招待論文

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# THE LATERAL DISPLACEMENT OF PILES FROM EMBANKMENT LOADS

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## ABSTRACT

Lateral loads on piles arise from a variety of causes. In the case of an embankment adjacent to a piled foundation, the lateral loads acting on the piles are induced by the horizontal movement of the soil as a result of the embankment load. Allowances have to be made to ensure that the piles have the capacity to resist the applied bending moments. In this paper, a simple method of estimating the applied bending moments in piles is proposed, based on the finite element method and the Winkler foundation method. Various charts are provided for estimating the lateral displacements of the piles. In addition, analyses have been carried out to study the behaviour of piles under the effects of different loading conditions and boundary conditions.


## 1. INTRODUCTION

The piles below an existing building can be damaged by the lateral displacement of the soil (ground movements) caused by for examples, the construction of a fill or an embankment next to the building as illustrated in Fig. 1. The underlying soft clay may be forced in under the structure as the soil settles under of the weight of the fill. The resulting lateral displacements due to creep can be large when the ultimate bearing capacity of the soft clay is approached. Several cases have been reported in the literature where this has actually occurred (Broms, 1964, Broms, 1972, Huder and Bucher 1981).

Another common case is shown in Fig. 2 where the piles supporting a crane way has been displaced laterally due to the increased weight caused by e. g. an adjacent fill or by a stock pile of coal, iron ore, or by any other heavy material stored nearby. The soft clay has in this case moved outwards away from the stock pile. Before any consolidation and volume changes of the soft clay the total lateral displacement will be the largest at the edge of the fill where the total lateral displacements corresponds to the total settlement of the stock pile. The settlements will gradually increase when the soft clay consolidates. During this stage the soft clay moves inwards the centre of the fill or of the stockpile. However when the factor of safety is low and the applied load is close to the ultimate bearing capacity of the soft clay, the soil

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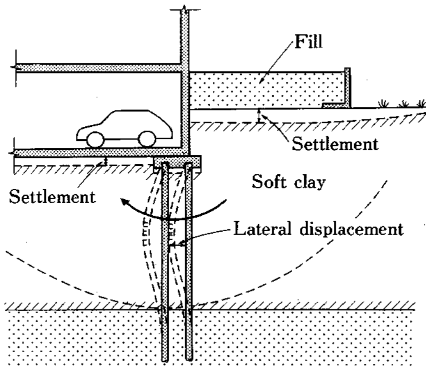



Fig. 1 Displacement of piles below a building.

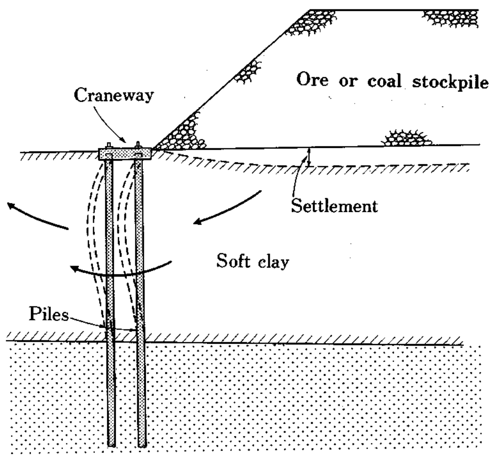



Fig. 2 Pile displacement caused by an adjacent stockpile.

will continue to move outwards away from the fill. The direction of the lateral movements in the soil is normally reversed as the soil consolidates.

Mainly the piles located close to the edge of the fill will be affected by the lateral displacements of the soil. The resulting maximum bending moment in the piles can under unfavourable conditions be so high that the yield strength of the reinforcement or the compressive strength of the concrete will be exceeded and the piles may fail. A number of cases has been reported in the literature where the lateral displacements of the piles have been excessive, e. g. Heyman and Boersma (1961), Peck and Raamot (1964), Heyman (1965), Leussink and Wenz (1969), Powell and Harris (1977) and de Beer (1977).

## 2. METHOD OF ANALYSIS

The lateral displacements in a clay layer located below a fill or a stock pile are investigated in the following. The clay has been assumed to behave as an ideal elastic material with a constant modulus of elasticity ($E_s$) and a constant Poisson's ratio ($\nu$). The lateral displacements have been calculated by the finite element method (FEM) for a trapezoidally distributed load. The influence of the distribution and the inclination of the load from the fill, the thickness of the underlaying compressible layer and the Poisson's ratio has been analyzed.

This case corresponds approximately to an overconsolidated clay with a constant shear strength. In a normally consolidated clay or in loose silt the modulus of elasticity generally increases with depth as the confining pressure increases. The Poisson's ratio normally decreases with time as the soil consolidates. Immediately after loading Poisson's ratio $\nu=0.50$ when the soil is saturated, before any significant consolidation or changes of the water content. In a partially saturated soil $\nu<0.50$.

It is generally possible to get some indications of the change of the Poisson's ratio $\nu$ with time by measuring the lateral effective stress $p'_h$ in the ground since

$$p'_h=K_o p'_v \qquad (1)$$

where $K_o$=coefficient of lateral earth pressure at rest and

$p'_v$=effective overburden pressure

From elasticity theory

$$v=\frac{K_o}{1+K_o} \qquad (2)$$

Brooker and Ireland (1965) have found that $K_o$ for a normally consolidated clay increases with increasing plasticity index $I_p$. At $I_p\leq 40$ the following relationship was proposed :

$$K_o=0.40+0.007\ I_p \qquad (3)$$

Table 1 Relationship between Poisson's Ratio and Plasticity Index $I_p$.

| Plasticity index, $I_p$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| Poisson's ratio, $\nu$ | 0.32 | 0.35 | 0.38 | 0.404 | 0.408 | 0.411 | 0.415 | 0.418 |

At a value on $I_p$ between 40 to 80

$$K_o = 0.68 + 0.001\ (I_p - 40) \quad (4)$$

Several methods have been developed to determine the lateral earth pressure directly in situ by measuring e. g. the pressure causing hydraulic fracturing of the soil (Bjerrum and Andersen, 1972). Thin earth pressure cells (Massarsch et al., 1975; Massarsch, 1975) selfboring pressuremeters (Wroth and Hughes, 1973) and dilatometers have also been used. These methods indicate similar values on $K_o$ as those that have been reported by brooker and Ireland (1965).

The relationship between $\nu$ and the plasticity index as evaluated by Eqs. (1) through (4) is shown in Table 1 for a normally consolidated clay. For most soils the maximum variation is between 0.32 and 0.42. These values have been used in the following analysis.

The equivalent modulus of elasticity of clay ($E_s$) increases in general with increasing shear strength. For very soft clay, it is often in the range 300 kPa to 3 000 kPa while for a medium to hard clay $E_s$ varies normally between 5 000 kPa to 20 000 kPa. The modulus of elasticity is, however, affected by a large number of factors such as the loading conditions, the stress history of the clay and the water content.

The lateral displacements caused by a nearby fill have been calculated in the following for $E_s = 1\,000$ kPa and a Poisson's ratio of 0.25, 0.35, 0.40 and 0.49. A Poisson's ratio of 0.49 corresponds to the conditions immediately after loading before any consolidation of the soil as mentioned above. The values $\nu = 0.25$, 0.35 and 0.40 correspond to the conditions after consolidation when the water content has been reduced.

The lateral displacements can also be evaluated for other values of $E_s$ since the displacements are proportional to $1/E_s$. For example, at $E_s = 2\,000$ kPa the displacements will be only half of those at $E_s = 1\,000$ kPa.

## 3. BENDING MOMENTS IN PILES

The distribution of bending moment in a pile caused by the lateral displacements of the soil has been calculated as illustrated in Fig. 3. It has been assumed that the behaviour can be analyzed as if the pile is supported on a series of elastic springs (Winkler foundation). The spring constant corresponds to the coefficient of horizontal subgrade reaction $k_h$. It has been suggested (Broms, 1964) that $k_h$ can be evaluated for a normally consolidated clay at <u>short</u> term loading from the relationship

$$k_h = 120\ c_u/D \quad (5)$$

where $D$ is the diameter of the pile and $c_u$ is the undrained shear strength. At <u>long</u> term conditions the following equation has been used

$$k_h = 20\ c_u/D \quad (6)$$

In the following calculations of the bending moment distribution the lateral load on the piles has first been evaluated from the lateral displacements of the soil assuming that the piles are fixed and do not move. The resulting lateral load $Q$ (kN/m) on the pile will in this case be proportional to the lateral displacement $\delta_h$.

$$Q = qD = k_h \delta_h D \quad (7)$$

where $q$ (kPa) is the load per unit area of the piles and $D$ is the diameter.

At <u>short</u> term loading before any consolidation of the soil ($k_h = 120\ c_u/D$).

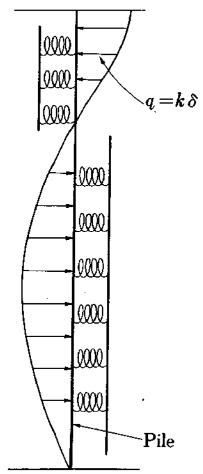


Fig. 3 Winkler foundation.

$Q=120\ c_u\delta_h$ ............ (8)

At long term loading ($k_h=20\ c_u/D$)

$Q=20\ c_u\delta_h$ ............ (9)

Thus the lateral load on the piles has been assumed to be proportional to the lateral displacement of the surrounding soil.

In the next step of the analysis the piles are released. The lateral deflections of the spring supported piles caused by the lateral load calculated above are then evaluated. The resulting lateral displacement of the pile and the calculated bending moments correspond to the actual deflections of the pile and the actual bending moments.

## 4. FINITE ELEMENT ANALYSIS

The finite element mesh that has been used to calculate the lateral displacements of the soft clay is shown in Fig. 4. The mesh is supported on a series of vertical rollers 200 m from the centre line and on horizontal rollers at a depth of 170 m. The size of mesh has been varied in order to utilise the available storage capacity of the computer. The horizontal ground surface is loaded by a trapezoidally distributed load with a total width of 160 m. Only half of the loaded area is shown because of symmetry. The lateral displacement of the soft clay along the centerline of the loaded area is thus equal to zero.

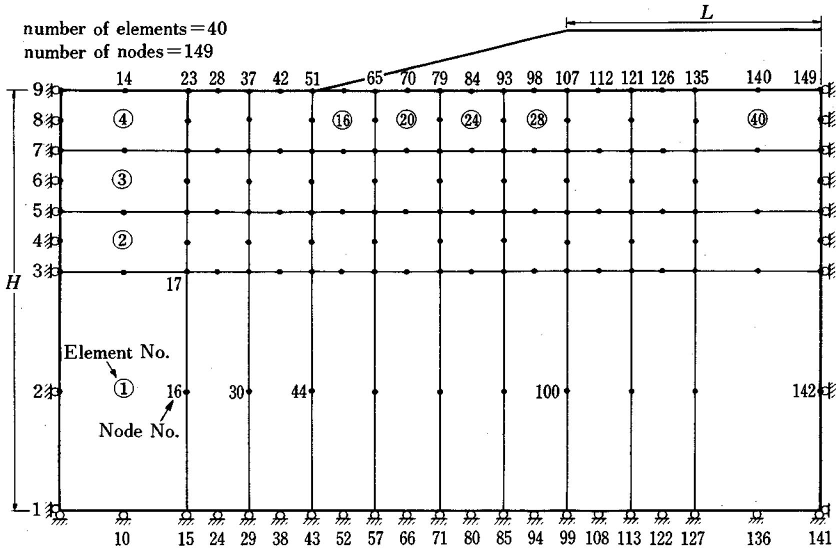



Fig. 4 Finite element mesh.

## 5. RESULTS

In Fig. 5 is shown the lateral displacement caused by a uniformly distributed load (1 kPa) at different distances from the centre line of the loaded area (a through d). It can be seen from the figure that the lateral displacements will be the largest at Point b just below the edge of the loaded area as expected. The soft clay moves towards the centre down to a depth of about 40 m. Below this depth the soil is displaced in the opposite direction away from the loaded area. The lateral displacement of a pile installed at this location (Point b) and the resulting bending moment can be calculated by assuming that the pile is supported on a series of elastic springs (Winkler foundation) as described above.

Fig. 5 can be used to determine the lateral displacement at different values of the applied load, the width of the loaded area, the Poisson's ratio $\nu$ and the equivalent modulus of elasticity, since the lateral

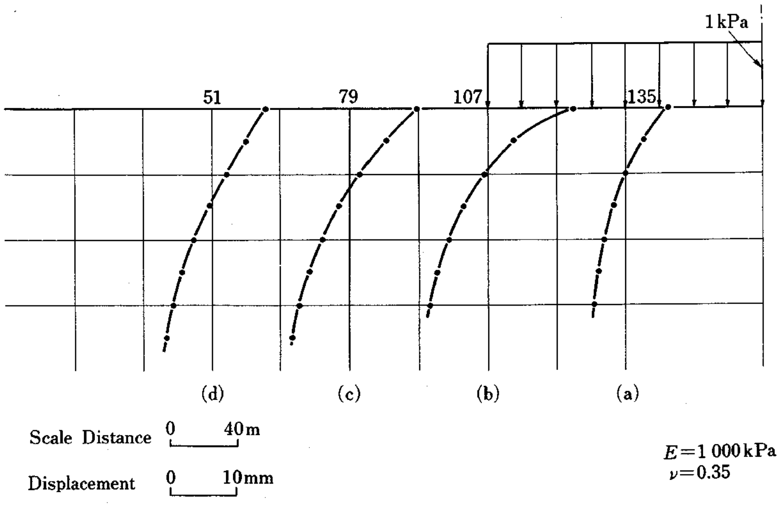



**Fig. 5** Lateral displacememt for a uniformly distributed vertical load.

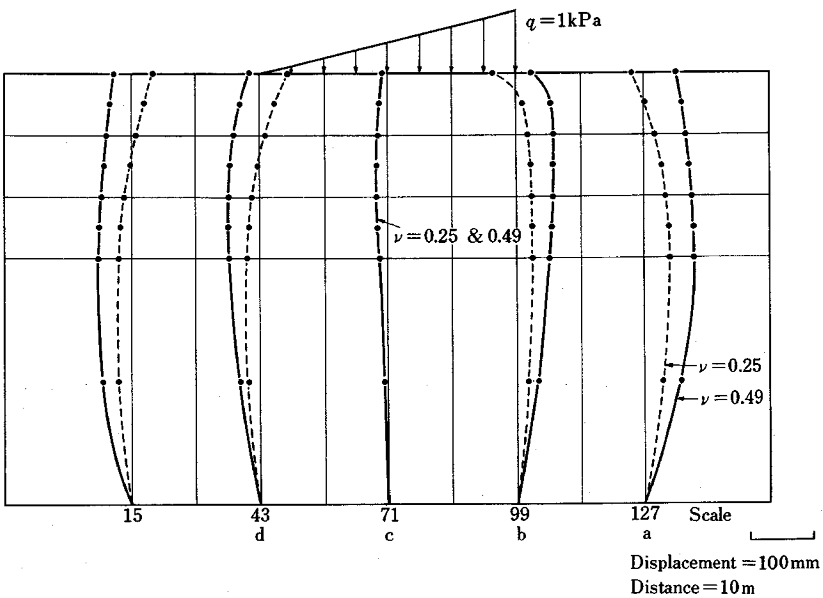



**Fig. 6** Lateral displacements for a triangularly distributed vertical load.

displacements are proportional to the applied load and the width of the loaded area at a given $L/H$-ratio. In the case $q=3$ kPa then the lateral displacements will be three times the indicated displacements. If the width of the loaded area is only 10 m instead of 40 m the lateral displacements will be 25 % (1/4) of the indicated values at the same $L/H$-ratio.

The lateral displacements caused by a triangular load are shown in **Fig. 6** for a Poisson's ratio of 0.49 and 0.25. The upper part of a pile located at a, b, c or d will move outwards away from the loaded area when $\nu=0.49$ while at $\nu=0.25$ the displacement of the pile at the surface is inwards in the opposite direction. Deeper down the piles will move outwards even when $\nu=0.25$.

At the edge of an embankment or a coal or a ore stock pile there is a horizontal component of the load from the stockpile due to the lateral pressure in the stock pile. When the angle of internal friction ($\phi'$) of the

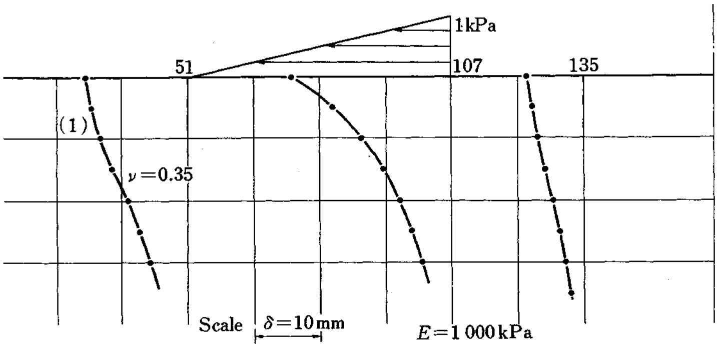



Fig. 7 Lateral displacements for a triangularly distributed horizontal load.

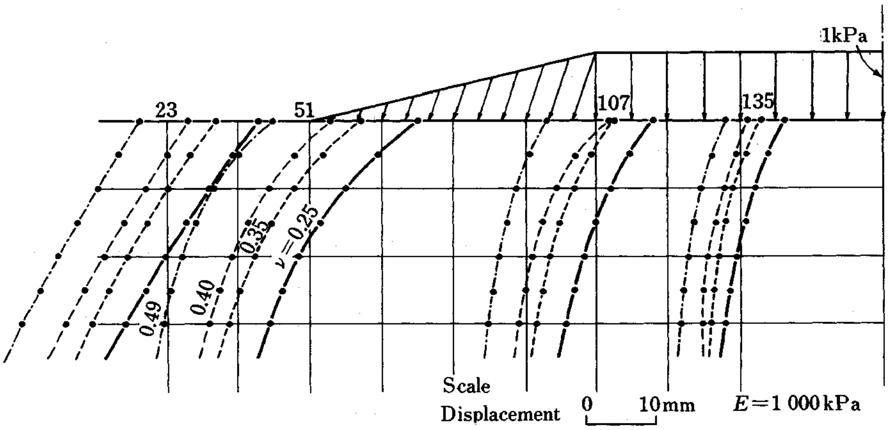


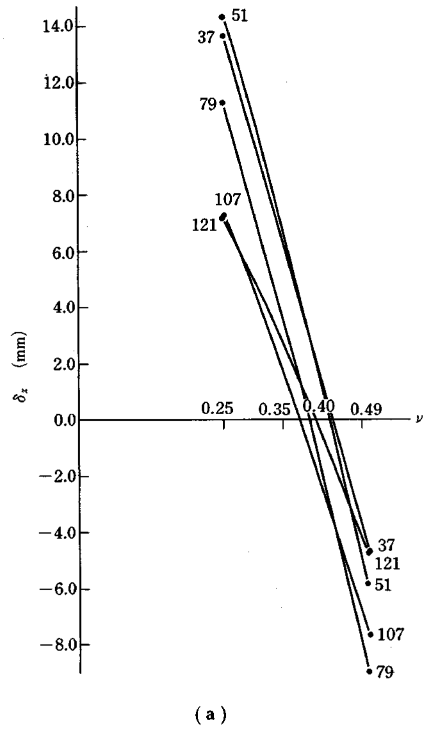

Fig. 8 Lateral displacements below a coal or ore pile.

material in the stockpile is 30° then the lateral pressure active will be one-third of the vertical pressure. This leteral pressure causes a shear force along the interface between the fill and the underlying soil which will increase the lateral displacements of the soil and of any adjacent piles. In Fig. 7 the lateral displacements have been calculated for a linearly increasing shear force. The resulting shear stress distribution corresponds approximately to that below the sloping sides of a coal or ore stock pile. It can be seen that the lateral displacements of the underlying soil will be large, much larger than the displacements for a corresponding vertical load. The lateral displacements will extend to considerable depth.

The lateral displacements below an embankment or a coal or ore pile are shown in Fig. 8 at different values of the Poisson's ratio ($\nu$=0.49, 0.40, 0.35 and 0.25). The lateral displacements have been calculated at different locations of the piles. It has been assumed in the analysis that the load distribution is trapezoidal and that the horizontal shear force at the edge of the embankment is one-third the vertical load. The analysis indicates that the lateral displacements within the soft clay are the largest at the edge of the loaded area. The lateral displacements are also large away from the edge. It can be seen that the piles rotate in the soil rather than bend.

The influence of Poisson's ratio $\nu$ has been evaluated in Fig. 9 for node points 37, 51, 79, 107 and 121 located at the ground surface. It is interesting to note that the lateral displacements vary almost linearly with $\nu$. This means that the lateral displacements at any value on $\nu$ can be evaluated by interpolation of the displacements at $\nu$=0.49 and $\nu$=0.25.

At the ground surface, the lateral displacement will be small outside the loaded area when $\nu$=0.43. This value of Poisson's ratio corresponds approximately to that of a normally consolidated clay with a plasticity



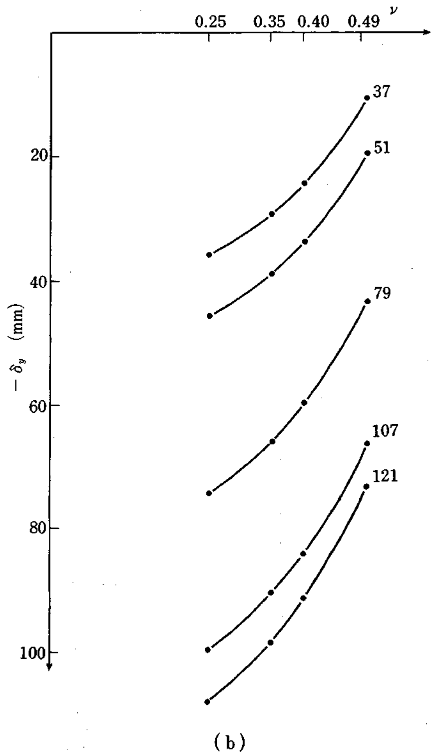




Fig. 9 Influence of Poisson's ratio.

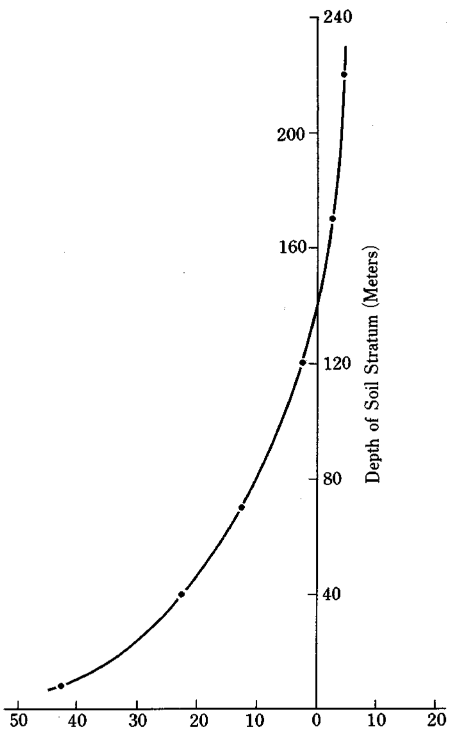



Fig. 10 Lateral movement of node 51 (millimeters).

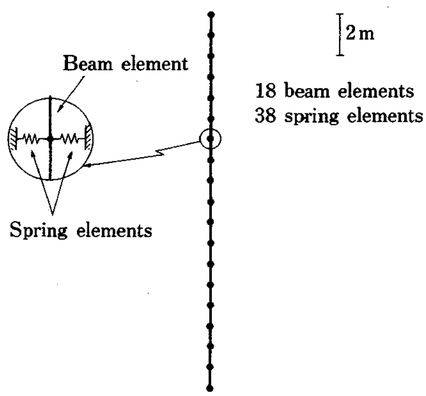



Fig. 11 Pile elements.

index ($I_p$) of about 100 to 120. However most clays have a plasticity index less than 100. Therefore the lateral displacement after consolidation will often be inwards towards the loaded area.

The effect of the thickness of the compressible layer below a coal or ore stockpile on the lateral displacements is illustrated in Fig. 10. It can be seen that the piles are displaced laterally without bending when the thickness $D$ of the compressible layer is small compared with the width of the loaded area $L$ when the $L/D$-ratio is large. High bending moments will occur when the thickness is large compared to the width.

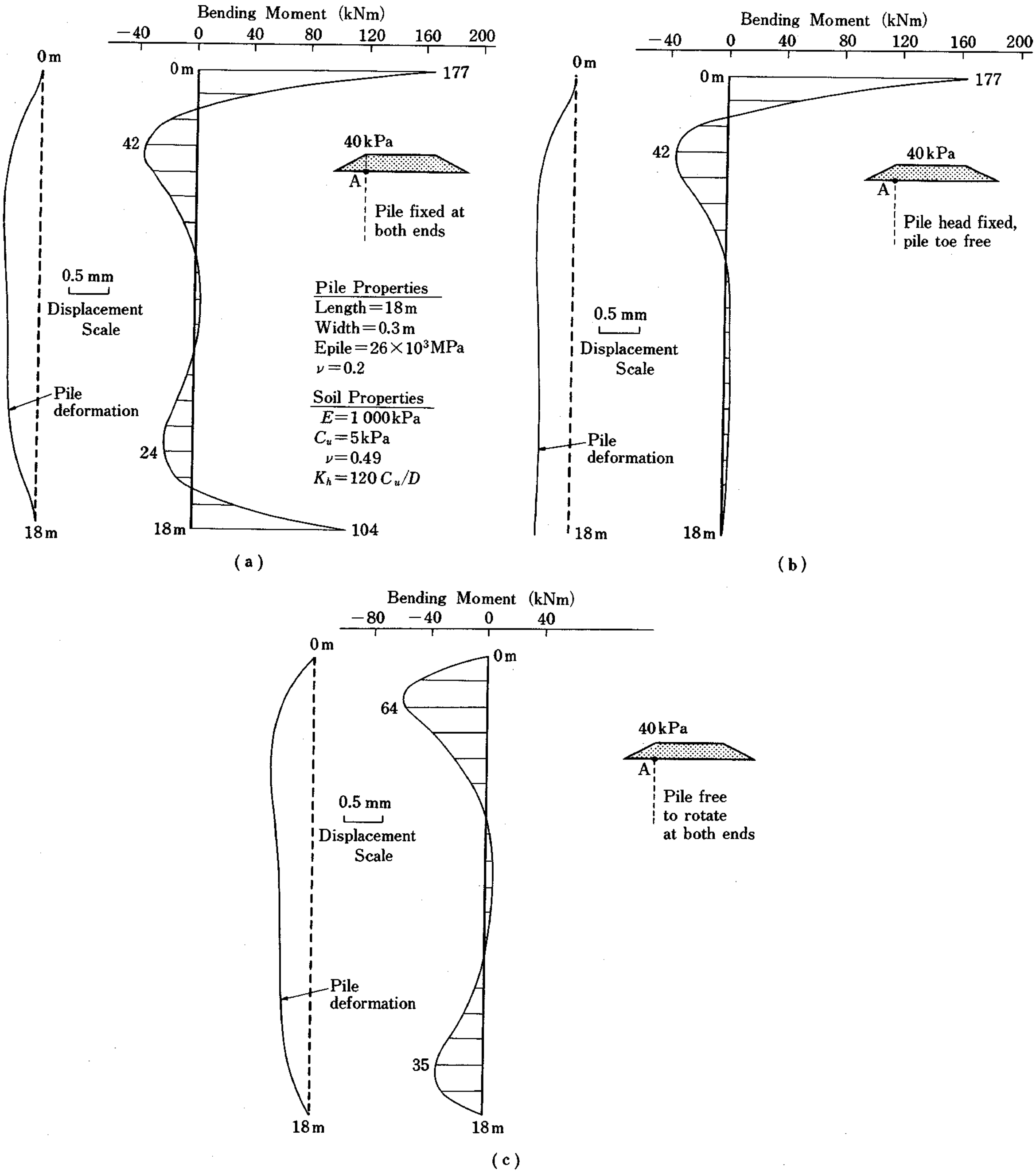



Fig. 12 Disteribution of bending moment in a pile located below an embankment.

## 6. ANALYSIS OF PILES

The finite element mesh which has been used to determine the distribution of the bending moment in an 18 m long square reinforced concrete pile, using the method described earlier, is shown in Fig. 11. The pile which has been driven through an 18 m thick layer of clay ($E$=1 000 kPa, $c_u$=5 kPa, $\nu$=0.49), has been displaced by an adjacent 2 m high embankment with a unit weight $\gamma$=20 kN/m$^3$.

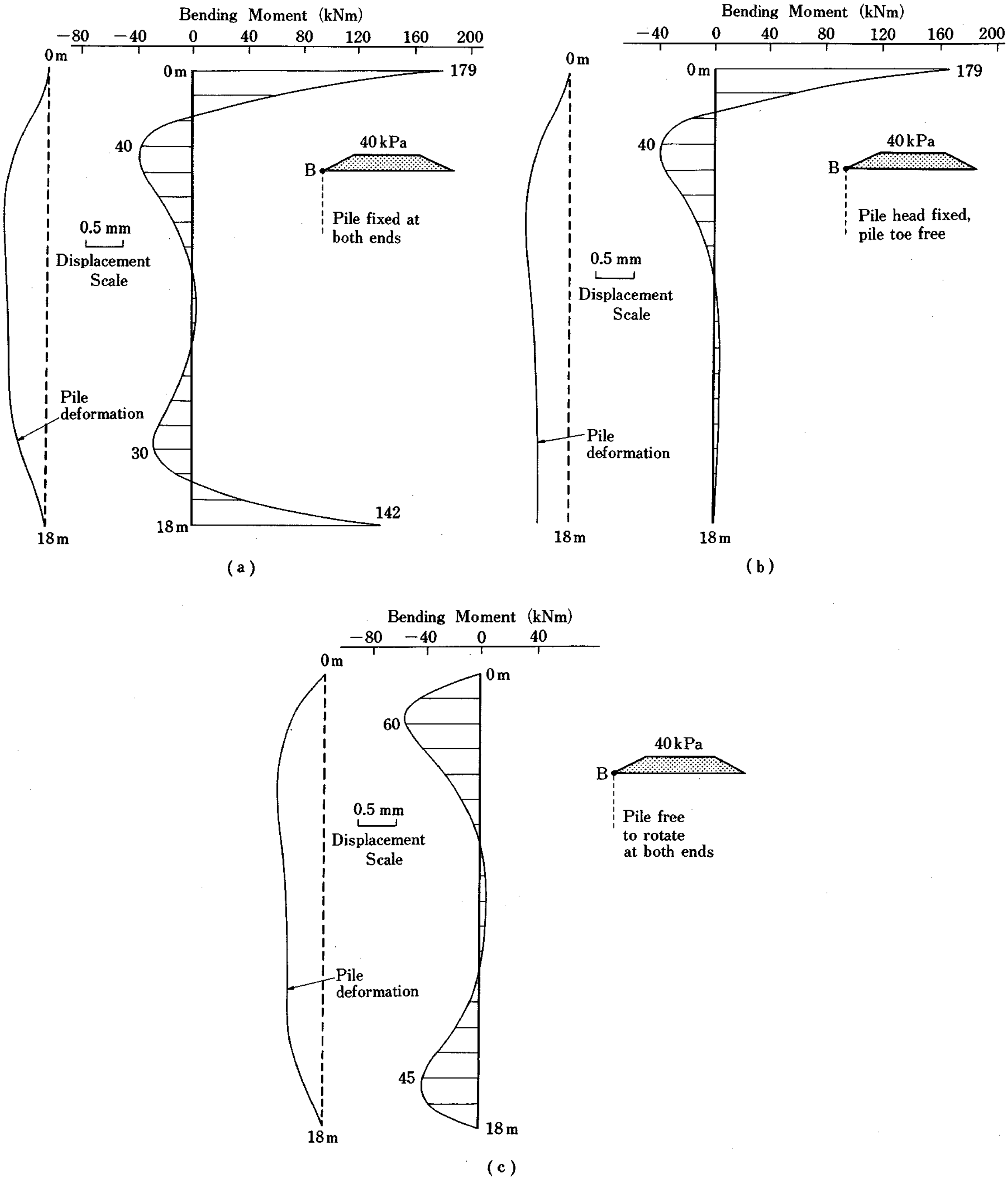



Fig. 13 Distribution of bending moment in a pile located next to an embankment.

Fig. 12 illustrates lateral deformations and the resulting bending moments in a pile installed at location A for three different end conditions. A comparison of Fig. 12 ( a ) and Fig. 12 ( b ) indicates that the end conditions have only a negligible effect on the peak bending moments in the pile, and in the pile cap. However, a comparison with Fig. 12 ( c ) shows that the pile bending moments are significantly different, if the pile head and the toe are not fixed and are able to rotate. This is also the case for a pile located at B, at the edge of the loaded area, Fig. 13.

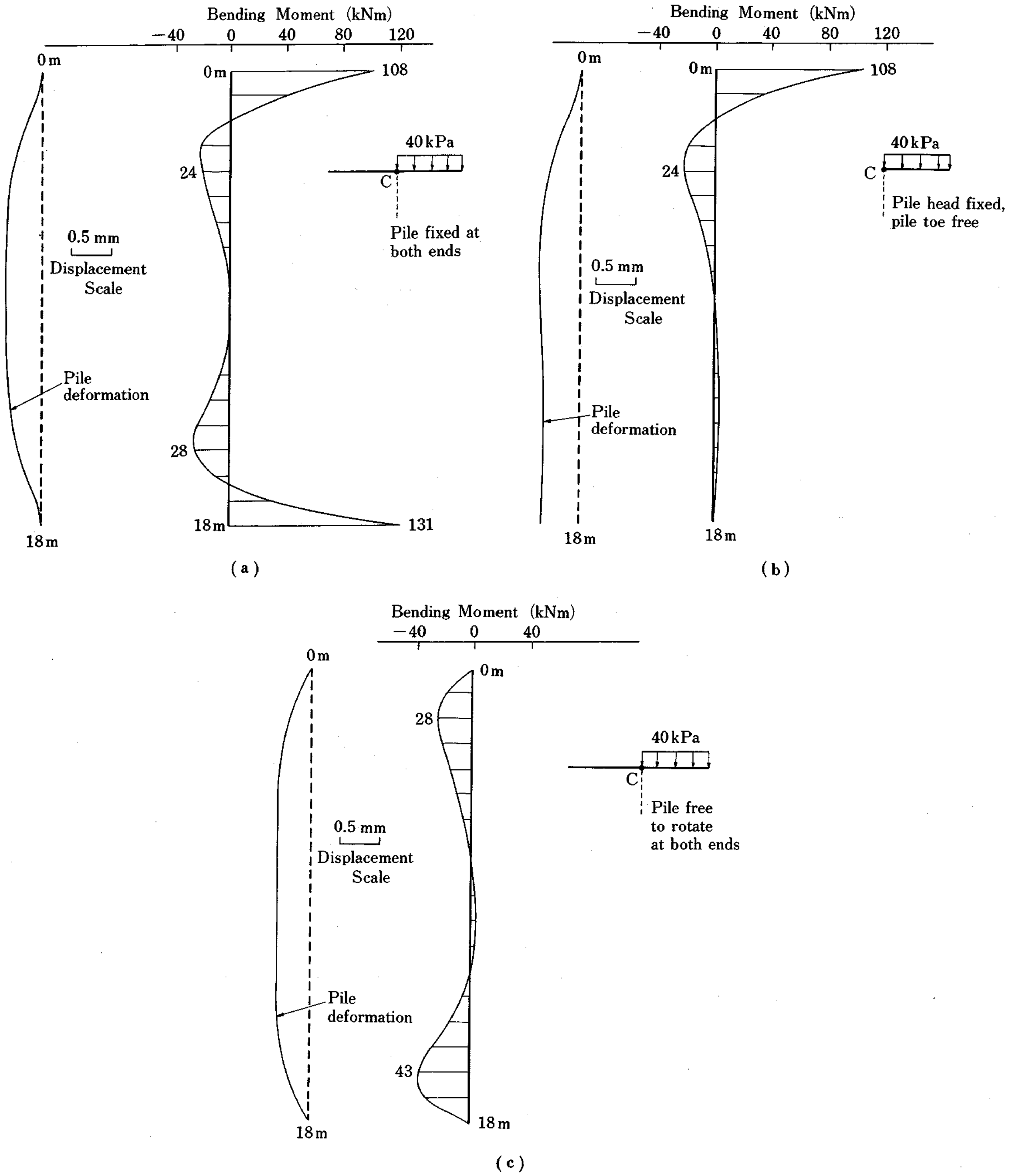



Fig. 14 Distribution of bending moment in a pile located next to a fill.

A comparison of the bending moments in the piles located at A and B indicates that the bending moment in the pile at A is higher in the top half of the pile compared with a pile at B. It can also be seen that a pile at B experiences a higher bending moment in the bottom half of the pile than a pile at A. The vertical pressure component from the sloping sides of the embankment thus restrains the lateral deformation of the pile at location A.

Fig. 14 shows the lateral displacements for a rectangular pressure distribution i. e. when the horizontal and vertical components of the sloping sides of the embankment are ignored. In this case the lateral displacements are underestimated in the top half of the pile as well as the bending moments in the pile and in the pile cap (see Fig. 12, 13 and 14). On the other hand, the vertical component of the pressure caused by the sloping embankment side (Fig. 12) reduces the lateral deformation in the lower half of the pile at location A. The bending moments are thereby reduced in comparison with those shown in Fig. 14.

## 7. SUMMARY

Large lateral displacements can occur in the soil below, for example, a coal or ore stock pile which may damage the piles supporting e. g. an adjacent craneway. The lateral displacements have been calculated for an ideal elastic material with a constant modulus of elasticity ($E_s$) and Poisson's ratio ($\nu$). For a normally consolidated clay the modulus $E_s$ normally increases with depth.

The lateral displacements have been evaluated for $E_s$=1 000 kPa, and a maximum applied load of 1 kPa. Different values of (0.49, 0.40, 0.35 and 0.25) have been assumed in the analysis. The results indicate that Poisson's ratio ($\nu$) has a large effect on the lateral displacements particularly at the ground surface. Figs. 4 and 9 can be used to calculate the displacements at other values on the modulus of elasticity ($E_s$), the intensity of the applied load ($q$) and size of the loaded area ($L$) since the displacements will increase linearly with $1/E_s$, $q$ and $L$.

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