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NONSTATIONARY RANDOM RESPONSE OF HIGHWAY BRIDGES UNDER A SERIES OF MOVING VEHICLES

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Taking account of the roughness of roadway surface, the authors have theoretically investigated nonstationary random responses of highway bridges to moving vehicles. On the basis of the assumption that a sequence of the surface roughness is a stationary random process, the simultaneous nonstationary random vibrations of both bridge and moving vehicles have been analysed by means of the theory of random vibration. It can be considered that the two-degree-of-freedom sprung-mass system with both front and rear axles is a more realistic model of each heavy dump truck than the one-degree-of-freedom system usually used. The effect of some important factors in the whole system on the root mean square of the random responses of highway bridges are discussed.

Keywords: *Nonstationary random response, roadway roughness, moving vehicles, power spectral density, highway bridges*

1. INTRODUCTION

The statistical characteristics of the dynamic response of highway bridges to moving vehicles have been receiving considerable attention of many researchers[1)~11)]. A sequence of roughness of the roadway surface can be regarded as a stationary random process, while the dynamic response of the whole system treated herein shows nonstationary randomness. No exact analysis for such a phenomenon has been presented till quite recent, but some approximate solutions have been given by some researchers[1)~5)]. Though a simulation being on Monte Carlo method[6)~8)] is availed of for a direct solution, it is considered as an unrefined way. Skillful analytical methods taking both simultaneity and nonstationary randomness into account have recently been developed[9)~11)].

One of them[10)] was a powerful method for solving the coupled nonstationary random response of the highway bridge under a single moving vehicle that was developed by the authors on the basis of the random vibration theory[12)]. A spectral density of surface roughness directly measured on the real roadway could be effectively utilized as the input data without any modification in that method. It has been found in Ref. 10) that the two-degree-of-freedom sprung-mass system with front and rear axles was a more realistic model of a heavy dump truck than the one-degree-of-freedom system usually used.

Real highway bridges normally experience the influence of several vehicles except for the case of very short span. Dynamic effects of live loads on bridges can be estimated validly under the condition that several heavy trucks move on them. In this paper, a statistical approach for analysing the nonstationary

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random response of bridge to a single moving vehicle is extended to the case of the response to a series of moving vehicles.

To verify the rationality of the present approach, some numerical results are compared with those given by other authors. Then, the variation of the statistical characteristics of the nonstationary random response according to some major factors as to the whole system are investigated through numerical calculations.

2. FORMULATION OF THE PROBLEM

(1) Fundamental equations of motion

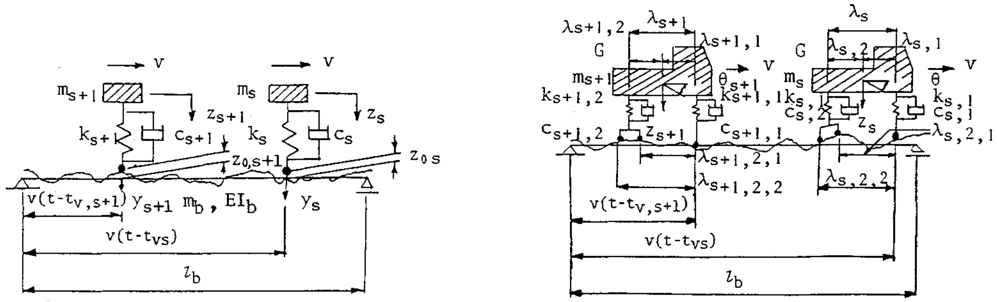
Fundamental equations of motion for a simple girder bridge subjected to loadings of a series of moving vehicles are firstly derived under the following assumptions :

i) A simple girder bridge is a plane system, having a certain cross section over the whole length.

ii) Each moving vehicle is either a one-degree-of-freedom damped sprung-mass system or a two-degree-of-freedom system which has front and rear axels, as shown in Fig. 1.

iii) A sequence of the roughness of roadway surface is a stationary random process.

The dynamic deflection $y(t, x)$ of the girder axis at an arbitrary point x and at any time t is expressed by the following equation, employing the well-known technique of modal analysis.



(a) For one-degree-of-freedom system of vehicle (b) For two-degree-of-freedom system of vehicle

Fig. 1 Analytical models of a bridge and moving vehicles system.

$$y(t, x)=\sum_i q_i(t)\,\phi_i(x) \qquad (1)$$

where $\phi_i(x)$ is the i-th mode of free vibration of the bridge and $q_i(t)$ is the generalized co-ordinate corresponding to ϕ_i. The simultaneous differential equations of motion for the bridge subjected to loadings of a series of moving vehicles, each of which is idealized as the one-degree-of-freedom system as shown in Fig. 1 (a), can be derived in the following form.

$$\left.\begin{array}{l}\ddot{q}_i(t)+2h_i\omega_i\dot{q}_i(t)+\omega_i^2 q_i(t)=\dfrac{1}{M_i}\displaystyle\sum_{j=1}^{h}\phi_i(x_{vj})\,m_j\{g-\ddot{z}_j(t)\} \quad (i=1, 2, \cdots\cdots, n)\\ \ddot{z}_j(t)+2h_j\omega_j\dot{z}_j(t)+\omega_j^2 z_j(t)=2h_j\omega_j\{\dot{y}_j(t)-\dot{z}_{0j}\}+\omega_j^2\{y_j(t)-z_{0j}\} \quad (j=1, 2, \cdots\cdots, h)\end{array}\right\} \qquad (2)$$

where ω_i is the i-th natural circular frequency of the undamped bridge system, h_i and M_i are the i-th damping constant and the i-th generalized mass of the bridge system, respectively, g is the acceleration of gravity, h is the number of moving vehicles, $x_{vj}=v(t-t_{vj})$ and the dot denotes the derivative with respect to time t. The other symbols are referred to Fig. 1 (a).

Assuming responses of $q_i(t)$ and $z_j(t)$ to be nonstationary processes with the mean values, $\bar{q}_i(t)$ and $\bar{z}_j(t)$, and with the centred random ones, $\tilde{q}_i(t)$ and $\tilde{z}_j(t)$, respectively,

$$q_i(t)=\bar{q}_i(t)+\tilde{q}_i(t), \quad z_j(t)=\bar{z}_j(t)+\tilde{z}_j(t) \qquad (3)$$

The dynamic deflections of $y(t, x)$ and $y_j(t)$ are also expressed in the similar forms as follows :

$$y(t, x)=\bar{y}(t, x)+\tilde{y}(t, x), \quad y_j(t)=\bar{y}_j(t)+\tilde{y}_j(t) \qquad (4)$$

where $\bar{y}(t,x)=\sum_i \bar{q}_i(t)\phi_i(x)$, $\tilde{y}(t,x)=\sum_i \tilde{q}_i(t)\phi_i(x)$

Substitution of Eqs. (3) and (4) into Eq. (2) formulates the differential equations for the deterministic mean values, $\bar{q}_i(t)$ and $\bar{z}_j(t)$, as well as those for the centred random ones, $\tilde{q}_i(t)$ and $\tilde{z}_j(t)$. The former system of equations, which represents the deterministic response of an ideal bridge with completely smooth roadway surface to a series of moving vehicles, can be easily solved[13]. The latter differential equations for the centred nonstationary random response can be written as follows :

$$\left.\begin{aligned}&\ddot{\tilde{q}}_i(t)+2h_i\omega_i\dot{\tilde{q}}_i(t)+\omega_i^2\tilde{q}_i(t)=-\frac{1}{M_i}\sum_{j=1}^{h}\phi_i(x_{vj})m_j\ddot{\tilde{z}}_j(t)\quad(i=1,2,\cdots\cdots,n)\\&\ddot{\tilde{z}}_j(t)+2h_j\omega_j\dot{\tilde{z}}_j(t)+\omega_j^2\tilde{z}_j(t)=2h_j\omega_j\left\{\sum_{g=1}^{n}\dot{\tilde{q}}_g(t)\phi_g(x_{vj})-\dot{z}_0(x_{vj})\right\}\\&\qquad+\omega_j^2\left\{\sum_{g=1}^{n}\tilde{q}_g(t)\phi_g(x_{vj})-z_0(x_{vj})\right\}\quad(j=1,2,\cdots\cdots,h)\end{aligned}\right\}\cdots\cdots\cdots\cdots(5)$$

Knowledges of the random differential equation are required for solving these equations.

(2) System of linear differential equations

The state vector $\boldsymbol{w}(t)$ associated with both bridge and moving vehicles as well as the external force vector $\boldsymbol{z}(t)$ are defined as follows :

$$\left.\begin{aligned}\boldsymbol{w}(t)=&\{\tilde{q}_1;\tilde{q}_2;\cdots\cdots;\tilde{q}_n;\dot{\tilde{q}}_1;\dot{\tilde{q}}_2;\cdots\cdots;\dot{\tilde{q}}_n;\tilde{z}_1(t-t_{v1});\dot{\tilde{z}}_1(t-t_{v1});\cdots\cdots;\\&\tilde{z}_h(t-t_{vh});\dot{\tilde{z}}(t-t_{vh})\}\\=&\{w_1;w_2;\cdots\cdots;w_n;w_{n+1};w_{n+2};\cdots\cdots;w_{2n};w_{2n+1};w_{2n+2};\cdots\cdots;w_{2n+2h-1};w_{2n+2h}\\\boldsymbol{z}(t)=&\{z_0[v(t-t_{v1})];\dot{z}_0[v(t-t_{v1})];\cdots\cdots;z_0[v(t-t_{vh})];\dot{z}_0[v(t-t_{vh})]\}\end{aligned}\right\}\cdots\cdots\cdots\cdots(6)$$

Using Eq. (6), the simultaneous equations of motion, Eq. (5), can be expressed in a matrix form.

$$\dot{\boldsymbol{w}}(t)=\boldsymbol{A}(t)\,\boldsymbol{w}(t)+\boldsymbol{B}(t)\,\boldsymbol{z}(t)\cdots\cdots(7)$$

Equation (7) is regarded as a linear differential equation, and has the following initial condition at time t_{v1}, t_{v2}, $\cdots$ and t_{vh}.

$$\left.\begin{aligned}&\boldsymbol{w}(t_{v1})=\boldsymbol{w}_{01}=\{0;\cdots\cdots;0;w_{2n+1};w_{2n+2};0;\cdots\cdots\cdots\cdots;0\}\\&\boldsymbol{w}(t_{v2})=\boldsymbol{w}_{02}=\{0;\cdots\cdots;0;w_{2n+1};w_{2n+2};w_{2n+3};w_{2n+4};0;\cdots\cdots;0\}\\&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,\\&\boldsymbol{w}(t_{vh})=\boldsymbol{w}_{0h}=\{0;\cdots\cdots;0;w_{2n+1};\cdots\cdots;w_{2n+2h}\}\end{aligned}\right\}\cdots\cdots\cdots\cdots(8)$$

According to the theory of differential equation[14], the solution of Eq. (7) can be given under the initial condition of Eq. (8) as follows :

$$\boldsymbol{w}(t)=\boldsymbol{\Phi}(t,t_0)\,\boldsymbol{w}_{0j}+\int_{t_0}^{t}\boldsymbol{\Phi}(t,\tau)\,\boldsymbol{B}(\tau)\,\boldsymbol{z}(\tau)\,d\tau\cdots\cdots(9)$$

where $\boldsymbol{\Phi}(t,\tau)$ is a transition matrix.

(3) Covariance matrix of response

The covariance matrix $\boldsymbol{R}_w(t_1,t_2)$ of the state vector $\boldsymbol{w}(t)$ can be written as follows :

$$\begin{aligned}\boldsymbol{R}_w(t_1,t_2)=&E[\boldsymbol{w}(t_1)\,\boldsymbol{w}^T(t_2)]\\=&\boldsymbol{\Phi}(t_1,t_0)\,\boldsymbol{W}_0\boldsymbol{\Phi}^T(t_2,t_0)\\&+\int_{t_0}^{t_1}\boldsymbol{\Phi}(t_1,\tau)\,\boldsymbol{B}(\tau)\,E[\boldsymbol{z}(\tau)\,\boldsymbol{w}_{0,k_2}^T]\,\boldsymbol{\Phi}^T(t_2,t_0)\,d\tau\\&+\int_{t_0}^{t_2}\boldsymbol{\Phi}(t_1,t_0)\,E[\boldsymbol{w}_{0,k_1}\boldsymbol{z}^T(s)]\,\boldsymbol{B}^T(s)\,\boldsymbol{\Phi}^T(t_2,s)\,ds\\&+\int_{t_0}^{t_1}\int_{t_0}^{t_2}\boldsymbol{\Phi}(t_1,\tau)\,\boldsymbol{B}(\tau)\,E[\boldsymbol{z}(\tau)\,\boldsymbol{z}^T(s)]\,\boldsymbol{B}^T(s)\,\boldsymbol{\Phi}^T(t_2,s)\,d\tau ds\cdots\cdots(10)\end{aligned}$$

where $\boldsymbol{W}_0=E[\boldsymbol{w}_{0,k_1}\boldsymbol{w}_{0,k_2}^T]$, E [ ] is the linear operator of the mean value and the superscript T denotes the transpose of a vector or a matrix.

The initial conditions for the bridge and each moving vehicle are assumed as follows :

i) The bridge remains at rest till the first vehicle enters its span.

ii) Each vehicle has been moving on the roadway having the statistically same surface roughness as those of the bridge roadway. Consequently, it can be expected that a stationary random vibration has been already produced in each vehicle before entering the span of the bridge.

Under the initial conditions mentioned above, the covariance matrix of the state vector can be given by the following equation based on the Wiener-Khinchine relations between the spectral density and the covariance of a stationary random process.

$$R_w(t_1,t_2)=E[w(t_1)w^T(t_2)]$$

$$=\Phi(t_1,t_0)\begin{bmatrix} 0 & 0 \\ 0 & \begin{matrix} E[w_{10}w_{10}^T] & & & 0 \\ & \ddots & & \\ & & E[w_{k0}w_{k0}^T] & \\ & & & \ddots \\ 0 & & & E[w_{h0}w_{h0}^T] \end{matrix} \end{bmatrix}\Phi^T(t_2,t_0)$$

$$+\int_{-\infty}^{\infty}\{H^*(t_1,\omega)_{\tau 1}\omega_{\tau 1,k}^{(T)}+\cdots\cdots+H^*(t_1,\omega)_{\tau k}\omega_{\tau k,k}^{(T)}\}\hat{S}_{z_0}\begin{bmatrix} 0 & \begin{matrix} H_1^T(t_{v1},\omega) & & 0 \\ & \ddots & \\ 0 & & H_k^T(t_{vk},\omega)\end{matrix}\end{bmatrix}d\omega\Phi^T(t_2,t_0)$$

$$+\Phi(t_1,t_0)\int_{-\infty}^{\infty}\begin{bmatrix} 0 \\ \begin{matrix} H_1(t_{v1},\omega) & & 0 \\ & \ddots & \\ 0 & & H_k(t_{vk},\omega)\end{matrix}\end{bmatrix}\hat{S}_{z_0}\{\omega_{k,s1}H^{*T}(t_2,\omega)_{s1}+\cdots\cdots+\omega_{k,sk}H^{*T}(t_2,\omega)_{sk}\}d\omega$$

$$+\int_{-\infty}^{\infty}\{H(t_1,\omega)_{\tau 1}[\omega_{\tau 1,s1}H^{*T}(t_2,\omega)_{s1}+\cdots+\omega_{\tau 1,sk}H^{*T}(t_2,\omega)_{sk}]+\cdots\cdots$$
$$+H(t_1,\omega)_{\tau k}[\omega_{\tau k,s_1}H^{*T}(t_2,\omega)_{s1}+\cdots+\omega_{\tau k,sk}H^{*T}(t_2,\omega)_{sk}]\}\hat{S}_{z_0}d\omega \quad \cdots\cdots (11)$$

where

$$\left.\begin{aligned} &H(t_1,\omega)_{\tau m}=\int_{t_{vm}}^{t_{vm+1}}\Phi_k(t_1,\tau)B(\tau)e^{-j\omega\tau}d\tau \\ &E[w_{k0}w_{k0}^T]=\int_{-\infty}^{\infty}H_k(t_{vk},\omega)\omega_{ekk}\hat{S}_{z0}H_k^{*T}(t_{vk},\omega)d\omega \\ &H_k(t_{vk},\omega)=\int_{-\infty+t_{vk}}^{t_{vk}}\Phi_k(t_{vk},\xi)B_k e^{-j\omega\xi}d\xi \\ &\hat{S}_{z_0}=\frac{1}{2\pi v}S_{z_0}\left(\frac{\omega}{2\pi v}\right) \\ &\omega_{epq}=\begin{pmatrix} 1 & j\omega \\ -j\omega & \omega^2\end{pmatrix}\exp\{j\omega(t_{vp}-t_{vq})\} \end{aligned}\right\} \cdots\cdots (12)$$

The superscript $*$ denotes the conjugate complex number and $j=\sqrt{-1}$. B_k and $\Phi_k(t_{vk},\xi)$ are a coefficient matrix for a external force vector and a transition matrix included in the equation for each vehicle moving on a rigid rough road, respectively.

From Eq. (11), the mean square values of the deflections of the bridge at an arbitrary point x can be expressed as follows :

$$R_y(t,t)=\sum_i\sum_k\phi_i(x)\phi_k(x)R_{q_iq_k}(t,t) \cdots\cdots (13)$$

where $R_{q_iq_k}(t,t)=E[\tilde{q}_i(t)\tilde{q}_k(t)]$

(4) Two-degree-of-freedom system model of vehicle

If each moving vehicle is idealized in a two-degree-of-freedom sprung-mass system with the front and rear axles, the motion of the vehicle can be completely described in terms of the vertical dispacement $z_j(t)$ of the centre of gravity G and the rotational angle $\theta_j(t)$ about it as shown in Fig. 1 (b). In this case, the simultaneous differential equations of motion for the system under consideration can be written for the centred random responses of $\tilde{q}_i(t)$, $\tilde{z}_j(t)$ and $\tilde{\theta}_j(t)$, as follows :

$$\left.\begin{array}{l}\ddot{\tilde{q}}_i(t)+2h_i\omega_i\dot{\tilde{q}}_i(t)+\omega_i^2\tilde{q}_i(t)=\dfrac{1}{M_i}\sum_{j=1}^{h}\sum_{s=1}^{2}\sum_{k=1}^{ax(s)}\phi_i(x_{jsk})\dfrac{1}{ax(s)}\tilde{v}_{js}(t)\quad(i=1,2,\cdots\cdots,n)\\ m_j\ddot{\tilde{z}}_j(t)+\sum_{s=1}^{2}\tilde{v}_{js}(t)=0,\quad m_jr_j^2\ddot{\tilde{\theta}}_j(t)-\sum_{s=1}^{2}(-1)^s\lambda_{js}\tilde{v}_{js}(t)=0\quad(j=1,2,\cdots\cdots,h)\end{array}\right\}\quad\cdots\cdots\cdots\cdots\cdots\ (14)$$

where

$$\left.\begin{array}{l}\tilde{v}_{js}(t)=k_{vjs}\left\{\tilde{z}_j-(-1)^s\lambda_{js}\tilde{\theta}_j-\dfrac{1}{ax(s)}\sum_{m=1}^{ax(s)}\tilde{y}_{vjsm}\right\}+c_{vjs}\left\{\dot{\tilde{z}}_j-(-1)^s\lambda_{js}\dot{\tilde{\theta}}_j-\dfrac{1}{ax(s)}\sum_{m=1}^{ax(s)}\dot{\tilde{y}}_{vjsm}\right\}\\ \tilde{y}_{vjsm}=\sum_g\tilde{q}_g(t)\,\phi_g(x_{jsm})-z_0(x_{jsm}),\quad x_{jsm}=v(t-t_{vj})-\lambda_{jsm}\end{array}\right\}\quad\cdots\cdots\ (15)$$

$m_jr_j^2$ is the mass moment of inertia of vehicle, and $ax(s)$ are the consecutive numbers of the front and rear axles ; for example, $ax(1)=1$ and $as(2)=2$ in a case shown in Fig. 1 (b).

Instead of Eq. (6), the state vector $w(t)$ and the external force vector $z(t)$ for the two-degree-of-freedom systems are defined as follows :

$$\left.\begin{array}{l}w(t)=\{\tilde{q}_1;\tilde{q}_2;\cdots;\tilde{q}_n;\dot{\tilde{q}}_1;\dot{\tilde{q}}_2;\cdots;\dot{\tilde{q}}_n;\tilde{z}_1;\dot{\tilde{z}}_1;\tilde{\theta}_1;\dot{\tilde{\theta}}_1;\cdots;\tilde{z}_h;\dot{\tilde{z}}_h;\tilde{\theta}_h;\dot{\tilde{\theta}}_h\}\\ z(t)=\{z_0[v(t-t_{v1})];z_0[v(t-t_{v1})-\lambda_{121}];z_0[v(t-t_{v1})-\lambda_{122}];\\ \qquad\dot{z}_0[v(t-t_{v1})];\dot{z}_0[v(t-t_{v1})-\lambda_{121}];\dot{z}_0[v(t-t_{v1})-\lambda_{122}];\\ \qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots;\\ \qquad z_0[v(t-t_{vh})];z_0[v(t-t_{vh})-\lambda_{h21}];z_0[v(t-t_{vh})-\lambda_{h22}];\\ \qquad\dot{z}_0[v(t-t_{vh})];\dot{z}_0[v(t-t_{vh})-\lambda_{h21}];\dot{z}_0[v(t-t_{vh})-\lambda_{h22}]\}\end{array}\right\}\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (16)$$

For the two-degree-of-freedom systems of vehicles, the linear differential equations of motions, its solution and the covariance matrix can be also expressed in the same forms as those shown in Sections (2) and (3), except that the expression of the fifth equation in Eq. (12) must be replaced by the following equation,

$$\omega_{epq}=\begin{pmatrix}\omega_{pq} & j\omega\omega_{pq}\\ -j\omega\omega_{pq} & \omega^2\omega_{pq}\end{pmatrix}\exp\{j\omega(t_{vp}-t_{vq})\}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (17)$$

where

$$\omega_{pq}=\begin{bmatrix}1 & \exp\left\{-j\dfrac{\omega}{v}\lambda_{q21}\right\} & \exp\left\{-j\dfrac{\omega}{v}\lambda_{q22}\right\}\\ \exp\left\{j\dfrac{\omega}{v}\lambda_{p21}\right\} & \exp\left\{j\dfrac{\omega}{v}(\lambda_{p21}-\lambda_{q21})\right\} & \exp\left\{j\dfrac{\omega}{v}(\lambda_{p21}-\lambda_{q22})\right\}\\ \exp\left\{j\dfrac{\omega}{v}\lambda_{p22}\right\} & \exp\left\{j\dfrac{\omega}{v}(\lambda_{p22}-\lambda_{q21})\right\} & \exp\left\{j\dfrac{\omega}{v}(\lambda_{p22}-\lambda_{q22})\right\}\end{bmatrix}\cdots\cdots\cdots\cdots\cdots\ (18)$$

3. NUMERICAL ANALYSIS

(1) Analytical model

The structural quantities of typical simple girder highway bridges used for numerical calculations are shown in Table 1[2]. These quantities are equivalent to the values per unit lane for the typical composite girder bridges designed in Japan. For bridge models, the damping constants are equally assumed to be 0.02.

Necessary physical quantities concerning the moving vehicles are also given in Table 2. The dimensions of the vehicle idealized in the two-degree-of-freedom system are shown in Fig. 2. These physical and geometrical values are fixed on the basis of heavy dump trucks available in Japan.

The power spectral densities of surface roughness adopted here are shown in Fig. 3. In this figure, the solid curve obtained by a formula, $S_{z_0}(\Omega)=A/(\Omega^2+a^2)$, indicates a power spectral density function, based on the original curve measured on Yasugawa Bridge of Meishin Highway just after completion[15]. In Fig. 3 the broken lines indicate the boundary of ISO estimate of roadway roughness[16].

Table 1 Structural quantities of highway bridges[2].

Span length	l_b (m)	40	50
Weight per unit length	m_b (t/m)	2.67	2.72
Moment of inertia	I_b (m⁴)	0.1124	0.2010
Fundamental natural frequency	f_1 (Hz)	2.94	2.45
Damping constant	h_1	0.02	0.02

Table 2 Physical quantities of moving vehicles.

Total weight	$m_j g$ (t)	20.0, 15.0 33.2, 24.2, 18.9 27.4, 26.7, 29.0
Natural frequency	f_j (Hz)	3.0
Damping constant	h_j	0.03
Vehicle speed	v (m/sec)	10.0
Headway	$v(t_{vj}-t_{v,j-1})$ (m)	14.0

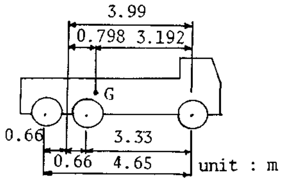


Fig. 2 Dimensions of vehicle.

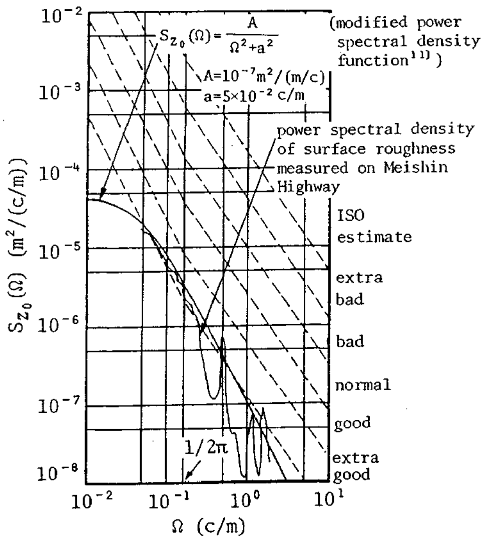


Fig. 3 Power spectral density of surface roughness.

As it is the main purpose of this paper to demonstrate the validity and availability of the present theory, only the fundamental mode of the bridge vibration is taken into account in the numerical calculations.

(2) Numerical computation

The transition matrices $\Phi(t, \tau)$ and $\Phi_k(t_{vk}, \xi)$ can be obtained in the way of a direct numerical integration of the linear differential equations. The step-by-step integration is carried out by means of Runge-Kutta-Gill method.

The covariance matrix $R_w(t_1, t_2)$ can be got by means of a numerical integration based on Simpson formula in the frequency domain. In the numerical integration, the frequency interval $\Delta\omega$ and the maximum frequency ω_{max} in the range of integration are determined on the basis of the confirmation that the covariance matrix can be obtained with the required accuracy in engineering.

(3) Variation of the root mean square of deflection in time domain

In order to illustrate the response of the bridge in detail, the root-mean-square values (RMS values) of the deflection at the span centre of simple girder bridges subjected to loadings of a series of moving vehicles are plotted against time as shown in Fig. 4 to Fig. 7.

a) Comparison with other analytical methods

The RMS values obtained by using several analytical methods such as the present theory, Okabayashi's method and the simulation technique by Monte Carlo method are together shown in Fig. 4. The RMS value given by the present theory is slightly greater than that calculated by Okabayashi's method and corresponds to the envelope curve of the RMS value by means of the simulation technique. From these results, the RMS value calculated by the present theory indicates the safety side estimate of the random response of bridges.

b) Effect of modification of the original spectrum

Fig. 5 facilitates the comparison between the RMS values of the deflection for the original and modified spectra of the surface roughness. The RMS value for the original spectrum is always smaller than that for the modified one. This result indicates that a modelling of the power spectral density of roadway roughness is an important factor in the random vibration analysis. This fact also shows the validity and usefulness of the present theory, in which the original spectral density of surface roughness measured on the real

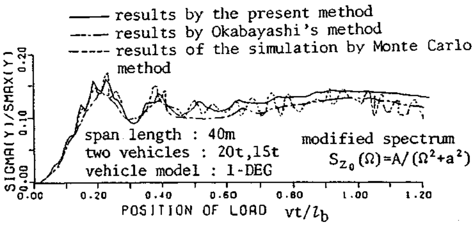


Fig. 4 Comparison of the present method with other analytical methods.

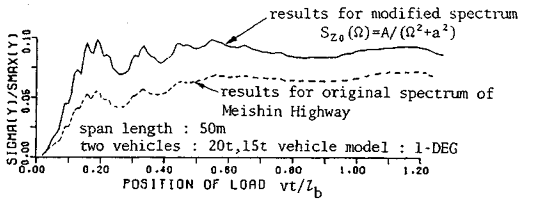


Fig. 5 Effect of modification of the original spectrum.

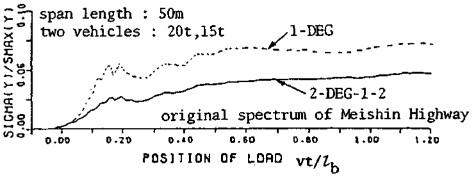


Fig. 6 Effect of modelling of vehicle.

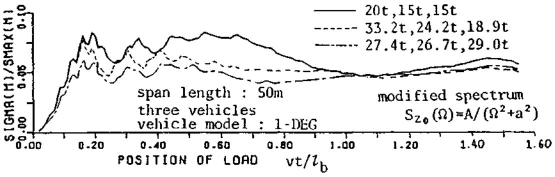


Fig. 7 Effect of the average weight of moving vehicles.

roadway can be used.

c) Effect of modelling of vehicle

The effects of modelling of vehicles on the RMS values of deflection are shown in Fig. 6. It should be noted that a great difference in the RMS values of deflection can be seen between two kinds of model of vehicle as shown in Fig. 6, where 1-DEG and 2-DEG-1-2 denote one-degree-of-freedom sprung-mass system and two-degree-of-freedom system with one front and two rear axles, respectively. Consequently, it is preferable that each heavy dump truck is idealized in the two-degree-of-freedom system having actual number of axles.

d) Effect of the average weight of moving vehicles

The RMS values of deflection for several series of three moving vehicles are shown in Fig. 7. One of them, 20t-15t-15t, is a series of moving vehicles equivalent to the design live loads L-20 in the Japanese Specification for Highway Bridges. As the examples corresponding to heavy trucks, the other two series are chosen from the measured data of road traffic flow[17]. The average weight values of the series of moving vehicles are 16.7t, 25.4t and 27.7t, respectively. The nondimensional RMS values increase gradually with decreasing average weight. After the first vehicle is running out from the bridge, that is, $vt/l_b>1.0$, the variation of the RMS values according to the average weight becomes small.

4. CONCLUSIONS

In the present paper, the simultaneous nonstationary random responses of highway bridges to a series of moving vehicles have been studied by means of an efficient method developed by the authors, which is based on the random vibration theory taking account of the roughness of roadway surface. The following conclusions are summarized from the results mentioned earlier.

(1) The RMS value obtained by the analytical method proposed herein was slightly greater than those given by Okabayashi's method and by Monte Carlo simulation method. The analytical results by the present theory indicated the safety side estimate of the random vibration.

(2) Numerical results showed both usefulness and superiority of the authors' method, in which the original spectral density of surface roughness determined by measurement on the real roadway can be used successfully.

(3) So long as the span length of bridge is not so large, each heavy dump truck may be idealized in the two-degree-of-freedom system having actual number of axles for the numerical analysis of the random vibration.

(4) The greater the average weight of moving vehicles, the smaller the nondimensional RMS values of random response of the bridge.

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