

## A PROVISION ON INTERMEDIATE DIAPHRAGM SPACING IN CURVED STEEL-PLATED BOX-BRIDGE-GIRDERS

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The normal stress distribution and cross-sectional deformation by distortion in single span curved box-bridge-girders have been investigated. An analytical modeling by so-called Beam on Elastic Foundation (BEF) analogy for the distortion is verified experimentally by testing programs. On the basis of the parametric studies by the BEF analysis, a spacing provision by which the distortional stress can be kept within the level of secondary stress is developed. Design examples specified by the proposed provision are also presented.

*Keywords: steel-plated box girder, curved girder, intermediate diaphragms, cross-sectional stiffening*

### 1. INTRODUCTION

A steel-plated box girder is particularly well suited for use in curved bridges since its high torsional stiffness enables it to effectively resist the torsional deformations encountered in curved members. However, loads causing a curved box girder section to undergo torsional deformation will distortionally deform its cross section and give a normal stress distribution by the cross-sectional distortion (herein termed as "distortional warping stress") if the cross-sectional stiffening supplied by intermediate diaphragms are insufficient. Namely, the intermediate diaphragms in box girders play a very important role on shear resistance for the distortional warping of the girder. Several research results on design procedures of the intermediate diaphragms for the curved girders have been reported by Dabrowski<sup>1)</sup>, Oleinik and Heins<sup>2)</sup>, Sakai and Nagai<sup>3)</sup>, and by Nakai and Murayama<sup>4)</sup>. However, the previous reports seem to be inconvenient for practical designers, and not to define design philosophy on so-called allowable stress level for the distortional warping stress and characteristics of structural parameter for distortional stiffness.

This paper presents a spacing provision which could be more readily utilized for steel-plated intermediate diaphragms in the curved box-bridge-girders based on the analytical results of an extensive parametric study on the distortion. A parameter for the distortional warping stiffness, which is used in this paper, is clearly characterized by a nondimensional value of stiffness uniformly distributed by the intermediate diaphragms. The principal factors to be considered in the design of the intermediate diaphragms are to provide adequate rigidity for the distortional deformation and to lessen the distortional warping stress. Namely, the provision proposed herein are evolved by limiting the distortional warping

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deformation and stress to the order of secondary magnitude for bridge structures<sup>5,6</sup>. The distortional warping response, herein, is analyzed by an analogical concept of Beam on Elastic Foundation (BEF) originally proposed by Wright, Abdel-Samad and Robinson<sup>7</sup> for straight girders and initially applied to the curved girders by Dabrowski<sup>1</sup>. In order to confirm the validity of the BEF analogical concept the analytical results are compared with experimental results.

## 2. EVALUATION OF DISTORTION

Distortional warping response of the curved box girders could be properly and briefly modeled by so-called BEF analogy<sup>1,3</sup>. Expressed mathematically with nondimensional manner using span length  $L$  and distortional warping rigidity  $EI_{dw}$ , it is as follows<sup>1,3</sup> :

$$V'''' - (kL^4/EI_{dw}) V = (L^4/EI_{dw})(p_v + M/R) \dots \dots \dots (1)$$

where ' = successive derivative of the function with respect to nondimensionalized longitudinal coordinate along the curvilinear girder axis divided by the span length  $L$ ;  $E$  = Young's modulus;  $V$  = angle due to distortion;  $k$  = stiffness for frame behavior provided by the cross section;  $I_{dw}$  = distortional warping constant;  $p_v$  = distributed torsional load per unit length of the girder axis;  $M$  = transverse plane bending moment of the girder;  $R$  = radius of curvature; i. e. :

$$I_{dw} = [\beta_1^2 A_t (1 + 2a/b)^2 + \beta_2^2 A_b + 2A_w (\beta_1^2 - \beta_1 \beta_2 + \beta_2^2)] / 3$$

$$\beta_1 = [e / (e + f)] (bd/2), \quad \beta_2 = [f / (e + f)] (bd/2), \quad e = I_{by} / b + bdt_w/4, \quad f = I_{ty} / b + bdt_w/4$$

$$k = 48 E (b/I_t + b/I_b + 6d/I_w) / [3(d/I_w)^2 + 2bd/(I_t I_w) + 2bd/(I_b I_w) + b^2/(I_t I_b)]$$

$$I_t = t_t^3 / [12(1 - \nu^2)], \quad I_b = t_b^3 / [12(1 - \nu^2)], \quad I_w = t_w^3 / [12(1 - \nu^2)], \quad A_t = (b + 2a)t_t, \quad A_w = dt_w, \quad A_b = bt_b$$

In the above equations :  $\nu$  = Poisson's ratio;  $t_t, t_b, t_w$  = plate thickness of top flange, bottom flange, and web;  $a, b, d$  = top flange plate length extending across the width of the box, and width and depth of the box girder;  $I_{ty}, I_{by}$  = moment of inertia with respect to  $y$  axis of top flange, and bottom flange, as shown in Fig.1. The governing differential equation for the distortional warping response, Eq. (1), can be solved using a finite difference numerical technique and this approach is used herein. The intermediate diaphragms are idealized as additional spring support attached to the structurally-mechanically modeled beam. The distortional warping stress,  $\sigma_{dw}$ , due to the cross-sectional distortion is given as  $\sigma_{dw} = E\phi V'' / L^2$  in which  $\phi$  = a generalized coordinate for the distortional warping displacement defined by Vlasov<sup>8</sup>.

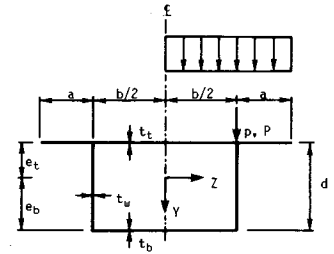


Fig.1 Cross-Section, Applied Live Load and its Idealized Load.

The accuracy of the solutions to the differential equation obtained from the finite difference technique is checked by comparing with the experimental test run. Two of the steel-plated test model girders with various number of intermediate diaphragms (6 mm in thickness) are fabricated for comparative purpose. In the first test model (Model M-1) an intermediate diaphragm is assembled at the center of the span. The second test model (Model M-2) consists of five intermediate diaphragms which partition the test model girder into six equal-intervals along the curvilinear axis. Both test models also have rigid end diaphragms at their supports. Section properties of the test model girders are shown in Fig. 2, where high depth girder model is adopted in order to grasp the effect of narrow box proportion on the distortional warping behavior. A concentrated load is applied on the middle of the box width at the center-span point. A general view of the test model and the test set-up is shown in Figs. 3. Some typical results on the analytical and experimental stress distributions in normal directions at the center-span points due to 9 742 kg or 95.47 kN for the Model M-1 are shown in Fig. 4 and 9 855 kg or 96.58 kN for the Model M-2 in Fig. 5, respectively. In these normal stress distributions, the effects of bending, and torsional and distortional warpings are all summed up. The effects of bending and torsional warping can be evaluated by well-known load-deformation response model for a curved box girder with undeformable rigid cross section as theoretically developed by

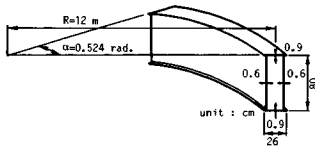


Fig. 2 Details of Tested Box Girder.

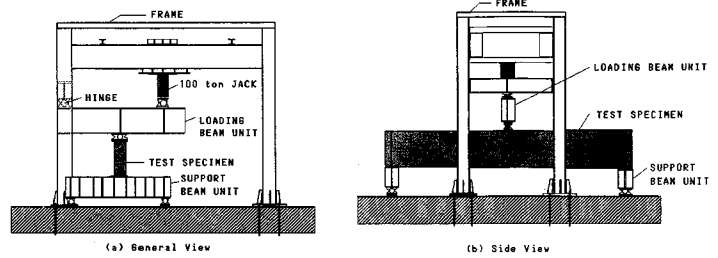


Fig. 3 Setup of a Test Beam.

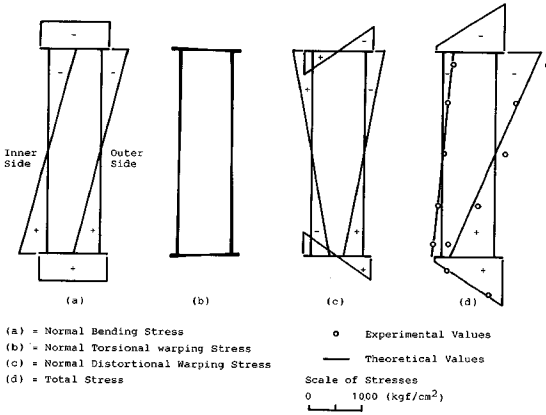


Fig. 4 Normal Stress Distributions at the Center-Span Cross-Section (for Model M-1).

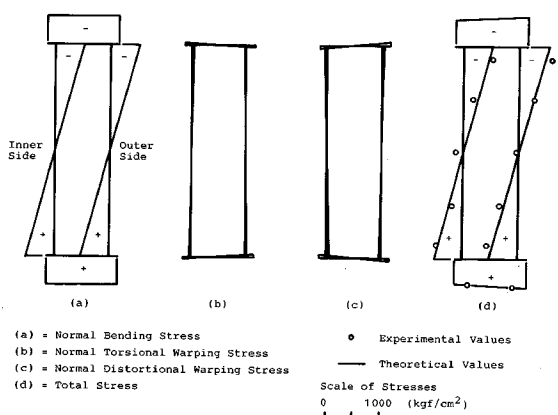


Fig. 5 Normal Stress Distributions at the Center-Span Cross-Section (for Model M-2).

Vlasov<sup>8)</sup> and this approach is used herein. Moreover, the effect of distortional warping is calculated by the aforementioned BEF analogy. The analytical girder boundary conditions are idealized as simply-supported relative to bending and fixed-supported to torsion and cross-sectional distortion. It seems to be probable, however, that the boundary conditions for torsion and cross-sectional distortion are not exactly as idealized, due to stiffness offered by the test apparatus. The difference between the normal stress distributions of the Model M-1 and M-2 is mainly caused by numbers of the assembled intermediate diaphragms, i. e., by the distortional warping phenomenon. As can be seen from Figs. 4 and 5 the analytical and experimental results are in good agreement, especially considering that the analysis is not able to exactly duplicate the boundary conditions encountered in the tests.

### 3. DISTORTIONAL CHARACTERISTICS

Parametric studies on the distortional warping response for the curved box girder are performed using the aforementioned BEF analysis. The data obtained from this study is then used to develop a spacing provision of the intermediate diaphragms for use in directly evaluating the distortional warping effects in curved box-girders as the second order response of bridge-structures. Figs. 6, 7 and 8 show typical results on the absolute maximum value of the distortional warping stress on the reference girder,  $\sigma_{dw}$ , by the parametric study as a function of distortional warping stiffness parameter,  $\mu$ , for various number of intervals equally partitioned along the girder axis by the intermediate diaphragms,  $n=5, 10$  and  $15$  under the uniformly distributed load, the eccentrically distributed load and a concentrated load, respectively. If it can be assumed that the intermediate diaphragm plates are distributed uniformly over their intervals, the distortional stiffness  $k$  in Eq. (1) is estimated as  $k=K_D/L_D$ , where  $K_D$ =shear stiffness of the diaphragm plate and  $L_D$ =interval of the diaphragms, because that the stiffness for frame behavior provided by the

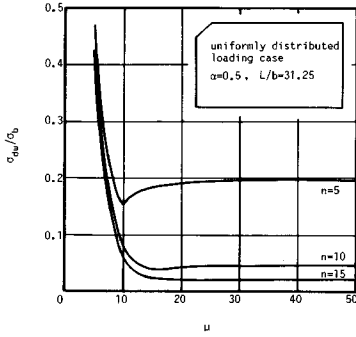


Fig. 6 Relationships between Distortional Warping Stress and Distortional Stiffness Parameter.

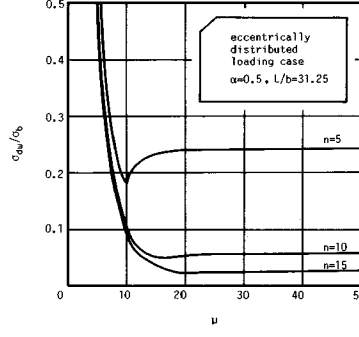


Fig. 7 Relationships between Distortional Warping Stress and Distortional Stiffness Parameter.

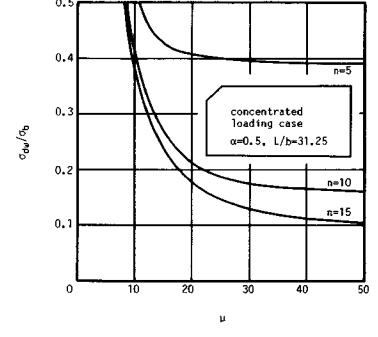


Fig. 8 Relationships between Distortional Warping Stress and Distortional Stiffness Parameters.

cross section, itself, of the girder might be practically negligible compared with the shear stiffness of the intermediate diaphragm plates. In this case, the nondimensional stiffness factor in Eq. (1) is expressed as follows :

$$kL^4/(EI_{dw}) = K_D L^4/(EI_{dw} L_D) = n K_D L^3/(EI_{dw}) = \mu^4; \quad (n = L/L_D) \dots\dots\dots (2)$$

Actually, a truss or framework may occasionally constitute interia intermediate diaphragm element in the curved steel plated box-bridge-girders. Since the diaphragm plays the role on shear stiffness for the cross-sectional distortion, such a truss or framework may be replaced in the Eq. (2) by an equivalent wall element<sup>(9),10)</sup>. The Ref. 9) or 10) could be refered on these fictitious wall thicknesses.

The  $\mu$  formulated by Eq. (2) is selected in this paper as the parameter of distortional warping stiffness provided by the intermediate diaphragms. The uniformly distributed loads are applied over the girder width. The eccentrically distributed loads are applied to the half of the girder width and idealized, herein, as corresponding torsional loads applied on the web plate as shown in Fig. 1. The concentrated load is also applied on the web plate. The  $\sigma_{dw}$ , herein, is the maximum value of distortional warping stress distribution searched along the curvilinear axis of the girder and is nondimensionalized by bending stress distribution,  $\sigma_b$ , which occurs in the girder under the same applied load as the  $\sigma_{dw}$ . As can be seen from these figures the distortional warping stress,  $\sigma_{dw}$ , decreases as the stiffness parameter,  $\mu$ , increases, and eventually the stress is merged into a certain value in the range of more than 20 of  $\mu$ -value. Therefore, this range can be proposed as a required stiffness for the distortional warping stress as follows :

$$\mu_{required} \geq 20 \text{ for distortional warping stress} \dots\dots\dots (3)$$

Fig. 9 shows some selected,  $\sigma_{dw}$ , as a function of central angle of the curved girder,  $\alpha$  in radian, for

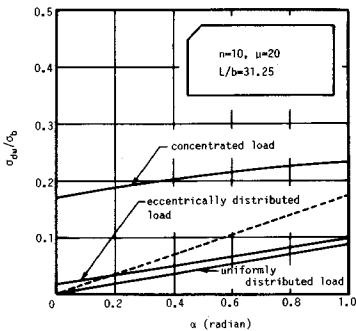


Fig. 9 Relationships between Distortional Warping Stress and Central Angle.

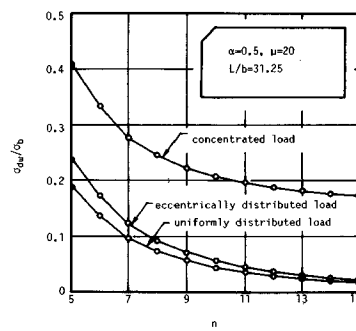


Fig. 10 Relationships between Distortional Warping Stress and Number of the Diaphragm Intervals.

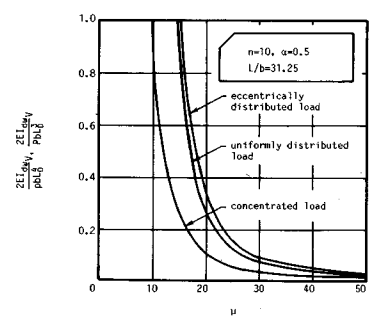


Fig. 11 Relationships between Distortional Displacement and Distortional Stiffness Parameter.

various loading cases, i. e., the uniformly distributed loading, the eccentrically distributed loading and the concentrated loading. From the figure, it can be observed that the distortional warping stress,  $\sigma_{dw}$ , increases as the central angle,  $\alpha$ , increases. Especially, for the distributed loading cases, the  $\sigma_{dw}$  increases linearly. Fig. 10 shows typical relationship between the distortional warping stress,  $\sigma_{dw}$ , and number of the diaphragm-intervals,  $n$ , for the various case of loadings. From the figure, obviously, the  $\sigma_{dw}$  decreases successively for increase of  $n$  and is eventually merged into a certain value. Some typical results for distortional warping deformation at the center of the span are plotted in Fig. 11 as a function of the distortional warping stiffness parameter,  $\mu$ , for various case of the loadings. It can be seen from the figure that the distortional deformation decreases rapidly as the distortional warping stiffness increases when in the range of  $\mu < 30$  and is practically merged into a certain value when  $\mu \geq 30$ .

#### 4. A PROPOSAL FOR SPACING PROVISION

In designing curved bridges with box-girders, the distortional warping stress may be expected to keep within the level of secondary stress,  $\sigma_{2nd}$ , for bridge-structures. By referring a presentable available specification<sup>6)</sup>, the secondary stress for compressive members shall not exceed 210 kg/cm<sup>2</sup> or 20.6 MPa. A stress factor 2.0, in designing, might be used to account for uncertainties that might result from inadequacies in the analysis assumptions regarding member behavior<sup>5)</sup>. Furthermore, a factor of safety 1.67 against yield stress,  $\sigma_y$ , is actually often used in practice to establish the allowable design stress<sup>5),6)</sup>. Therefore, a ratio of the allowable secondary stress to the allowable design stress could be taken into account within  $(\sigma_{2nd}/2.0)/(\sigma_y/1.67) = (210/2)/(2520/1.67) = 0.07$  in which structural mild steel material<sup>6)</sup> is used. Considering conservatively, 5% of normal bending stress under design loadings [= dead load + (1 + impact fraction) × (uniformly distributed live load + line load)],  $\sigma_{design}$  (hereafter termed as “design bending stress”), is adopted in this paper as the allowable distortional warping stress, namely:

$$[(\sigma_{aw})_d + (\sigma_{aw})_p + (\sigma_{aw})_l] / \sigma_{design} = \sigma_{aw} / \sigma_{design} \leq 0.05 \dots \dots \dots (4)$$

where the symbols,  $d$ ,  $p$  and  $l$  means the distortional stress distribution by the dead load of curved bridge with thin-walled box-girders, and by the specified distributed live load and line load, respectively.

In the above equation,  $p = 350(1+i)$  kg/m (m) or 3.43(1+i) kN/m (m) is adopted for the distributed live load with impact effect<sup>5)</sup> and is idealized as the aforementioned eccentrically or uniformly distributed loading.  $l = 5000(1+i)$  kg (m) or 49.03(1+i) kN (m) for the line load with impact effect<sup>5)</sup> is adopted as the concentrated load applied on the web plate by considering the most conservative manner. The  $i$  is so-called impact fraction and determined by the formula of  $20/(50+L)$  in which  $L$  is a metered span-length<sup>5)</sup>. In Fig. 9 the dashed line shows the distortional warping stress under the uniformly distributed load, however, the stress is nondimensionalized by the  $\sigma_b$  under the eccentrically distributed load. As can be seen from comparing with a solid line for the eccentrically distributed loading in Fig. 9 and

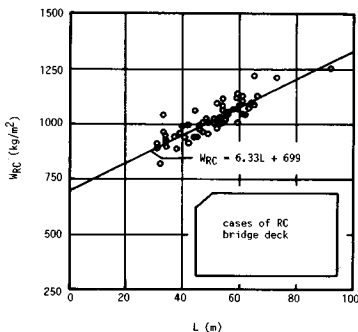


Fig.12 Relationships between Dead Load and Span Length for Reinforced Concrete Bridge Deck.

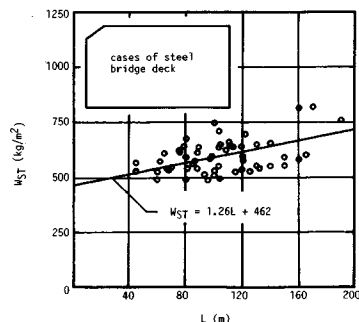


Fig.13 Relationships between Dead Load and Span Length for Steel Bridge Deck.

**Table 1 Ratios of Bending Stress Distribution under Dead, Live or Line Loading to Design Bending Stress.**

(a) R.C. Bridge Deck				
L (m)	F <sub>d</sub>	F <sub>p</sub>		F <sub>p</sub>
		(1)	(2)	
30	0.5106	0.1257	0.2513	0.1191
40	0.5655	0.1270	0.2540	0.0903
50	0.6066	0.1254	0.2508	0.0713
60	0.6390	0.1225	0.2450	0.0580
70	0.6655	0.1190	0.2379	0.0483
80	0.6878	0.1152	0.2304	0.0410

(b) Steel Bridge Deck				
L (m)	F <sub>d</sub>	F <sub>p</sub>		F <sub>p</sub>
		(1)	(2)	
60	0.4687	0.1803	0.3606	0.0854
80	0.5070	0.1819	0.3638	0.0646
100	0.5472	0.1740	0.3480	0.0525
120	0.5828	0.1646	0.3292	0.0440
140	0.6089	0.1581	0.3162	0.0375
160	0.6259	0.1550	0.3099	0.0321
180	0.6406	0.1517	0.3033	0.0280
200	0.6541	0.1484	0.2967	0.0246

Note : (1) = eccentrically distributed live load,  
 (2) = uniformly distributed live load.

**Table 2 Section Properties of Box Girders.**

No.	SPAN (m)	b (cm)	a (cm)	d (cm)	t <sub>t</sub> (cm)	t <sub>w</sub> (cm)	t <sub>b</sub> (cm)	L/b	BRIDGE DECK TYPE
1	75.0	240	10.0	280	1.46	1.18	1.21	31.25	R.C.
2	54.5	190	10.0	220	1.77	1.11	1.47	28.68	R.C.
3	24.5	180	10.0	120	1.42	0.90	1.19	13.61	R.C.
4	37.0	240	10.0	177	1.30	1.00	1.27	15.42	R.C.
5	34.0	220	10.0	150	1.75	0.96	1.49	15.45	R.C.
6	30.0	350	10.0	170	1.27	0.96	1.03	8.57	R.C.
7	32.5	180	10.0	120	1.94	1.00	1.79	18.06	R.C.
8	34.5	180	10.0	125	1.94	1.00	1.79	19.17	R.C.
9	41.6	150	10.0	173	2.15	0.95	2.13	27.73	R.C.
10	41.1	150	10.0	173	1.95	0.95	1.93	27.40	R.C.
11	33.5	150	10.0	160	1.80	0.96	1.54	22.33	R.C.
12	140.0	600	10.0	303	2.66	1.40	2.41	23.32	R.C.
13	79.0	400	10.0	360	2.64	1.53	2.07	19.75	R.C.
14	80.0	210	10.0	250	1.62	1.11	1.53	38.10	R.C.
15	67.0	230	10.0	260	1.84	1.09	1.44	29.13	R.C.
16	61.3	270	10.0	260	2.86	1.31	2.80	22.70	R.C.
17	114.0	260	11.0	290	3.30	1.30	3.40	43.85	R.C.
18	76.5	470	10.0	275	2.14	1.09	2.34	16.28	R.C.
19	50.0	280	10.0	240	1.28	1.17	1.30	17.86	R.C.
20	41.3	150	75.0	160	2.90	0.96	1.86	27.53	R.C.
21	72.0	200	100.0	250	1.75	1.09	1.44	36.00	STEEL
22	91.2	340	170.0	440	2.18	2.09	2.68	26.82	STEEL
23	170.0	340	170.0	345	2.38	1.58	5.34	50.00	STEEL
24	165.0	500	152.5	406	2.02	1.42	3.14	33.00	STEEL
25	110.0	253	126.5	340	1.70	1.31	1.62	43.48	STEEL
26	130.0	500	250.0	330	2.48	1.28	3.23	26.00	STEEL
27	120.0	260	130.0	300	1.96	1.18	3.11	46.15	STEEL
28	103.0	260	130.0	320	2.06	1.19	2.34	39.62	STEEL
29	129.0	300	150.0	300	3.00	1.32	3.31	43.00	STEEL
30	60.0	410	80.0	200	1.98	1.00	1.29	14.63	STEEL
31	90.0	490	110.0	250	1.82	1.00	1.39	18.75	STEEL
32	120.0	594	199.0	199	1.89	1.30	2.97	20.20	STEEL
33	150.0	550	225.0	400	1.86	1.20	2.77	27.27	STEEL

the dashed line, the  $\sigma_{dw}$ -value of the solid line is higher than the value of the dashed line when  $\alpha$  is less than a critical value, while the solid line is lower than the dashed line when  $\alpha$  is larger than the critical value. Therefore, in this paper, it is decided in calculating the distortional warping stress that the distributed live load should be applied either eccentrically or uniformly by examining the abovementioned relationship in each case. The dead loads of actually constructed curved bridges with box-girders in Japan are examined herein and shown in Fig. 12 for reinforced concrete bridge decks and in Fig. 13 for steel bridge decks, as the function of their span length,  $L$ . By applying a regression analysis on statistics to the actual data prediction formulas on the dead loads of steel-plated, curved box-bridge-girders can be obtained as follows:

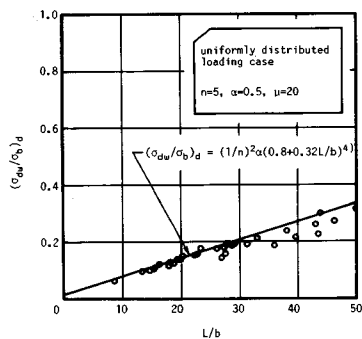
$$W_{RC} = 6.33 L + 699 \text{ (kg/m}^2 \text{ or } \times 9.8 \text{ kN/m}^2\text{) for reinforced concrete deck ;}$$

$$W_{ST} = 1.26 L + 462 \text{ (kg/m}^2 \text{ or } \times 9.8 \text{ kN/m}^2\text{) for steel deck ..... (5)}$$

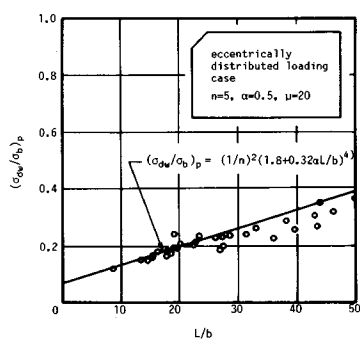
Those formulas are also shown in Figs. 12 and 13 and used in this paper to predict the dead loads. The stress distributions  $(\sigma_{dw})_d$  and  $(\sigma_b)_d$  can be calculated using Eqs. (5). Some typical results on the ratio of bending stress distribution under the dead, live or line loading to the design bending stress are summarized in Table 1, where :

$$F_d = (\sigma_b)_d / \sigma_{design}, F_p = (\sigma_b)_p / \sigma_{design}, F_p = (\sigma_b)_p / \sigma_{design} ..... (6)$$

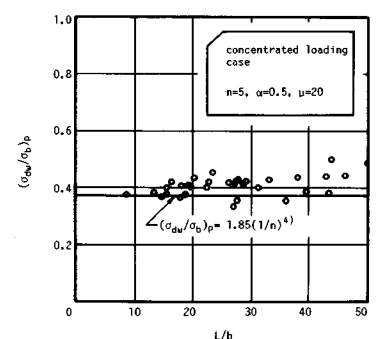
Some calculated results on ratios of the distortional warping stress distribution to the bending stress distribution are shown in Fig. 14 for the dead loading, Fig. 15 for the eccentrically distributed live loading and in Fig. 16 for the line loading, as the function of  $L/b$ . In calculating the typical structural properties



**Fig. 14 Relationships between Distortional Stress and  $L/b$  for Uniformly Distributed Load.**



**Fig. 15 Relationships between Distortional Stress and  $L/b$  for Eccentrically Distributed Load.**



**Fig. 16 Relationships between Distortional Stress and  $L/b$  for Concentrated Load.**

of the curved bridges with box-girders actually constructed in Japan are adopted and tabulated in Table 2. The extensive numerical results obtained from the parametric study show that the distortional warping stress can be formulated by making the variables  $L/b$ ,  $\alpha$  and  $n$  separate. Similar results were obtained by Nakai and Murayama<sup>4)</sup> and practical formulas proposed by them might be available for predicting the distortional stress, as follows :

$$(\sigma_{dw}/\sigma_b)_a = \alpha(0.8 + 0.32 L/b)/n^2; (\sigma_{dw}/\sigma_b)_p = (1.8 + 0.32 \alpha L/b)/n^2; (\sigma_{dw}/\sigma_b)_P = 1.85/n \dots\dots\dots (7 \cdot a)$$

These formulas are also shown in Figs. 14, 15 and 16, respectively. As can be seen from these figures, the predicted and analyzed results are in good agreement. However, in the case of uniformly distributed live load, the  $(\sigma_{dw}/\sigma_b)_P$  should be formulated as follows :

$$(\sigma_{dw}/\sigma_b)_P = \alpha(0.8 + 0.32 L/b)/n^2 \dots\dots\dots (7 \cdot b)$$

Referring Eqs. (6) and (7) the distortional warping stress on a standard girder with the structural properties as shown in the No.1 data of Table 2 is expressed as :

$$\left[ \frac{(\sigma_{dw})_a + (\sigma_{dw})_P + (\sigma_{dw})_P}{\sigma_{design}} \right]_{standard} = \left[ \frac{\sigma_{dw}^*}{\sigma_{design}} \right]_{standard} + \left[ \frac{\sigma_{dw}^S}{\sigma_{design}} \right]_{standard} \dots\dots\dots (8)$$

where  $\sigma_{dw}^*$  = term for effect of curvature of the curved girder on the distortional stress ;  $\sigma_{dw}^S$  = distortional warping stress distribution for the straighten box girder ( $\alpha=0$ ). The  $\sigma_{dw}^S$  is also calculated by using the BEF analogy, namely, by substituting  $R=\infty$  into Eq. (1), and is unrelated with  $L/b$ . Some analytically calculated results on the  $(\sigma_{dw}^S/\sigma_{design})_{standard}$ ,  $(\sigma_{dw}^*/\sigma_{design})_{standard}$  and  $(\sigma_{dw}/\sigma_{design})_{standard}$  are tabulated in Table 3 for the case of  $\alpha=0.5$  and  $n=12$ , as the function of span-length. As can be seen from the table, the uniformly distributed live loading should be adopted in this case because it causes higher magnitude of the distortional warping stress than the eccentrically distributed live loading.

Table 3 Results on  $(\sigma_{dw}^S/\sigma_{design})_{standard}$ ,  $(\sigma_{dw}^*/\sigma_{design})_{standard}$  and  $(\sigma_{dw}/\sigma_{design})_{standard}$ .

(a) eccentrically distributed live load								
L (m)	R.C. Bridge Deck				Steel Bridge Deck			
	30	50	70	80	80	100	150	200
$\sigma_{dw}^S/\sigma_{design}$	0.0212	0.0135	0.0097	0.0084	0.0133	0.0112	0.0081	0.0063
$\sigma_{dw}^*/\sigma_{design}$	0.0210	0.0224	0.0234	0.0238	0.0206	0.0213	0.0224	0.0231
$\sigma_{dw}/\sigma_{design}$	0.0422	0.0359	0.0331	0.0322	0.0339	0.0325	0.0305	0.0294
(b) uniformly distributed live load								
L (m)	R.C. Bridge Deck				Steel Bridge Deck			
	30	50	70	80	80	100	150	200
$\sigma_{dw}^S/\sigma_{design}$	0.0192	0.0115	0.0078	0.0066	0.0104	0.0085	0.0056	0.0040
$\sigma_{dw}^*/\sigma_{design}$	0.0256	0.0271	0.0278	0.0280	0.0273	0.0276	0.0282	0.0285
$\sigma_{dw}/\sigma_{design}$	0.0448	0.0386	0.0356	0.0346	0.0377	0.0361	0.0338	0.0325

Note :  $\alpha=0.5$ ,  $n=12$ ,  $L/b=31.25$ .

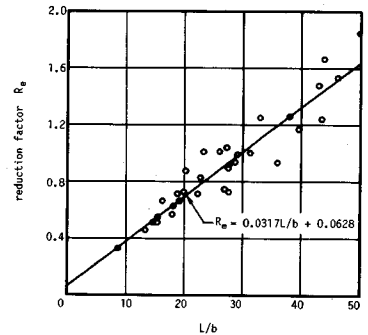


Fig. 17 Reduction Factor for the Case of Eccentrically Distributed Live Load.

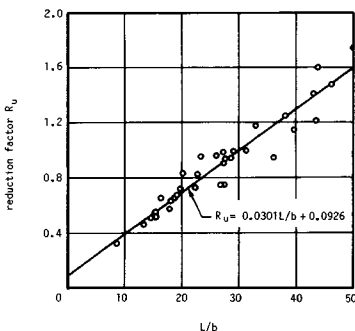


Fig. 18 Reduction Factor for the Case of Uniformly Distributed Live Load.

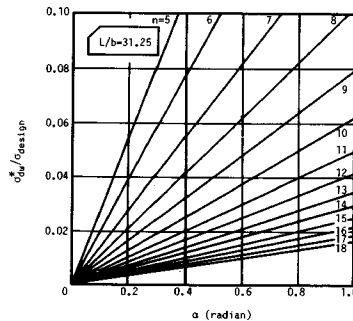


Fig. 19 Relationships between  $\sigma_{dw}^*/\sigma_{design}$  and Central Angle for Eccentrically Distributed Live Load.

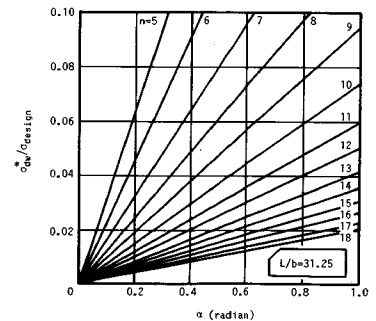


Fig. 20 Relationships between  $\sigma_{dw}^*/\sigma_{design}$  and Central Angle for Uniformly Distributed Live Load.

The ratios of the  $\sigma_{dw}^*/\sigma_{design}$  for the structural properties of curved bridges with steel-plated box-girders actually constructed in Japan as shown in Table 2 to the aforementioned standard one  $(\sigma_{dw}^*/\sigma_{design})_{standard}$  is calculated to grasp the effect of cross-sectional properties. The ratio is, herein, defined with  $R_e$  for the case of eccentrically distributed live loading and with  $R_u$  for the case of uniformly distributed live loading. These ratios are shown in Figs. 17 and 18, as the function of  $L/b$ . By applying the regression analysis to the results prediction formulas for the ratios,  $R_e$  and  $R_u$ , can be obtained as follows :

$$R_e = 0.0317 L/b + 0.0628 ; R_u = 0.0301 L/b + 0.0926 \dots \dots \dots (9 \cdot a, b)$$

These formulas are also illustrated in Figs. 17 and 18. It is proposed to use these formulas to define the effect of cross-sectional properties for evaluating the distortional warping stress. The  $\sigma_{dw}^*/\sigma_{design}$  for an arbitrary proportion of cross section can be evaluated, in practical sense, as follows :

$$\sigma_{dw}^*/\sigma_{design} = (R_e \text{ OR } R_u) [\sigma_{dw}^*/\sigma_{design}]_{standard} \dots \dots \dots (10)$$

Fig. 19 shows typical relationship between  $\sigma_{dw}^*/\sigma_{design}$  and  $\alpha$  under the eccentrically distributed live loading for various value of  $n$  and Fig. 20 under the uniformly distributed live loading, respectively. Hereafter, comparing with the two kinds of figures for various values of  $\alpha$ ,  $n$  and  $L/b$  it is decided whether  $R_e$  should be adopted or  $R_u$ , in each case.

Substituting Eqs. (8·a) and (10) into Eq. (4) gives :

$$\sigma_{dw}/\sigma_{design} = (R_e \text{ OR } R_u) [\sigma_{dw}^*/\sigma_{design}]_{standard} + [\sigma_{dw}^s/\sigma_{design}] \leq 0.05 \dots \dots \dots (11)$$

This relationship becomes the key step in establishing the spacing provision for the intermediate diaphragms of curved box-bridge-girders. The conditions be satisfied Eq. (11) are examined for the ranges of  $\alpha = 0 \sim 1.0$ ,  $n = 5 \sim 18$ , and  $L/b = 10 \sim 50$ . The examined results are summarized in Figs. 21, 22, and 23 for the  $L/b$ -value of 10, 30 and 50, respectively. From these figures the required value of  $n$  can be obtained for various values of  $\alpha$  and  $L/b$ . For example from Fig. 22 the 5 % level of the calculated value of  $\sigma_{dw}/\sigma_{design}$  as shown in Eq. (11) is corresponding to  $\alpha = 0.215$  for  $n = 8$  and  $L/b = 30$ . Thus, the condition be satisfied Eq. (11) in this case is  $\alpha \leq 0.215$ . Namely the required number of  $n$  should be 8 for the case of  $\alpha = 0.2$  and  $L/b = 30$ . The required number of  $n$  as shown in Table 4 is obtained by the abovementioned manner for various values of  $\alpha = 0 \sim 1.0$  and  $L/b = 10 \sim 50$ , respectively.

The design of the diaphragms should be considered not only the distortional stress requirement but also the distortional deformation characteristics in order to provide adequate stiffness for the cross-sectional deformation by the distortion. The distortional displacement is also examined for the various values of  $\mu$ ,  $n$ ,  $\alpha$  and  $L/b$ . Some typical results are, heretofore, shown in Fig. 11. The results to hand show that the ratio of vertical deflection by the distortion,  $bV$  to the

Table 4 Required Number of  $n$ .

L/b \ $\alpha$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
10	5	6	6	7	8	8	9	9	10	10	10
20	5	6	7	8	9	10	11	11	12	13	13
30	5	7	8	9	11	11	12	13	14	15	15
40	5	7	9	10	12	13	14	15	15	16	17
50	5	8	10	11	13	14	15	16	17	18	19

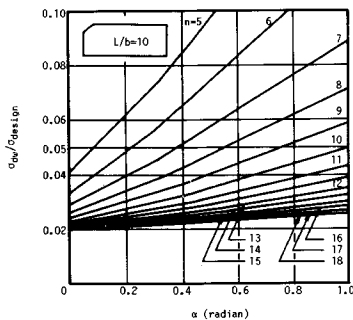


Fig. 21 Relationships between  $\sigma_{dw}/\sigma_{design}$  and Central Angle. (for  $L/b=10$ )

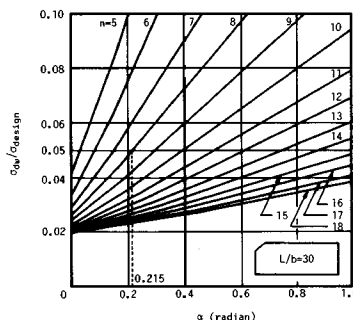


Fig. 22 Relationships between  $\sigma_{dw}/\sigma_{design}$  and Central Angle. (for  $L/b=30$ )

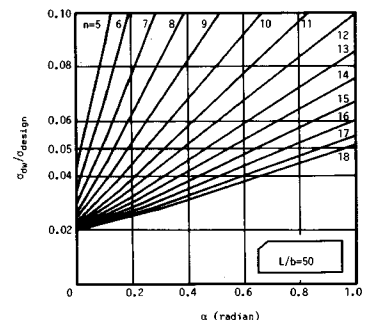


Fig. 23 Relationships between  $\sigma_{dw}/\sigma_{design}$  and Central Angle. (for  $L/b=50$ )



interval-length of intermediate diaphragms,  $L_D$ , is within the order of  $1/1\ 000$  for  $\mu \geq 30$  and the required  $n$  as shown in Table 4. Initial geometrical imperfections permitted for compressive members shall not exceed  $1/1\ 000$  of the member length<sup>5),6)</sup> and might be adopted as so-called secondary magnitude of the deflection of bridge structures. From the above discussion it can be proposed that  $\mu \geq 30$  is a required stiffness for the distortional deformation, namely :

$$\mu_{\text{required}} \geq 30 \text{ for distortional deformation} \dots\dots\dots (12)$$

Depending on a design specification for highway bridges<sup>5)</sup>, when a bridge deck is supported by three plated girders or more with span-length over 10 m the girders shall be provided with and rigidly interconnected with cross beams spaced at intervals not to exceed 20 m. The cross beams or lateral bracings, or both should be placed in the exterior bays between the intermediate diaphragms. Therefore the diaphragms also should be spaced at intervals,  $L_D$ , not to exceed 20 m. Furthermore, it is specified in most of the bridge specifications<sup>5),6)</sup> that structural steel shall be not less than 8 mm in thickness. Summarizing these specifications, Eqs. (2), (3), (9), (11) and (12) spacing provision for the intermediate diaphragms in steel-plated, curved, box-bridge-girders can be formulated as follows :

$$n \geq L/20 \text{ (L in m)}; \quad K_D \geq 810\ 000 \text{ } EI_{dw}/(nL^3), \text{ provided that } t_D \geq 8 \text{ mm}; \dots\dots\dots (13 \cdot a)$$

$$(R_e \text{ or } R_u)[\sigma_{dw}^*/\sigma_{\text{design}}]_{\text{standard}} + \sigma_{dw}^s/\sigma_{\text{design}} \leq 0.05 \text{ or Table 4} \dots\dots\dots (13 \cdot b)$$

Note :  $K_D = 4 GA t_D$ ;  $R_e = 0.0317 L/b + 0.0628$ ;  $R_u = 0.0301 L/b + 0.0926$

in which  $G$ =shear modulus= $8.1 \times 10^5$  kg/cm<sup>2</sup> or 7.94 N/mm<sup>2</sup>,  $t_D$ =thickness of diaphragm plate. For an example case of  $L/b=30$ ,  $n$  is given as 5, 11 and 15 for  $\alpha=0, 0.5$  and  $1.0$ , respectively by Table 4. Since  $n=L/L_D$ , the proposed critical interval-length of the intermediate diaphragm  $L_D$  is equal to  $L/5, L/11$  and  $L/15$  for  $\alpha=0, 0.5$  and  $1.0$ , respectively as shown in Fig. 24. However,  $n$  should be larger than and equal to  $L/20$  by Eq. (13·a), namely,  $L_D$  should be less than and equal to 20 m. Therefore, the critical  $L_D$  is equal to 20 m in the range of  $L \geq 100$  m for the case of  $\alpha=0$ .

5. DESIGN EXAMPLES

Two of design examples for the intermediate diaphragms in steel-plated, curved box-bridge-girders are examined. Example TYPE A-Consider the steel-plated rectangular box girder with mid-high depth shown schematically in Fig. 25 (a). Given :  $L=65$  m;  $L/b=32.5$ ;  $\alpha=0.65$

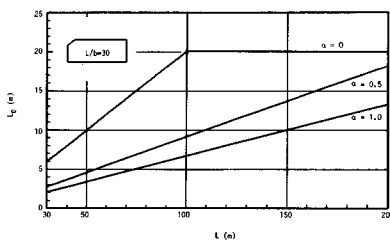


Fig. 24 Proposed Intervals of the Intermediate Diaphragms.

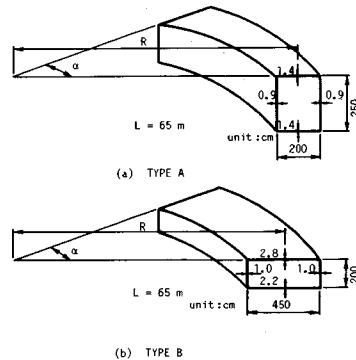


Fig. 25 Details of Box Girder for Design Examples.

Table 5 Required Numbers and Thicknesses of Intermediate Diaphragm, and Stress and Deflection Ratios.

	L (m)	$\alpha$ (rad.)	L/b	n	$t_D$ (cm)	$(\sigma_{dw}/\sigma_{\text{design}})$		$v_{dw}/L_D$	
						(1)	(2)	(1)	(2)
						TYPE A	65	0.65	32.50
		1.00	32.50	17	0.8(>0.118)	0.0335	0.0352	1/3260	1/5085
TYPE B	65	0.65	14.44	11	0.857	0.0368	0.0404	1/11230	1/16716
		1.00	14.44	13	0.8(>0.725)	0.0383	0.0412	1/6514	1/9952

Note : (1) = for R.C. bridge deck, (2) = for steel bridge deck.

and 1.0. Example TYPE B—Consider the steel-plated rectangular box girder with flat proportion shown schematically in Fig. 25(b). Given :  $L=65$  m ;  $L/b=14.44$  ;  $\alpha=0.65$  and 1.0.

For the types of A and B, the required numbers and thicknesses of the intermediate diaphragms proposed by Eqs. (13) are summarized in Table 5. Let us take a case of  $\alpha=0.65$  radian in TYPE A as an example.  $n=15$  shall be decided by  $\alpha=0.7$  ( $>0.65$ ) and  $L/b=40$  ( $>32.50$ ) in Table 4.  $t_d=0.134$  cm is evaluated by the required stiffness  $\mu=30$ , i. e.,  $K_D=810\,000 EI_{aw}/(nL^3)$ . So,  $t_a=0.8$  cm ( $>0.134$  cm) shall be provided. The ratio of the distortional warping stress  $\sigma_{aw}$  under the design loads to the design bending stress  $\sigma_{design}$  and the ratio of the distortional deflection  $v_{aw}=bV$  to the interval length of the intermediate diaphragm  $L_D$  are also tabulated in Table 5. It can be seen from the table that the  $\sigma_{aw}/\sigma_{design}$  for the provided diaphragms is within 5 % and the  $v_{aw}/L_D$  is under 1/1 000. From the results it could be mentioned that the spacing provision herein can provide adequate intermediate diaphragms to the steel-plated, curved box-bridge-girders with both kinds of cross-sectional proportions, i. e., high depth and flat depth.

## 6. CONCLUDING REMARKS

First of all the combined analytical result ; in which the bending and torsional warpings are evaluated by Vlasov theory<sup>8)</sup> and the distortional warping is mechanically modeled by Beam on Elastic Foundation (BEF) analogy, is verified experimentally by test runs. Then, distortional characteristics on steel-plated, curved box-bridge-girders are parametrically examined using the BEF analysis method.

Based on the parametric study, a spacing provision which could be readily utilized for intermediate diaphragms in steel-plated, curved bridges with box girders is developed. The proposed requirements are simple and suitable for direct use in practical design. The design examples show that the proposed provision can provide adequate spacing for the intermediate diaphragms which are limiting the distortional deformation and stress to the order of secondary magnitude for bridge structures, i. e., 1/1 000 of the interval-length of intermediate diaphragms for the secondary deformation and 5 % of the design bending stress for the secondary stress.

## ACKNOWLEDGMENTS

The authors are greatly indebted to Monorail-Preparatory Office in Okinawa Prefectural Government for many cooperations with planning experimental work. Sakurada Iron Works Co., Ltd., Tokyo Steel Rib & Bridge Const. Co., Ltd., Harumoto Iron Works Co., Ltd. and Miyaji Iron Works Co., Ltd. are greatly appreciated for providing the test model girders and apparatus used in the testing.

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(Received April 25 1988)