ON BENDING TORSION OF SIMPLY SUPPORTEE BEAMS WITH ⊓ SHAPE SECTION

*By Dr. Eng., Sumio NOMACHI, C.E. Member**

Synopsis. In the present paper, some formulae concerning the bending torsion of the simply supported beam with ⊓ shape section, are described from H. Wagner's[1] and Goodier-Barton's[2)3)] theory; and then other formulae which coincide, in particular case where the span length is much longer than the breadth, with those from H. Wagner's theory, are obtained by the precise analysis. Lastly, the effects of the local torsion due to the side girders are discussed and some numerical values of stresses are given for comparison.

Introduction. In solving the elastic problems of the beam under the bending or the torsion, we consider the equilibrium of the resultant forces and moments caused by the internal stresses and by the external loads, with respect to any section, wether the local equilibrium of forces are fulfilled or not. However, the above opinion may generally be satisfactorily applicable for the actual cases and makes the analysis simple.

According to the same opinion there are, on the beam under the torsion, H. Wagner's and Goodier-Barton's theory of bending torsion which are now being used in our country for computing the stresses in the slab bridge with two box girders.[4)5)]

Consequently it would be interesting and important to compare the results evaluated from both theories with those from the precise analysis, though it is for the special section.

If the beam with ⊓ shape section, has two simply supported ends and is under the torsion caused by a couple of forces $P\left(=\sum_{m} q_m \cdot \sin\frac{m\pi}{a}x : m=1,2,3\cdots\cdots\right)$ acting on both sides of the breadth b, then the torsional moment may be Pb.

We shall deal with the above problem by the three methods namly, H. Wagner's theory, Goodier-Barton's theory and precise analysis. The dimensions of the section are shown in **Fig. 1**.

H. Wagner's theory. In this case we assume that the section of the beam under the torsion, rotates by angle φ around the center of shear c; and the condition of equilibrium for the beam is expressed by the well-known differential equation as follows :

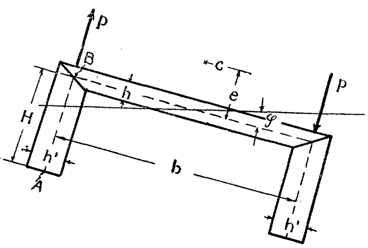
$$D\varphi'''' - C\varphi'' = -Pb. \quad\cdots\cdots(1)$$

where C denotes the torsional rigidity of the beam and D the rigidity of bending torsion :

$$D = E'\frac{b^2H^3h}{12}\,\frac{3\,Hh' + 2\,nbH}{6\,Hh' + nbh} \quad\cdots\cdots(2)$$

where $n = E/E'$, E and E' are the Young's modulus of the slab and the side girders respectively.

Fig. 1



Since the two ends are simply supported, we have

* Professor, Dept. of Civil Eng., Muroran Tech. University

$$\varphi=\sum_m -\frac{bq_m \sin\dfrac{m\pi}{a}x}{D\left(\dfrac{m\pi}{a}\right)^4+C\left(\dfrac{m\pi}{a}\right)^2} \qquad \cdots\cdots(3)$$

If we let σ_{xA} and σ_B be the normal stress in x direction at A and shearing stress at B as shown in **Fig. 1**, they are written in the forms

$$\sigma_{xA}=E'\frac{b}{2}(e-H)\varphi''=E'\frac{b^2}{2}(e-H)\sum_m\frac{q_m \sin\dfrac{m\pi}{a}x}{D\left(\dfrac{m\pi}{a}\right)^2+C}, \qquad \cdots\cdots(4)$$

$$\tau_B=E'\frac{bH}{2}\left(e-\frac{H}{2}\right)\varphi'''=E'\frac{b^2H}{2a}\left(e-\frac{H}{2}\right)\sum_m\frac{m\pi q_m \cos\dfrac{m\pi}{a}x}{D\left(\dfrac{m\pi}{a}\right)^2+C}, \qquad \cdots\cdots(5)$$

where e means the distance from the center of shear to the middle plane in the slab, that is, $H^2h/(2Hh'+nbh/3)$.

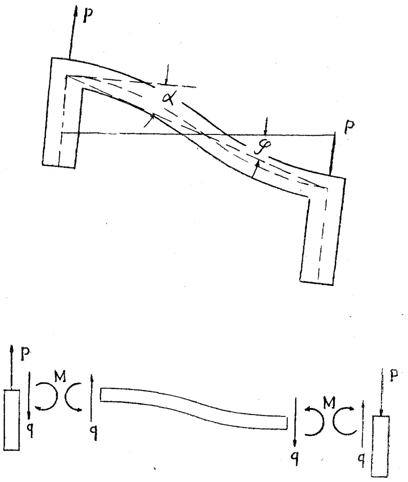
Goodier-Barton's theory. Goodier and Barton proposed the analysis of I-beams in torsion considering the effects of web deformation. We shall apply the same analysis to the beam with ┌─┐ shape section, neglecting the shear lag in the slab.

Let φ and α be the main torsional angle and the local one respectively, then from **Fig. 2** we have at once

$$\left.\begin{aligned}
&bE'I\varphi''''=-p+q,\\
&G'J'(\varphi''-\alpha'')=M,\\
&GJ\varphi''=-2M+qb,\\
&M=-\frac{6}{b}N\alpha
\end{aligned}\right\} \qquad \cdots\cdots(6)$$

where $E'I$ and $G'J'$ denote the flexial rigidity and the torsional rigidity of the side girders; N and GJ also the flexial rigidity and the torsional rigidity of the slab.

Fig. 2



Hence, by eliminating q, α and M, we obtain the ordinary differential equation in six degrees with respect to x, from which we finally have

$$\left.\begin{aligned}
&\varphi=-\sum_m\frac{q_m b\left\{1+\dfrac{G'J'}{6Nb}\left(\dfrac{m\pi}{a}\right)^2\right\}\sin\dfrac{m\pi}{a}x}{\left(\dfrac{m\pi}{a}\right)^2\left\{\left(\dfrac{m\pi}{a}\right)^4A+\left(\dfrac{m\pi}{a}\right)^2B+\dfrac{C}{2}\right\}},\\
&\sigma_{xA}=\frac{E'bH}{4}\varphi'',\\
&M=\frac{1}{2}\left(\frac{GJ}{b}\varphi''-\frac{b}{2}E'I\varphi''''-p\right),\\
&m=1,\ 2,\ 3,\cdots\cdots,
\end{aligned}\right\} \qquad \cdots\cdots(7)$$

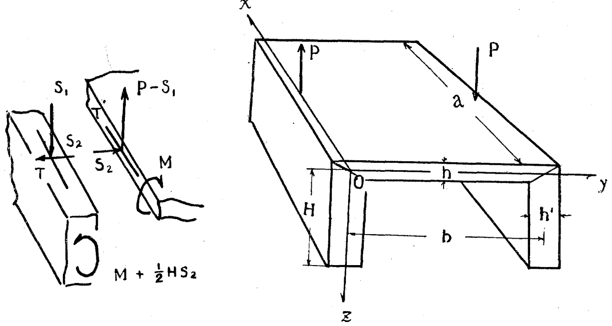
where

$$A=\frac{G'J'E'Ib^3}{24N}, \quad B=\frac{b^2}{2}\left(\frac{E'I}{2}-\frac{G'J'(1-\nu)}{3}\right), \quad C=2G'J'+GJ,$$

and ν denotes Poisson's ratio in the slab.

Precise analysis. We now take the point, where the middle line in the thickness of the one side girder and that in the thickness of the slab meet, as the origin whose rectangular Cartesian co-ordinates may be chosen as shown in **Fig. 3**, namely, the span length along the above side girder corresponds to x direction, the middle line in the thickness of the slab y direction and that in the thickness of the side girder z direction.

Fig. 3



Then supposing the beam to be divided into the side girder whose upper surface accords with the line ox and the slab whose one side corresponds to the same line, there occur some unknown forces and moment at the cutting, that is, S_1 denotes the normal force in z direction, S_2 the same force in y direction, T the shear along ox, and M the bending moment in y direction. Again supposing the beam to be cut along the line bx, where we may consider similar unknown forces indicating the directions opposite to the former unknown forces.

Let the couple P acts on both line ox and bx, then from these forces we may make the combinations as follows :

a) Bending of the side girders in xz plane : $(S_1.\ T)$,

b) Bending of the side girder in xy plane : (S_2),

c) Plane stress of the slab : $(S_2.\ T)$,

d) Bending of the slab : $(P-S_1.\ M+HS_2/2)$.

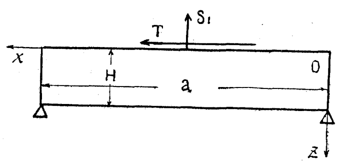
According to the above we shall solve the problems one by one.

a) Bending of the side girder in xz plane. In this case, on the upper surface there act the normal and tangential tractions :

$$S_1=\sum_m N_m \sin\frac{m\pi}{a}x,\quad T=\sum_m R_m \cos\frac{m\pi}{a}x$$

as shown in **Fig. 4**, and both ends are simply supported.

Fig. 4



If we let σ_x, σ_z and τ_{xz} be the normal stress in x and z directions respectively, and the shearing stress, after tedious calculations similar to those in Love's moderately thick plate[6], by assuming the plane conservation, we obtain

$$\left.\begin{aligned}\sigma_z&=\sum_m \sin\frac{m\pi}{a}x\left\{\frac{N_m}{H^3}(H-z)^2(H+2z)+\frac{R_m}{H^2}\frac{m\pi}{a}z(z-H)^2\right\},\\ \tau_{xz}&=\sum_m \cos\frac{m\pi}{a}x\left(\frac{a}{m\pi}\right)\left\{\frac{N_m}{H^3}6z(H-z)+\frac{R_m}{H^2}\frac{m\pi}{a}(H-z)(H-3z)\right\},\\ \sigma_x&=\sum_m \sin\frac{m\pi}{a}x\left(\frac{a}{m\pi}\right)^2\left\{\frac{N_m}{H^3}6(2z-H)+\frac{R_m}{H^2}\frac{m\pi}{a}(4H-6z)\right\},\end{aligned}\right\}\quad\cdots\cdots(8)$$

And the displacement in x and z directions are denoted by u_1 and w_1 respectively, as follows :

$$
\left.\begin{aligned}
E'u_1 &= -\sum_m \cos\frac{m\pi}{a}x\left(\frac{a}{m\pi}\right)^3\left\{\frac{N_m}{H^3}6\,(2z-H)+\frac{R_m}{H^2}\frac{m\pi}{a}(4H-6z)\right\},\\
E'w_1 &= \sum_m \sin\frac{m\pi}{a}x\left(\frac{a}{m\pi}\right)^4\left\{\frac{12N_m}{H^3}-\frac{6R_m}{H^2}\frac{m\pi}{a}\right\},\\
&m=1,\ 2,\ 3,\cdots\cdots
\end{aligned}\right\}\qquad(9)
$$

b) Bending of the side girder in xy plane. If v_1 and $E'I'$ denote the displacement and the flexial rigidity of the side girder, in y direction, we have at once

$$
v_1=\frac{a^4}{E'I'}\sum_m\frac{N_m'}{(m\pi)^4}\sin\frac{m\pi}{a}x,\quad \left(S_2=\sum_m N_m'\sin\frac{m\pi}{a}x\right).\qquad(10)
$$

c) Plane stress of the slab. Let u_2 and v_2 be the displacement in x and y directions respectively, then we may write the differential equations governing the equilibrium of forces in the slab, neglecting its body forces, as follows :

$$
(1-\nu)\Delta(u_2\cdot v_2)+(1+\nu)\left(\frac{d}{\partial x}\cdot\frac{d}{\partial y}\right)\varepsilon=0,\qquad(11)
$$

where Δ denotes Laplacian operator and $\varepsilon=\dfrac{\partial u_2}{\partial x}+\dfrac{\partial v_2}{\partial y}$.

In this case, at the boundaries $y=0$ and b, the function u_2 and v_2 must satisfy the conditions

$$
\left.\begin{aligned}
h\tau_{xy} &= hG\left(\frac{\partial u_2}{\partial y}+\frac{\partial v_2}{\partial x}\right)=\pm T=\pm\sum_m R_m\cos\frac{m\pi}{a}x,\\
h\sigma_y &= \frac{hE}{1-\nu^2}\left(\frac{\partial v_2}{\partial y}+\nu\frac{\partial u_2}{\partial x}\right)=\pm S_2=\pm\sum_m N_m'\sin\frac{m\pi}{a}x,
\end{aligned}\right\}\qquad(12)
$$

and at the other two boundaries $x=0$ and a, the conditions are satisfied by

$$
\sigma_x=0,\quad v_2=0.\qquad(13)
$$

By using the double finite Fourier transforms from eq. (11) the solutions which satisfy the conditions (12) and (13), are written in the forms

$$
\left.\begin{aligned}
u_2 &= \sum_m \cos\frac{m\pi}{a}x\left\{A_m\left(Q_{m\eta}-\frac{(1+\nu)}{2}P_{m\eta}\right)-\frac{b(1+\nu)^2}{Eh\pi}\frac{N_m'}{2\beta_m}P_{m\eta}\right\},\\
v_2 &= \sum_m \sin\frac{m\pi}{a}x\left[\frac{A_m}{2}\left\{(1-\nu)V_{m\eta}-(1+\nu)U_{m\eta}\right\}+\frac{b(1+\nu)}{Eh\pi}\frac{N_m'}{2\beta_m}\left\{(3-\nu)V_{m\eta}-(1+\nu)U_{m\eta}\right\}\right],
\end{aligned}\right\}\qquad(14)
$$

where

$$
\frac{\pi A_m}{2}=\frac{b}{\beta_m\pi Eh}\left\{N_m'\frac{(1-\nu)\,\mathrm{sh}\,\pi\beta_m+(1+\nu)\pi\beta_m}{\mathrm{sh}\,\pi\beta_m-\pi\beta_m}+2R_m\frac{\mathrm{ch}\,\pi\beta_m-1}{\mathrm{sh}\,\pi\beta_m-\pi\beta_m}\right\},
$$

$$
P_{m\eta}=\frac{\beta_m\pi\{\eta\,\mathrm{sh}\,\pi\beta_m(1-\eta)-(1-\eta)\,\mathrm{sh}\,\pi\beta_m\eta\}}{\mathrm{ch}\,\pi\beta_m-\pi\beta_m},
$$

$$
Q_{m\eta}=\frac{\mathrm{ch}\,\pi\beta_m(1-\eta)-\mathrm{ch}\,\pi\beta_m\eta}{\mathrm{ch}\,\pi\beta_m-\pi\beta_m},
$$

$$
U_{m\eta}=\frac{\beta_m\pi\{\eta\,\mathrm{ch}\,\pi\beta_m(1-\eta)-(1-\eta)\,\mathrm{ch}\,\pi\beta_m\eta\}}{\mathrm{ch}\,\pi\beta_m-\pi\beta_m},
$$

$$
V_{m\eta}=\frac{\mathrm{sh}\,\pi\beta_m(1-\eta)-\mathrm{sh}\,\pi\beta_m\eta}{\mathrm{ch}\,\pi\beta_m-\pi\beta_m},
$$

$$
\beta_m=\frac{b}{a}m,\quad \eta=\frac{y}{b},\quad m=1,\ 2,\ 3,\cdots\cdots.
$$

So that the normal stress in x direction σ_x, in y direction σ_y, and the shearing stress τ_{xy} are easily given by the expressions

$$
\sigma_x=\frac{E}{1-\nu^2}\left(\frac{\partial u_2}{\partial x}+\nu\frac{\partial v_2}{\partial y}\right),
$$

$$\sigma_y=\frac{E}{1-\nu^2}\left(\frac{\partial v_2}{\partial y}+\nu\frac{\partial u_2}{\partial x}\right),$$

$$\tau_{xy}=G\left(\frac{\partial u_2}{\partial y}+\frac{\partial v_2}{\partial x}\right).$$

d) Bending of the slab. If w denotes the deflection of the slab in z direction, the condition of equilibrium for the bending of the slab, without any load on it, is expressed by

$$N\Delta^2 w=0, \qquad (15)$$

and the boundary conditions at $x=0$ and a, in this case, are both satisfied by

$$\Delta w=w=0,$$

while the boundary conditions at $y=0$ and b, are given by

$$N\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)=\mp\left(G'J'\frac{\partial^3 w}{\partial x^2\partial y}+\frac{1}{2}HS_2\right),$$

$$N\left(\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\right)=\mp(\rho-S_1).$$

From eq. (15) and the above boundary conditions, we finally obtain

$$w=\frac{b^2}{N\pi^2}\sum_m \sin\frac{m\pi}{a}x\left\{\left(Q_m\eta+\frac{(1-\nu)}{2}P_{m\eta}\right)\frac{B_m}{\beta_m{}^2}+\frac{P_{m\eta}}{2}\frac{F_m}{\beta_m{}^2}\right\}, \qquad (16)$$

where

$$\left.\begin{aligned}F_m&=-\frac{b}{N}\frac{T_m(q_m-N_m)}{\beta_m\pi(H_mK_m+T_m{}^2)}-\frac{Hb}{2G'J'}\frac{K_mN_m'}{\beta_m\pi(H_mK_m+T^2{}_m)},\\ B_m&=-\frac{b}{N}\frac{H_m(q_m-N_m)}{\beta_m\pi(H_mK_m+T_m{}^2)}+\frac{Hb}{2G'J'}\frac{T_mN'_m}{\beta_m\pi(H_mK_m+T_m{}^2)},\end{aligned}\right\} \qquad (17)$$

$$\left.\begin{aligned}H_m&=\frac{1}{4}\frac{\mathrm{sh}\,\pi\beta_m-\pi\beta_m}{\mathrm{ch}\,\pi\beta_m-1}+\frac{Nb}{2G'J'\pi\beta_m},\\ K_m&=\frac{1}{4}\frac{(3+\nu)(1-\nu)\mathrm{sh}\,\pi\beta_m+(1-\nu)^2\pi\beta_m}{\mathrm{ch}\,\pi\beta_m-1},\\ T_m&=\frac{1}{4}\frac{(1+\nu)\mathrm{sh}\,\pi\beta_m+(1-\nu)\pi\beta_m}{\mathrm{ch}\,\pi\beta_m-1}.\end{aligned}\right\} \qquad (18)$$

Accordingly, we may evaluate the bending moment in x direction M_x, in y direction M_y, and the torsional moment M_{xy} from the formulae; i.e.

$$M_x=-N\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right),\quad M_y=-N\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right),\quad M_{xy}=-(1-\nu)N\frac{\partial^2 w}{\partial x\partial y}.$$

e) Conditions of compatibility. The displacements of the side girders and the slab are all given in term of the unknown forces S_1, S_2 and T, as we already described. Hence, making use of equalities for the displacements along ox line, namely

$$v_1=v_2, \qquad (19)$$

$$u_1=u_2, \qquad (20)$$

$$w_1=w, \qquad (21)$$

we find the three unknowns as follows :

from the condition (19)

$$N_m'=\frac{R_{mn}\dfrac{(1-\nu)\mathrm{sh}\,\pi\beta_m+(1+\nu)\pi\beta_m}{\pi\beta_m(\mathrm{sh}\,\pi\beta_m-\pi\beta_m)}}{n\dfrac{a^4h}{I'b}\dfrac{1}{(\mathrm{m}\,\pi)^4}+\dfrac{2(\mathrm{ch}\,\pi\beta_m+1)}{\beta_m\pi(\mathrm{sh}\,\pi\beta_m-\pi\beta_m)}}, \qquad (22)$$

from the condition (20)

$$R_m\left(\beta_m\pi\frac{Hh'}{nbh}\frac{\mathrm{ch}\,\pi\beta_m-1}{\mathrm{sh}\,\pi\beta_m-\pi\beta_m}+2\right)=\frac{3bN_m}{H\beta_m\pi}+\frac{N_m'}{2}\frac{(1-\nu)\mathrm{sh}\,\pi\beta_m+(1+\nu)\pi\beta_m}{\mathrm{sh}\,\pi\beta_m-\pi\beta_m}, \qquad (23)$$

and from the condition (21)

$$B_m=\frac{12N}{EH^2h'}\left(\frac{2N_m}{H(\beta_m\pi)^2}-\frac{R_m}{b\beta_m\pi}\right). \quad\cdots\cdots(24)$$

By aid of the above, we can determine any stress and displacement in the beam now discussed. For instance, the unknown moment at ox line is given in the form

$$M=\sum_m F_m \sin\frac{m\pi}{a}x. \quad\cdots\cdots(25)$$

In the case where the span length is much longr than the breadth.

In this case, the quantity $\beta_m\pi$ may become very small, so that we write

$$\mathrm{sh}\,\pi\beta_m-\pi\beta_m=(\pi\beta_m)^3/6, \qquad \mathrm{sh}\,\pi\beta_m=\pi\beta_m,$$

$$\mathrm{ch}\,\pi\beta_m-1=(\pi\beta_m)^2/2, \qquad \mathrm{ch}\,\pi\beta_m=1.$$

Accordingly, putting the above results in eq. (22), we have

$$N_m'=\frac{R_m(\pi\beta_m)}{\dfrac{nb^4h}{12I'}+2}, \quad\cdots\cdots(26)$$

from which we easily see that S_2 is usually much smaller than S_1 or T. So that we may transform eqs. (23) and (24), neglecting S_2, into the simple forms

$$R_m=3n\frac{N_m}{\beta_m\pi}\frac{b^2h}{H^2h'}\frac{Hh'}{3Hh'+2nbh}, \quad\cdots\cdots(27)$$

$$\frac{q_m}{C}=\frac{N_m}{(\beta_m\pi)^2}\left(\frac{12}{E'H^3h'}\frac{6Hh'+nbh}{3Hh'+2nbh}+\frac{(\beta_m\pi)^2}{C}\right). \quad\cdots\cdots(28)$$

Again using the notation D that is defined by the formula (2), from the relation (28) we finally find

$$N_m=\frac{(\beta_m\pi)^2Dq_m}{Cb^2+D(\beta_m\pi)^2}. \quad\cdots\cdots(29)$$

By virtue of the above, after some evaluations we write the rotating angle φ of the plane which contains both side of the slab, as follows :

$$\varphi=\sum_m\frac{bq_m\sin\dfrac{m\pi}{a}x}{D\left(\dfrac{m\pi}{a}\right)^4+C\left(\dfrac{m\pi}{a}\right)^2},$$

and putting the relations (27) and (29) into the formula (8), we have

$$\sigma_{xA}=\frac{E'}{2}b^2(e-H)\sum_m\frac{q_m\sin\dfrac{m\pi}{a}x}{C+D\left(\dfrac{m\pi}{a}\right)^2},$$

$$\tau_B=\frac{E'}{2}\frac{b^2}{a}H\left(e-\frac{H}{2}\right)\sum_m\frac{q_m m\pi\cos\dfrac{m\pi}{a}x}{C+D\left(\dfrac{m\pi}{a}\right)^2}.$$

The above results are evidently identical with those derived from H. Wagner's theory, which are seen in the formulae (3), (4), and (5) ; consequently, we may conclude that the smaller the ratio b/a becomes, the more Wagner's theory becomes exact.

The secondary stresses due to the side girders. In order to simplify the problems of bending torsion, H. Wagner proposed to let the stress distributions in such sections as thin tubular shape, be uniform to its thickness ; and he was, therefore, obliged to neglect the stress distributions which cause the bending to the middle line in the thickness. To the thin tubular sections, the above opinion may indeed be adequate; however, in such sections as ⊓ shape, there may occur, with considerable magnitude, the stresses showing the latter stress distributions. On other words, in the beam with ⊓ shape section, the secondary stresses which bend the slab in y direction, may occur due to the side girders.

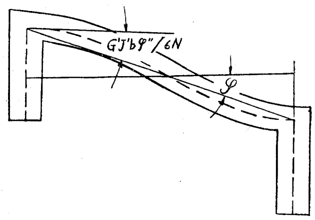
We shall evaluate the secondary stresses before mentioned, by virtue of the precise results. On letting $\beta_m\pi$ be the small quantity, we find that the relation (18) is replaced by $F_m=-\frac{G'J'}{C}(q_m-N_m)$. Then substituting the above into the formula (25), we have

$$M=-\sum_m \frac{G'J'bq_m}{C+D\left(\frac{m\pi}{a}\right)^2}\sin\frac{m\pi}{a}x, \qquad (30)$$

which may be transformed in term of φ, to the form $M=-G'J'\frac{d^2\varphi}{dx^2}$.

Hence, in the case where the span length is much longer to the breadth, there occur in the slab along ox, the bending stress that is $6G'J'\varphi''/h$, and the secondary slope amounting to $G'J'b\varphi''/6N$ radians, as shown in **Fig. 5**.

Fig. 5



Numerical examples. For the numerical examples, we now choose the three sections as shown in Fig. 6 namely, the sections (a) and (b) are assumed to be made of a uniform material whose ν is 0.15, on the other hand the section (c) consist of the same slab as in (a) or (b), and side girders with Young's modulus ten time as much as that of the slab; in addition to it, we let Poisson's ratio of the side girder in (c) be 1/3.

Tables 1, 2 and **3** give the maximum values of σ_{xA}, σ_{yB} and τ_B in the case $P=q\sin\pi x/a$, respectively; and the same values of σ_{xA} and σ_{yB} in the range b/a from zero to one are shown in **Fig. 7, 8** and **9** where the curves 1, 2 and 3 represent the stresses obtained by H. Wagner's theory, Goodier-Barton's theory and the precise analysis respectively.

Fig. 6

section (a) **section (b)** **section (c)**

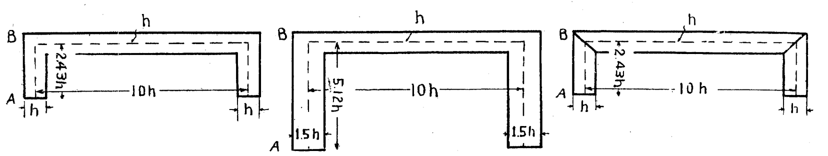


Table 1 Values of σ_{xA}, in which columns 1, 2, and 3 give the values by means of H. Wagner's theory, Goodier-Baston's theory and the precise solution respectively. (unit q/h)

b/a	Section (a)			Section (b)			Section (c)		
	1	2	3	1	2	3	1	2	3
0	43.35	30.82	43.35	27.27	23.13	27.27	108.89	102.09	108.89
1/20	42.65	30.60	42.80	25.79	22.72	26.25	106.04	100.02	106.20
1/8	39.53	29.35	39.13	20.06	19.55	20.80	93.43	89.62	94.75
1/6	37.02	28.27	36.35	16.64	17.27	17.45	84.15	78.80	85.88
1/4	31.33	25.60	30.22	11.19	12.86	11.85	65.54	58.39	67.20
1/3	25.79	22.59	24.46	7.67	9.06	7.94	50.04	40.41	51.20
1/2	17.13	16.90	15.88	4.04	5.21	4.27	30.57	19.05	30.34
1/1	6.09	7.59	5.59				8.96	4.01	8.89

Table 2 Values of σ_{yB} (unit q/h)

b/a	Section (a)			Section (b)			Section (c)		
	1	2	3	1	2	3	1	2	3
0	0.792	0.792	0.792	22.15	22.15	22.15	22.68	22.68	22.68
1/20	0.782	0.789	0.773	20.94	21.24	20.22	22.09	22.31	21.22
1/8	0.724	0.760	0.688	16.29	17.18	15.77	19.46	20.57	17.25
1/6	0.678	0.736	0.624	13.51	14.35	12.60	17.53	18.29	14.80
1/4	0.574	0.689	0.469	9.08	9.57	6.20	13.65	15.18	9.69
1/3	0.473	0.635	0.382	6.23	7.47	4.01	10.42	14.05	6.28
1/2	0.314	0.555	0.233	3.28	7.52	1.67	6.37	14.85	2.74
1/1	0.166	0.615	0.112				4.87	17.88	0.90

Table 3 Values of τ_B (unit q/h)

b/a	Section (a)			Section (b)			Section (c)		
	1	2	3	1	2	3	1	2	3
0	0	0	0	0	0	0	0	0	0
1/20	0.44	0	0.43	0.28	0	0.27	0.25	0	0.24
1/8	1.08	0	1.00	0.63	0	0.54	0.55	0	0.54
1/6	1.33	0	1.26	0.70	0	0.61	0.66	0	0.65
1/4	1.65	0	1.61	0.71	0	0.64	0.77	0	0.76
1/3	1.77	0	1.79	0.63	0	0.59	0.78	0	0.73
1/2	1.69	0	1.82	0.49	0	0.48	0.72	0	0.65
1/1	1.09	0	1.37				0.42	0	0.32

Fig. 7 In case of section (a)
(curve 1, 2 and 3 being derived from H. Wagner's theory, Goodier-Barton's theory, and the precise solution respectively.)

values of σ_{xA} values of σ_{yB}

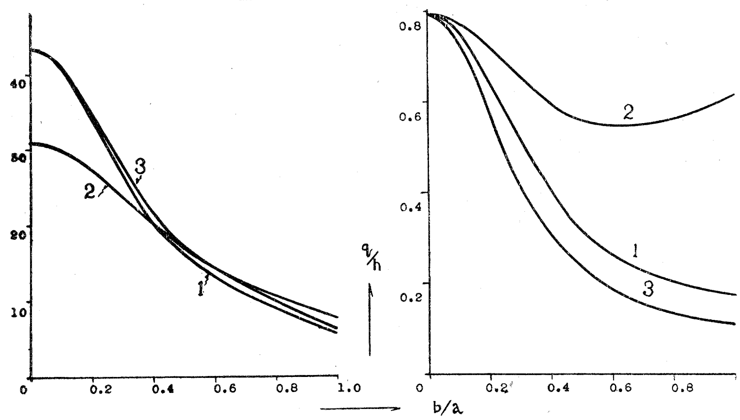


Fig. 8 In case of section (b)

values of σ_{xA} values of σ_{yB}

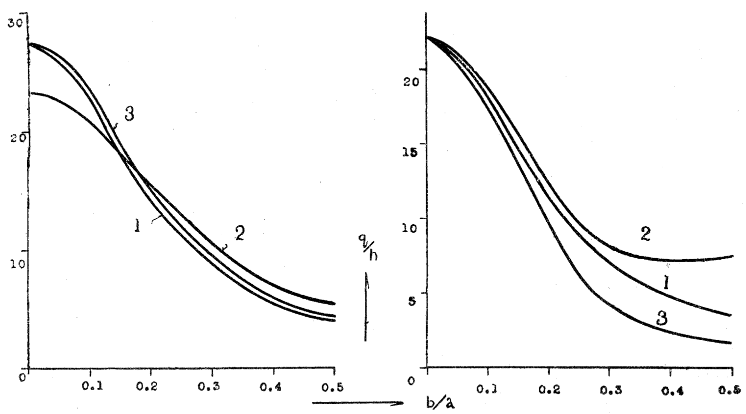
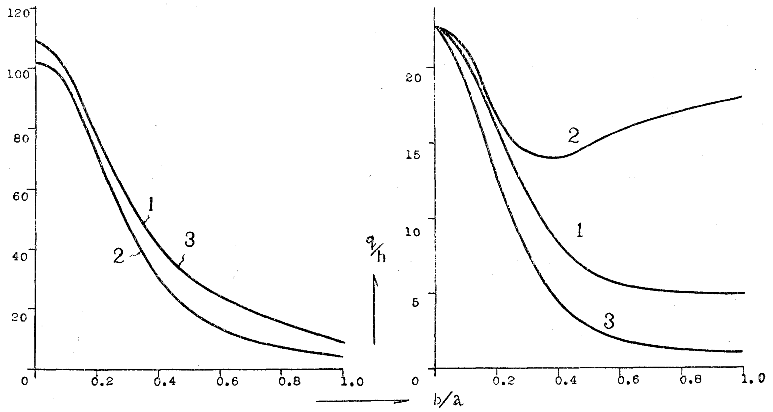


Fig. 9 In case of section (c)

values of σ_{xA} **values of σ_{yB}**



Summary. According to the numerical results in the range of b/a from zero to one, the stresses from H. Wagner's theory excluding σ_{yB} remarkably coincide with those from the precise analysis; whereas the stresses from Goodier-Barton's theory show considerable difference. As for σ_{yB}, so long as the ratio b/a is smaller than 1/8, the values from the formula (30), are within 10% larger than those from the exact formula; we may, therefore, say that the formula (30) has the practical value.

The effects caused by the secondary torsion of the side girders, as already discussed, can easily be obtained from the formula (30); and in case of the beam whose side girders take the section as box shape, so long as it has both simply supported ends, the secondary stresses in it may similarly be obtained by the same formula.

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