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OPTIMAL ALLOCATION OF THE SAFETY FACTORS IN A LONG-SPAN SUSPENSION BRIDGE DESIGN

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The optimal allocation of safety factors is discussed for long-span suspension bridge. Not only statistical uncertainties but also uncertainties due to unknown factors or gross errors such as human errors are taken into account. The optimal allocation of the safety factors for cables and girder in suspension bridge has been obtained by the simple reliability optimization of structural system. The results suggest to increase the safety factor of girders for wind load and to decrease the safety factor of cables for dead load. The current code appears to be not balanced and revision is suggested.
Keywords : safety factor, design code, suspension bridge

1. INTRODUCTION

In general, civil engineering structures consist of different components and have various failure modes. Some of the failure modes lead to the direct collapse of the whole structure, while others result in the partial structural failure because of, for example, redundancy. Hence the civil engineering structures are to be treated as structural systems. In the case when the safety of structural system is discussed, it is necessary to apply the system reliability analysis[1)~4)]. And the optimal allocation of safety factors or reliability levels for each component should be determined in full consideration of the following three factors ; namely initial construction cost, effects of the failure of each component on the total structural reliability, and the possibility of its repair.

Long-span suspension bridge is certainly one of the typical structural systems. Its superstructure is composed of towers, cables and stiffened girder. Consider the case that we design a long-span suspension bridge as the same scale as Akashi Strait Bridge in conformity to the current design specifications. In towers and cables, dead load effect exceeds 90 % of the total design load effect. On the other hand, the design of the principal members of stiffened girder is controlled by wind load, at least in Japanese practice. This is especially true for the truss type of girders because of the large wind force. Dead load has a very small variation under elaborate quality and construction controls. According to the investigation about the ratio of calculated dead load at the design stage to actual weight of superstructure[5)], mean value of this ratio is approximately 1.0 and its coefficient of variation is 0.7 % only. Design value of wind load has been determined taking account of the correction of span length, altitude of superstructure and gust response. For lack of statistical data used to determine the design wind velocity, however, there exists the

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possibility that a strong typhoon whose wind velocity exceeds the design value may hit the bridges[6]. Therefore the wind load and the performance of suspension bridge under wind action is very much uncertain. If we determine the safety levels of each component based on only statistical uncertainty, this observation leads to smaller safety factor for towers and cables and to larger safety factor for stiffened girder.

Consider the possibility of repair of each component when it is destroyed or damaged. Both breakdown of cable and failure of tower lead to the collapse of suspension bridge. And it is considered to be impossible to repair them. This collapse also causes considerably large loss related to social and economical effects. On the other hand, failure of the stiffened girder due to strong wind effect does not necessarily mean the collapse of the whole structure and its damage is easily repaired. These facts show that larger safety factor for towers and cables and smaller safety factor for the stiffened girder should be assigned. This situation regarding to safety factor does not agree with the one obtained from the point of view of statistical variation of design load.

In the current design specification for long-span suspension bridges[7], the following safety factors for cable, tower and stiffened girder are specified.

cable : primary load is dead load and the safety factor of 2.0 for the 0.7 % tensile strength is specified.
tower : primary load is dead load and the safety factor of 1.7 for yield and buckling is specified.
stiffened girder : primary load is wind load effect and the safety factor of 1.14 for yield and buckling is specified.

The value of 1.14 for girder is derived after consideration of 50 % increase of allowable stress for wind load. It is, however, considered questionable whether these values may be appropriate when we take account of the mutually contradictory situations mentioned above. It has been pointed out that the safety factor for cables of Akashi Strait Bridge is too high for us to design it economically[8].

Ref. 8) attempted to decrease the value of safety factor for cable by introducing Load and Resistance Factor Design Method. However this approach may not be proper because only the variations of loads acting on the cables are considered. In other words, the superstructure of suspension bridge is not treated as a structural system. Although the authors have already discussed about optimal allocation of safety levels of a long-span suspension bridge[9),10)], we do not consider the effect of uncertainties due to unknown factors and/or gross errors on structural reliability. The words "unknown factor" mean that whole of the dynamic and static characteristics of the structure has not been completely recognized yet. Because, for example, erosion of cables has not been well understood at present, we have to consider the effect of erosion of cables on the structural safety as unknown factor at design stage. Although the construction control for long-span suspension bridge might be well established, there might exist the possibility of non-skilled welding or a failure to tighten bolts. Therefore we have to take account of the uncertainties due to unknown factors and/or gross errors.

The purpose of this paper is to discuss the balance of safety factor allocation in a structural system from an economical point of view. Effect of uncertainties due to unknown factors and/or gross errors on the allocation is also taken into consideration. A long-span suspension bridge is used as an example. Only two components in the suspension bridge, namely cables and stiffened girder are investigated here. Tower is disregarded in this study for both the reason that tower is considered to have the same characteristics as cables in system reliability analysis and the reason of computation convenience.

2. OPTIMAL ALLOCATION OF SAFETY FACTORS

(1) Evaluation of total cost

Total cost C_T of civil engineering structure may be expressed by

$$C_T = C_I + P_F \cdot C_F \quad (1)$$

where C_I and C_F represent the construction plus maintenance cost and failure cost of the structure respectively[11], while P_F is the probability that structure reaches its limit state (s). In a similar manner as in Eq. (1), the total cost of each component is expressed as

$$C_{TC}=C_{IC}+P_{FC}\cdot C_{FC}, \qquad C_{TG}=C_{IG}+P_{FG}\cdot C_{FG}$$

in which subscripts 'C' and 'G' stand for the cables and girder, respectively. Then the total cost of the superstructure of suspension bridge is given by

$$\begin{aligned} C_T&=(C_{IC}+P_{FC}\cdot C_{FC})+(C_{IG}+P_{FG}\cdot C_{FG}) \\ &=(C_{IC}+C_{IG})+(P_{FC}\cdot C_{FC}+P_{FG}\cdot C_{FG}) \\ &=(C_{IC}+C_{IG})+C_{FG}(P_{FG}+P_{FC}\cdot \alpha) \end{aligned} \qquad (2)$$

where $\alpha=C_{FC}/C_{FG}$.

In order to find the optimal allocation of safety factors, it is assumed here that the sum of C_{IC} and C_{IG} keeps constant. Under this assumption, we only have to minimize the value of

$$(P_{FG}+P_{FC}\cdot\alpha) \qquad (3)$$

and we do not have to evaluate the absolute values of C_{FC} and C_{FG}. It should be noted that absolutely optimal values of safety factors can not be obtained under this assumption.

(2) Probabilistic model

It is well recognized that structural safety depends not only on statistical uncertainties but also on uncertainties due to unknown factors and/or gross errors such as human errors. Then the probabilistic model taking account of the effect of the latter uncertainties on structural reliability[12] is used in this study. Both the structural resistance R and the load effect S are treated here as random quantities which are log-normally distributed. When we consider both statistical uncertainties and uncertainties due to unknown factors and/or gross errors, the probability that the structure (or structural element) reaches the limit state is given as follows[12].

$$P_F=\mathrm{Prob}[R<S]=p\cdot P_{FU}+(1-p)\cdot P_{Fn}$$

in which

$$P_{FU}=\Phi[-\beta_n-\ln\phi/\sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}}]$$

$$P_{Fn}=\Phi(-\beta_n)$$

$$\beta_n=\{\ln(\overline{\theta}\sqrt{1+V_S^2}/\sqrt{1+V_R^2})\}/\sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}}$$

$\overline{\theta}=\overline{R}/\overline{S}$: central safety factor

$\overline{R}$: mean of R, V_R : coefficient of variation of R,

$\overline{S}$: mean of S, V_S : coefficient of variation of S,

$\Phi(\cdot)$: cumulative distribution function of the standard normal variable

'p' is the parameter which represents the probability of occurrence of structural resistance deterioration.

(3) Evaluation of initial cost

The following design format is assumed to be used :

$$R_d/\nu\geq S_d \qquad (4)$$

where R_d and S_d represent the design strength and design load effect respectively, while ν is the so-called safety factor accounting for the importance of the structure, social and economical effect caused by the failure of structure, and so on. In general, initial construction cost C_I increases with the increase of the safety factor ν. Regarding to the superstructure of steel bridge, the following function proposed in Ref. 13) is adopted.

$$C_I(\nu)=C_I(\nu_0)\{1+b(\nu/\nu_0-1)\} \qquad (5)$$

where ν_0 is the value of safety factor adopted in the current code and 'b' is a constant[12]. We also assume that Eq. (5) is applicable in case of cables and stiffened girder of long-span suspension bridge. Then the initial construction costs of both elements are given by

$$C_{IC}(\nu_C)=C_{IC}(\nu_{0C})[1+b_C(\nu_C/\nu_{0C}-1)] \qquad (6\cdot a)$$

$$C_{IG}(\nu_G)=C_{IG}(\nu_{0G})[1+b_G(\nu_G/\nu_{0G}-1)] \qquad (6\cdot b)$$

The increase of weight of girder generally leads to the increase of design sectional area of cable. However the rate of increase of the latter is extremely small[14].

We attempt here to estimate the reasonable range of parameters b_C and b_G from the actual design calculation used in the proposed Akashi Strait Bridge. Fig. 1 shows the relation between the tension of cable and dead load factor in the case that cables of Akashi Strait Bridge are designed by the following design format[15].

$\sigma_C/\gamma_m \geq (\gamma_D \cdot D + \gamma_L \cdot L + \gamma_T \cdot T)$

in which

σ_C : limit resistance of cable (=160 kgf/mm^2),

γ_m : resistance factor, γ_D : dead load factor,

γ_L : live load factor, γ_T : thermal effect factor,

D : dead load, L : live load, T : thermal effect

As mentioned before, dead load exceeds 90 % of the total design load effect. Hence it is acceptable to consider dead load factor to be equal to the safety factor included in Eq. (4). Furthermore it is reasonable to assume that the increase rate of cable tension is nearly the same as that of steel weight. The reason is that the relation between stress resultant and sectional area of cable is approximately linear. Under these observations we can obtain the values of 1.1~1.3 for parameter b_C from Fig. 1.

Table 1 indicates the relations among design wind velocity, safety factor for wind load and weight of stiffened girder in the design of Akashi Strait Bridge[14]. In this table, safety factors of 1.14 and 1.25 for design wind velocity of 43 m/s correspond to the following cases.

1.14 : case that the steel of SM 50 Y is used

1.25 : case that the steel of HT 70 is used

The values of b_G in Eq. (6·b) ranges 0.2 to 0.3 as shown in Table 1. The value of b_G is several times as small as that of b_C. The parameter b for superstructure of composite girder bridge takes the value between 0.4 and 0.6[13),16)].

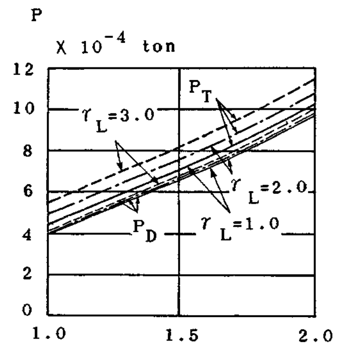

Fig. 1 Relations between dead load factor and tension of cable[15].
P_D : Tension of cable caused by dead load
P_T : Total tension of cables.

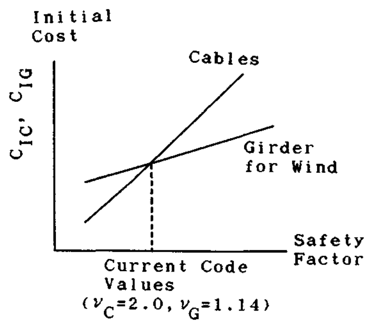

Fig. 2 Schematic diagram of the relation between initial construction cost and safety factor for cables and stiffened girder.

Table 1 Relations among design wind velocity, safety factor for wind load effect and weight ratio of stiffened girder[14].

Design wind velocity [m/s]	40	43	45
Safety factor for SM50Y	0.99	1.14	1.25
Safety factor for HT70	1.08	1.25	1.37
Weight ratio of stiffened girder	0.962	1.00	1.023

3. NUMERICAL RESULTS AND DISCUSSIONS

(1) Assumptions

Numerical computations are carried out in order to find the optimal set of safety factors ν_C and ν_G. On referring to these observations as well as the design calculation used in Akashi Strait Bridge[5),6),14),15)], the following assumptions and values of parameters are chosen in the calculations.

1) Breakdown of the cable leads to the collapse of the suspension bridge. Failure of the stiffened girder does not necessarily mean the collapse of whole structure and its damage is easily repaired. Considering this, it is assumed as the first step that $\alpha = C_{FC}/C_{FG} = 2$. It should be noted that the assumption of this value is very difficult at present.
2) Design of cables depends only on dead load. The coefficient of variation (C. O. V.) of dead load V_D is 0.1. According to Ref. 5), C. O. V. of dead load is 0.007, a extremely small value. However considering both the possibility of the change of loading condition after completion and the variation of superimposed dead load, C. O. V. of dead load is assumed 0.1.
3) Design of stiffened girder is controlled by wind load. C. O. V. of wind load V_W is 0.3. Although C. O. V. of annual maximum wind velocity is about 0.1, variation of data which lie around the design wind velocity is considerably large[6)]. Considering this we assume here that C. O. V. of wind load is 0.3. Furthermore we assume that V_W takes the value between 0.2 and 0.5 in order to grasp the influence of V_W on the results.
4) C. O. V. of resistance of cables and that of girder are both 0.1[17)].
5) According to the design calculations, the ratios of construction cost of cables and stiffened girder to that of superstructure are about 0.4 and 0.37, respectively. Therefore the ratio of initial construction cost of cables to that of stiffened girder is assumed to be 1.0.
6) The safety factor of 2.0 for the 0.7 % tensile strength of cable is used as the value adopted in the current specifications (i. e. ν_{0C}).
7) Taking account of 50 % increase of allowable stress for wind load, the safety factor of 1.14 (= 1.71/1.5) for yield and buckling caused by wind load effect is used as the value adopted in the current specification (i. e. ν_{0G}).
8) The values of parameters b_C and b_G are 1.2 and 0.25, respectively (see section 2. (3)).
9) All of the random variables, namely resistances of cables and stiffened girder, dead load effect and wind load effect are log-normally distributed[12)]. These assumptions are made for the calculation convenience.
10) The following characteristic values are adopted as the design values.

$$R_a = \overline{R} \cdot \exp[-k_R\sqrt{\ln(1+V_R^2)}]/\sqrt{1+V_R^2}$$
$$S_a = \overline{S} \cdot \exp[k_S\sqrt{\ln(1+V_S^2)}]/\sqrt{1+V_S^2}$$

Both of the parameters k_R and k_S, which represent the probabilistic level of design values, take the value of 1.28. The value of 1.28 means the safety-side 10 %-fractile.

A schematic diagram of the relations between initial construction cost and safety factor for both cables and stiffened girder based on the above assumptions is given in Fig. 2. From this figure, we can recognize that initial construction cost of cables is more sensitive to safety factor than that of girder.

(2) Results and discussions

The value of $(P_{FG} + P_{FC} \cdot \alpha)$ was calculated as the function of ν_C and ν_G as shown in Fig. 3(a). Fig. 3(b) presents the relations between safety factor and safety index for cables and girders. The following facts can be found.

a) The optimal safety factor $\nu_{C,opt}$ is noticeably smaller than $\nu_{G,opt}$. This means that larger safety factor for cables and smaller one for girder is not reasonable from the economical standpoint.
b) The optimal safety index of cables β_C corresponding to $\nu_{C,opt}$ is larger than or nearly equal to optimal

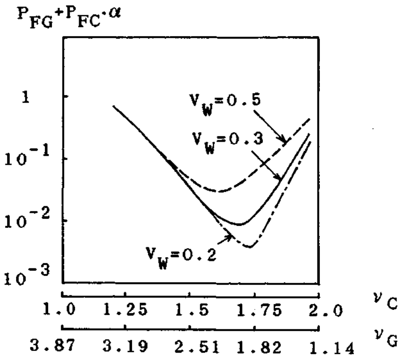

Fig. 3 (a) Safety factors ν_C and ν_G vs. $(P_{FG}+P_{FC}\cdot\alpha)$.

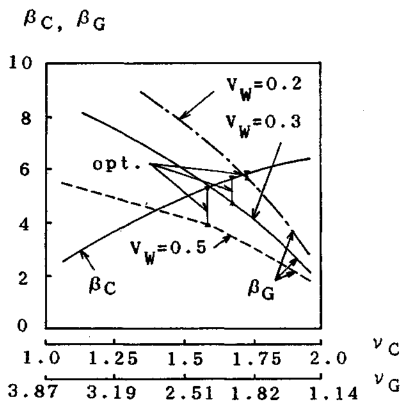

Fig. 3 (b) Safety factors ν_C and ν_G vs. safety indices β_C and β_G.

β_G corresponding to $\nu_{G,opt}$. In other words, the higher safety level for the cable and the lower for the stiffened girder is optimal.

c) The safety index β_C increases remarkably as the increase of ν_C, while β_G does not so.

d) As expected, the optimal ν_G becomes larger as the increase of the variation of wind load effect.

The allocation of ν_C and ν_G in the current design specifications for long-span suspension bridges contradicts with the above finding a).

Fig. 4 shows the influence of considering the uncertainties due to unknown factors and/or gross errors. In this figure, the abscissa is C. O. V. of wind load effect and the ordinate is optimal safety factor. The words 'Case A', and 'Case B' correspond to the following cases.

Case A : safety factor in case that uncertainties due to unknown factors and/or gross errors are considered

Case B : safety factor in case that uncertainties due to unknown factors and/or gross errors are not considered

Comparing Case A with Case B, the differences between the optimal safety factors and the current used values in the former case are smaller than those in the latter case. However $\nu_{G,opt}$ is always larger than $\nu_{C,opt}$. In other words, reconsideration of the selection of safety factors might be desirable even if we take account of the effect of uncertainties due to unknown factors and/or gross errors.

Next we discuss how the assumed parameter values influence on the optimal allocation of safety factors. Figs. 5 to 10 show the relations between optimal safety factors and parameters introduced in the probabilistic model. Each figure presents the following relations.

Fig. 5 : V_D vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

Fig. 6 : b_C vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

Fig. 7 : k_S vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

Fig. 8 : p vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

Fig. 9 : ϕ vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

Fig. 10 : α vs. $\nu_{C,opt}$ and $\nu_{G,opt}$

The definitions of parameters p and ϕ are mentioned in detail in Ref. 12). Considerably precise estimation of the parameters except for α may be possible by collecting enough data. On the other hand, it is very difficult to evaluate α at present. Therefore we consider here that α takes the value between 2 and 10^6.

From these figures, we can find that the optimal allocation of safety factors is not so sensitive to the parameters except for ϕ and α. And the same fact as above finding a) is recognized for the parameters V_D, b_C, k_S and p. In case that ϕ is less than 0.6, $\nu_{C,opt}$ becomes larger than $\nu_{G,opt}$. However there exist considerable differences between $\nu_{C,opt}$ and ν_{0C} and between $\nu_{G,opt}$ and ν_{0G}. In case that α becomes larger than about 30, $\nu_{C,opt}$ becomes larger than $\nu_{G,opt}$, too. And both $\nu_{C,opt}$ and $\nu_{G,opt}$ approach their current used values as the increase of α. When α takes the value of about 10^4, both optimal safety factors become equal to the current specified values. In other words, the allocation of safety factors adopted in the current design specifications for long-span suspension bridge becomes reasonable only when α is about 10^4. The value of 10^4 for α seems to be extremely large.

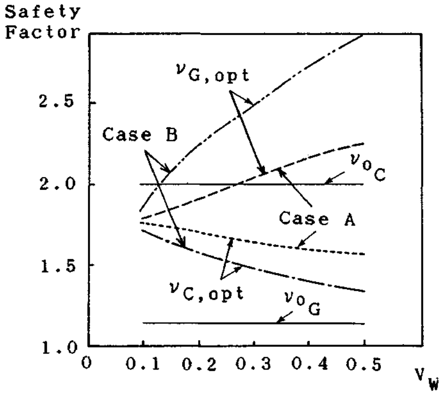

Fig. 4 Effect of uncertainties due to gross errors on the optimal safety factors.

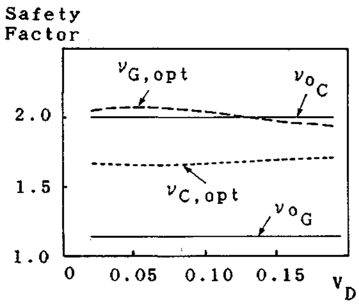

Fig. 5 Optimal safety factors vs. C. O. V. of dead load.

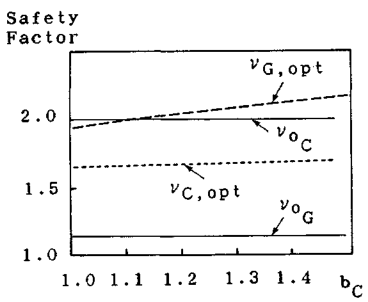

Fig. 6 Optimal safety factors vs. parameter b_C.

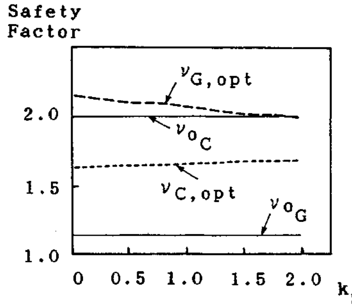

Fig. 7 Optimal safety factors vs. parameter k_S.

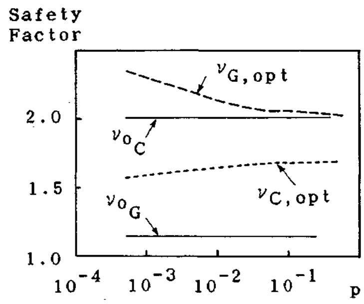

Fig. 8 Optimal safety factors vs. parameter p.

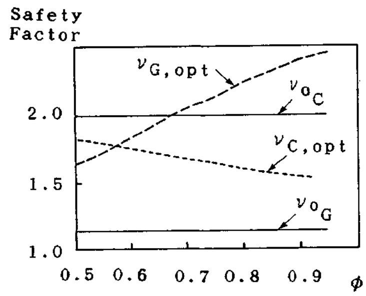

Fig. 9 Optimal safety factors vs. parameter ϕ.

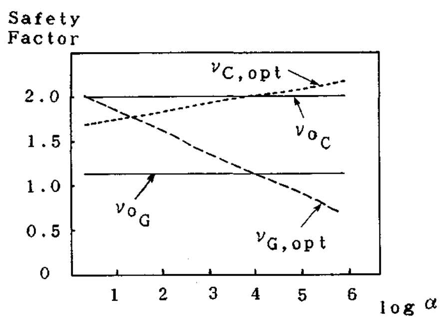

Fig. 10 Optimal safety factors vs. parameter α.

The numerical results obtained here are summarized as follows. Although it is necessary to select the parameter values more carefully, most of the numerical results suggest that reconsideration of the selection of safety factors might be desirable.

4. CONCLUDING REMARKS

Employing a long-span suspension bridge as a structural system example, the optimal allocation of the safety factors for different components was studied from an economical standpoint. The effect of uncertainties due to unknown factors and/or gross errors on structural reliability was also taken into account in this study. Erosion of cables is considered as one of unknown factors here because it has not been well known yet.

The results show that the numerically higher safety factor for the components subject to larger variable load effects is optimal. In other words, the higher safety factor for stiffened girder and the lower for the cable are optimal. It should be noted that the reliability level of cables is still higher than that of stiffened girder even if we adopt this allocation of safety factors. Although the parameter values need to be more carefully chosen and redundancy after reaching limit state is not considered here, the reconsideration of the safety factors in the current design specification for long-span suspension bridges is suggested from these findings.

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