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NONLINEAR DISCRETE STRUCTURAL OPTIMIZATION

By Hossain M. AMIR and Takashi HASEGAWA***

Structural design optimization will be more convenient to formulate the design problem with discrete variable than it would be if the variables were assumed to be continuous. In order to solve a structural design problem with discrete variable only, two completely different techniques, 1) Integer gradient direction, which is later supported by subsequential search interval technique and 2) Modified Rosenbrocks orthognalization techniques have hybridized. Rosenbrocks original procedure is a well established method to solve continuous variable optimization problem ; but to suit to discrete variable problem solution some modifications are needed and reported here. By this hybridizing most of the practical difficulties, usually, encountered in the discrete optimization can be overcome. Details of the techniques are discussed and by their combination a solution code has been generated. A constrained problem is first converted into a sequence of unconstrained problem by use of interior penalty function and then solved by the generated code. The efficiency of the generated code is revealed by solving several test problems.

Keywords : Integer Gradient Direction, IGD; Subsequential Search Interval, SSI unit neighborhood, NI(X) ; principal neighborhood, NP(X) ; resolution, discrete points

1. INTRODUCTION

A shortcoming of much structural optimization work has been the concentration on continuous variable problems. Frequently structural engineers in civil practice are confronted with only a limited set of discrete alternatives. Rolled steel beams are generally available in standard sizes, as are reinforcing bars, and both are used only in integral numbers. One popular approach to nonlinear discrete value programming problems in practice has been to treat the variables as continuous. Once the continuous optimum has been determined by some means, the usual practice is to select a feasible set of discrete variable values near the continuous optimum point. It is well known that this procedure can lead to a point which may or may not represent the discrete optimum ; also feasibility of the solution may be destroyed. Thus from a practical stand point it is more convenient to formulate the design problem with discrete variable than it would be if the variables were assumed to be continuous.

Several researchers have attempted to develop methods for discrete optimization of structures. Reiter[1] used modified gradient method for discrete variable problems where both the objective function and constraints are quadratic. Gisvold and Moe[2] adopted the continuous penalty function method by using the discretized penalty functions with modified weighting factors; Weinstein and Yu[3] also developed a generalized Lagrange multiplier approach using dynamic programming. Toakley[4] formulated the optimal plastic design of frame structures as a discrete problem in which the solution space consisted of the available standard sections and solved it by using a mixed integer-continuous programming algorithm. Lai and Achenbach[5] developed a direct search technique to solve discrete nonlinear optimization problems. Grierson et al.[6] solved discrete optimization problem by an iterative procedure using the generalized

* Student member of JSCE, M. of Eng., Dept. of Agr. Eng., Kyoto University, Kitashirakawa-Oiwakecho, Kyoto 606, Japan.
** Member of JSCE, Dr. Agr., Dept. of Agr. Eng., Professor, Kyoto University.

optimality criteria technique. In which a sensitivity analysis technique is employed at a given design to approximate the service and ultimate performance constraints as linear functions of the member sizing variable. Glankwhamdee[7] developed a method to solve unconstrained nonlinear discrete variable optimization problems using the concepts of integer gradient method. Liebman et al.[8]~[10] later used Glankwhamdee's method to solve the discrete structural optimization problems.

A reliable discrete search technique is not easily devised due to resolution ridge difficulties, and no identical procedure demonstrated to be reasonably reliable has appeared in the literature. A discrete variable optimization code must have the following properties : i) Each search direction contains discrete points in addition to the base point ; ii) When the first discrete point in the direction of movement starting from the base point overshoots the optimum, the search must be able to recover ; iii) The method must be able to identify a principal resolution valley in order to move away from a false local optimum or from a discrete local optimum to a better point.

To meet the all three requirements, the method presented here is divided into two main parts, namely, 1) Integer Gradient Direction method, IGD, which is supported by Subsequential Search Interval technique, SSI, (these two techniques are developed in ref. 7)), specially design to meet the i) and ii) requirements narrated before. These are reasonably efficient and is convenient, since it utilizes concepts developed for problems with continuous variables to solve problems with discrete variable. 3) Modified Rosenbrocks orthogonalization procedure, choose to meet the iii) requirements. In the following sections all three techniques are presented with three example problems.

2. SOME BASIC DEFINITIONS AND DESCRIPTIONS OF DIFFERENT TECHNIQUES

Following basic definitions will conveniently be employed throughout the paper. A discrete point, X, is defined as a node of the lattice of the existing values of independent variables. The problem functional values, $F(X)$, exist only at the discrete points. A principal axis is a coordinate axis. Resolution in the x_i direction is represented by Δx_i, which is the shortest distance between two discrete points on a line parallel to x_i axis. A unit neighborhood, $NI(X)$, of the discrete point, X, contains all points that differ from X, $(X=x_i,\ i=1, 2, 3, \cdots, n)$ by no more than $\pm\Delta x_i$. The principle neighborhood, $NP(X)$, of X, contains all points in the intersection of the unit neighborhood with the axes parallel to the coordinate axes, and centered at x. The normalized form of the search direction vector V, defines the unit direction vector, S, i.e., $S=V/\|V\|$. A relative direction vector, $DR=(dr_i,\ i=1, 2, \cdots, n)$ is an n-dimensional vector representing relative movement in each coordinate direction in which the smallest non zero movement is set to unity and the other elements are scaled accordingly. An integer gradient direction, GM, is defined as $GM=(gm_i,\ i=1, 2, 3, \cdots, n)$, where gm_i is the nearest integer value of dr_i.

(1) The integer gradient direction, IGD

Due to the discrete nature of the optimization problem, the objective function exists only at discrete points. The gradient of the function at any discrete points must be approximated by evaluating the functions at the discrete points in the principal neighborhood. Calculation of the approximated gradient at any discrete point (X), produces an n-dimensional vector V representing a search direction. The normalized form of V defines the unit direction vector, S. For minimization take $S=-S$.

Since the components of S are not in general integer valued, it is necessary to transform these vector elements to the integer valued. In this respect, 2nd part of Weighted Perpendicular method (WP) used by Reiter[1] can be used and it is as follows :

let $s^*=(\min|s_i|,\ i=1, 2, \cdots, n)$; (where s^* is the-non zero minimum of $|s_i|$ ······················ (1)

Calculate the relative gradient direction vector at (X),

$$DR(X)=dr(x_i)=\frac{s_i}{s^*} \quad i=1, 2, \cdots, n \tag{2}$$

The IGD vector at (X),

$$GM(X)=gm(x_i)=\begin{cases}[dr(x_i)+0.5] \text{ if } dr(x_i)\geqq 0 \\ [dr(x_i)+0.5] \text{ if } dr(x_i)\leqq 0\end{cases} \quad (i=1,\ 2,\ \cdots,\ n) \tag{3}$$

where $[\omega]$ denotes the largest integer not exceeding ω. $GM(X)$, generated is the approximated steepest descent direction in a discrete design space, and any improve point (XT) along $GM(X)$ can be generated from the Eq. (4), where Δx is a diagonal matrix of discrete step sizes in which the ith diagonal component is the step sizes of the ith design variable, λ is the optimal step length along $GM(X)$.

$$(XT)=(X)+\lambda\cdot\Delta x\cdot GM(X) \tag{4}$$

It is possible to modify most of the one dimensional search techniques designed for continuous variable problem to find out λ of Eq. (4). There are very few points inside the interval of interest along $GM(X)$, therefore, any of the established method of one dimensional search will be burden some instead. Hence, a modified one dimensional search is used which is very simple in nature and as follows.

Taking successively the values $\lambda=1,\ 2,\ 3,\ \cdots$, test the corresponding (XT) of Eq. (4), necessarily a discrete point for feasibility of (XT) and improvement of the objective function, $F(X)$. Eventually, either locate a improved feasible point or not. The 'λ' value where unimprovement or non feasibility noted, stop the one dimensional search and take the just previous λ value as the optimal step length.

(2) The subsequential search interval, SSI

The Integer Gradient Direction, $GM(X)$, is an approximated steepest descent direction in a discrete design space, therefore, may deviate from the true steepest descent direction. This could possibly cause a premature termination of search at a non optimal point or may overshoot the optimum, especially in the near optimum region. To over come this difficulty, The SSI, described by Glankwhamdee[7] in his Ph. D dissertation can be used. Physically, the SSI means a technique that can handle the points in the vicinity of (X) and the direction $GM(X)$ that do not fall precisely on the line of search. The SSI in a n-dimensional space, for (X) with respect to the 'IGD vector', $GM(X)$, is defined as an interval exclusively bounded by XL and XU,

where $XU=X+\Delta X.GM(X)$ and $XL=X-\Delta X.GM(X)$ ⋯⋯ (5)

let $GM(X)$ in its component form is $GM(X)=(gm_i,\ i=1,\ 2,\ 3,\ \cdots,\ n)$, the number of points, K, inside the subsequential search interval is determined by

$$K=2.[(\max|gm_i|,\ i=1,\ 2,\ 3,\ \cdots,\ n)]^{-2} \tag{6}$$

$$X(k)_i=X_i-I\langle(K/2-k+1)dr_i/dx\rangle \text{ for } i=1,\ 2,\ \cdots,\ n\ ;\ k=1,\ 2,\ \cdots,\ K/2 \tag{7}$$

$$X(k)_i=X_i-I\langle(k-K/2)dr_i/dx\rangle \text{ for } i=1,\ 2,\ \cdots,\ n\ ;\ k=K/2+1,\ K/2+2,\ \cdots,\ K \tag{8}$$

where $dx=\max(|dr_i|,\ i=1,\ 2,\ \cdots,\ n)$. Recall that $(X)=(x_i,\ i=1,\ 2,\ \cdots,\ n)$, the points of interest, i. e., points inside the SSI are $X(k)=[X(k)_i],\ i=1,\ 2,\ \cdots,\ n\ ;\ k=1,\ 2,\ 3,\ \cdots,\ K_j$ and can be computed by Eq. (7) for the lower half of the interval and Eq. (8) for the upper half. Where $I\langle\cdot\rangle$ denotes the nearest integer value of the arguments. The SSI in a two dimensional search space is illustrated in Fig. 1, in which points $X(1)$ and $X(2)$ are in the lower half and $X(3)$ and $X(4)$ are in the upper half of the interval. Use only the upper half of the interval if $GM(X)$ is irreversible ; use the entire interval if reversible.

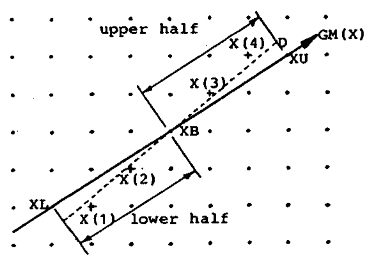


Fig. 1 The subsequential search interval. (after Glankwhamdee)

(3) Modified rosenbrocks orthogonalization procedure, ROP

In Rosenbrocks method[11], the coordinate system is rotated in each stage of minimization in such a manner that the first axis is oriented towards the locally estimated direction of the valley and all other axes are made mutually orthogonal and normal to first one. This criteria made the process to follow curved and steep valleys when encounter in a search process in the n-dimensional space. To suit to discrete variable problem, it needs some modifications and are described below for a function of n variables.

a) The set of $S_1^{(j)},\ S_2^{(j)},\ \cdots,\ S_n^{(j)}$ and the base point are known at the beginning of the j-th stage (usually, the coordinate direction, and the base point is the point extracted through IGD and SSI).

A step of length $\lambda_1=1$ is taken in the directions $S_1^{(j)}$ from the known base point. If the step is successful, λ_1 is multiplied by an integer constant α (usually $\alpha=2$), the new point is retained, and a success is recorded. If the step is a failure, λ_1 is multiplied by $-\beta$, an integer parameter, chosen in such a way that λ_1 will be -1, 2, -2, 3, -3, $\cdots$ in the successive cycles of the j-th stage. In this way, up to a certain distances, all points in both positive and negative sides of $S_1^{(j)}$ can be inspected. A failure is recorded.

b) Continue the search sequentially along the directions $S_1^{(j)}, S_2^{(j)}, \cdots, S_{n-1}^{(j)}, S_n^{(j)}, \cdots, S_1^{(j)}, S_2^{(j)}, \cdots$ until at least one step has been successful and one step has been failed in each of the n directions.

c) Compute the new set of directions $S_1^{(j+1)}, S_2^{(j+1)}, \cdots, S_n^{(j+1)}$ for use in the next or $(j+1)$th stage of minimization by using the Gram-Schmidt orthogonalization procedure. For this,

i) Compute a set of independent directions $P_1, P_2, \cdots, P_n$ as

$$\underset{(n\times n)}{P}=[P_1, P_2, \cdots, P_n]=\underset{(n\times n)}{[S_1^{(j)}, S_2^{(j)}, \cdots, S_n^{(j)}]}\begin{bmatrix} \nabla_1 & & & & \\ \nabla_2 & \nabla_2 & & 0 & \\ \vdots & & & & \\ \nabla_n & \nabla_n & \nabla_n & \cdots & \nabla_n \end{bmatrix}_{(n\times n)} \qquad (9)$$

where ∇_k is the algebraic sum of all the successful step lengths in the corresponding directions, $S_k^{(j)}$.

Set $Q_1=P_1$; and $S_1^{(j+1)}=Q_1/\|Q_1\|$ $\qquad (10)$

$$\left.\begin{array}{l} \text{Compute } Q_{i+1}=P_{i+1}-\sum_{m=1}^{i}[P_{i+1}^T\, S_m^{(j+1)}]S_m^{(j+1)} \quad (i=1, 2, \cdots, n-1) \\ \text{with } S_i^{(j+1)}=\dfrac{Q_i}{\|Q_i\|} \quad (i=1, 2, \cdots, n) \end{array}\right\} \qquad (11)$$

ii) The new search directions, $S_1^{(j+1)}, S_2^{(j+1)}, \cdots, S_n^{(j+1)}$, which are mutually orthogonal to each other are transformed to the integer directions by the same procedure described in section 2(1).

d) Take the point obtained in the j-th stage as the base point for the $(j+1)$-th stage, set the new iteration number as $(j+1)$, and repeat the procedure from step a) onwards.

e) Assume convergence either after completing a specified number of stages or after satisfying the conditions $\nabla_i=0$ for all i; for discrete variable problem ∇_i is either zero or a integer value.

Unlike continuous variable problem, any of ∇_i in discrete variable problem may become zero, which implemented any two of P_i of Eq. (9) to be identical. As a result, the process described above failed to provide orthogonal directions. To prevent this, the following procedure is used,

i) Suppose that in the j-th stage of Rosenbrocks method ∇_p and ∇_r are zero, then the order of the unit directions is rearranged from

$S_1^{(j)}, S_2^{(j)}, \cdots, S_p^{(j)}, \cdots, S_r^{(j)}, \cdots, S_n^{(j)}$ $\qquad (12)$

to,

$S_1^{(j)}, S_2^{(j)}, \cdots, S_n^{(j)}, S_p^{(j)}, S_r^{(j)}$ $\qquad (13)$

ii) Also change the (∇_i, $i=1, 2, \cdots, n$) arrangement of Eq. (9) accordingly,

iii) Orthogonalization operation and transformation to integer direction carried out as usual

iv) The new unit directions which are in the order of

$S_1^{(j+1)}, S_2^{(j+1)}, \cdots, S_n^{(j+1)}, S_p^{(j+1)}, S_r^{(j+1)}$ $\qquad (14)$

rearranged back to the original order,

$S_1^{(j+1)}, S_2^{(j+1)}, \cdots, S_p^{(j+1)}, \cdots, S_r^{(j+1)}, \cdots, S_n^{(j+1)}$ $\qquad (15)$

By the nature of the orthogonalization procedure, the directions $S_p^{(j+1)}$, and $S_r^{(j+1)}$ are the same as $S_p^{(j)}$ and $S_r^{(j)}$ and the first $n-2$ directions generated are mutually orthonormal with no component in the directions $S_p^{(j+1)}$ and $S_r^{(j+1)}$; therefore, the new search directions remain orthonormal.

3. DESCRIPTION OF THE OPTIMIZATION PROBLEM

In general, a constrained structural optimization problem subject to the behavior (stresses, displacements, etc.) constraints and side constraints can be expressed as :

Minimize $F(X)$ ········ (16)

subject to $g_i(X) \geqq 0 \quad i=1, 2, \cdots, m$ ········ (17)

$X \geqq 0$ ········ (18)

where, $X=$ the n-dimensional independent design variable vector; $g_i(X)$ are the constraints imposed upon the structural behavior and $F(X)$ is the objective function. Present paper aims to the discrete optimization, therefore, $F(X)$ and all constraint functions exist only at discrete points.

If the constraints $g_i(X)$ are explicit functions of the variables, x_i, it is possible to make a transformation of the independent variables such that the constraints are automatically satisfied. One way, by which constrained problems can be solved is converting them in to a sequence of unconstrained problems by use of interior penalty function. The problem formulation given in Eq. (16) through Eq. (18) becomes

$\min F(X, r) \equiv F(X) + rG(X)$ ········ (19)

where, $F(X, r)$=parametric objective function, called penalty function ; r=a positive constant known as the penalty parameter ; $G(X)$=some function of the constraints $g_i(X)$. The second term in the right hand side of Eq. (19) is called the penalty term. There are many popular form of $G(X)$, but the one initially introduced by Carroll[12] and frequently used by others is as follows :

$$G(X)=\sum_{i=1}^{m} \frac{1}{g_i(X)} \quad \cdots\cdots \quad (20)$$

It is necessary for the penalty function approach to reach a locally optimal solution, the penalty parameter, r_k, in Eq. (19) must assume a monotonicaly decreasing sequence of positive numbers, approaching zero in the limit, i. e., if r_k is the initial value, the subsequent value r_{k+1} will be

$r_{k+1} < r_k$ ········ (21)

Subsequent values of r are chosen according to the relation

$r_{k+1} = C \cdot r_k$ ········ (22)

where $C<1.0$. Usually, $C=0.1$ or 0.25 were found satisfactory for smaller and larger problems, respectively.

4. STRUCTURAL DESIGN

Proposed algorithm is applied to the design of steel building frames in which the beams and columns are prismatic and are made of standard sections. The constraints limit the maximum stresses due to the combination of bending moments and axial forces. Slenderness of the members can conveniently be taken into account but this has discarded in this paper. Structural analysis is carried out by the well known Stiffness Method[14].

(1) Objective function

The objective function which is minimized is taken to be the weight of the structures and is given by

$$F(X)=\sum_{i=1}^{NG} \rho_i A_i l_i \quad \cdots\cdots \quad (23)$$

where the subscript 'i' denotes the group number ; ρ_i is the weight density ; A_i is the cross sectional area, l_i is the sum of lengths of all elements belonging to group i ; NG is the total number of groups.

(2) Constraint functions

The constraint functions are formulated for a typical beam and column under different loading conditions in accordance with the AISC specifications[13]. There is one stress constraint for each member for each load ; thus $g_{ji}(x)$ indicates the stress constraint corresponding to the j-th member subjected to i-th loading conditions. A typical beam of a steel building frame subjected to a particular load conditions has three joint forces at each end of the beam, namely, axial force, P_a, vertical or shear force, P_v, and the bending moment, M. The governing moment $(M_{max})_{ji}$ is the largest of the three moments $(M_1)_{ji}$, $(M_2)_{ji}$ and $(M_m)_{ji}$; where, 1, 2 and m indicate the left end, right end and mid of the beam, respectively. The j-th constraints for the beam (neglecting axial forces) can be expressed as follows :

$$g_{ji}(x)\equiv 1-\frac{(M_{max})_{ji}}{(F_b)_j z_j}\geqq 0 \qquad (24)$$

in which z_j=the section modulus of beam j. For column, considering axial forces too

$$g_{ji}(x)\cong 1-\frac{(f_a)_{ji}}{(F_a)_j}-\frac{(f_b)_{ji}}{(F_b)_j}\geqq 0.0 \quad \text{when } \frac{(f_a)_{ji}}{(F_a)_j}\leqq 0.15 \qquad (25)$$

or

$$g_{ji}(x)\cong \min\left[1-\frac{(f_a)_{ji}}{(F_a)_j}-\frac{C_m (f_b)_{ji}}{\left(1-\frac{(f_a)_{ji}}{(F_e)_j}\right)F_b}\,;\,1-\frac{(f_a)_{ji}}{0.6\,F_y}-\frac{(f_b)_{ji}}{(F_b)_{ji}}\right]\geqq 0.0\,;\ \text{when } \frac{(f_a)_{ji}}{(F_a)_j}<0.15 \qquad (26)$$

in which

$(f_a)_{ji}$ and $(F_a)_j$=the computed and allowable axial stresses in member j under the loading condition 'i', respectively ;

$(f_b)_{ji}$ and $(F_b)_j$=the computed and allowable bending stresses in member 'j' under the loading condition 'i', respectively ; There are numerous ways of calculation of F_b. Here $F_b=0.66\,F_y$[13)]

$(F_e)_j$=the Eular critical stresses divided by factor of safety ; $F_e=\dfrac{12\,\pi^2 E}{23\,(K_f l/\gamma)^2}$

C_m=a coefficient applied to the bending term ; $C_m=0.85$;

F_y=the yield point stress.

$(F_a)_j$ and $(F_b)_j$ etc., may be increased one third above the values provided for gravity loads[13)].

Allowable stresses, (F_a) can be calculated by the relations given below[13)], [cf. Ref. 13)] ; also the effective length factor, K_f, for buckling, were decided according to ref. 13).

$$(F_a)_j=\frac{\left[1-\frac{(K_f l/\gamma)^2}{2\,C_c^2}\right]F_y}{F.S} \quad \text{when } \frac{(K_f l)}{\gamma}\leqq C_c \qquad (27)$$

$$(F_a)_j=\frac{12\,\pi^2 E}{23\left(\frac{K_f l}{\gamma}\right)^2} \quad \text{when } \frac{(K_f l)}{\gamma}>C_c \qquad (28)$$

$$F.S=\text{the factor of safety}=5/3+\frac{3\left(\frac{K_f l}{\gamma}\right)}{8\,C_c}-\frac{\left(\frac{K_f l}{\gamma}\right)^3}{8\,C_c^3} \qquad (29)$$

$$C_c=2\,\pi^2 E/F_y\,;\ \gamma=\text{radius of gyration}=(I/A)^{\frac{1}{2}} \qquad (30)$$

5. TOTAL PROPOSED ALGORITHM

Total algorithms may be summarized as follows :

I) Transform the original constrained optimization problem into the unconstrained optimization problem by use of interior penalty function. Select the penalty parameter, r_k (k=1 to start with), and the value of C, a parameter that reduce the value of r_k in the successive iteration.

II) Form the n-dimensional design space made up of the available standard sections (cf. Table 1) and select point, XS, let $XB=XS$.

III) Calculate the gradient approximation at (XB) and produce the IGD vector, $GM(XB)$.

IV) Search along $GM(XB)$ using the discrete one dimensional search of section 2(1). If there is an optimal step length ; produce the improved point (XT) from Eq. 4. Let $(XB)=(XT)$ and go to step III. If $\lambda=0$ go to step V.

V) Apply the SSI technique to locate an improved point. If a new point, (XT), can be located, test (XT) for optimality, if optimum go to step Ⅶ. If not optimum, let $(XB)=(XT)$; and go to step Ⅲ. Otherwise go to step Ⅵ.

Ⅵ) Use the modified ROP procedure, and denote the point obtained through ROP as the optimum point of the function $F(X, r_k)$.

Ⅶ) Change the value of r_k to r_{k+1} $(r_{k+1}=Cr_k)$; Start the new iteration, and form the new penalty function $F(X, r_{k+1})$ and go to step Ⅲ. Terminate the whole process when $F(X)\equiv F(X, r_k)$, or after a predetermined number of iteration.

6. EXAMPLE PROBLEMS

Example problems presented in this section are the specific of several problems designed by this approach, and three of those are reported. Design data common to all the example problems are as follows : For each element of the frame, the modulus of elasticity, E, the specific weight, ρ, and the yield point of stress, F_y, are 3×10^4 ksi, 0.2836 lbs./in^3, and 36.0 ksi, respectively. A total of 137 standard W sections is available to form the solution space and is listed in Table 1. Standard sections of the design space are numbered in the order of increasing cross sectional area, because objective function is directly dependent to the cross sectional area. There is no sidesway limitation to these problems and only stress constraints have been considered.

A computer code has been generated with the proposed algorithm and all calculations have been done on FACOM M-380/382 at the Data Processing Center, Kyoto University.

(1) Four-bay, one storey plane frame

Fig. 2 represents the dimensions and the loading conditions of the problem. There are three loading conditions, and nine members, and because of unsymmetrical frame, there will be nine design variables,

Table 1 Design space for the test problems.

X	Designation	A, in inch^2	I, in inch^4	Z, in inch^3	X	Designation	A, in inch^2	I, in inch^4	Z, in inch^3	X	Designation	A, in inch^2	I, in inch^4	Z, in inch^3
1	W6x8.5	2.51	14.8	5.1	47	W10x66	19.40	382.0	73.7	93	W27x145	42.70	5430.0	404.0
2	W8x10	2.96	30.8	7.8	48	W8x67	19.70	272.0	60.4	94	W36x150	44.20	9030.0	504.0
3	W10x11.5	3.39	52.0	10.5	49	W24x68	20.00	1820.0	153.0	95	W33x152	44.80	8160.0	487.0
4	W6x12	3.54	21.7	7.3	50	W18x70	20.60	841.0	129.0	96	W14x158	46.50	1900.0	253.0
5	W8x13	3.83	39.6	9.9	51	W16x71	20.90	841.0	116.0	97	W36x160	47.10	9760.0	542.0
6	W12x14	4.12	88.0	14.8	52	W12x72	21.20	597.0	97.5	98	W12x161	47.40	1540.0	222.0
7	W10x15	4.41	68.9	13.8	53	W21x73	21.50	1600.0	151.0	99	W14x167	49.10	2020.0	267.0
8	W6x15.5	4.56	30.1	10.0	54	W14x74	21.80	797.0	122.0	100	W36x170	50.00	10500.0	580.0
9	W6x16	4.72	31.7	10.2	55	W24x76	22.40	2100.0	176.0	101	W30x172	50.70	7910.0	530.0
10	W12x16.5	4.87	105.0	17.6	56	W18x77	22.70	1290.0	142.0	102	W14x176	51.70	2150.0	282.0
11	W10x17	4.99	81.9	16.2	57	W16x78	23.00	1050.0	128.0	103	W27x177	52.20	6740.0	494.0
12	W5x18.5	5.43	25.4	9.9	58	W12x79	23.20	663.0	107.0	104	W36x182	53.60	11300.0	622.0
13	W12x19	5.59	130.0	21.3	59	W21x82	24.20	1760.0	169.0	105	W14x184	54.10	2270.0	296.0
14	W8x20	5.89	69.4	17.0	60	W27x80	24.80	2830.0	212.0	106	W30x190	56.00	8850.0	587.0
15	W10x21	6.20	107.0	21.5	61	W18x85	25.00	1440.0	157.0	107	W14x193	56.70	2400.0	310.0
16	W14x22	6.49	198.0	28.9	62	W14x87	25.60	967.0	138.0	108	W36x194	57.20	12100.0	665.0
17	W8x24	7.06	82.5	20.8	63	W16x88	25.90	1220.0	151.0	109	W33x200	58.90	11100.0	671.0
18	W10x25	7.37	133.0	26.5	64	W10x89	26.20	542.0	99.7	110	W14x202	59.40	2540.0	325.0
19	W16x26	7.67	300.0	38.3	65	W12x92	27.10	789.0	125.0	111	W30x210	61.90	9890.0	651.0
20	W12x27	7.95	204.0	34.2	66	W27x94	27.70	3270.0	243.0	112	W14x211	62.10	2670.0	339.0
21	W8x28	8.23	97.8	24.3	67	W14x95	27.90	1060.0	151.0	113	W14x219	64.40	2800.0	353.0
22	W10x29	8.54	158.0	30.8	68	W21x96	28.30	2100.0	198.0	114	W33x220	64.80	12300.0	742.0
23	W14x30	8.83	290.0	41.9	69	W30x99	29.10	4000.0	270.0	115	W14x228	67.10	2940.0	368.0
24	W16x31	9.13	374.0	47.2	70	W24x100	29.50	3000.0	250.0	116	W36x230	67.70	15000.0	837.0
25	W10x33	9.71	171.0	35.0	71	W27x102	30.00	3610.0	267.0	117	W14x237	69.70	3080.0	382.0
26	W14x34	10.00	340.0	48.6	72	W14x103	30.30	3610.0	164.0	118	W33x240	70.60	13600.0	813.0
27	W18x35	10.30	513.0	57.9	73	W18x105	30.90	1850.0	202.0	119	W36x245	72.10	16100.0	894.0
28	W16x36	10.60	447.0	56.5	74	W12x106	31.20	931.0	145.0	120	W14x246	72.30	3230.0	397.0
29	W14x38	11.20	386.0	54.7	75	W30x108	31.80	4470.0	300.0	121	W36x260	76.50	17300.0	952.0
30	W10x39	11.50	210.0	42.2	76	W24x100	32.50	3330.0	276.0	122	W14x264	77.60	3530.0	427.0
31	W18x40	11.80	612.0	68.4	77	W14x111	31.70	1270.0	300.0	123	W36x280	82.40	18900.0	1030.0
32	W14x43	12.60	429.0	62.7	78	W21x112	33.00	2620.0	250.0	124	W14x287	84.40	3910.0	465.0
33	W21x44	13.00	843.0	81.6	79	W27x114	33.60	4090.0	300.0	125	W36x300	88.30	20300.0	1110.0
34	W18x45	13.20	705.0	79.0	80	W30x116	34.20	3930.0	329.0	126	W14x314	92.30	4400.0	512.0
35	W14x48	14.10	485.0	70.2	81	W33x118	34.80	5900.0	359.0	127	W14x320	94.10	4140.0	493.0
36	W21x49	14.40	971.0	93.3	82	W14x119	35.00	1370.0	189.0	128	W14x342	101.00	4910.0	559.0
37	W18x50	14.70	802.0	89.1	83	W24x120	36.00	650.0	300.0	129	W14x370	109.00	5450.0	608.0
38	W14x53	15.60	542.0	77.8	84	W40x124	36.50	5360.0	355.0	130	W14x398	117.00	6010.0	657.0
39	W10x54	15.90	306.0	60.4	85	W21x127	37.40	3020.0	284.0	131	W14x426	125.00	6610.0	707.0
40	W24x55	16.20	1340.0	114.0	86	W33x130	38.30	6710.0	406.0	132	W14x455	134.00	7220.0	758.0
41	W16x58	17.10	748.0	94.4	87	W30x132	38.90	5760.0	380.0	133	W14x500	147.00	8250.0	840.0
42	W18x60	17.70	986.0	108.0	88	W12x133	39.10	1220.0	183.0	134	W14x550	162.00	9540.0	933.0
43	W24x61	18.00	1450.0	130.0	89	W36x135	39.80	7820.0	440.0	135	W14x605	178.00	10900.0	1040.0
44	W21x62	18.30	1330.0	127.0	90	W14x136	40.00	1590.0	216.0	136	W14x665	196.00	12500.0	1150.0
45	W18x64	18.90	1050.0	118.0	91	W33x141	41.60	7460.0	448.0	137	W14x730	215.00	14400.0	1280.0
46	W12x65	19.10	533.0	88.0	92	W21x142	41.80	3410.0	317.0					

Note: 1 in. = 25.44 mm; 1 in^2 = 645 mm^2; 1 in^3 = 16400 mm^3; 1 in^4 = 416200 mm^4.

and nine constraints for each of the three loading conditions. Each of the three loading conditions is treated as the independent alternate loads acting on the frame, therefore, there are total of 27 constraints in this problem. The effective length factors, K_y, for all columns of this problem are assumed to be 2.5.

Results : The solution has started from the initial base point given in Eq. (31). The convergence parameter, $C=0.25$ is used to reduce the penalty parameter, r_k, sequentially. The Penalty Function, $PF(X,\ r)$, and the objective Function, $F(X)$, values at the initial design is 111 923.687 lbs. and 67 613.0 lbs., respectively; with $r_1=4\,550.0$. Using one sided gradient approximation in calculating the IGD vector. The optimal solution is reached when $r_{12}=0.00108$, after 12 iterations and $PF(X,\ r)^*=$ 12 430.22 lbs. and $F(X)^*=12\,430.01$ lbs. Total calculation time, CPU=14.49 sec.

$$(X)_1=\begin{bmatrix}125\\ \vdots\\ 125\end{bmatrix}=\begin{bmatrix}W36\times300\\ \vdots\\ W36\times300\end{bmatrix}_{(1\times9)} \qquad\cdots\cdots (31)$$

A summary of the results is given in Table 2, and Fig. 3 shows the profiles of values of penalty function, $PF(X,\ r)$ and objective function, $F(X)$, at each iteration of the search.

The optimal solution is also shown in Fig. 4. In Ref. 10), the same problem has also been solved by a solution code mainly made of IGD and the results reported is 14 375.0 lbs. with no mentioned of CPU time.

(2) One-bay, two-storey plane frame

This problem has been taken from Ref. 15); and the dimensions and loading conditions are as shown in Fig. 5. The frame has designed under two loading conditions, namely, a) uniformly distributed loads of 0.5 kips/in on elements 2, 4, 5, 7; and b) point loads of 45 kips each at node 2 and 3; and are treated as the

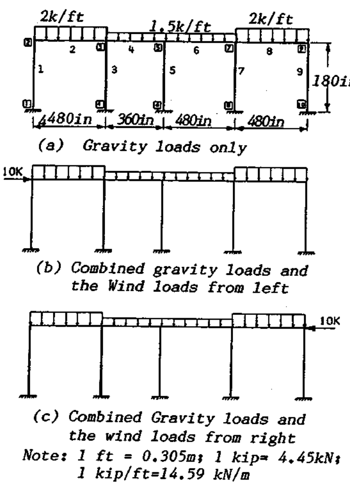


Fig. 2 Four bay, One storey plane frame.

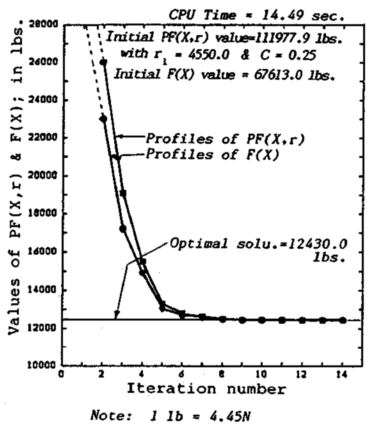


Fig. 3 Profiles of $PF(X,\ r)$ and $F(X)$ function.

Table 2 Results for 4-bay, 1-storey frame.

Iteration (k)	Value of r_k	Starting $F(X)_k$ in (lb.)	Starting $F(X,r)_k$ in (lb.)	Optimum $F(X)^*$ in (lb.)	Optimum $F(X,r)^*$ in(lb.)
1	4.55×10^{3}	67613.0	111923.7	38257.1	86171.9
2	1.14×10^{3}	38257.1	50235.8	26990.8	41199.8
3	2.84×10^{2}	26990.0	30543.0	17228.7	24624.7
4	7.11×10^{1}	17228.7	19077.7	14919.6	17177.2
5	1.77×10^{1}	14919.6	15484.0	13051.3	13935.2
6	4.44	13501.3	13277.0	12700.6	13111.8
7	1.11	12700.6	12803.0	12595.9	12741.3
8	2.77×10^{-1}	12595.9	12632.0	12489.7	12536.5
9	6.9×10^{-2}	12489.7	12501.4	12430.0	12447.9
10	1.7×10^{-2}	12430.0	12434.5	12430.0	12434.5
11	4.34×10^{-3}	12430.0	12431.1	12430.0	12431.1
12	1.08×10^{-3}	12430.0	12430.2	12430.0	12430.2
Reduction factor, C= 0.25; CPU Time =14.492 sec. Note: 1 lb. = 4.45N					

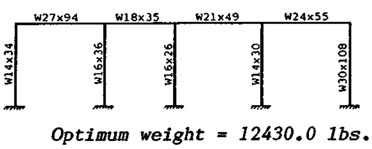


Fig. 4 Optimal solutions.

independent alternate loads acting on the frame. The frame is considered as a symmetrical one, with elements 1 & 8, 2 & 7, 3 & 6 and 4 & 5 form the group 1, 2, 3 and 4, respectively. Thus, there are 4 design variables, one for each groups, and 8 constraints, one for each elements of the structure under each loading conditions. Altogether, there are 16 constraints in this problem. $K_f=2.0$, for all columns.

$$(X)_1=\begin{bmatrix}100\\ \vdots \\ 100\end{bmatrix}=\begin{bmatrix}W36\times170\\ \vdots \\ W36\times170\end{bmatrix}_{(1\times4)} \qquad (32)$$

Results : The solution has started from the initial base point given in Eq. (32). At $(X)_1$, the $PF(X, r)$ and $F(X)$ values are 42 394.714 lbs. and 17 015 lbs., respectively; with $r_1=2\,450.0$. The optimal solution is reached when $r_{12}=0.000584$ after 12 iterations and is $PF(X, r)^*=6\,275.7$ lbs. and $F(X)^*=6\,275.51$ lbs. CPU time=5.275 sec. Profiles of values of $PF(X, r)$ and $F(X)$, at each iteration is shown in **Fig. 6**; and a summary of the results is given in **Table 3**. The optimal solution is also shown in **Fig. 7**.

In ref. 15, it is solved by using different solution codes namely, CONMIN, OPTDYN, LINRM, M-3, M-4, M-5, etc., based on different theory and algorithms; and solved as a continuous variable problem. Best optimum result reported is 6 460.0 lbs. with CPU time of 148.0 sec. under M-3 code.

Compare to these results, present code shows sufficient improvements, though there is slight discrimination in the constraint equations formulation and use of different computer.

(3) Two-bay, six-storey plane frame

The dimensions of the problem is as shown in **Fig. 8**. This problem has also been taken from Ref. 15); and is designed for two loading conditions : a) Uniformly distributed loads of 4.0 kips/ft on elements 1, 7, 11, 17, 21 and 27, and 1.0 kips/ft on elements 2, 6, 12, 16, 22 and 26; b) uniformly distributed loads of 1 kips/ft on elements 1, 2, 6, 7, 11, 12, 16, 17, 21, 22, 26 & 27 and point loads of 9.0 kips each at

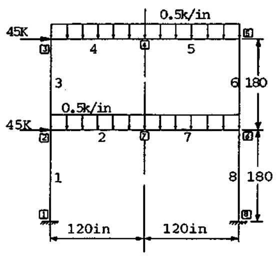


Fig. 5 One bay, Two storey plane frame.

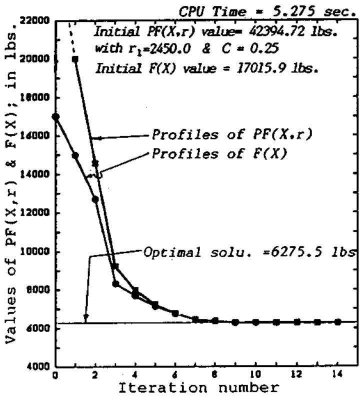
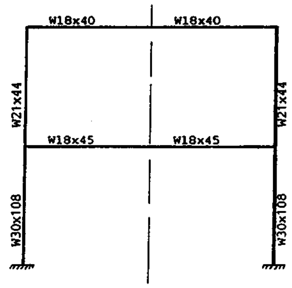


Fig. 6 Profiles of $PF(X, r)$ and $F(X)$ function.

Table 3 Results for 1-bay, 2-storey frame.

Iter-ation (k)	Value of r_k	Starting $F(X)_k$ in (lb.)	Starting $F(X,r)_k$ in (lb.)	Optimum $F(X)^*$ in (lb.)	OPtimum $F(X,r)^*$ in (lb.)
1	2.54×10^{3}	17015.9	42394.7	14994.5	41435.2
2	6.12×10^{2}	14994.5	21604.6	12707.5	20149.6
3	1.53×10^{2}	12707.5	14568.1	8314.1	11954.3
4	3.83×10^{1}	8314.0	9224.1	7691.2	8858.1
5	9.57	7691.2	7982.9	7129.7	7599.7
6	2.39	7129.7	7247.2	6745.1	6866.2
7	5.98×10^{-1}	6745.1	6775.4	6438.8	6508.6
8	1.49×10^{-1}	6438.8	6456.3	6377.6	6392.9
9	3.74×10^{-2}	6377.6	6381.4	6275.5	6638.7
10	9.35×10^{-3}	6275.5	6291.3	6275.5	6291.3
11	2.33×10^{-3}	6275.5	6279.4	6275.5	6279.4
12	5.84×10^{-4}	6275.5	6275.7	6275.5	6275.7

Reduction factor, C=0.25; CPU Time = 5.275 sec.
Note: 1lb. = 4.45N



Optimum weight =6275.7 lbs.

Fig. 7 Optimal solutions.

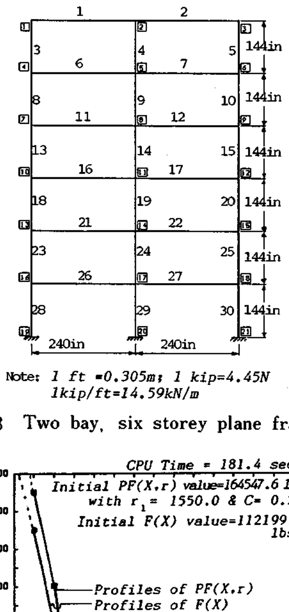
nodes, 1, 4, 7, 10, 13, and 16.

$$(X)_1 = \begin{bmatrix} 120 \\ \vdots \\ 120 \end{bmatrix} = \begin{bmatrix} W14\times246 \\ \vdots \\ W14\times246 \end{bmatrix}_{(1\times18)} \qquad (33)$$

All point loads act in the direction of the x_1 axis. The frame has 30 members and 54 degrees of freedom. The elements of the structure have been combined into 18 groups so that the optimum structure is symmetric. Groups 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, and 17 consist of the pairs of elements of 1 & 2, 3 & 5, 6 & 7, 8 & 10, 11 & 12, 13 & 15, 16 & 17, 18 & 20, 21 & 22, 23 & 25, 26 & 27, 28 & 30, respectively. Group 3, 6, 9, 12, 15, and 18 consists of the single elements 4, 9, 14, 19, 24 and 29, respectively. Thus there are 18 design variables, one for each groups and 30 stress constraints for each of the two loading conditions. Therefore, altogether there are 60 constraints.

Results : The solution has started from the initial base point, $(X)_1$, given in Eq. (33). The $PF(X, r)$ and $F(X)$ value at the base point, $(X)_1$, are 164 547.6 lbs and 112 199.3 lbs., respectively ; with the initial penalty parameter, $r_1=1\,550$. The optimal solution of this constrained problem is $PF(X, r)^*=$ 18 537.4 lbs. ; and $F(X)^*=18\,537.215$ lbs ; and is reached after 11 iteration with $r_{11}=0.001478$.

A summary of the results is given in **Table 4** ; and the profiles of values of $PF(X, r)$ and $F(X)$ at each iteration is shown in **Fig. 9**. The optimal solution is also shown in **Fig. 10**. CPU=181.3 sec. The optimum results reported in ref. 15 is 22 832 lbs. with CPU time 330.0 sec. under LINRM Code.



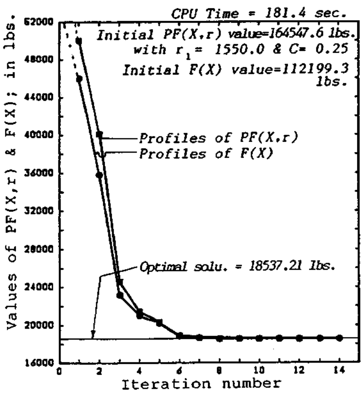
Note: *1 ft =0.305m; 1 kip=4.45N*
1kip/ft=14.59kN/m

Fig. 8 Two bay, six storey plane frame.

Table 4 Results for 2-bay, 6-storey frame.

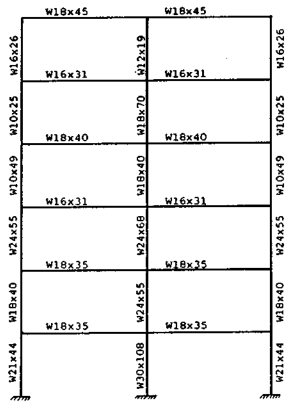
Iter-ation (k)	Value of r_k	Starting $F(X)_k$ in (lb.)	Starting $F(X,r)_k$ in (lb.)	Optimum $F(X)^*$ in (lb.)	Optimum $F(X,r)^*$ in (lb.)
1	1.5×10^3	112199.3	164547.6	56291.6	112488.6
2	3.85×10^2	56291.6	70340.7	35822.1	52865.9
3	9.68×10	35822.1	40082.9	23197.7	28822.8
4	2.42×10	23197.7	24603.9	21017.5	22740.9
5	6.055	21017.5	21448.3	20176.2	20747.5
6	1.51	20176.2	20318.9	18859.8	19010.2
7	3.78×10^1	18859.8	18897.4	18655.6	18713.9
8	9.46×10^2	18655.6	18670.2	18537.2	18551.4
9	2.36×10^2	18537.2	18540.7	18537.2	18540.7
10	5.9×10^3	18537.2	18538.0	18537.2	18538.0
11	1.48×10^3	18537.2	18537.3	18537.2	18537.4

Reduction factor, C =0.25; CPU Time =181.0 sec.
Note: 1 lb. = 4.45N



Note: 1 lb = 4.45N

Fig. 9 Profiles of $PF(X, r)$ and $F(X)$ function.



Optimum weight= 18537.4 lbs.

Fig. 10 Optimal solutions.

7. CONCLUSIONS

Discrete way of structural optimization described here is robustly worthy. Based on the results presented above, the followings can be pointed out for the appeal of the proposed code.

a) Results of all three test problems reveal that very good optimal points have been obtained.

b) Deviation of IGD from the true steepest descent directions can mostly be recovered by the addition of SSI to IGD. Therefore, combination of these two almost produced a true steepest descent direction.

c) Modified ROP proposed here can also follow curved and steep valleys when encounter in a search process for the followings.

i) The modified step lengths taken during the one dimensional search along each of the directions of orthogonal set can claimed to be helpful to follow or move away from the discrete resolution valley to better points. A discrete resolution valley can be described by a series of discrete local optima. A discrete local optimum is a local optimum with respect to the unit neighborhood, $NI(X)$.

ii) Like generalized ROP, modified ROP can also change the direction of searches in each stage and the first direction is oriented towards the locally estimated direction of the valley and all other axes are made mutually orthogonal and normal to first one.

d) Preparation of discrete design space made it possible to use the designation number of standard sections as the design variables, and in this way each elements of a structure always has only one design variable while using all the design properties in the analysis.

e) Coupling of two different techniques with different characteristics made it possible to claim that proposed code can be used to solve varieties of problem with reasonable advantages.

Present code can also be applied to mixed discrete variable problems with slight modifications.

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