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AVAILABILITY OF RELIABILITY INDEX FOR STRUCTURAL DESIGN

*By Takashi CHOU**

The reliability index is usually utilized for structural design problems. Relations between design results, such as configurations, and the reliability indices in accordance with the Advanced First Order Second Moment (AFOSM) method, are numerically investigated with the help of some fundamental problems and practical examples. In general, the AFOSM method contains some intrinsic errors. When the index value is large the error in the failure probability increases significantly, even if the error in the index is relatively small. It is explained that the design result is not affected to the same degree as the error in the failure probability by the error in the index, but it is usually affected to the same or somewhat larger degree as the error in the index. It is preferable to use the reliability index rather than the probability by which the design results are indirectly affected.

Keywords : design, failure, probability, reliability index, structural engineering

1. INTRODUCTION

The reliability index, β, in accordance with the Advanced First Order Second Moment (AFOSM) method[1)-3)] (see **Appendix I**) is considered to be a very powerful tool for the structural reliability problems which can have any nonlinear or non-closed algebraic performance function with nonnormal basic random variables. This is because the calculation of the failure probability, p_f, from $\Phi(-\beta)$ usually provides a reasonable estimate, i. e.,

$$p_f \simeq \Phi(-\beta) \cdots\cdots (1)$$

without much computational effort, where $\Phi(.)$ denotes the standard normal cumulative distribution function.

In general, the AFOSM method contains some intrinsic errors. According to the writer's experience[3)], the error in β to $-\Phi^{-1}(p_f)$ is usually estimated to be less than 5% for many practical engineering problems, where $\Phi^{-1}(.)$ denotes the inverse function of $\Phi(.)$. As the value of β increases, however, the error of $\Phi(-\beta)$ to p_f increases significantly, even if the error in β is small. In order to observe these circumstances, the following probability effect ratio, γ, is defined as shown below :

$$\gamma = \Phi(-\alpha\beta)/\Phi(-\beta) \cdots\cdots (2)$$

where α is a coefficient that takes values of 0.95-1.05 corresponding to the extent of the error in β. The results of γ are shown in **Fig. 1**, from which it can be seen that γ amounts, for example, to 3.5 when $\beta=5$ and $\alpha=0.95$. This means that only a 5% error in β produces a 250% error in the failure probability. Therefore, it may safely be said that the reliability index, β, is a reasonably accurate estimation of

* Member of JSCE, Dr. Eng., Professor, Shinshu University (500 Wakasato, Nagano, 380)

$-\Phi^{-1}(p_f)$, i.e.,

$$\beta \simeq -\Phi^{-1}(p_f) \quad (3)$$

in comparison with Eq. (1).

A question may be raised how the availability of the reliability index is due to improper correspondence to the failure probability in case of a large β value. It is surely undesirable to use this index with relation to such problems with which the probability itself is important and its variance brings about directly some serious consequences. The reliability index is usually utilized for structural design problems. In this paper, the relations between design results, such as configurations, and the reliability indices will be numerically demonstrated with the help of some fundamental problems and practical examples. It will be indicated that it is preferable to use the reliability index rather than the failure probability, and the reliability index is available for the structural design problem, even if the value of β is large and its compatibility to the failure probability is not so good.

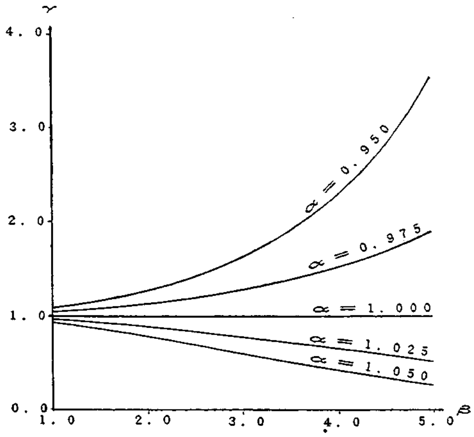

Fig. 1 Probability Effect Ratio, γ.

2. INFLUENCE OF ERROR IN RELIABILITY INDEX ON DESIGN RESULTS

Case 1.

Let us consider a performance function

$$g(\underline{x}) = x_1 - x_2 \quad (4)$$

where $\underline{x}$ is a vector of the random variables having a strength term, x_1, and a load term, x_2. $g(\underline{x}) \leq 0$ identifies failure, and $g(\underline{x}) > 0$ corresponds to a safe state. When x_1 and x_2 are statistically independent and they are normally distributed variables, the central safety factor, ν, required by the reliability index, β_D, can be expressed as:

$$\nu(\beta_D) = \left[1 + \sqrt{1 - (1 - \beta_D^2 V_1^2)(1 - \beta_D^2 V_2^2)}\right] \Big/ (1 - \beta_D^2 V_1^2) \quad (5)$$

where V_1 and V_2 are coefficients of variation (COV) of x_1 and x_2, respectively. In order to observe the influence of the error of β_D on ν, the following central safety factor ratio, δ, representing the effect of the error in the reliability index on the design result, is defined:

$$\delta = \nu(\alpha\beta_D) / \nu(\beta_D) \quad (6)$$

Calculating results of δ with relation to some combinations of the COVs are shown in Figs. 2-7 and Table 1, compared with γ which is depicted with broken lines. From these results it is revealed that the values of δ are not affected to the extent of the values of γ by α. When the value of β is close to the reciprocal of the COV of the strength term, the design result is considerably affected by the error of the reliability index. Under such a condition, however, the reliability can not be increased beyond some fixed level by strengthening the structure, and therefore it can hardly be said to be realistic. This means that the probability distribution of the strength term, x_1, which is usually expressed as a nonlinear function of the basic random variables, may not be considered as the normal distribution, even if they are normally distributed variables. It is noticed that the central safety factor may not represent the design result for some rare cases.

Case 2.

When x_1 and x_2 in Eq. (4) are lognormally distributed variables, the central safety factor, ν, can be expressed as:

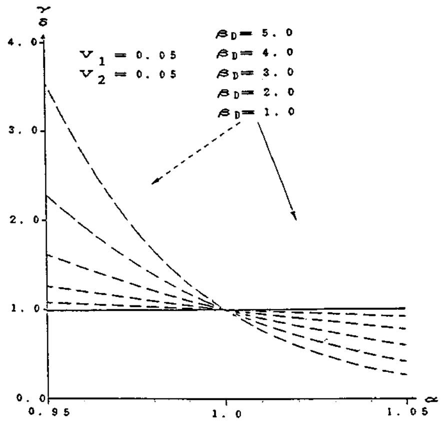

Fig. 2 δ and γ for Case 1 (1).

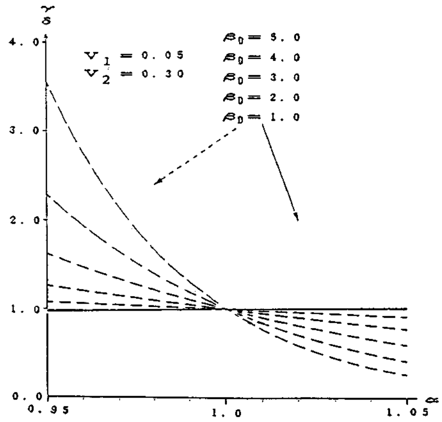

Fig. 3 δ and γ for Case 1 (2).

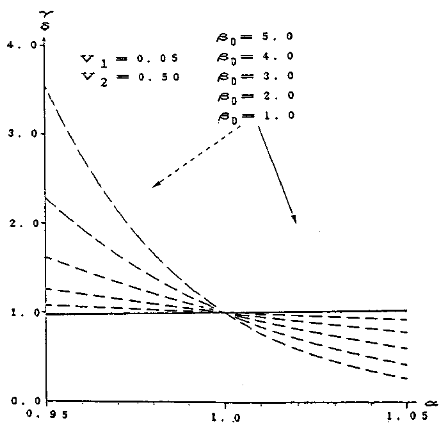

Fig. 4 δ and γ for Case 1 (3).

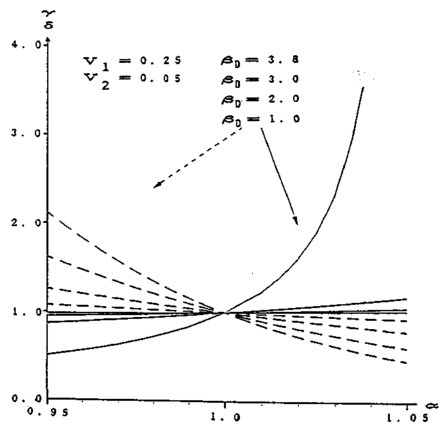

Fig. 5 δ and γ for Case 1 (4).

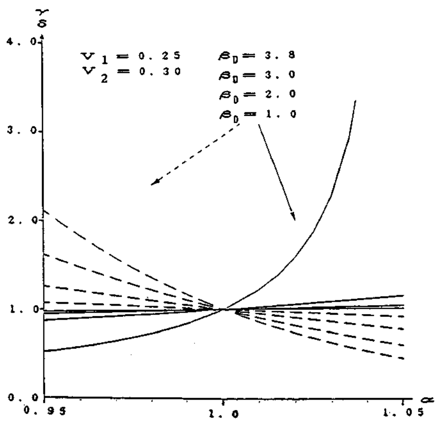

Fig. 6 δ and γ for Case 1 (5).

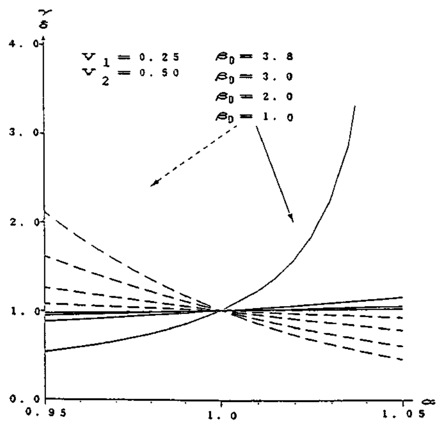

Fig. 7 δ and γ for Case 1 (6).

Table 1 Examples of δ and γ for Case 1 with $V_1=0.25$ and $V_2=0.30$.

	β_D	1.0		2.0		3.0		3.8	
		δ	γ	δ	γ	δ	γ	δ	γ
	0.950	0.98	1.08	0.95	1.26	0.88	1.62	0.53	2.12
α	0.975	0.99	1.04	0.98	1.12	0.93	1.28	0.69	1.46
	1.025	1.01	0.96	1.03	0.89	1.08	0.78	1.88	0.68
	1.050	1.02	0.93	1.05	0.79	1.16	0.60	1.84	0.46

Table 2 Examples of δ and γ for Case 2 with $V_1=0.25$ and $V_2=0.30$.

	β_D	1.0		2.0		3.0		4.0		5.0	
		δ	γ	δ	γ	δ	γ	δ	γ	δ	γ
	0.950	0.98	1.08	0.96	1.26	0.94	1.62	0.93	2.28	0.91	3.55
α	0.975	0.99	1.04	0.98	1.12	0.97	1.28	0.96	1.52	0.95	1.90
	1.025	1.01	0.96	1.02	0.89	1.03	0.78	1.04	0.65	1.05	0.52
	1.050	1.02	0.93	1.04	0.79	1.06	0.60	1.08	0.42	1.10	0.27

$$\nu(\beta_D)=\exp\left[\beta_D\sqrt{\ln\{(1+V_1^2)(1+V_2^2)\}}-\ln\sqrt{(1+V_2^2)/(1+V_1^2)}\right] \cdots\cdots (7)$$

It is noticed that Eq. (7), which is led with the use of the reliability index, holds approximately. This will also be noticed in the succeeding cases. Figs. 8-13 and Table 2 show results from Eq. (6) using Eq. (7) as the central safety factor for some examples, compared with γ from Eq. (2). It can be observed that the design results are affected to the same or somewhat larger extent as the errors in reliability indices and none of the unrealistic situations such as in Case 1 can be seen.

Case 3-9.

With regard to the performance function in Eq. (4), let us consider various types of combinations of the distribution shown in Table 3. Notations denote in the table that N=Normal, L=Lognormal, E=Type I asympototic, W=Weibull with a lower limit, x_l, as:

$$x_l=x_m-2\sigma \cdots\cdots (8)$$

where x_m is a mean value and σ is a standard deviation, B=Beta with a lower limit the same as Eq. (8) and an upper limit, x_u, as:

$$x_u=x_m+3\sigma \cdots\cdots (9)$$

and NB, for example, denotes that x_1=Normal and x_2=Beta. Table 3 shows the central safety factors, which are obtained numerically by an algorithm shown in Appendix II, and the central safety factor ratios for Cases 3-9 with Cases 1 and 2, where $\beta_D=3.0$, $\alpha=0.95$, 1.00, 1.05, $V_1=0.25$, and $V_2=0.3$. To examine the compatibility of the reliability indices for the central safety factors in Table 3 with β_D, the Monte Carlo simulations were executed and the results are shown in Table 4. These tables suggest that in case x_1=Normal, even x_2=Nonnormal, the influence of the error in the reliability index is similar to that in Case 1, whereas in case x_1=Lognormal, even x_2=Nonlognormal, it is similar to that in Case 2, and the realized reliabilities, β, are nearly equal to the design target, β_D.

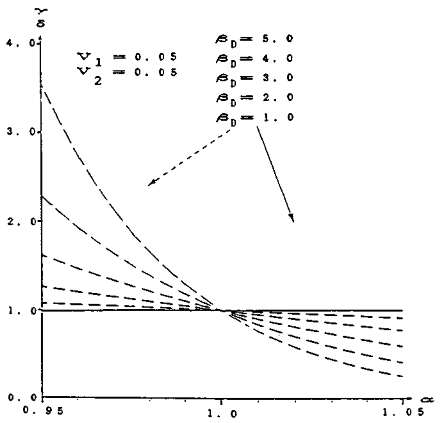

Fig. 8 δ and γ for Case 2 (1).

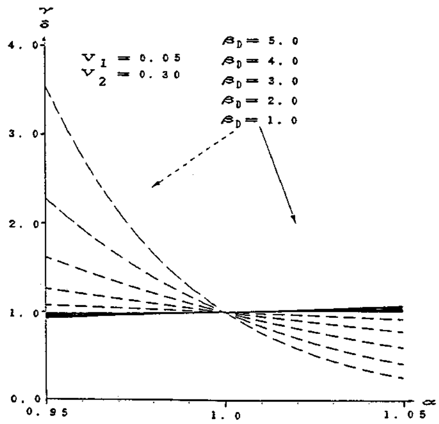

Fig. 9 δ and γ for Case 2 (2).

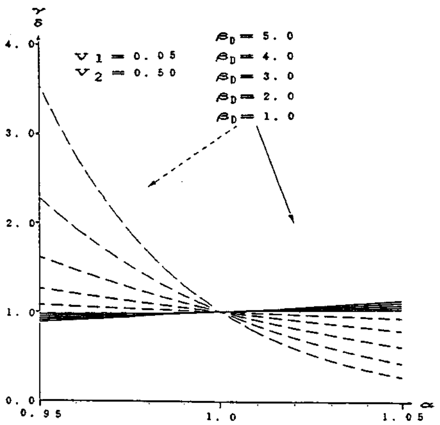

Fig. 10 δ and γ for Case 2 (3).

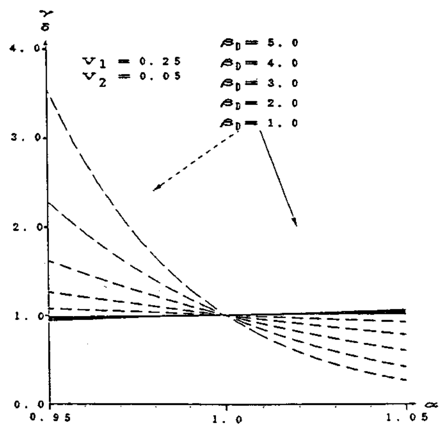

Fig. 11 δ and γ for Case 2 (4).

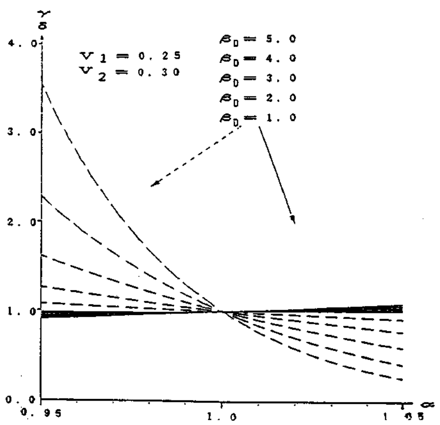

Fig. 12 δ and γ for Case 2 (5).

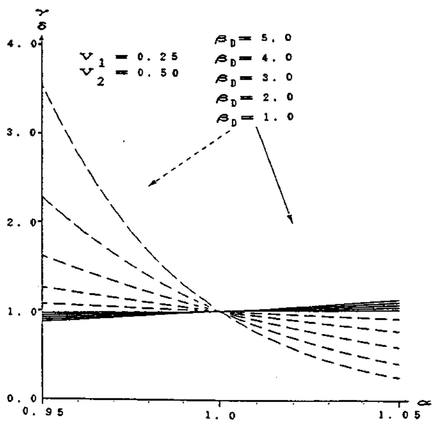

Fig. 13 δ and γ for Case 2 (6).

Table 3 ν and δ for Cases 1-9.

Case No.			1	2	3	4	5	6	7	8	9
Distributions			NN	LL	NL	LN	NE	LE	WE	NB	BB
	0.95		3.92	2.94	3.84	2.73	3.84	3.00	3.05	4.00	2.80
α	1.00	ν	4.47	3.12	4.38	2.87	4.34	3.19	3.23	4.56	2.89
	1.05		5.21	3.30	5.06	3.01	5.06	3.39	3.42	5.31	2.98
	0.95		0.88	0.94	0.88	0.95	0.88	0.94	0.92	0.88	0.97
α	1.00	δ	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05		1.16	1.06	1.16	1.05	1.17	1.06	1.08	1.16	1.03

Table 4 Monte Carlo Simulation Results for Cases 1-9 with β_D=3.0.

Case No.		1	2	3	4	5	6	7	8	9
Distributions		NN	LL	NL	LN	NE	LE	WE	NB	BB
	β	3.00	2.98	2.94	3.04	2.93	2.98	3.06	3.01	3.16

Case 10-11.

The performance function of the plastic flexural problem of a steel beam section may be given as :

$$g(\underline{x})=x_1x_2x_3-(x_4+x_5)x_6 \quad (10)$$

where x_1=plastic section modulus of the beam, x_2=yield strength of steel, x_3=correction variable for resistance, x_4=bending moment due to dead loads, x_5=bending moment due to live loads and x_6= correction variable for load effect. Determine mean value of x_1, for β_D=4.0 and α=0.95-1.05, supposing the following mean vector, $\underline{m}$, in which elements are given by dimensionless values, COV vector, $\underline{V}$, and distribution vector, $\underline{D}$. It is assumed that all random variables are statistically independent.

$$\underline{m}=(x_{m1}\ 2.0\ 1.0\ 1.0\ 1.0\ 1.0) \quad (11)$$

$$\underline{V}=(0.1\ 0.15\ 0.1\ 0.05\ 0.35\ 0.1) \quad (12)$$

$$\underline{D}=(N\ N\ N\ N\ N\ N)\ \text{for Case 10},\ (L\ L\ L\ L\ L\ L)\ \text{for Case 11} \quad (13)$$

The results are listed in Table 5. It can be deduced from the table that the influence, i. e., δ, of the error in the reliability index, i. e., α, on the design result, i. e., x_{m1}, is somewhat larger than α. When the strength term, i. e., $x_1x_2x_3$, and the load effect term, i. e., $(x_4+x_5)x_6$, are normally distributed variables, 0.80 (for α=0.95) and 1.34 (for α=1.05) as the values of δ are produced from Eqs. (5)-(6). The corresponding values in Case 10 are 0.93 and 1.08. As mentioned in Case 1, it is not advisable to assume that the strength term is a normally distributed variable.

Case 12-13.

The performance function of a singly reinforced concrete rectangular section subjected to bending moment can be expressed as :

$$g(\underline{x})=[x_1x_2\{x_3-(x_1x_2)/(1.7\,x_4x_5)\}x_6]-(x_7+x_8)x_9 \quad (14)$$

where x_1=area of steel reinforcement, x_2=yield strength of steel, x_3=effective depth, x_4=width of section, x_5=compressive strength of concrete, x_6=correction variable for strength term, x_7=bending moment due to dead loads, x_8=bending moment due to live loads and x_9=correction variable for load effect term. Assume that the variables are uncorrelated, and means, COVs, and distributions are as follows :

Table 5 x_{m1}, α, γ and δ for Cases 10 and 11.

β_D		3.8	3.9	4.0	4.1	4.2
α		0.950	0.975	1.000	1.025	1.050
γ		2.28	1.52	1.0	0.65	0.42
Case 10	x_{m1}	3.17	3.28	3.40	3.53	3.66
	δ	0.93	0.96		1.04	1.08
Case 11	x_{m1}	3.15	3.26	3.37	3.48	3.60
	δ	0.93	0.97		1.03	1.07

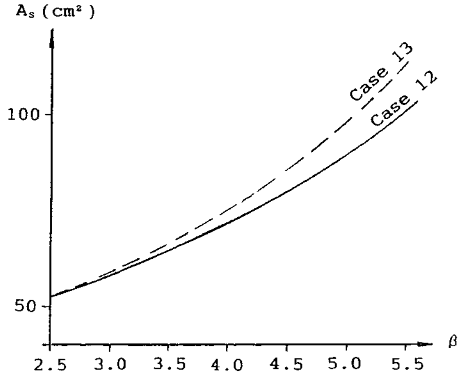

Fig. 14 Design Results of Cases 12 and 13.

$\underline{m}=(x_{m1}$ 323.4 MPa 84.92 cm 100 cm 28.22 MPa 1.0 59.4 MN-cm 25.1 MN-cm 1.0$)$ ············ (15)

$\underline{V}=(0.03\ 0.04\ 0.05\ 0.05\ 0.2\ 0.1\ 0.05\ 0.35\ 0.1)$ ············ (16)

$\underline{D}=(N\ N\ N\ N\ N\ N\ N\ N\ N)$ for Case 12, $(L\ L\ L\ L\ L\ L\ L\ L\ L)$ for Case 13 ············ (17)

Fig. 14 shows the design results for the area of steel reinforcement required by $\beta_D=2.5$-5.5. If a changing range of β_D is limited to be the extent of, say, 1, the design results are almost proportional to β_D, while the failure probabilities associated with the extent of β_D are logarithmically affected.

3. CONCLUSIONS

Some fundamental and practical examples of influence of the error in the reliability index on the design result are presented, from which the following conclusions can be drawn :

(1) The design result is not affected to the same degree as the error in the failure probability by the error in the reliability index, but it is usually affected to the same or somewhat larger degree as the error in the index, probably less than 5 %. This level of accuracy in the index seems to be practically sufficient, with the lack of some statistical data taken into consideration.

(2) When the probability distribution of the strength term is assumed as the normal variate and the value of the reliability index is close to the reciprocal of the COV of the term, the design result is considerably affected by the error in the index. Such a condition, however, is not realistic because the reliability is not increased beyond a certain fixed level by strengthening the structure. It is not advisable to assume that the strength term is a normally distributed variable.

(3) It is preferable to use the reliability index rather than the failure probability, because the design result is not directly affected by the probability.

(4) The reliability index is available for structural design problems, even if the value of the index is large and its compatibility to the failure probability is not so good.

APPENDIX I : AFOSM METHOD

A flow chart of the procedure in accordance with the AFOSM method is shown in Fig. A-1. Symbols used in the figure are as shown below :

$\underline{F}$: a vector of the cumulative distribution function having elements F_i,

$\underline{f}$: a vector of the density function having elements f_i,

$\underline{\rho}$: a correlation matrix having elements ρ_{ij},

$\underline{x}^0$: a vector having the elements as initial values of n basic random variables,

ϕ : the density function of the standard normal variate.

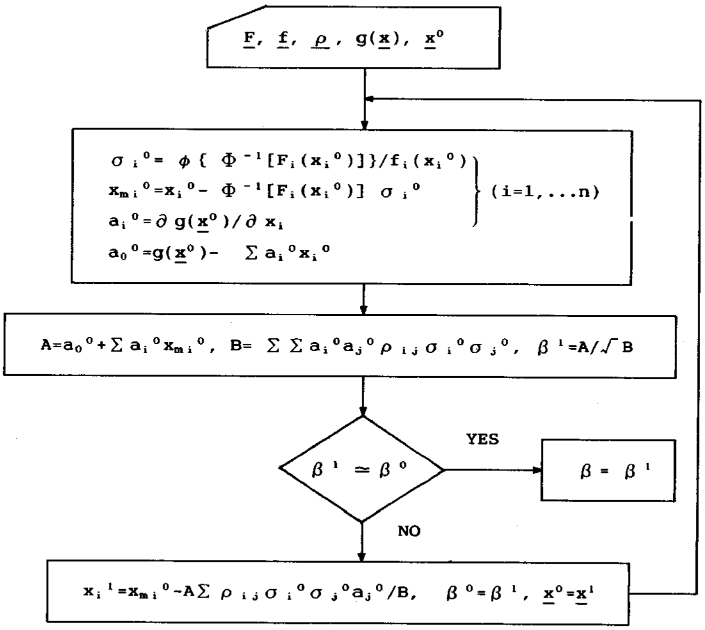

Fig. A·1 Flow Chart of AFOSM Method.

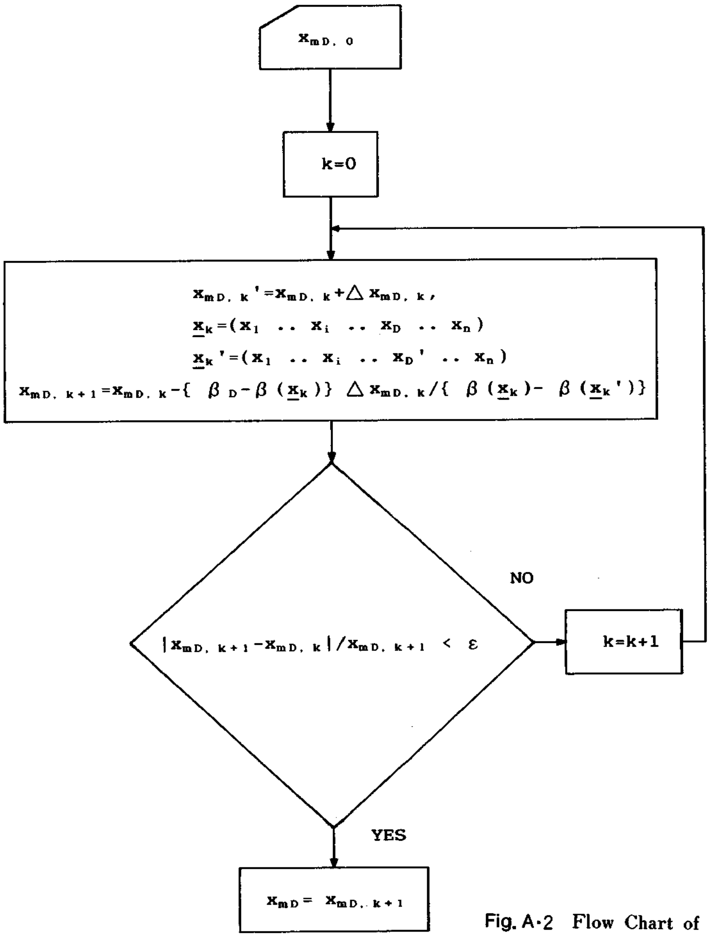

Fig. A·2 Flow Chart of Design.

It should be emphasized that F_i, f_i and $g\ (\underline{x})$ are not necessarily expressed as a closed form and can be just evaluated numerically for the fixed random values. In the sequence procedure, vibration may occur. This can be avoided by decreasing a step taken.

APPENDIX Ⅱ : DESIGN BY NEWTON-RAPHSON METHOD USING NUMERICAL DIFFERENTIATING

Fig. A・2 shows a flow chart to determine a mean value of a design variable, x_{mD}, by the Newton-Raphson method with which the numerical differentiating is used. Meanwhile, $x_{mD,0}$ is a initial value of x_{mD} and $\triangle x_{mD,k}$ is a infinitesimal increasing quantity of $x_{mD,k}$, say, $x_{mD,k}/100$. Furthermore, $\beta\ (\underline{x})$ is the reliability index for $\underline{x}$, and ε is a small positive value, say, 0.0001. In accordance with the writer's experience, convergence of this sequential procedure is obtained by less than 5 or 6 iterations.

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