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NATURAL FREQUENCY OF A FILL DAM BY MEANS OF TWO DIMENSIONAL TRUNCATED WEDGE TAKING SHEAR AND BENDING MOMENT EFFECTS INTO ACCOUNT

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The discussion, in this paper, is confined to the natural vibration of the fill dam which can be described by the truncated wedge of inhomogeneous rigidity : $G=G_m \cdot (z/h)^n$ where G : modulus of rigidity, z : distance from the top of dam, h : dam height, n : rigidity index, and with rectangular canyon shape. Equilibrium of the bending moment and the shearing force in the vertical direction, together with the shearing force and twisting moment in the transverse direction, leads to the fundamental dynamic equations of the prescribed model of dam.

Finite difference method is used to determine the natural frequencies and natural modes of the dam. The frequencies and the modes are computed for the various rigidity index n, the canyon width and the bottom width, the results are put in order in graphical or tabular forms.

Keywords : fill dam, natural frequency, inhomogeneous beam

1. INTRODUCTION

Many researches have so far been carried on the dynamic response of fill type dams, however, the theoretical results of fundamental modes by the shear beam model of a constant rigidity[1] does not always agree to experimental ones. On this viewpoint, Okamoto[2] pointed out to estimate the dynamic response by taking the varying rigidity in the direction of dam height and in the direction of up-down stream into account. Sawada et al.[3] studied on dynamic characteristics of a few large scale rock fill dams in Japan by carrying out field measurement of stress wave velocity and forced and / or ambient vibration tests. They have concluded that 1) the shear wave velocity increases in proportion to the 1/3 power of the distance from the crest, 2) lateral vibration modes are apparently influenced by the canyon shape. That is to say, fundamental mode of the rock fill dam located on U shape canyon is similar to the mode of shear beam, whereas the mode of the dam located on the V shape canyon is different from the former one and has an amplitude strongly partial near to the crest of the dam. Gazetas[4] demonstrated that shear rigidity varied in proportion to the 2/3 power of the distance from the crest, and it quite agreed to the field test results measured by Abdel-Ghaffar et al.[5] And they have developed the new inhomogeneous shear beam theory for the fill dam from the above results. Thus introduced power index is the same as proposed by Sawada et al.[3] Oka et al.[6] analyzed the natural vibration of earth dams by means of two dimen-

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sional multi-degrees of lumped mass system. They found that natural frequency of the dam on the narrow canyon is larger than the one for the dam having infinite width. It is because that the canyon wall more or less restrains the deformation of the dam.

In order to estimate the dynamic behaviors of the fill dams on any arbitrary canyon shape more easily and exactly, Ohmachi et al.[7),8)] proposed a new method of computation. They modeled the dam as the spatial shear beam with varying cross section and assumed that shape function along the crest is linear variation and the one along the depth is the same variation as the mode of the dam with infinite width. Kokusho et al.[9)] indicated that the dam such as fill type ones vibrates from shear action predominantly and bending a little. They analyzed the vibration of homogeneous infinite wedge beam considering shear and bending by Rayleigh-Ritz technique.

As the materials composing fill dams have been adequately modeled, the corresponding a few methods of dynamic analysis for fill type dams have been developed. If the bottom width of the dam in the up-down stream direction or the length in the crest-wise direction become smaller than the dam height, bending component might take more place in the vibration modes than Kokusho et al.[9)] indicated. However theories developed so far, are almost based on only shearing action and few, on bending action as well.

This paper attempts to introduce the frequency equations of the dam by a rectangular inhomogeneous truncated wedge accounting the bending moment as well as shearing force in the vertical direction and the twisting moment as well as the shearing force in the crest-wise direction. Furthermore, the rigidity variation with the depth is put in the equations. The objective is to make clear whether the dynamic analysis of inhomogeneous shear theory is able to fit the vibration of the fill type dams.

2. ANALYTICAL PROCEDURE

(1) Formulation of the frequency equations

The cross sectional shape of the dam on the rectangular canyon is shown in **Fig. 1** as a truncated wedge of which crest height is h, wedge height is H, bottom width is B and canyon width is L. The x, y and z coordinates are taken in the crest-wise, up-down stream and vertical directions respectively. And the displacements in the above mentioned directions are u, v and w. Furthermore, the displacement components are assumed to be linear variation of y, namely

$$u=0 \quad \cdots\cdots (1\cdot a)$$

$$v=v(x,\ z) \quad \cdots\cdots (1\cdot b)$$

$$w=y\cdot\phi(x,\ z) \quad \cdots\cdots (1\cdot c)$$

The forces on the cross sections of the infinitesimal body and its inertia forces are shown in **Fig. 2**, and they must be in equilibrium in the y and x directions,

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_z}{\partial z}=\rho\cdot B_z\frac{\partial^2 v}{\partial t^2} \quad \cdots\cdots (2)$$

$$\frac{\partial M_{zx}}{\partial x}-Q_z+\frac{\partial M_z}{\partial z}=\rho\cdot I_z\frac{\partial^2 \phi}{\partial t^2} \quad \cdots\cdots (3)$$

where ρ : unit mass of the dam, B_z : dam width along up-down stream at z,

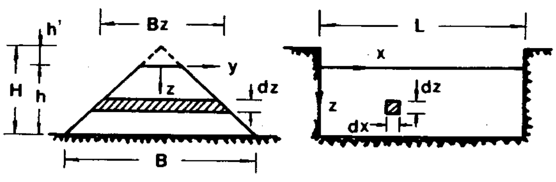

Fig. 1 Two-dimensional truncated wedge modeling of dam geometry.

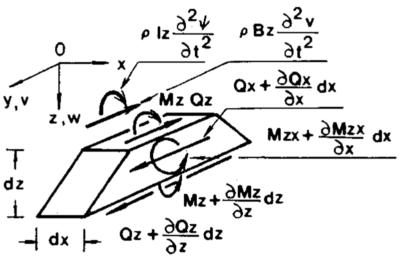

Fig. 2 All forces acting on infinitesimal body.

Q_z, Q_x : shearing forces acting on the planes $x-y$ and $y-z$ respectively,
M_z, M_{zx} : the bending and twisting moments by σ_z, τ_{zx} respectively.
The forces of the cross sections are related to the displacements as follows;

$$Q_z = G_z \cdot B_z \cdot \left(\psi + \frac{\partial v}{\partial z}\right) \qquad (4\cdot a)$$

$$Q_x = G_z \cdot B_z \cdot \frac{\partial v}{\partial x} \qquad (4\cdot b)$$

$$M_z = E_z \cdot I_z \cdot \frac{\partial \psi}{\partial z}, \quad M_{zx} = G_z \cdot I_z \cdot \frac{\partial \psi}{\partial x} \qquad (4\cdot c,\ d)$$

which is demonstrated by Nomachi et al.[10] for assuming that the rigidity indices of Q_z and also Q_x are 1.0 to simplify formulating procedure.

Assuming that two vertical sides of the truncated wedge are fixed to the rectangular canyon through out the harmonic motion, it follows that v and ψ must be zero at all times for each point of the wedge on these boundaries. Then supposing that the displacements and sectional forces vibrate with the m-th trigonometric distribution in the x direction and angular velocity p, they are represented as follows;

$$v = V \sin\left(\frac{m\pi x}{L}\right)\cdot \sin pt, \qquad \psi = \boldsymbol{\Psi} \sin\left(\frac{m\pi x}{L}\right)\cdot \sin pt \qquad (5\cdot a,\ b)$$

$$Q_z = \boldsymbol{Q}_z \sin\left(\frac{m\pi x}{L}\right)\cdot \sin pt, \quad Q_x = \boldsymbol{Q}_x \cos\left(\frac{m\pi x}{L}\right)\cdot \sin pt \qquad (5\cdot c,\ d)$$

$$M_z = \boldsymbol{M}_z \sin\left(\frac{m\pi x}{L}\right)\cdot \sin pt, \quad M_{zx} = \boldsymbol{M}_{zx} \cos\left(\frac{m\pi x}{L}\right)\cdot \sin pt \qquad (5\cdot e,\ f)$$

To make further discussion simple, we introduce the new function regarding ψ, the non-dimensional parameters of coordinates and aspect ratios of the dam;

$$z^2 \boldsymbol{\Psi} = \theta \qquad (6)$$

$$\xi = x/h, \qquad \eta = z/h \qquad (7\cdot a,\ b)$$

$$\gamma_h = h'/h, \qquad \gamma_L = h/L \qquad (7\cdot c,\ d)$$

$$\gamma_B = H/B, \qquad K' = h'/H \qquad (7\cdot e,\ f)$$

$$\mu = 1/(12\cdot \gamma_B^3) \qquad (7\cdot g)$$

Hence,

$$B_z = (\eta + \gamma_h)\cdot h/\gamma_B, \quad I_z = (\eta + \gamma_h)^3 \cdot \mu \cdot h^3 \qquad (8\cdot a,\ b)$$

We assume rigidity indices for shear and Young's moduli as n.

Then

$$G_z = G_m \cdot \eta^n, \quad E_z = E_m \cdot \eta^n \qquad (9\cdot a,\ b)$$

where G_m and E_m are shear and Young's moduli at the bottom of the dam.

In Eqs. (9·a) and (9·b), $n=0$ means constant rigidity distribution, $n=1/3$ is the value predicted by Iida[11] for grains of sand, and $n=2/3$ is the value that Sawada et al.[3] and Abdel-Ghaffar et al.[5] obtained experimentally and also Gazetas[4] has used in inhomogeneous shear beam model.

We denote displacement amplitudes and circular frequency by following form;

$$\boldsymbol{V} = G_m \cdot V, \quad \boldsymbol{\Theta} = G_m \cdot \theta / h \qquad (10\cdot a,\ b)$$

$$\beta = p \cdot h / C_S \qquad (10\cdot c)$$

where $C_S = \sqrt{G_m/\rho}$

Substitution of Eqs. (4)~(10) into Eqs. (2) and (3) yields the following two differential equations regarding $\boldsymbol{Q}_z$, and $\boldsymbol{Q}_x$;

$$\frac{d^2 \boldsymbol{Q}_z}{d\eta^2} - \left(\frac{\eta}{\eta+\gamma_h}\right)\frac{1}{\eta}\frac{d\boldsymbol{Q}_z}{d\eta} + \beta^2 \frac{1}{\eta^n}\boldsymbol{Q}_z = \beta^2 \frac{1}{\gamma_B}\left(\frac{\eta+\gamma_h}{\eta}\right)\frac{1}{\eta}\boldsymbol{\Theta} + (m\pi\gamma_L)\left\{\frac{d\boldsymbol{Q}_x}{d\eta} - \left(\frac{\eta}{\eta+\gamma_h}\right)\frac{1}{\eta}\boldsymbol{Q}_x\right\} \qquad (11)$$

$$\frac{d^2 \boldsymbol{\Theta}}{d\eta^2} - \left\{(1-n) + \frac{3\gamma_h}{\eta+\gamma_h}\right\}\frac{1}{\eta}\frac{d\boldsymbol{\Theta}}{d\eta} + \left\{\beta^2 \frac{G_m}{E_m}\frac{1}{\eta^n} - \left(2n - \frac{6\gamma_h}{\eta+\gamma_h}\right)\frac{1}{\eta^2} - (m\pi\gamma_L)^2 \frac{G_m}{E_m}\right\}\boldsymbol{\Theta}$$

$$=\frac{G_m}{E_m}\frac{1}{\mu}\left(\frac{\eta}{\eta+\gamma_h}\right)^3\frac{1}{\eta^{n+1}}Q_z \cdots\cdots (12)$$

The relationship among Q_x, Q_z and Θ is written as

$$\frac{dQ_x}{d\eta}-\left(n+\frac{\eta}{\eta+\gamma_h}\right)\frac{1}{\eta}Q_x=(m\pi\gamma_L)\left[Q_z-\frac{1}{\gamma_B}\left(\frac{\eta+\gamma_h}{\eta}\right)\eta^{n-1}\Theta\right] \cdots\cdots (13)$$

Especially, for $n=0$ and $\gamma_h=0$, Eqs. (11) ~ (13) are reduced to as follows;

$$\frac{d^2Q_z}{d\eta^2}-\frac{1}{\eta}\frac{dQ_z}{d\eta}+\bar{\beta}^2Q_z=\bar{\beta}^2\frac{1}{\gamma_B}\frac{1}{\eta}\Theta \cdots\cdots (14)$$

$$\frac{d^2\Theta}{d\eta^2}-\frac{1}{\eta}\frac{d\Theta}{d\eta}+\bar{\beta}^2\frac{G_m}{E_m}\Theta=\frac{G_m}{E_m}\frac{1}{\mu}\frac{1}{\eta}Q_z \cdots\cdots (15)$$

where $\bar{\beta}^2=\beta^2-(m\pi\gamma_L)^2$ ⋯⋯ (16)

Eqs. (14) and (15) have the same expressions as two-dimensional shear beam theory with $n=0$ derived by Ohmachi et al. [8] That is to say, natural vibration modes by them for the dam on the rectangular canyon with any width L are almost same with the one for the dam on infinite width canyon. And their natural frequencies are easily found by Eq. (16).

Eqs. (11) ~ (16) are frequency equations relating to two-dimensional truncated wedge beam taking bending moments and shearing forces into account.

Putting $\Theta=0$ in these equations leads to the frequency equations of two-dimensional inhomogeneous truncated shear beam theory.

(2) Determination of eigen value equations using finite difference method

In order to solve Eqs. (11) ~ (16), we will take the method of finite difference. As for the boundary conditions, natural boundary condition is to be taken in the x direction as discribed in Eqs. (5) and the conditions in the z direction are satisfied by

for $\eta=0$, (at the crest)

$Q_z=0$, $M_z=0$ ⋯⋯ (17·a, b)

for $\eta=1$, (at the bottom)

$\Theta=0$, $V=0$ ⋯⋯ (18·a, b)

The division number along the depth is taken as k, reverse of which becomes interval $(\Delta\eta)$ between lattice points. Let f_i be the value of function f at any lattice point i, then we find that truncation error is less than $\frac{1}{5}\Delta\eta^4\frac{\partial^5 f}{\partial\eta^5}$ and each differential scheme is described as follows;

for $2\leq i\leq k-2$

$$d^2f/d\eta^2=(-f_{i-2}+16f_{i-1}-30f_i+16f_{i+1}-f_{i+2})/(12\Delta\eta^2) \cdots\cdots (19)$$

$$df/d\eta=(f_{i-2}-8f_{i-1}+8f_{i+1}-f_{i+2})/(12\Delta\eta) \cdots\cdots (20)$$

for $i=0$

$$df/d\eta=(-25f_0+48f_1-36f_2+16f_3-3f_4)/(12\Delta\eta) \cdots\cdots (21)$$

for $i=1$

$$d^2f/d\eta^2=(11f_0-20f_1+6f_2+4f_3-f_4)/(12\Delta\eta^2) \cdots\cdots (22)$$

$$df/d\eta=(-3f_0-10f_1+18f_2-6f_3+f_4)/(12\Delta\eta) \cdots\cdots (23)$$

for $i=k-1$

$$d^2f/d\eta^2=(-f_{k-4}+4f_{k-3}+6f_{k-2}-20f_{k-1}+11f_k)/(12\Delta\eta^2) \cdots\cdots (24)$$

$$df/d\eta=(-f_{k-4}+6f_{k-3}-18f_{k-2}+10f_{k-1}+3f_k)/(12\Delta\eta) \cdots\cdots (25)$$

for $i=k$

$$df/d\eta=(3f_{k-4}-16f_{k-3}+36f_{k-2}-48f_{k-1}+25f_k)/(12\Delta\eta) \cdots\cdots (26)$$

The boundary values V_0 and $Q_{z,k}$ are derived by using above differential coefficient equations (21) and (26) for Eqs. (17) and (18) as follows;

$$V_0=\frac{(48\ V_1-36\ V_2+16\ V_3-3\ V_4)}{25}+\frac{12\ \Delta\eta}{625}\left(\frac{48\ \Theta_1}{\eta_1^2}-\frac{36\ \Theta_2}{\eta_2^2}+\frac{16\ \Theta_3}{\eta_3^2}-\frac{3\ \Theta_4}{\eta_4^2}\right)\cdots\cdots (27)$$

$$Q_{z,k}=(-3\ Q_{z,k-4}+16\ Q_{z,k-3}-36\ Q_{z,k-2}+48\ Q_{z,k-1})/25\cdots\cdots (28)$$

Transforming Eqs. (11) ~ (13) into finite difference equations by using Eqs. (19) ~ (28), frequency equations are represented by the following matrix equations

$$[K_{zz}]\{Q_z\}-\beta^2[M_{zz}]\{Q_z\}=-\beta^2[M_{z\theta}]\{\Theta\}+(m\pi\gamma_L)[K_{zx}]\{Q_x\}\cdots\cdots (29)$$

$$[K_{\theta\theta}]\{\Theta\}-\beta^2[M_{\theta\theta}]\{\Theta\}=[K_{\theta z}]\{Q_z\}\cdots\cdots (30)$$

$$[K_{xx}]\{Q_x\}=(m\pi\gamma_L)([K_{xz}]\{Q_z\}-[K_{x\theta}]\{\Theta\})\cdots\cdots (31)$$

Solving the above equations simultaneously, we have

$$[K]\{\Delta\}-\beta^2[M]\{\Delta\}=0\cdots\cdots (32)$$

The fact that $\{\Delta\}\neq 0$ leads to the eigen value equation as follows;

$$det\ |[K]-\beta^2[M]|=0\cdots\cdots (33)$$

Now we can obtain eigen value and eigen vector from Eq. (33), in which

$$[K]=\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix},\quad [M]=\begin{bmatrix} [M_{\theta\theta}] & 0 \\ -[M_{z\theta}] & [M_{zz}] \end{bmatrix}\cdots\cdots (34,\ 35)$$

$$\{\Delta\}=[\Theta \quad Q_z]^T\cdots\cdots (36)$$

where $k_{11}=[K_{\theta\theta}]$, $k_{12}=-[K_{\theta z}]\cdots\cdots$ (37・a, b)

$$k_{21}=(m\pi\gamma_L)^2[K_{zx}][K_{xx}]^{-1}[K_{x\theta}]\cdots\cdots (37\cdot c)$$

$$k_{22}=[K_{zz}]-(m\pi\gamma_L)^2[K_{zx}][K_{xx}]^{-1}[K_{xz}]\cdots\cdots (37\cdot d)$$

Eqs. (33) ~ (37) yield the vectors $\{\Theta\}$ and $\{Q_z\}$ and they are associated with the displacement vector $\{V\}$ by Eq. (4) which is expressed by

$$[K_{sz}]\{Q_z\}=[K_{s\theta}]\{\Theta\}+[K_{sv}]\{V\}\cdots\cdots (38)$$

we will leave out the detail of the matrices in Eq. (38) here.

(3) Convergency of the numerical results

Convergency check of eigen values is carried on by comparing the results for various division numbers. Thus we calculated the eigen value of the dam modeled by the truncated wedge beam which is specified as $B/H=4$, $L/H=3$, $K'=0.2$ and rigidity power index $n=2/3$. The results are tabulated on Table 1, from which we determine to take 20 for k in numerical examples.

Table 1 Convergence of eigen value β for division number on the case of $n=2/3$, $B/H=4$, $L/H=3$, $K'=0.2$.

m	m = 1			m = 3		
k	1	2	3	1	2	3
5	2.253	4.444	5.949	2.915	4.931	6.351
10	2.240	4.461	5.941	2.888	4.913	6.310
15	2.241	4.460	5.904	2.894	4.906	6.243
20	2.241	4.463	5.885	2.889	4.908	6.238
25	2.241	4.465	5.875	2.893	4.909	6.224
30	2.241	4.468	5.869	2.890	4.911	6.218
40	2.241	4.470	5.865	2.891	4.913	6.211

3. NUMERICAL RESULTS AND DISCUSSION

(1) The effects of bottom width of the dam

Non-dimensional natural frequency β of the wedge with different rigidity index n for infinite canyon width are shown in Fig. 3, in which β is coordinated by the abscissa $B/5\ H$ and $5\ H/B$, to cover all the value of $B/5\ H$. The straight line standing from the origin corresponds to the result of the wedge for $n=0$, by the bending beam theory and every β tends to the result for $n=1$ when $B/5\ H$ is smaller than 0.1. Another limit where $5\ H/B=0$, coincides with the natural frequency referring to shear stratum of the rigidity index n. In case of $n=1$, the first and second eigen values may be related to both bending and shearing actions within $B/5\ H\fallingdotseq 0.1 \sim 5\ H/B\fallingdotseq 0.5$. In the case of $n\neq 1$, both actions take place for $B/5\ H\fallingdotseq 0.1 \sim 5\ H/B\fallingdotseq 0.8$ in the first eigen values and the region for both actions seems to expand in the second eigen values compared with the first ones. Consequently, being treated the dam as wedge beam of infinite canyon width, especially the dam for $B/H=1.0$ like a conerete gravity type, we must design it by taking bending moments and shearing forces into account.

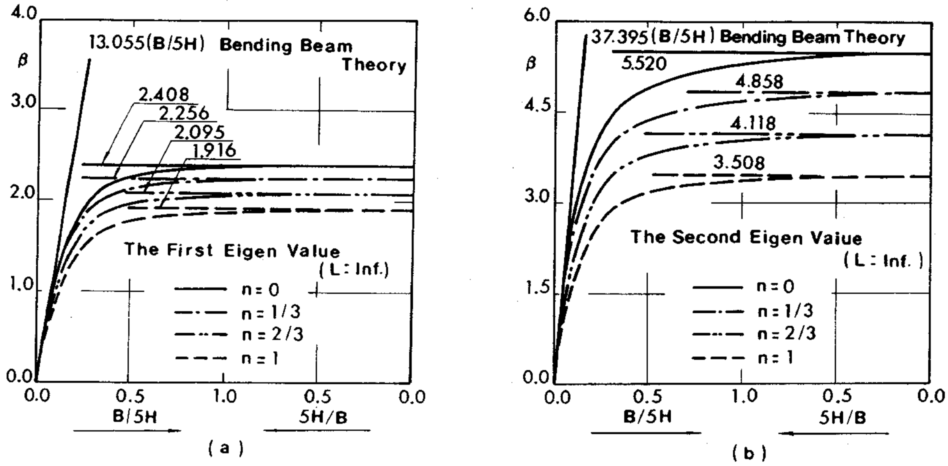

Fig. 3 Eigen value β of wedge with infinite length.

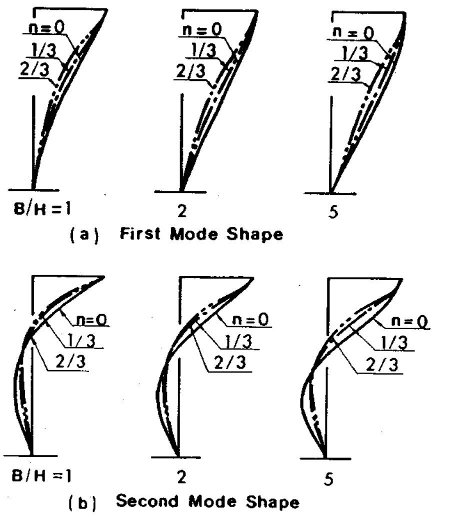

Fig. 4 V mode shapes along depth of wedge in case of $L/H=3$.

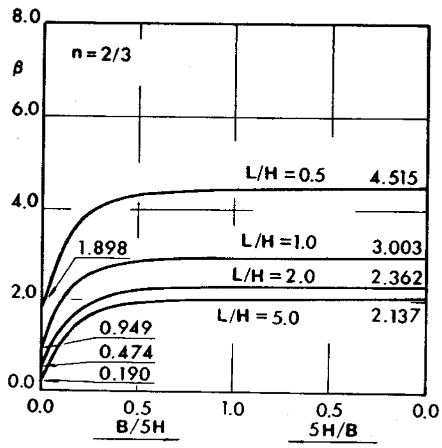

Fig. 5 Variation of the first eigen value β of wedge with $B/5H$ or $5H/B$ in case of $n=2/3$.

Fig. 4 shows how the modes of V vary along the depth with the variation of the bottom width B and the power index n in the rigidity G for $L/H=3$. The figure also illustrates that the mode distribution is more partial to the crest, as n becomes larger or B, narrower.

Fig. 5 shows how the first eigen values vary for various bottom width B with the different values of L/H in case of $n=2/3$. It may be clear from the figure, even if the bottom width becomes infinitely small, eigen value β still has a finite value in this calculation. In the range of $L/H>5$, the eigen value closes to the one for infinite canyon width.

(2) The effects of canyon width of the dam

Fig. 6 shows the distributions of the non-dimensional natural period $T\cdot C_s/H$ with the variation of $L/5H$ and its inverse number, in case of $n=2/3$ and $B/H=4.0$. It is directly seen in Eq. (16) that the natural circular frequency p is proportional to inverse of canyon width. We choose the non-dimensional parameter relating to the natural period T as abscissa ordinate, which can cover all the range of the canyon width. Thus we can effectively plot the eigen values in the figure. The solid lines are the results obtained by the wedge beam model and broken lines are, by the truncated wedge beam model. The natural period

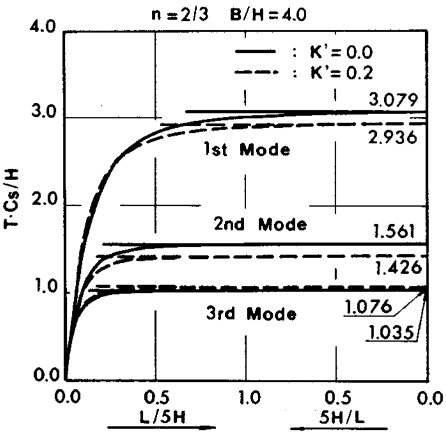

Fig. 6 Variation of non-dimensional period with $L/5H$ or $5H/L$ in case of $n=2/3$ and $B/H=4$.

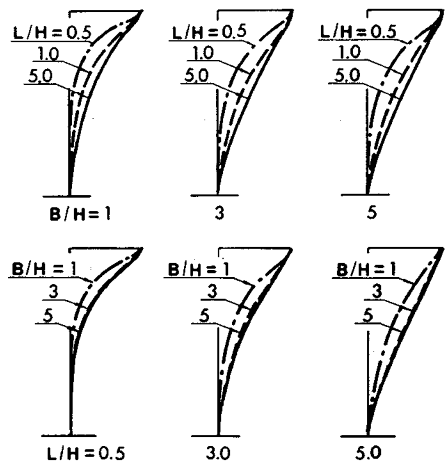

Fig. 7 Comparison of mode shapes along depth of wedge with aspects L/H and B/H in case of $n=2/3$.

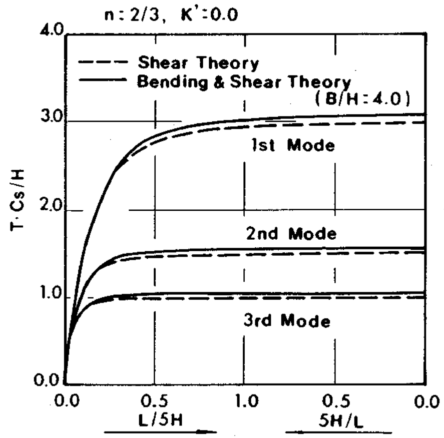

Fig. 8 Variation of non-dimensional period with $L/5H$ or $5H/L$ in case of $n=2/3$ and $K'=0.0$.

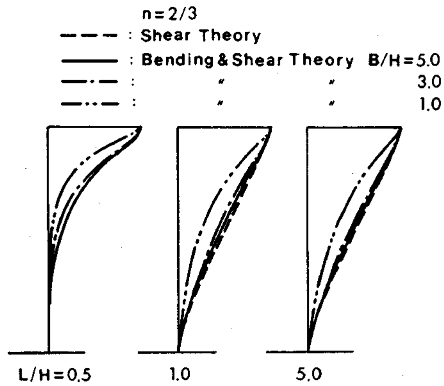

Fig. 9 Comparison of mode shapes along depth of dam between two two-dimensional beam theories.

concerning the first mode for $5H/L=0.5$ closes to the natural period for the dam of infinite canyon width. Thus, the canyon width plays an important role in not only the first eigen values but also the 2 nd and 3 rd ones. That is to say, the eigen values of the 2 nd order mode almost equal to the one for the dam of infinite canyon width in the range $L/5H \geqq 0.75$ while the eigen values of the third order mode, in the range $L/5H > 0.5$.

The mode distributions of V for various B/H or L/H are shown in Fig. 7, which also illustrates that the smaller the ratios L/H or B/H become, the more partial to the crest the shape of the mode becomes, and the numerical results for $B/H=3.0$ and $B/H=5.0$ are almost the same values.

Two kinds of non-dimensional natural periods, the one is from the theory considering bending moment together with shearing forces, and the other is from the theory only by shearing force, are compared with each other in Fig. 8. The figure shows that the results obtained by two theories are in good agreement.

Fig. 9 shows the comparison of the shape of V mode in the vertical direction due to the above two theories. We see again that the modes of two theories have almost the same distribution for $B/H > 3.0$.

Fig. 10 shows the shape of the 1 st and 2 nd lowest modes in the vertical direction for various L/H obtained by two-dimensional inhomogeneous shear beam theory. We see in the figure that both modes are

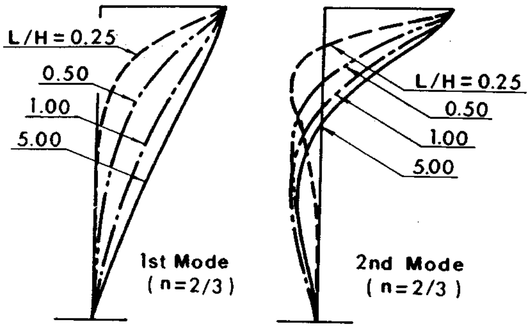

Fig. 10 The lowest two mode shapes along depth of dam by two-dimensional inhomogeneous shear beam theory.

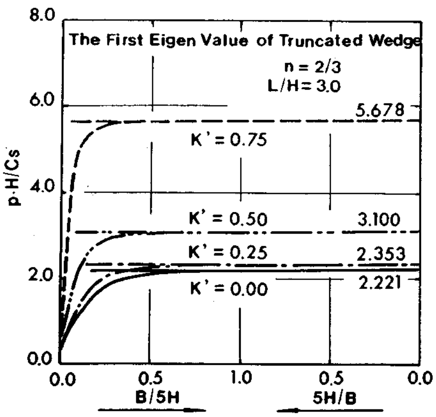

Fig. 11 Variation of non-dimensional first circular frequency of truncated wedge with $B/5H$ or $5H/B$ in case of $n=2/3$ and $L/H=3$.

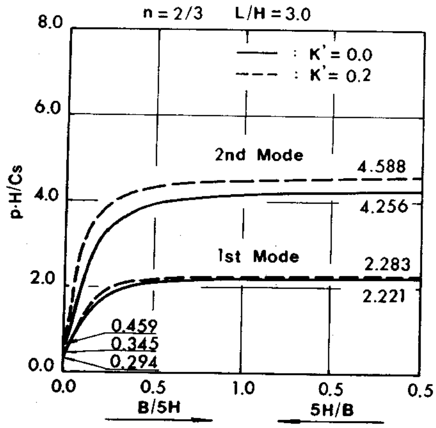

Fig. 12 Variation of non-dimensional the lowest two circular frequencies of truncated wedge with $B/5H$ or $5H/B$ in case of $n=2/3$ and $L/H=3$.

partial to the crest. It is similar as we see in Fig. 7. So long as the parameter B/H is larger than 3, the theory only by shearing force yields almost the same results as the theory by bending moment together with shearing forces does, and so the shear theory can be used for the evaluation of the dynamic characteristics of the dam for $B/H>3.0$, instead of the theory by bending moment together with shearing forces.

(3) The effects of the truncation of the dam crest

Fig. 11 shows the distribution of non-dimensional first circular frequency $p\cdot H/C_s$ of truncated wedge with the variation of $B/5H$ or $5H/B$ for $n=2/3$ and $L/H=3$ in the range from $K'=0.0$ to $K'=0.75$ which is like a bank. From the figure, we can find that larger K' corresponds to higher circular frequency and the part predominating shear effect extends in the direction of decreasing B. The distributions of the 1 st and 2 nd circular frequency for $K'=0.0$ and 0.2 are drawn in Fig. 12. The actual dams usually have the crest with 0.02~0.03 truncation. The difference of the results between apex wedge and truncated one is very small and both results are not so much as the one indicated by Ambraseys[1] for $n=0$, we can therefore assume the actual fill type dam as the apex wedge.

4. CONCLUSIONS

Modeling the fill type dam located on U shape canyon as the two-dimensional inhomogeneous truncated wedge beam with shear modulus which is constant in the up-down stream direction and varying n-th power in the depth direction, the differential equations for natural frequency of the dam are derived from equilibrium of the bending moment and shearing force in the vertical direction and the shearing force and twisting moment in the crest-wise direction. Formulating those equations in the eigen value matrix equations by using the finite difference technique taking the boundary conditions into account, the effects of rigidity power index, bottom width, canyon width and truncation of the wedge shape on the eigen value have been fully discussed.

The results obtained in this paper are summarized as follows :

(1) As the rigidity power index n becomes larger or bottom width and canyon width become narrower, the vibration modes in the up-down stream direction tend to partially amplify nearer to the crest.

(2) As the canyon width becomes narrower, the two-dimensional inhomogeneous shear beam theory gives almost the same results as those by the theory taking the bending moment and shearing forces into account.

(3) We can apply the two-dimensional inhomogeneous shear beam theory to the evaluation of the dynamic characteristics of the fill type dam with B/H larger than 4.0.

(4) As the effects of the crest truncation on the dynamic characteristics of the actual dams seem to be little, we can apply apex wedge model to the dynamic analysis of the fill type dam.

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