

## AN ANALYSIS OF FATIGUE CRACK GROWTH FROM BLOWHOLES IN LONGITUDINAL WELDED JOINTS

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An analysis of fatigue crack growth, using a fracture mechanics concept, and fatigue tests are performed to quantitatively estimate the effect of weld defect (blowhole) size and shape on the fatigue strength of partially penetrated longitudinal welded joints. In this analysis, a blowhole is regarded as an ellipsoidal cavity with small initial cracks along its wall. Number, interval, size and shape of initial cracks are determined by closely observed results on the fatigue crack initiation and propagation from the blowhole. The equivalent circular crack size for a blowhole is presented to predict fatigue life by using a simplified approach. The predicted fatigue life is compared with experimental results.

*Keywords : fatigue crack, blowhole, fracture mechanics*

### 1. INTRODUCTION

Fatigue test results of large sized truss structural models, which were conducted by the Honshu-Shikoku Bridge Authority, indicated the significance of weld defects such as blowholes in partially penetrated longitudinal welded joints<sup>1)</sup>. With this as a start, many studies had been performed on the fatigue strength of the joints<sup>2)-8)</sup>. These studies made clear that a blowhole at the weld root is often the origin of fatigue failure, and the fatigue strength decreased with increasing in the size of the blowhole. Based on these results, allowable size of blowholes was specified in the code for the construction of Honshu-Shikoku bridges<sup>9)</sup>. However, the effect of blowhole shape on the fatigue strength is not so clear yet in previous studies, and also the shape effect is not considered in the code.

Previous studies also indicated that fatigue cracks initiate and propagate at an early stage of fatigue life. It is therefore effective that a fracture mechanics approach is used to estimate the fatigue strength of joints containing blowholes. In this approach, fatigue crack initiation life is neglected and the joint is assumed to contain a crack-like defect. In the ASME code<sup>10)</sup> for assessing defects in boiler and pressure vessels, a blowhole is transformed into an elliptical crack circumscribed in the blowhole on the section perpendicular to the stress direction. Hirt et al.<sup>11)</sup> discussed on the crack model for estimating the influence of blowholes on fatigue strength. In Hirt's study, a blowhole was transformed into an inscribed circular crack, a circumscribed circular crack, a circumscribed elliptical crack and circular crack of which diameter is calculated from the equivalence of the stress intensity factor with the circumscribed elliptical crack. The

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first author et al.<sup>5)</sup> showed that the inscribed circular crack model was appropriate on the basis of the comparison between the experimental and analytical results of the fatigue life. However, blowholes are generally roundish in shape and some semi-elliptical fatigue cracks originate from their wall. These cracks then grow to coalesce with one another as shown in the previous studies. Until the crack grows to be circular or elliptical, the major part of fatigue life is consumed. Consequently, to estimate fatigue life more exactly, further discussions are needed on the transformation blowholes into cracks.

The purpose of this study is to quantitatively estimate the effect of size and shape of a blowhole on the fatigue strength of a partially penetrated longitudinal welded joint by using a fracture mechanics approach. In this approach, a blowhole is regarded as an ellipsoidal cavity with small initial cracks on the wall of the cavity. The size and shape of the initial cracks are determined on the basis of observed results on fatigue crack initiation and propagation behavior. Validity of this method is ascertained by the comparison between analytical and experimental results. By using the conception of this approach, blowholes of various sizes and shapes are transformed into circular cracks of appropriate diameters.

## 2. TEST PROCEDURE AND RESULTS

The specimens were fabricated from 16 mm thick JIS SM 50 Y steel plate (yield point=420 MPa, tensile strength=560 MPa). Partially penetrated weldings were done by submerged arc processes (46 kJ). The weld groove was a single bevel of 8 mm depth and 60 degree flank angle. In these weldings, means, such as spreading paint on the face of the groove and/or sticking tape on the back face of the groove and so on, were done to produce blowholes of various sizes and shapes. Specimens are flat plates with longitudinal welds along the middle as in Fig. 1. Fatigue tests were performed using Amsler-type and electro-hydraulic-servo-type testing machines. Fatigue stresses were pulsating tension of 20~30 MPa in minimum stress. Some specimens were tested under two-stage multi stress ranges, in which the stress range was reduced to half every certain number of cycles, keeping the maximum stress constant (beach mark test). The beach mark test resulted in the failure surface with particular marks which corresponded to that of reduced cyclic stresses.

Fig. 2 shows two examples of failure surfaces with beach marks. In both cases, fatigue cracks originate from the wall of the blowhole. These cracks then propagate to coalesce with one another as enclosing the blowhole. The crack is semi-elliptical in form before the enclosing and elliptical after the enclosing. And then, the crack grows to approach a circular form. To predict fatigue life accurately, this phenomena should be considered in a fracture mechanics approach.

Fig. 3 shows the results of fatigue tests plotting stress range ( $S_r$ ) on the ordinate and number of stress cycles to failure ( $N_r$ , fatigue life) on the abscissa. The data in this figure are classified according to the width ( $W$ ) and shape ratio ( $H/W$ ) of a blowhole appeared on the failure surface. The  $W$ -size and  $H$ -size of a blowhole are defined in Fig. 4. These results indicate that fatigue strength decreases as the  $W$ -size

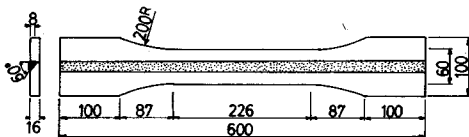


Fig. 1 Specimen configuration and dimensions.

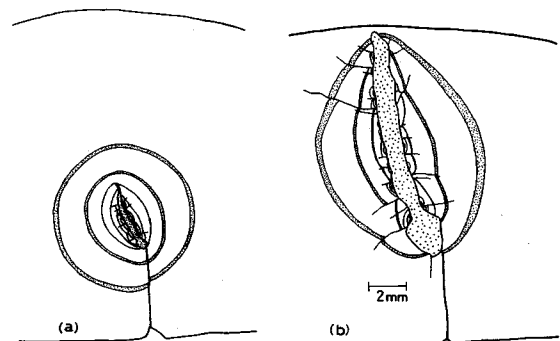


Fig. 2 Sketches of fracture surfaces with beach marks.

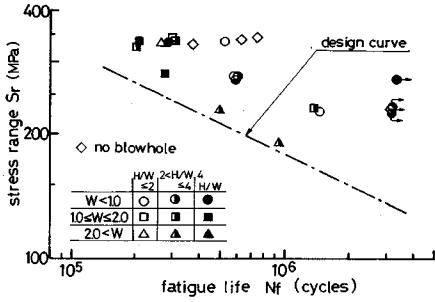


Fig. 3 Fatigue test results.

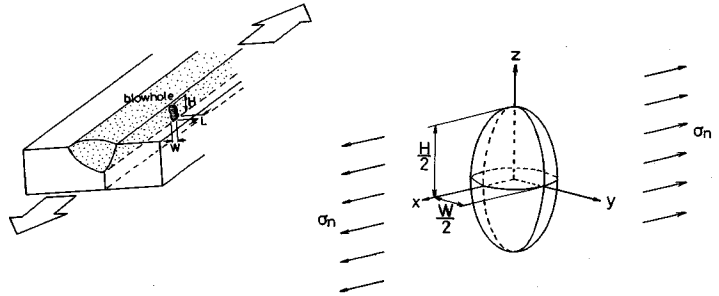


Fig. 4 Changing blowholes to ellipsoidal cavities.

becomes large and the shape ratio becomes high. The main purpose of this study is to verify the cause and degree of blowhole size and shape effect through a fracture mechanics approach.

### 3. STRESS CONCENTRATION AROUND A BLOWHOLE

After completion of fatigue tests, specimens were broken into two pieces along the weld line, then the weld roots were appeared. So it is able to observe blowholes and verify the existence of partially propagated cracks on the face of blowhole. Observed results show that a blowhole is a roundish cavity and its shape on a horizontal section is almost circular. Fatigue cracks initiated along the middle of the blowhole and are propagated on the plane perpendicular to the stress direction.

To estimate the stress concentration caused by a blowhole, which is modeled as a cavity in the shape of an ellipsoid of revolution as shown in Fig. 4. The stress distribution around the ellipsoidal cavity in an infinite body had been analyzed by Sadowsky et al.<sup>12)</sup> In case of uniaxial tension (*x*-direction) as shown in Fig. 4, significant stress concentration occurs on the *y-z* plane and fatigue cracks originate and propagate on this plane. Fig. 5 shows the stress distribution on the *y-z* plane. The stress depends entirely upon the nondimensional coordinates (*y*/*W*, *z*/*W*) and on the shape ratio (*H*/*W*). The magnitude of stress concentration becomes larger as the shape ratio being high. And also, the region, where stress concentration occurs, becomes wider the *W*-size being large and the shape ratio being high.

### 4. ANALYSIS OF FATIGUE CRACK GROWTH

#### (1) Stress intensity factor range

Fatigue cracks originated from blowholes are divided into semi-elliptical surface cracks and elliptical embedded cracks as shown in Fig. 6 (see Fig. 2). The ranges of stress intensity factors at point A and B of the surface crack in Fig. 6(a) are expressed by<sup>13), 14)</sup>

$$\left. \begin{aligned} \Delta K_A &= F_{eA} F_{SA} F_{gA} S_r \sqrt{\pi a} \\ \Delta K_B &= F_{eB} F_{gB} S_r \sqrt{\pi a} \end{aligned} \right\} \dots\dots\dots (1)$$

in which *S<sub>r</sub>* is a nominal stress range. *F<sub>eA</sub>* and *F<sub>eB</sub>* are correction factors for crack shape and *F<sub>SA</sub>* is a correction factor for free surface (blowhole)<sup>15), 16)</sup>. *F<sub>gA</sub>* and *F<sub>gB</sub>* are correction factors for stress concentration caused by a blowhole. The correction factors for stress concentration are calculated by a superposition approach in which stresses in the crack free body (see Fig. 5) are used to estimate stress intensity factor<sup>14)</sup>. Fig. 7 shows the relationship between the value of *F<sub>gA</sub>* and the depth of a crack originated at the equator point of a cavity as in Fig. 6(a), in which the width of cavity is 0.5, 1.0 and 2.0 mm and the shape ratio of a cavity is 1, 2 and 4. The value of *F<sub>gA</sub>* becomes high with increasing the width and shape ratio, which is similar to the stress distribution.

The stress intensity factor ranges at point A and B of the embedded elliptical crack in Fig. 6(b) are given by

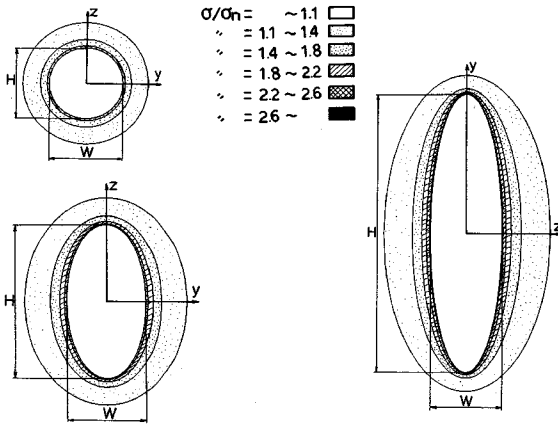


Fig. 5 Stress distribution around a blowhole.

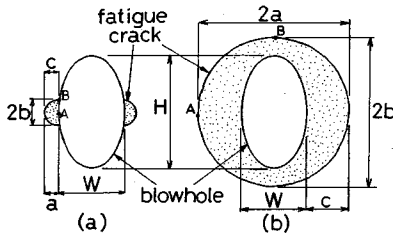


Fig. 6 Definition of crack size and shape.

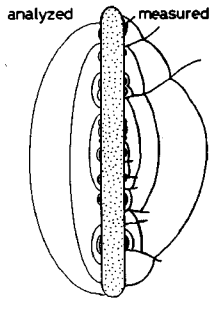


Fig. 9 Predicted result on fatigue crack growth.

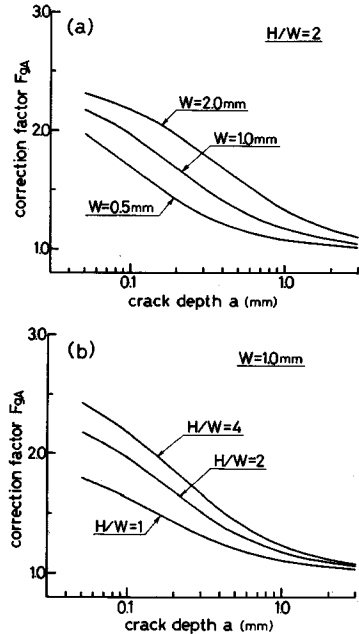


Fig. 7 Correction factor of stress intensity factor for stress concentration.

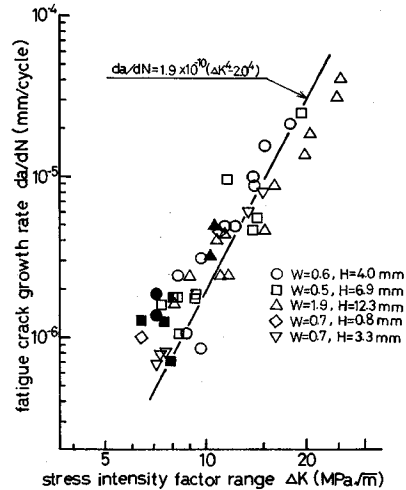


Fig. 8 Fatigue crack growth rate.

$$\left. \begin{aligned} \Delta K_A &= F_{eA} F_{tA} S_r \sqrt{\pi a} \\ \Delta K_B &= F_{eB} F_{tB} S_r \sqrt{\pi a} \end{aligned} \right\} \dots\dots\dots (2)$$

$F_{tA}$  and  $F_{tB}$  is the correction factor for finite plate width and thickness.

(2) Fatigue crack growth rate

Crack depth growth rate ( $da/dN$ ) is calculated as the space between beach marks (see Fig. 2) divided by the corresponding number of stress cycles. Stress intensity factor range at the deepest point of a crack ( $\Delta K_A$ ) is estimated by using equation (1) or (2). The values of  $da/dN$  and  $\Delta K_A$  are plotted in Fig. 8. The solid line in this figure shows typical  $da/dN-\Delta K$  relationship which was derived from many data of steel tested by using standard specimens such as CT types and CCT types<sup>17)</sup>. The solid marks are the data on small cracks with depths less than 0.1 mm.

The  $da/dN-\Delta K$  data obtained from a beach marks observation show good linear relationship on logarithmic coordinates and agree approximately with the solid line regardless of blowhole size and shape. This result indicates that the equation (1) and (2) are valid to calculate the stress intensity factor for a crack originated from a blowhole. The growth rates of the small cracks (solid marks) do not differ so much from those of large cracks (hollow marks). Therefore, the  $da/dN-\Delta K$  relationship indicated by the solid line is employed to calculate fatigue crack growth from a blowhole and is expressed by equation (3).

$$da/dN = 1.9 \times 10^{-10} (\Delta K^4 - \Delta K_{th}^4) \dots \dots \dots (3)$$

In this equation,  $\Delta K_{th}$  is threshold  $\Delta K$  for fatigue crack growth and is set at  $2 \text{ MPa} \sqrt{m}$  on the basis of the data obtained by welded joints<sup>(8)</sup>.

(3) Method of crack growth analysis

The process of fatigue crack growth analysis is done in the following.

- (i) Determine the number, intervals, sizes and shapes of initial cracks.
- (ii) Estimate  $\Delta K_A$  and  $\Delta K_B$  for the crack by equation (1) or (2).
- (iii) By substituting  $\Delta K_A$  and  $\Delta K_B$  into equation (3), calculate the increments of crack depth and width ( $\Delta a$ ,  $\Delta b$ ) caused by a certain number of stress cycles ( $\Delta n$ ).
- (iv) Determine the new crack size ( $a+\Delta a$ ,  $b+\Delta b$ ).
- (v) Return to (ii)

Crack growth life can be obtained by repeating this process until the crack reaches a critical size.

In case of several fatigue cracks initiating on the wall of a blowhole, adjacent two semi-elliptical cracks,  $(a_1, b_1)$  and  $(a_2, b_2)$ , are considered to coalesce when the edges of these two cracks come into contact. It is assumed that the coalesced crack is semi-elliptical in form, its depth  $a_3$  is equal to a larger one of  $a_1$  and  $a_2$ , and its width  $b_3$  is equal to the sum of  $b_1$  and  $b_2$ . The interaction effect of an adjacent crack on the stress intensity factor for the crack is neglected.

Fig. 9 shows the comparison of the predicted results with experimental results regarding crack growth behavior (Fig. 2(b)), and indicates that the fracture mechanics approach in this study is effective in estimating fatigue crack growth from a blowhole.

(4) Initial cracks

The size of initial cracks greatly influences the calculated life. In order to determine the appropriate size of an initial crack, the following two conditions need to be taken into consideration, (i) a fracture mechanics approach is applicable to an initial crack, that is, equation (3) is valid for the growth from the initial crack, and (ii) the fatigue crack growth life after the crack has been formed occupies the greater

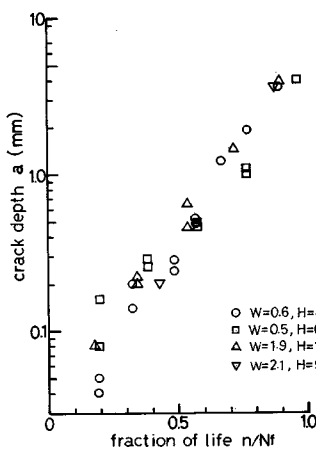


Fig. 10 Relationship between crack depth and fraction of life.

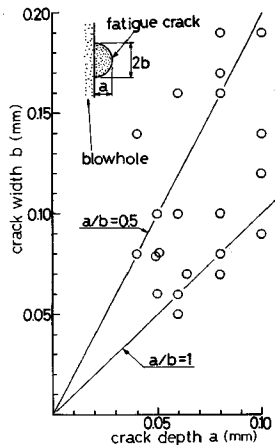


Fig. 11 Aspect ratio of small crack.

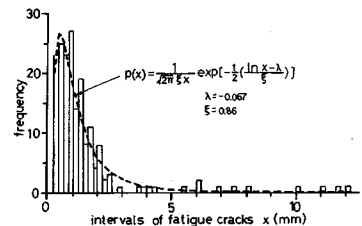


Fig. 12 Histograms for fatigue crack intervals.

part of the total fatigue life.

Fig. 10 shows the relationship between the depth of crack ( $a$ ) and the fraction of life ( $n/N_f$ ) obtained by beach mark observations. On the life fraction of 0.2, a 0.1 mm deep crack has been already formed. Therefore, most of the fatigue life can be predicted on the condition that the initial crack depth is less than 0.1 mm. Fig. 11 indicates the aspect ratios ( $a/b$ ) of fatigue cracks of which depths are less than 0.1 mm. Most of data in this figure are plotted within 0.5 to 1.0 of the aspect ratio. The smallest depth of these cracks is 0.04 mm, and the crack growth law (eq. (3)) is valid for such small cracks as shown in Fig. 8. Fig. 12 shows the histogram for intervals of fatigue cracks originating points on the wall of a blowhole. This histogram is approximately logarithmic normal distribution in form.

From these experimental results, initial cracks are assumed to be as follows. (i) The depth of a initial crack ranges from 0.05 mm to 0.1 mm and is uniformly distributed. (ii) The aspect ratio ranges from 0.5 to 1.0 and is uniformly distributed. (iii) The interval is log-normally distributed as in Fig. 12. Under these assumptions, the depths, aspect ratios, number and intervals of initial cracks are determined on the wall of a blowhole by using Monte Carlo simulation.

The critical size ( $2b_{cr}$ ) of a crack is set at 80% of joint thickness, which does not predominantly influence the predicted life.

### 5. INFLUENCE OF BLOWHOLE SIZE AND SHAPE ON FATIGUE LIFE

#### (1) Predicted results

Fatigue lives are predicted for joints containing blowholes of various sizes and shapes. The results are plotted in Fig. 13. In this prediction, a hundred simulations were performed to determine initial cracks on blowhole walls, so a hundred fatigue lives are calculated for a joint. The circular marks in Fig. 13 indicate the logarithmic mean values of predicted fatigue lives. In this figure, the points away from the mean life by one logarithmic standard deviation are also presented.

Predicted life decreases linearly as the width of a blowhole becomes large on logarithmic coordinates, and increasing the width of blowhole by two times reduces the fatigue life to half. When the width is constant, the life decreases as the shape of a blowhole becomes slender. Duplicating the shape ratio ( $H/W$ ) reduces

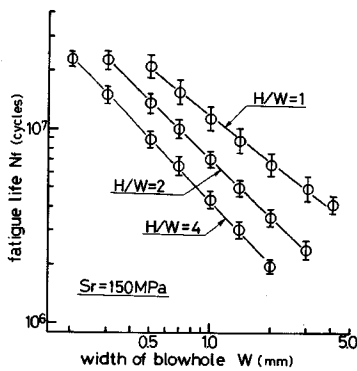


Fig. 13 Predicted fatigue life.

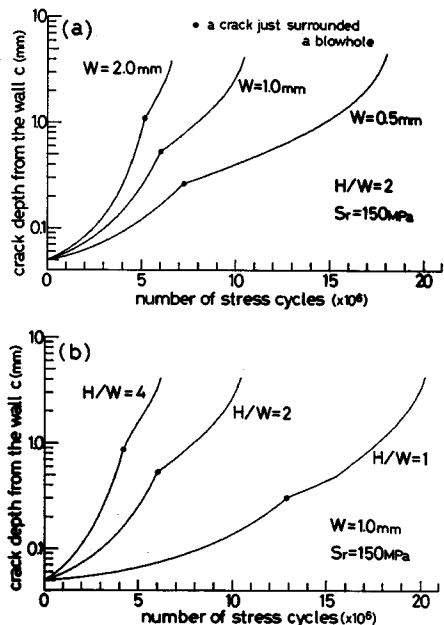


Fig. 14 Predicted fatigue crack growth curve.

the life to  $2/3$  to  $1/2$ .

The reason for this blowhole size effect is considered as follows. If the shape ratio of a blowhole ( $H/W$ ) is constant, the stress around a blowhole depends predominantly upon the ratio of crack depth to the blowhole width ( $a/W$ ), so the stress intensity factor for a given crack from a large blowhole is higher than for the same-sized crack from a small blowhole, which were shown in Fig. 5 and 7 before. Accordingly, the crack growth rate is higher and the fatigue life reduces with increasing blowhole size. Fig. 14(a) shows the relationship between the crack depth from the wall of a blowhole ( $c$ , see Fig. 6) and the number of stress cycles where initial cracks are assumed to exist on the both equator points of a blowhole as in Fig. 6(a). From this figure, the blowhole size effect is clearly seen. This blowhole size effect is similar to the plate thickness effect in geometrically similar transverse fillet welded joints<sup>(19), (20)</sup>.

Since the stress magnitude and distribution are also influenced by the shape ratio of a blowhole, the fatigue life varies with the shape ratio as shown in Fig. 14(b).

## (2) Comparison with experimental results

Fig. 15 shows the experimental results for the relationship between the fatigue life and the blowhole width under a stress range of about 335 MPa (see Fig. 3). In this figure, data related to partially propagated cracks are also plotted, in which the fatigue life is defined as the sum of stress cycles until and after the partially propagated crack being formed. The latter one is calculated by using the crack growth analysis. The number of partially propagated cracks larger than 1.0 mm in depth occupied about 50 % and the number of cracks larger than 0.5 mm was nearly 70 %. As shown in Fig. 10, the life after the formation of 1.0 mm and 0.5 mm deep crack occupied only 30 % and 45 % of the fatigue life. Fatigue crack growth behavior and fatigue life after a given crack can be calculated well as shown in Fig. 9. Therefore, the fatigue lives calculated on the partially propagated cracks are considered to be almost equal to the actual fatigue lives.

Three solid lines in Fig. 15 show the predicted  $N_f$ - $W$  relations for blowholes of which shape ratios are 1, 2 and 4. These lines express well the rate of fatigue life reducing as increasing in the size and shape ratio of a blowhole.

## 6. TRANSFORMATION OF BLOWHOLES INTO CIRCULAR CRACKS

In the ASME code, blowholes are transformed into circumscribed elliptical cracks and the cracks are assumed to grow in similar form. Fig. 16 shows the predicted fatigue life versus blowhole width relationship by means of the ASME code and by regarding blowholes as ellipsoidal cavities which was shown in the previous section. The predicted life by means of the ASME code is very susceptible to the  $W$ -size and the shape ratio, compared with the predicted life by using the method in this study.

As described, it is not appropriate for the blowhole to be transformed into a circumscribed elliptical crack when estimating the effect of blowhole size. But, for simplicity in crack growth analysis, it is useful that the blowhole is transformed into a simple shaped crack and the crack is growing in similar form. Accordingly, the diameter of initial circular crack ( $2a_e$ ) is calculated so that the predicted life in this condition is equal to that obtained by using the ellipsoidal cavity of which width is  $W$  and height is  $H$ . Doing multiple linear regression on the logarithms of  $2a_e$ ,  $W$  and  $H$ , the following equation is obtained.

$$2a_e = 0.90 \times W^{0.22} \times H^{0.47} \dots \dots \dots (4)$$

With regard to blowholes which caused failure or possessed partially propagated cracks, equivalent diameters of circular cracks ( $2a_e$ ) are calculated. Fig. 17(a), (b), (c) show the  $S_r$ - $N_f$  relationship arranged by  $2a_e$ . Predicted Results shown in solid lines approximately express the tendency of the plotted data, and also the data arranged by  $2a_e$  are not influenced by the blowhole shape ratio. Equation (4) is therefore considered to be effective in estimating the effect of blowhole size and shape on fatigue life by using a simplified fracture mechanics approach.

Many fatigue tests on large sized longitudinal welded joints of high tensile strength steel (600, 800 MPa)

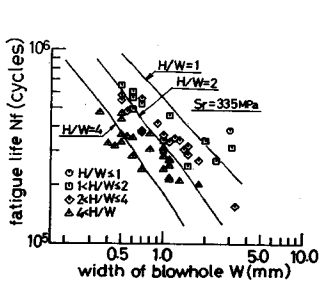


Fig. 15 Comparison of predicted fatigue life with experimental results.

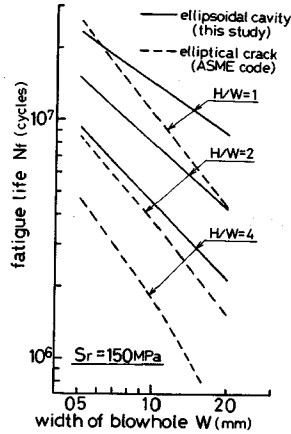


Fig. 16 Predicted fatigue life according to ASME code.

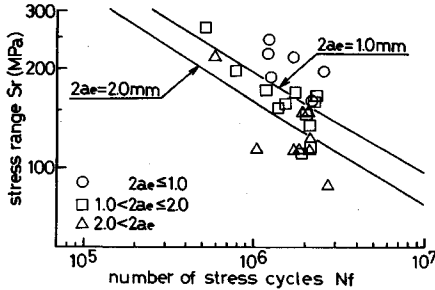


Fig. 18 Predicted and experimental fatigue lives (600 and 800 MPa class steel welds).

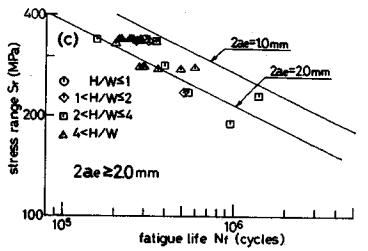
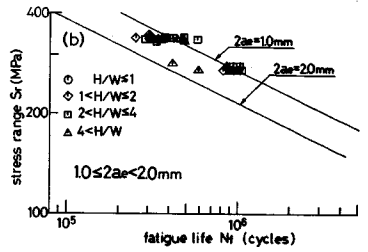
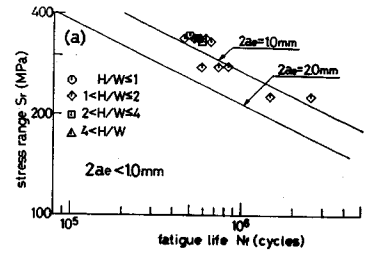


Fig. 17 Predicted and experimental fatigue lives (500 MPa class steel welds).

were conducted by the Honshu-Shikoku Bridge Authority. Analysis to change blowholes to circular cracks is also done about these joints, and the following relationship is obtained.

$$2 a_e = 0.94 \times W^{0.29} \times H^{0.48} \dots \dots \dots (5)$$

In this analysis, equation (6) was employed as the fatigue crack growth law<sup>17), 18)</sup>.

$$da/dN = 5.4 \times 10^{-9} (\Delta K^3 - 2^3) \dots \dots \dots (6)$$

Fig. 18 shows the predicted  $S_r-N_f$  relationship and corresponding experimental data. This figure shows the usefulness of equation (5).

7. CONCLUSION

(1) The fatigue life of a longitudinal welded joint decreases as the size and shape of a blowhole becomes large and slender. These are caused by that the magnitude of stress becomes higher with an increase in the shape ratio, and the high stress region becomes wider with increase in the size and shape ratio of a blowhole.

(2) Some semi-elliptical fatigue cracks are originated from the blowhole walls at early stage of fatigue life. These cracks grow to coalesce with one another, then grow to enclose the blowhole. The relationship between the rate of fatigue crack growth and the range of stress intensity factor is almost equal to the relationship obtained by using standard specimens for the examination of the fatigue crack growth rate.

(3) The fatigue crack growth is analysed by a fracture mechanics approach on the condition that blowholes are regarded as ellipsoidal cavities with small initial cracks. The analytical results coincide well with the experimental results.



(4) A blowhole of width  $W$  and height  $H$  can be changed to a circular crack of  $2 a_e$  diameter to predict fatigue life, using a fracture mechanics approach.

$$2 a_e = 0.90 \times W^{0.22} \times H^{0.47} \quad (500 \text{ MPa class steel joints})$$

$$2 a_e = 0.94 \times W^{0.29} \times H^{0.48} \quad (600 \text{ and } 800 \text{ MPa class steel joints})$$

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