

Invited Paper**STOCHASTIC ASPECTS OF DAM SAFETY ANALYSIS**

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A stochastic analysis is presented of the probability of overtopping of a dam. The discussion is based on the case of a dam for a small water storage reservoir which has recently been constructed in the Saar district in the FRG. The problem is first solved by means of a simulation method. However, it is possible to describe the result of the simulation method by means of a much simpler model which is based on a solution of the failure integral of Freudenthal for uncorrelated resistances and loads. It is shown that the actual safety of this dam against overtopping is extremely sensitive to both the operation rule for the reservoir, and the freeboard allowance. Some general conclusions are derived from this study for assisting in the ongoing discussion of dam safety.

Keywords: dam safety, failure probability, dam overtopping

1. INTRODUCTION

The recent failure of a small dam in the State of Baden-Württemberg of the Federal Republic of Germany has drawn attention to the fact that the present practice of assessing the safety of dams may need reconsideration. At present, the design engineer estimates the flood protection requirements for the downstream areas by allowing to store in the reservoir the volume of about the 100 year flood, and designing the spillway independent of the storage for a design flood. The practice in some countries (such as the USA) is to base the design flood for the spillway on a certain fraction of the maximum probable flood (pmf), whereas in other countries (such as the FRG) the design flood is the flood corresponding to a given exceedance probability P_E , which is usually expressed by a recurrence interval T (in years), with T the average time between flood events which exceed the design flood. The advantage of this method is that we need to know only the peak discharge of the flood with T year recurrence, which can often be determined from extreme value statistics. The safety of the dam then is improved by adding a freeboard, which primarily accounts for wind set up and wave run up, but which also serves the purpose of correcting for the uncertainty in the models used for the design, and the imperfections of the construction and the long term stability of the structures.

All these factors add to a safety of the dam which is satisfactory according to the experience of the engineers, but which usually is not quantified. If asked how safe a dam is, the engineer usually tends to think that a properly designed dam cannot fail. However, if pressed, he will only be able to state that the

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spillway was designed to be safe against the 1 000 year flood (in the FRG) -which implies that the failure rate should be about one failure every thousand years, or one failure every year among 1 000 dams. This certainly does not meet the safety requirements to be expected for large structures with high damage potential for which we tolerate failure probabilities of a minimum of 10^{-5} to 10^{-6} per year. In fact, the actual failure probability is much lower than 10^{-3} per year because of the reserves in safety which have been obtained by adding freeboard and other features. But the safety added by these measures cannot be expressed in terms of a probability. Because an increasingly safety-conscientious public wants to know the actual safety of the dams in terms of an expected probability of failure, it has been an objective of a research group at the University of Karlsruhe to develop methods by means of which the actual safety of a dam can be estimated. The general concept has been described by Meon et al. (1987). The first step in this program has been to develop a method for determining the probability of dam overtopping, which we shall call P_F in contrast to the exceedance probability P_E of the design flood. It is the purpose of this paper to describe our methods for the first step and to present some general conclusions derived from results obtained from a small reservoir in South West Germany.

The first result of this study was the development of a simple method based on the Freudenthal integral for determining P_F (Plate, 1984), which will be reviewed briefly in section 4. (1). However, it can be shown that the actual safety of the dam is extremely sensitive to the operation of the reservoir as reflected in the filling of the reservoir : a dam whose main purpose is to create potential energy for water power is much more prone to failure than a reservoir for irrigation or water supply, which was the case considered by Plate (1984), as is discussed in section 4. (2). Therefore, it seemed advisable to make a sensitivity study of the safety of an actual dam, in which both design and operational aspects are included. The best method of doing this appeared to be by means of a simulation model, which has been developed and applied by Meier (see Plate et al., 1985), and which will be reviewed in section 3.

The simulation model requires a large effort in data analysis, and it was therefore investigated if it is not possible to use the method of section 4. (1) in combination with findings from the simulation study. Such a combination is discussed in section 4. (3), and we show that this combined method could be used as a simple model for estimating the safety of a dam against overtopping. The paper will conclude with some remarks on the possibility of generalizing the results of the present study.

2. FORMULATION OF THE DAM SAFETY PROBLEM

The easiest way to formulate the problem of dam safety is by considering volumes (rather than discharges). Fig.1 is a schematic representation of a reservoir which is formed behind a dam. The different types of storage assigned by the design engineer are illustrated in Fig.1(a), S_{FBdes} is the freeboard storage not used by the design flood, S_{des} is the volume of the spillway design flood, which consists of two parts : the volume S_{STdes} retained in the basin, and the volume corresponding to the outflow $Q_{sdes}(t)$:

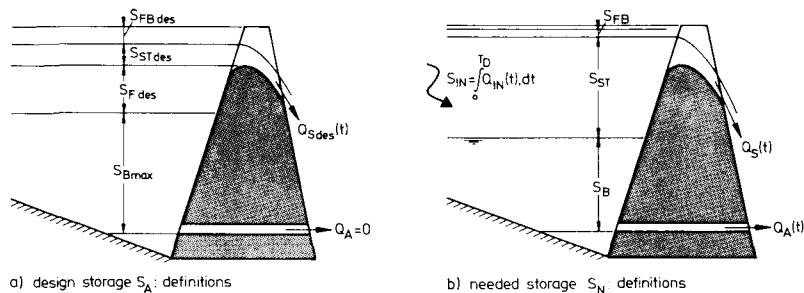


Fig.1 Definitions of dam storages.

$$S_{des} = S_{STdes} + \int_0^{T_D} Q_{sdes}(t) dt \dots\dots\dots (1)$$

where T_D is the duration of the design flood starting at $t=0$.

S_{Fdes} is the volume reserved for flood protection of the downstream area, and S_{Bmax} is the total storage available for serving the purpose of the reservoir, such as irrigation or water power. All these storages add up to the storage S_A which according to the design is available in the reservoir. From Fig. 1(a) one obtains :

$$S_A = S_{Bmax} + S_{Fdes} + S_{STdes} + S_{FBdes} \dots\dots\dots (2)$$

The available storage must be compared with the storage S_N needed for storing the inflowing flood volume. According to Fig. 1 (b) it consists of the part S_B which is already filled up before the beginning of the flood event, and which therefore is not available for flood storage, and the volume S_{ST} filled up by the extreme flood. In addition, there is the part S_{FB} of the freeboard that is taken up by other effects than the flood, such as uncertainty in the actual dam height, or the wind set up. We thus obtain :

$$S_N = S_B + S_{ST} + S_{FB} \dots\dots\dots (3)$$

which becomes, in terms of the flood volume S_{IN} of the extreme flood :

$$S_N = S_B + S_{IN} + S_{FB} - \int_0^{T_D} Q_{sdes}(t) dt - S_{HA} \dots\dots\dots (4)$$

where the flow volume S_{HA} is the volume in excess of the design flood volume which flows out of the reservoir during the duration of the extreme flood. S_{HA} consists of two contributions : one from the discharge difference $Q_s(t) - Q_{sdes}(t)$ between the actual flow over the spillway $Q_s(t)$ and the design discharge $Q_{sdes}(t)$ for the spillway. The other one results from the discharge $Q_A(t)$ through the bottom outlet :

$$S_{HA} = \int_0^{T_D} [Q_s(t) - Q_{sdes}(t)] dt + \int_0^{T_D} Q_A(t) dt \dots\dots\dots (5)$$

If failure is defined as the overtopping event, then failure occurs for the condition :

$$S_N > S_A \dots\dots\dots (6)$$

which after some rearranging yields the condition :

$$S_{IN} > (S_{Bmax} + S_{Fdes} + S_{FBdes} + S_{des}) - (S_B + S_{FB} - S_{HA}) \dots\dots\dots (7)$$

or :

$$S_{IN} > S_R \dots\dots\dots (8)$$

and :

$$S_R = S_{TOTdes} - (S_B + S_{FB} - S_{HA}) \dots\dots\dots (9)$$

where S_{TOTdes} is the total designed storage. Note that S_R has the following limits :

$$S_{Rmin} = S_{des} + DS, \quad \text{with } DS = S_{FBdes} - S_{FB} + S_{HA} \dots\dots\dots (10)$$

and :

$$S_{Rmax} = S_{TOTdes} + S_{HA} \sim S_{TOTdes} + DS \dots\dots\dots (11)$$

where the approximation $S_{Rmax} = S_{TOTdes} + DS$ is permissible because both S_{HA} and DS are very small in comparison to S_{TOTdes} .

Eq. (8) with Eq. (9) is well suited for stochastic analysis. In the terminology of reliability analysis, the quantity S_{IN} is the external load s , and the sum S_R of the terms on the right side of the inequality Eq. (8) is the resistance r , and the failure probability is given by :

$$P_F = P \{s > r\} = P \{S_{IN} > S_R\} \dots\dots\dots (12)$$

The probability P_F is governed by two natural random variables, as well as by the uncertainty of the design parameters. The random variables in Eq. (7) due to natural variability are the load s , i. e. the volume of the inflow S_{IN} , and the difference of S_{HA} and the freeboard S_{FB} needed by other sources than floods. These quantities are considered as stochastically independent -for example, an extensive investigation of the joint occurrence of strong winds and large floods has shown that there exists no correlation between these

effects, and we assume that other effects had occurred before the arrival of the flood, so that they are independent of the flood.

In addition to the natural variables there are the decision variables : S_B , which depends on the operation rule used for the reservoir, and S_{Fdes} which is the flood storage volume set aside for the protection of the downstream area. For simplicity, we shall consider only cases for which S_{Fdes} is set equal to zero, but it presents no problem to incorporate a flood storage volume S_{Fdes} into the analysis. Also, the uncertainty of the S_{HA} : -the gate may not be plugged, the flow over the spillway may depend both on the incoming flood and the used storage S_B -will be incorporated into the stochastic variable DS for the lower bound according to Eq. (10), and will be replaced by DS for the upper bound as described by Eq. (11). Other uncertainties shall not be considered.

With this analysis, the problem has been formulated, as shown schematically in Fig.2, and the next step is the determination of P_F . There are two different methods by which Eq. (12) can be solved. In principle, the easiest method is to use simulation techniques. However, the difficulty for obtaining P_F by simulation is the large amount of data required, which includes models for simulating all stochastic quantities. We find that the structure of the problem makes it easier to employ a method which involves the direct integration of the joint probability density $f_{sr}(s, r)$ for s and r , as shall be discussed in section 4.

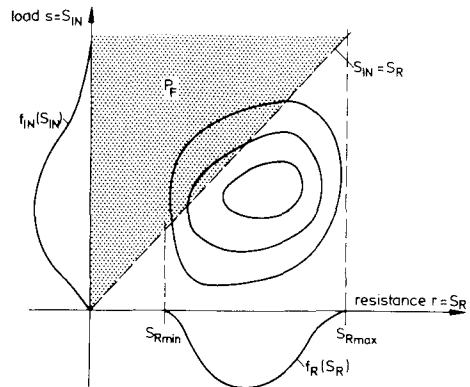


Fig.2 Defining the dam overtopping problem in probability space.

3. DETERMINATION OF P_F BY MEANS OF SIMULATION

A method which permits to determine P_F by means of simulation has been developed by Meier (Plate et al. 1985) and will be described briefly in this section. It was considered necessary to use a real world example. Accordingly, a small reservoir on the river Prims, which is a tributary of the Saar river in the Moselle region of the FRG, had been selected. The principal dimensions of the reservoir are listed in Table 1, the depth storage curve for the reservoir is shown in Fig.3.

The model consists of the following 4 major components :

- a. a generation model for the inflow into the reservoir,
- b. a model for the calculation of wind effects (wind set up, wave run up) during extreme floods,

S_{Fdes}	= 1.40 $M_{10} m^3$
S_{Bmax}	= 18.70 $M_{10} m^3$
S_{100} (S_{IN} for 100 year flood)	= 6.46 $M_{10} m^3$
$S_{des} = S_{1000}$ (S_{IN} for 1 000 year flood)	= 8.20 $M_{10} m^3$
S_{STdes}	= 1.10 $M_{10} m^3$
From S_{100} and S_{1000} : μ	= 1.320
S_0	= 2.971

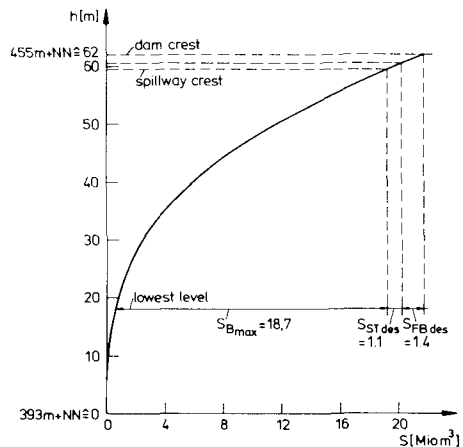


Fig.3 Depth storage curve for Prims reservoir.

- c. a model for the availability of the bottom outlet of the reservoir for releases during extreme floods,
- d. a model for the operation of the reservoir, by means of which the releases from the reservoir are calculated.

A generation model in which both flood storage and used part of the reservoir storage are to be simulated must consist of a continuum of all flows. The time scale must be that of the floods, which in this region is at most of a few days duration. Furthermore, since this flood must be routed through the reservoir, much shorter time increments are required, and it was found that a time step of one hour was required during flood conditions. Therefore, a two stage model was adopted. In a first step, the monthly inflow volumes were generated by using a modification of the Thomas-Fiering model (Fiering and Jackson, 1971, Schmidt and Treiber, 1980), in which the residuals were considered Weibull distributed monthly variates, and a small serial correlation was included between annual values. This model of monthly values was used in conjunction with an operation model, which was patterned after the actual use of the reservoir, to serve as a basis model for the storage in the reservoir. The monthly storage had been correlated with monthly maximum discharges through a linear regression plus a random term which accounts for natural variability. Whenever this monthly peak value exceeded a critical value (which was defined as the 50-year flood HQ_{50}), the model switched to a submodel with a time step of one hour. The disaggregation of the extreme flood volume was accomplished by means of a stochastic flood wave generation model. For these situations, the wind conditions were modelled also, and the maximum wave run up was generated. The water level is calculated by adding the wave run up to the depth calculated from the reservoir storage.

The probability of failure (=probability of dam overtopping) is calculated by routing the sequence of (observed or artificially generated) flow volumes through the reservoir according to prescribed operating rules, and by determining for every year if a failure occurs or not. The failure probability is then defined through a limiting process :

$$P_f = \lim_{n \rightarrow \infty} \frac{n_f}{n} \dots\dots\dots (13)$$

where n_f is the number of years in which failure occurred out of a total of n years. The number n has to be sufficiently large to obtain stable averages. The simulation technique can readily incorporate uncertainties, for example the uncertainty of having the flow $Q_A(t)$ available (the outlet may be plugged), or of the variability of the required freeboard, in drawing appropriate values for these quantities from suitable probability distributions.

The application of the method to the example of the Prims dam yielded a probability of failure P_f of about 10^{-4} , or a recurrence interval of 10 000 years, which clearly is not sufficient. It should be pointed out, that the actual safety of the dam is much higher, because the dam has been designed with a much larger spillway (for 90 m³/s) than the one used (30 m³/s) for the simulation. Also, the actual operation rule includes a safety storage S_{Faes} , which is ignored in this analysis.

4. SIMPLIFIED CALCULATION OF P_f FOR UNCORRELATED S_R AND S_{IN}

(1) Derivation of the failure integral

As had been argued before, resistance and load are uncorrelated in the case of dam overtopping, and therefore it is possible to write $f_{sr}(s, r) = f_s(s) \cdot f_r(r)$. Then P_f can be obtained by integrating $f_s(s) \cdot f_r(r)$ over the hatched region defined in Fig. 2. In this way Eq. (12) becomes (Ang and Tang, 1984) :

$$P_f = \int_{-\infty}^{\infty} \int_0^s f_s(s) f_r(r) dr ds \dots\dots\dots (14)$$

which can be integrated once to read (Freudenthal et al., 1966) :

$$P_f = \int_{-\infty}^{\infty} F_r(s) f_s(s) ds \text{ with } F_r(s) = \int_0^s f_r(r) dr \dots\dots\dots (15)$$

We shall consider this integral in this section. For clarity, we shall assume first, that DS is a known and

constant quantity. Then $s = S_{IN}$, and r is the random variable S_R with pdf $f_R(S_R)$ which varies only with S_B . In this case, Eq. (15) requires integration of the product of the functions shown schematically in Fig. 4, which yields :

$$P_F(DS) = \int_{-\infty}^{\infty} F_R(S_{IN}) f_{IN}(S_{IN}) dS_{IN} \dots \dots \dots (16)$$

where the argument DS in $P_F(DS)$ signifies that P_F is a conditional probability depending on the magnitude of the random variable DS , which we assume to follow a probability density distribution $f_{DS}(DS)$, which exists between DS_{min} and DS_{max} . The best estimate for the true probability of failure is then found by taking the expectation $E\{\cdot\}$ of $P_F(DS)$:

$$E\{P_F\} = \int_{-\infty}^{\infty} P_F(DS) f_{DS}(DS) dDS \dots \dots \dots (17)$$

which is a useful way of representing the uncertainty in DS and for taking it into account.

(2) Failure probability of an irrigation reservoir

The failure probability will depend on the mode of operation through the shape of the distribution function for S_B . When irrigation reservoirs are considered, one may assume that the reservoir is drawn down during the irrigation season, and filled up by the floods which occur in some random or seasonal fashion. An approximate method of solving Eq. (16) for the assessment of dam safety in such a case has been suggested by Plate (1984), who proposed to assume that the probability density $f_R(S_R)$ of the available storage is uniform over the range between maximum S_{Rmax} and minimum S_{Rmin} , i. e. :

$$f_R(S_R) = \frac{1}{S_{Rmax} - S_{Rmin}} \text{ for } S_{Rmin} < S_R < S_{Rmax} \dots \dots \dots (18)$$

Plate also showed some evidence that in the region of extreme floods, which are the only ones contributing to the failure probability, the probability density distribution $f_{IN}(S_{IN})$ could be represented by an exponential distribution :

$$f_{IN}(S_{IN}) = \mu \exp\{-\mu(S_{IN} - S_0)\} \dots \dots \dots (19)$$

where μ and S_0 are empirical constants found by fitting the exponential distribution to the extreme tail of the empirical (or fitted) extreme value distribution. They are determined for the case of the Prims reservoir from S_{100} and S_{1000} (see Table 1), which are the volumes of the 100-year and 1 000-year flood, respectively. With these results, the failure probability of Eq. (16) becomes after some transformations and integration :

$$P_F(DS) = P_F(0) \exp\{-\mu DS\} \text{ with } P_F(0) = \frac{\exp\{\mu(S_0 - S_{aes})\}}{\mu S_{Bmax}} [1 - \exp\{-\mu S_{Bmax}\}] \dots \dots \dots (20)$$

which is a first estimate useful for an assessment of the safety of a dam, and also, for the additional safety which can be obtained by adding a small amount to the freeboard. For this, the storage volume must be transformed into reservoir depth h through the depth storage curve of the reservoir (Fig. 3).

The probability distribution for DS is not known. As a first approximation, we can again make the assumption that DS is a uniformly distributed random variable between 0 and DS_{max} , where DS_{max} is likely of the order of S_{FBdes} . With this assumption, the expected value (Eq. (17)) becomes :

$$E\{P_F|DS_{max}\} = \frac{1}{DS_{max}} \int_0^{DS_{max}} P_F(DS) dDS \dots \dots \dots (21)$$

in which the notation $E\{P_F|DS_{max}\}$ implies an expected value as function of the upper limit of DS .

For the data of Table 1, P_F and $E\{P_F|DS_{max}\}$ have been calculated from Eq. (20), and from the combination of Eqs. (20) and (21), respectively. From them, the inverse $T(DS) = [P_F(DS)]^{-1}$ have been calculated and are plotted in Fig. 5 as functions of DS and of the corresponding height taken from the depth

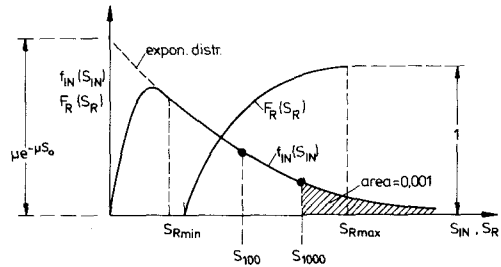


Fig. 4 Graphical display of functions for solving Eq. (16).

storage curve. Also, we calculated $T(DS_{max})=[E\{P_F|DS_{max}\}]^{-1}$. It is evident, that the failure probability is always lower than the value, which has been obtained by the simulation method, and which is supposed to be the best estimate, which can be obtained. The reason lies in the fact, that the uniform distribution has been used for $f_R(S_R)$, whereas the actual distribution is determined by the special operating rules for the reservoir under consideration. To improve the results, we have therefore incorporated some of the results of the simulation calculations into the model.

(3) Failure probability for improved distribution for S_B

The simplification introduced by using a uniform distribution for the sum of S_B and S_{FB} is not permissible in most cases. This distribution should be separated first of all into the two effects of S_B and DS . Furthermore, the probability distribution $F_R(S_R)$ of the available storage should be more realistic and reflect the operation rules used for the reservoir. This important statistical distribution can be derived from the simulation results. The model of Thomas and Fiering (1971) for streamflow generation used in conjunction with the reservoir operation rule, permits to determine the distribution $F_B(S_B)$ of the volumes in the reservoir, and thus of the reservoir levels at the beginning of each month. This generation model is rather conventionally employed for reservoir simulation, and its application without the complicated extreme flood generation subroutine can give a significant improvement for the determination of the failure probability P_F according to Eq. (16). The distribution $F_B(S_B)$ obtained from the simulation results of Meier (Plate et al., 1984) has been converted to the function $F_R(S_R)$ and is shown in Fig. 6. Because of the operation rule for the reservoir which required to keep the reservoir as full as possible for recreational purposes it is found, that the frequency of a flood occurring for a full reservoir is rather high : in fact, it is possible to fit an exponential curve to the probability distribution of the available storage S_R , which is given by the following relation :

$$F_R(S_R) = 1 - \exp\{-0.1965(S_R - 7.702)\} \dots (22)$$

which has been plotted in Fig. 6. Insertion of Eq. (19) and Eq. (22) into Eq. (16) yields :

$$P_F(DS) = \int_{S_{Rmin}}^{S_{Rmax}} [1 - \exp\{-0.1965(S_{IN} - 7.702)\}] [\mu \exp\{-\mu(S_{IN} - S_0)\}] dS_{IN} \dots (23)$$

and for the expected value $E\{P_F|DS_{max}\}$ we obtain :

$$E\{P_F|DS_{max}\} = \frac{1}{DS_{max}} \int_0^{DS_{max}} P_F(DS) dDS \dots (24)$$

Both the results of Eq. (23) and of Eq. (24) have

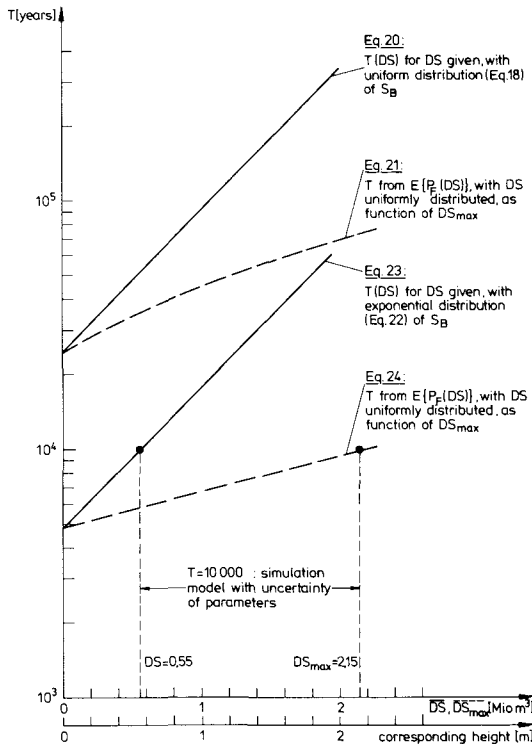


Fig. 5 Recurrence interval as function of DS and DS_{max} for Prims Reservoir.

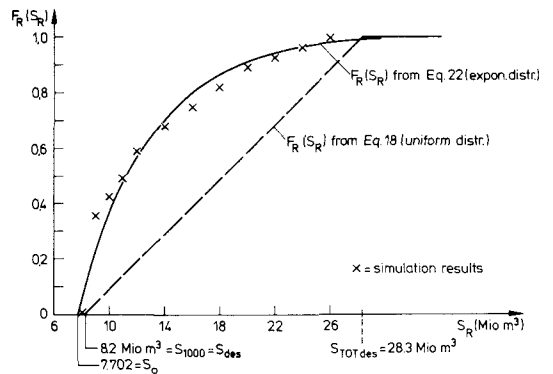


Fig. 6 Distribution $F_R(S_R)$ from simulation and fitted exponential, $DS=0$.

been plotted into Fig. 5. It is evident that the assumption $DS=0$ is too much on the safe side, but much closer to the simulation result of $T=10\ 000$ years than the result obtained for the uniformly distributed S_R . Agreement between the simulation result and Eq. (23) is found for a value of $DS=0.55 M_{10} \text{ m}^3$ (corresponding height taken from depth storage curve : 0.55 m), which is still unrealistically low. For Eq. (24) agreement is found for the expected value of DS corresponding to $DS_{\max}=2.15 M_{10} \text{ m}^3$ ($\sim 2.15 \text{ m}$). This value of DS_{\max} is very much in the range of about $1.5 S_{FBdes}$, which one would have assumed (without knowing the exact distribution of S_{HA} , or of the used part of the freebord S_{FB}).

5. CONCLUSIONS

The results which we have obtained give us some confidence to propose our method for general safety analysis of dams. It is simple to apply, because it requires only information which is usually obtained in the design phase of dams or for dams which have been operated for some time, from the operation records which usually are kept for each reservoir. The method is based on a combination of extreme value analysis of the incoming flood volumes - an analysis which is fairly standard practice - with the probability distribution of the storage in the reservoir. For the available storage, only the statistics of the storage in the reservoir is required, and this is obtained as a side result if simulation on monthly basis is used to determine optimum operation rules for the reservoir. The advantage of using the monthly time series of inflow volumes is that for the safety analysis the average distributions for the monthly inflow volumes are needed, and not short term volumes.

We shall continue to apply the method to other cases in order to obtain an indication if the method is generally useful, or can be improved to yield good results in general cases. The final objective is to have not only a design method, but also to provide a simple tool to assess the safety of existing dams.

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