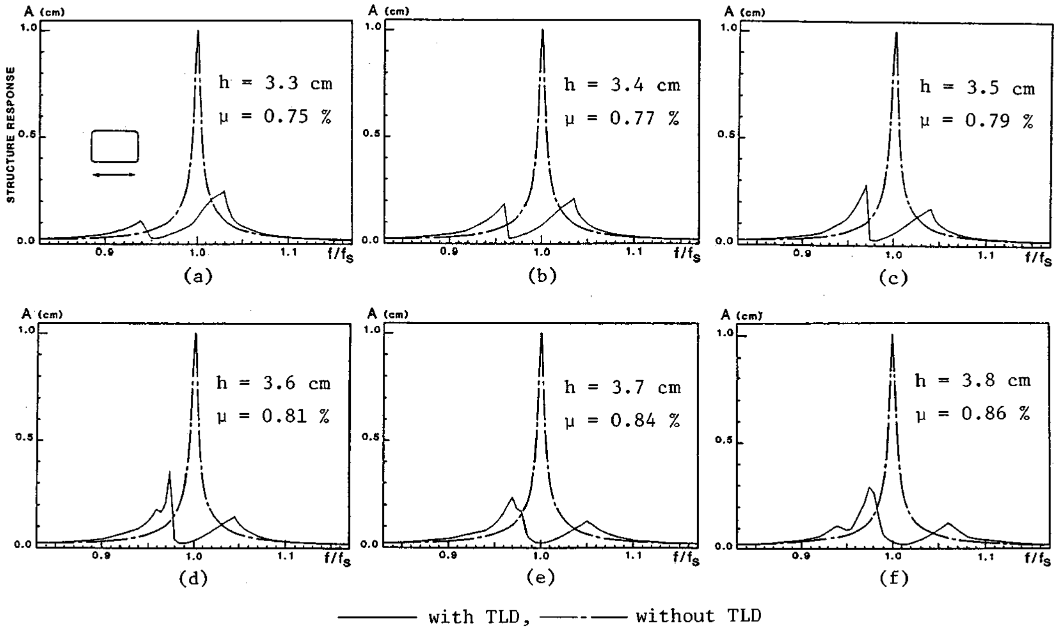


Fig. 10 Simulated frequency response curves; 600 32-cm tanks.

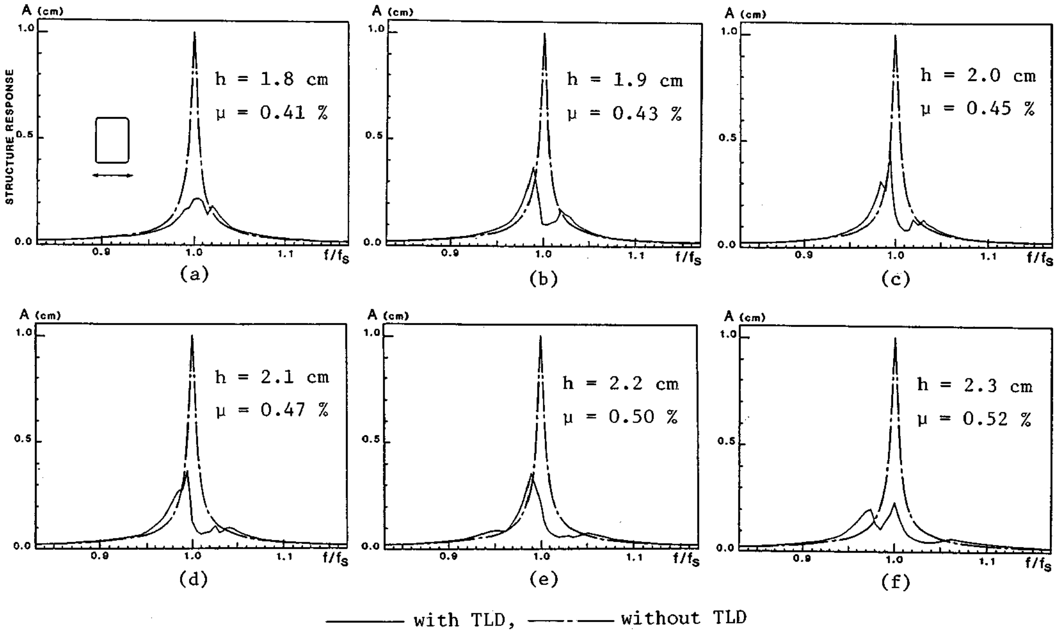


Fig. 11 Simulated frequency response curves; 600 25-cm tanks.

in Sect. 3(3). The model can be, therefore, used to conduct extensive numerical simulations with various water depth h.

Figs. 10(a) ~ (f) show the simulated frequency response curves of structure with 32-cm-tank TLD. The water depth varies from 3.3 cm (Fig. 10(a)) to 3.8 cm (Fig. 10(f)). The mass ratio μ ranges from 0.75 % to 0.86 % while the value of f_w/f_s varied from 0.95 to 1.02. Note that increase in the water depth, h, leads to increase of the corresponding mass ratio, μ. As the water depth h of 3.3~3.8 cm is not so shallow, the liquid damping of TLD is rather low (see Sect. 3(3) and Fig. 8(a)). This can explain two local peaks in the frequency response curves seen in Figs. 10(a) ~ (f). Within this range of water depth, the frequency response curves are not so different from one another.

Figs. 11(a) ~ (f) show the simulated frequency response curves when using 25-cm-tank TLD. Water depth h varies from 1.8 cm (Fig. 11(a)) to 2.3 cm (Fig. 11(f)); f_w/f_s ranges from 0.91 to 1.02. Compared with the previous cases, the frequency response curves in Fig. 11 are more sensitive to the change of water depth. The reductions of the response are relatively small.

Comparing the magnitudes of local peaks in the frequency response curves shown in Figs. 10 and 11, the water depth h of 3.4 cm with 32 cm side (Fig. 10(b)) appears to be the reasonable choice for this hypothetical problem.

5. CONCLUDING REMARKS

The interaction of a rectangular TLD with an SDOF structure was studied experimentally and theoretically under sinusoidal forced excitation. Focus of the study was mainly placed on a range of relatively small amplitude of structure vibration where TLD has no wave breaking. A commercially available rectangular plastic tank was used as a prototype TLD. The experimental results showed significant reduction of structural response, which can be achieved if the dimensions and liquid depth of TLD are properly selected.

Two-dimensional nonlinear model of liquid motion in TLD was utilized to study the TLD-structure interaction. The model includes the damping effect due to solid boundary friction and free surface contamination but not wave breaking effect. The proposed TLD-structure interaction model was verified by forced excitation experiment. Reasonable agreement between the experimental results and the numerical simulations was observed within the range of relatively small excitation amplitudes where no wave breaking existed.

The TLD-structure interaction model, although limited to no wave breaking condition, was shown to be useful in simulating the structure frequency response before deciding the proper TLD design. Modeling wave breaking in TLD is also of great importance and has to be developed next in order to widen the application of TLD to structure with larger excitation. A semi-empirical engineering approach is discussed in Ref. 6).

Beating of water surface elevation and structural response was found in the experiment in several cases under certain excitation frequencies and vibration amplitudes. Possible causes of this beating phenomenon were discussed. Further study on the behaviour and effect of beating in liquid motion has to be conducted experimentally as well as theoretically.

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