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GROUP THEORETIC STUDY OF BIFURCATION POINTS OF TRUSS DOME STRUCTURES

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This papar offers a group theoretic study of symmetry breaking bifurcation points of truss dome structures. A dominant role of symmetry in bifurcation phenomena has been demonstrated through case studies performed on bifurcation points of a series of reticulated regular, polygonal-shaped, truss domes subjected to axisymmetric loadings. Various characteristics of bifurcation points are described in relation with the level of symmetry of main and bifurcation paths. Several rules governing bifurcation points have been derived through these studies with an aim toward the development of a more complete theory in the future.

Keywords : bifurcation points, bifurcation behavior, group theory, truss dome structures.

1. INTRODUCTION

Bifurcation buckling serves as one of the typical collapse modes of structures. One can point out various structures having potential to display bifurcation buckling behavior, including : the beam-column members[1], the shallow arches[2], and the dome structures[3,4]. Naturally, the theoretical description and categorization of bifurcation points have drawn great engineering and academic interest.

Thompson and Hunt[5] have developed a general theory of bifurcation buckling behavior with the use of total potential energy function and perturbation technique. Bifurcation points were devided into the following major types : the symmetric bifurcation point, the asymmetric one, and so on. Niwa et al.[6] categorized bifurcation points based on a catastrophe theory to arrive at basically the same major types as those obtained by Thompson and Hunt. They defined a symmetric bifurcation point as what possesses bifurcation paths with a zero slope of the loading parameter, and an asymmetric point with a non-zero slope. Hosono[7] advanced an alternative way to define these two types of points. His definition can be restated as follows : for symmetric bifurcation points, bifurcation modes for a bifurcation path can be carried into those for the other bifurcation path branching toward an opposite direction through a rotation or reflection, thereby denoting the same physical behavior; however, such is not the case for asymmetric ones. This is one of the first attempt by engineers to describe bifurcation behavior based on symmetry. Fujii[8,9] divided bifurcation points into two types : symmetry preserving bifurcation points and symmetry breaking ones. The geometric symmetry of deformation modes is preserved for the formers, whereas reduced for the latters. In addition, he noticed that bifurcation points with double roots consist of two types : parametric ones and group theoretic ones. These occur by virtue of geometric symmetry ; those do

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as a consequence of a concidence of two critical points with a single root.

At present, bifurcation tracing analyses of dome structures rely primarily on an analytical standpoint. While it is not feasible to expect the evaluation of potential energy and its derivatives as a part of such analyses, the concept of symmetry may provide additional information. Currently, a lack of bacic understanding of bifurcation buckling behavior is impeding the development of systematic means of bifurcation tracing. Extending mathematical studies on bifurcation behavior performed by Fujii and others[8),9),10)], the authors[11),12)] performed a group theoretic categorization of bifircation 'paths' of regular polygonal-shaped truss domes subjected to axisymmetric loadings. This sutudy revealed the predominant influence of symmetry on bifurcation buckling phenomena. The domes' bifurcation phenomena were described completely by the subgroups of a dihedral group, which is a typical mathematical tool for representing the symmetry of regular polygons[13)]. We, however, somewhat disregarded the description of bifurcation points in favor of the study of bifurcation paths.

This investigation is undertaken in order to make the group theoretic method applicable to the description of bifurcation points of D_n-covariant truss dome structures under axisymmetric loadings. The bifurcation points studied herein belong to the symmetry breaking bifurcation points with a single root and those with group-theoretic double roots. We emphasize their qualitative and symmetric aspects, as Hosono did. The properties of bifurcation points are described in relation with the characteristics of main and bifurcation paths passing through these points. Several rules concerning bifurcation points are derived so as to achieve a deeper insight into bifurcation behavior.

2. SYMMETRY IN BIFURCATION BEHAVIOR OF A REGULAR-HEXAGONAL TRUSS DOME

This section offers a study of symmetric nature of bifurcation paths and points. For this purpose, bifurcation behavior of the reticulated, regular hexagonal, truss dome shown in Fig. 1 is investigated based on a group theoretic approach.

Dihedral groups, which have been employed extensively for representing symmetry of regular polygons in mathematics[8)], are utilized here and in the sequal to express symmetric nature of bifurcation modes (paths). As reported in Refs. 11 and 12 bifurcation modes (paths) of a regular n-gonal dome (n=3, 4, 5 · · ·) can be represented by subgroups of a dihedral group of degree n, D_n. Bifurcation paths of the regular hexagonal dome, for example, are categorized with the aid of the following subgroups of group D_6 : E, C_2, D_1^{2j-1}, D_1^{2j}, D_2^{j}, $D_{6/2}$ (or D_3^2 in Ref. 11), $D'_{6/2}$ (or D_3^1), C_6 and D_6[11)]. Group E denotes a completely asymmetric deformation mode ; D_1^{2j-1} and D_1^{2j} are modes with one axis of

Table 1 Vertical Loading Pattern.

Node Number	Loading Pattern
0	0.5
1	1.0
2	1.0
3	1.0
·	·
·	·
·	·
n	1.0

n=6 for the hexagonal dome.

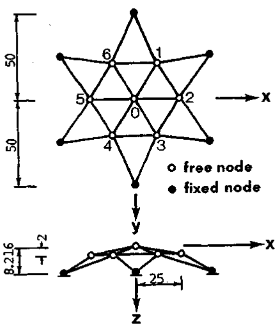
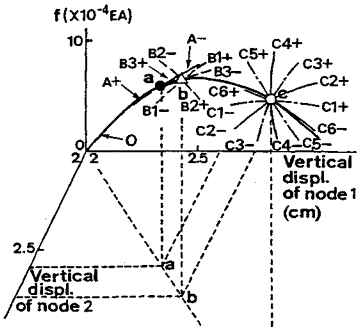
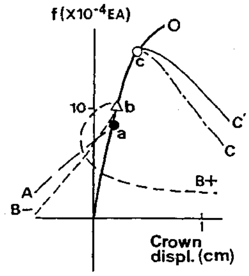


Fig. 1 Regular-Hexagonal Truss Dome. (unit in cm)



(a) Spatial Sketch



(b) Force Versus Crown Displacement Relationship

Fig. 2 Equilibrium Paths of the Hexagonal Dome.[14)] line symmetry.

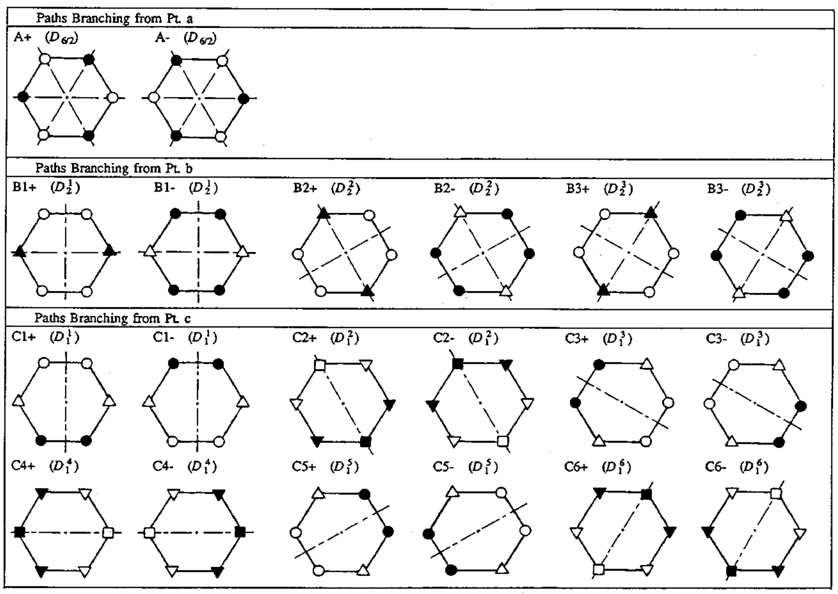
Groups C_6 and C_3 denote rotatory symmetric modes; $D'_{6/2}$ does a three-axes symmetric mode. These three modes were disregarded in Refs. 11 and 12 that ignored rotatory and radial displacements and considered only vertical displacements of truss domes. A (sub) group representing the deformation pattern of a path is called a symmetry group of the path[8].

Figure 2 shows equilibrium paths of the dome obtained by using a finite displacement analysis for the axisymmetric loading patten listed in Table 1. Figure 2(a) is a spatial sketch of the paths showing interrelationships among the loading parameter and vertical displacements of nodes 1 and 2 of the dome, while Fig. 2(b) shows the corresponding external force versus crown displacement relationship of the same bifurcation phenomena. Dark continuous lines express the path represented by the group D_6, light continuous ones do $D_1^{2,j}$ and so on. The symbol (●) denotes the symmetric bifurcation point with a single root; (○) expresses that with a double root; (△) is the asymmetric point with a double root. The authors followed the aforementioned Honoso's way to define symmetric and asymmetric points of bifurcation. The multiplicity of a bifurcation point denotes the number of zero eigenvalues of the tangent stiffness matrix of a structural system to be analyzed.

As can be seen, as many as twenty paths branched from the fundamental path O at bifurcation points, a, b, and c. Points a and c belonged to symmetric bifurcation points, while point b to an asymmetric one. Each bifurcation path is smoothly connected with another path branching toward the opposite direction, such as a pair of paths A^+ and A^-. The authors henceforth term such a case that the paths form a 'pair'. Paths that can form a pair are indicated in this and subsequent figures by means of the same name and with the symbol + or −.

Symmetry has strong influence on bifurcation modes corresponding to these bifurcation paths. Table 2 illustrates schematically these modes in terms of vertical displacements of nodes 1 through 6 of the dome, where dotted-dash lines express the axis of line symmetry and symbols (▲), (□), · · ·, and (▽), denote that nodes with the same symbol have an identical vertical and radial displacements. The nodes on axes of symmetry cannot rotate, whereas those on elsewhere can rotate in such a manner that the nodes

Table 2 Bifurcation modes of Several Bifurcation Paths of the Hexagonal Dome.



○, △, ···, ▽ : nodes with the same symbols possess identical vertical and radial displacements;
—·—·—·— : axis of line symmetry;
( ) : symmetry group of a bifurcation path.

satisfy the required line symmetries. The symmetry groups representing these modes are shown in this table with the use of the parenthesis. As can be seen, a 180 degree rotation about the origin carries the mode for A^+ into that for A^-. In this manner, the authors arrived at the folowing interrelationships among the modes :

$$x_{A+}=\sigma_4 x_{A-} \qquad x_{B1+}=\sigma_3 x_{B2+}=\sigma_5 x_{B3+} \qquad x_{B1-}=\sigma_3 x_{B2-}=\sigma_5 x_{B3-}$$
$$x_{C1+}=\sigma_2 x_{C3+}=\sigma_3 x_{C5+}=\sigma_4 x_{C1-}=\sigma_5 x_{C3-}=\sigma_6 x_{C5-} \qquad \cdots\cdots (1)$$
$$x_{C2+}=\sigma_2 x_{C4+}=\sigma_3 x_{C6+}=\sigma_4 x_{C2-}=\sigma_5 x_{C4-}=\sigma_6 x_{C6-}$$

where vector x_k denotes a deformation mode of the dome related to a bifurcation path; the subscript k expresses the name of relevant path; variable $\sigma_j (j=1, 2, \cdots, \text{ or } 6)$ indicates the $60 \cdot (j-1)$ degree rotation around the z-axis. The first equation of Eq. (1) indicates that the deformation mode for path A^+ can be obtained by rotating the mode for A^- around the z-axis through an angle of 180 degrees. Since such a rotation does not deform the dome but produces its rigid body displacement, these two modes represent the same physical behavior. In that equation, all the modes related by equalities with the aid of a rotation denote the same physical behavior. As a result of this, the twenty bifurcation paths in Fig. 2(a) denoted only five independent bifurcation phenomena.

Symmetries in these bifurcation paths can be divided into two general types, including : (1) the symmetry between two paths forming a pair and (2) the symmetry among paths belonging to different pairs. The former is attributable to line symmetry of the dome and the latter to its rotational symmetry. The first type symmetry determines if a bifurcation point is a symmetric or asymmetric one. Such symmetry can be seen in each pair of paths branching from the symmetric bifurcation points a and c; by contrast, a pair of paths branching from the asymmetric bifurcation piont b denoted different physical phenomena. The second type symmetry, arising from rotatory symmetry, was observed in paths $B1^+$, $B2^+$, and $B3^+$, whose symmetry groups are D_2^1, D_2^2, and D_2^3, respectively. The rotational symmetry among these groups resulted in symmetry among the paths. Likewise, the rotatory symmetry of groups D_1^1, D_1^3, and D_1^5 led to the symmetry among (1) paths $C1^+$, $C3^+$ and $C5^+$ and (2) $C1^-$, $C3^-$, and $C5^-$.

As a consequence of the two types of symmetries, some of the bifurcation paths shown in Fig. 2(a) represented the same external force versus crown displacement curves in Fig. 2(b). For example, a pair of paths A^+ and A^- degenerated into a single path A in these curves. Similarly, (1) paths $B1^+$, $B2^+$, and $B3^+$; (2) paths $B1^-$, $B2^-$, and $B3^-$; (3) $C1^+$, $C1^-$, $C3^+$, $C3^-$, $C5^+$, and $C5^-$; and (4) $C2^+$, $C2^-$, $C4^+$, $C4^-$, $C6^+$ and $C6^-$ degenerated into paths B^+, B^-, C and C', respectively.

As we have seen, the bifurcation paths had a lot of symmetries associated with the geometric symmetry of the regular hexagonal dome. In post-bifurcation analyses of highly symmetric structures with many branching paths, one needs to trace only a few paths since most of them can be automatically obtained from the condition of geometric symmetry (Mathematically, such symmetry can be monitored through the derivation of bifurcation equations[8),9)].) The concept of symmetry, in this regard, should be of great importance not only in theory but also in practice.

3. BIFURCATION POINTS OF REGULAR-POLYGONAL DOMES

(1) General

This section offers case studies on bifurcation points in the equilibrium paths of reticulated regular polygonal-shaped truss domes with an emphasis on their symmetric nature. The domes studied include the regular five through ten-gonal, single-layered, truss domes (see Fig. 3 for their typical configuration) subjected to the symmetric vertical loading pattern listed in Table

Table 3 Vertical Loading Patterns Applied to the Triple-Hexagonal Dome.

Layer Number	Node Number	Loading Pattern (a)	Loading Pattern (b)
0	0	1/2	0
1	1 2 . . n	1 1 . . 1	0 0 . . 0
2	1 2 . . n	1 1 . . 1	1 1 . . 1
3	1 2 . . n	1 1 . . 1	0 0 . . 0

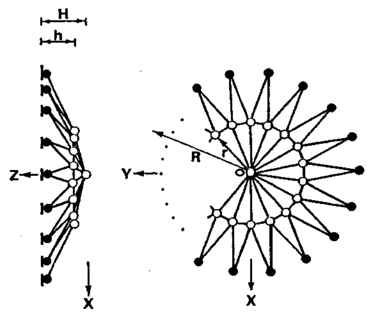


Fig. 3 Single-Layer Regular Polygonal Dome (R=50; r=25; H=8.216; h=6.216 cm).

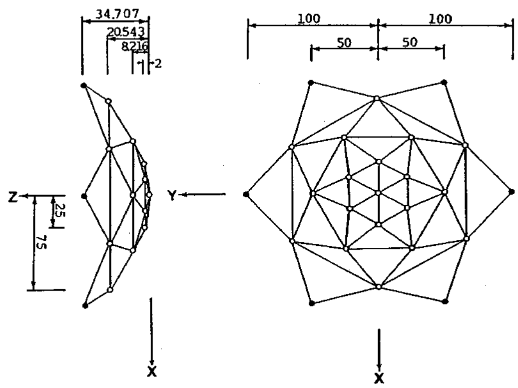
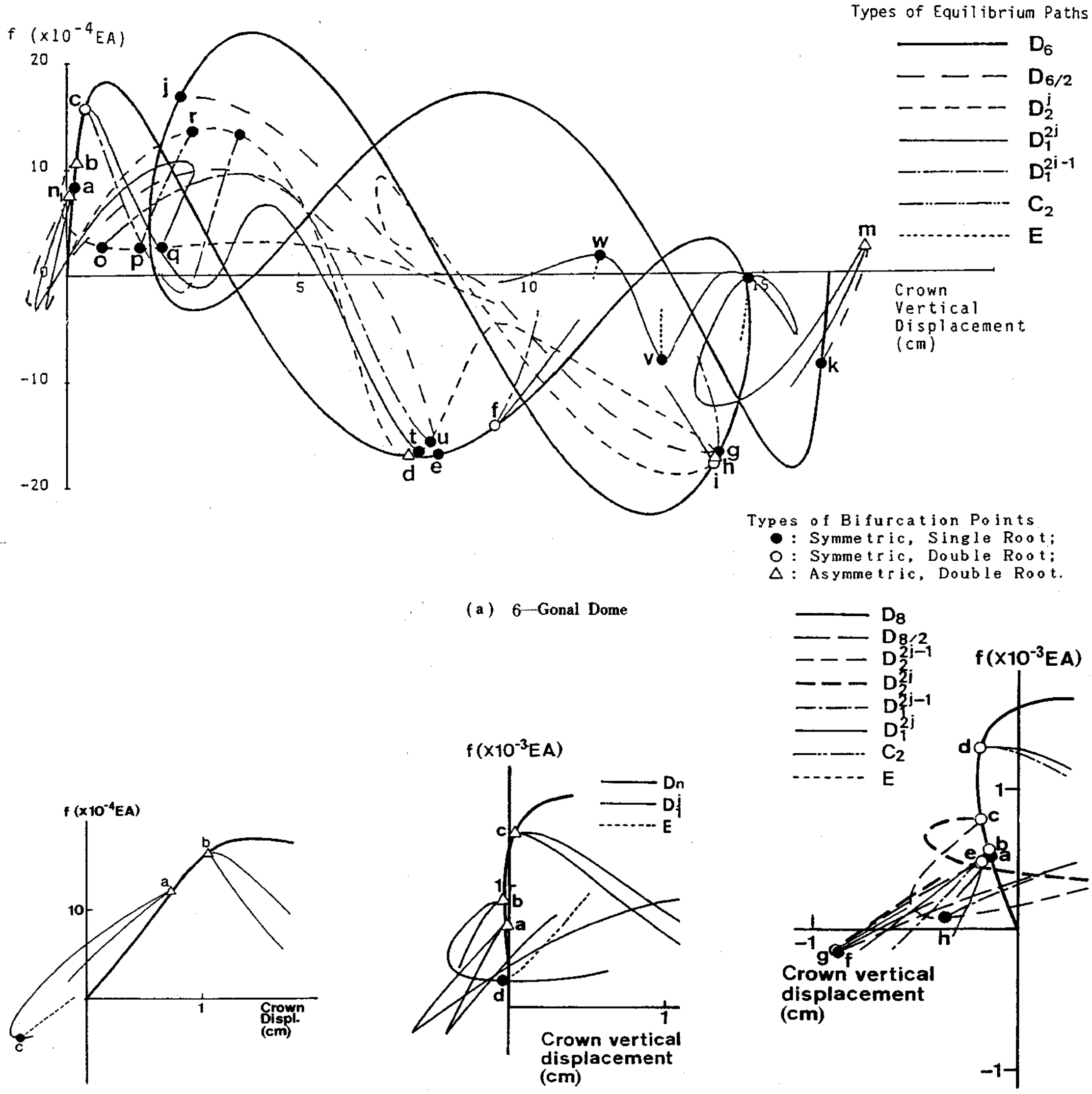


Fig. 4 Triple-Hexagonal Dome. (unit in cm)



(a) 6—Gonal Dome

(b) 5—Gonal Dome

(c) 7—Gonal Dome

(d) 8—Gonal Dome

Fig. 5 Equilibrium Paths of Various Types of Regular Polygonal Domes.

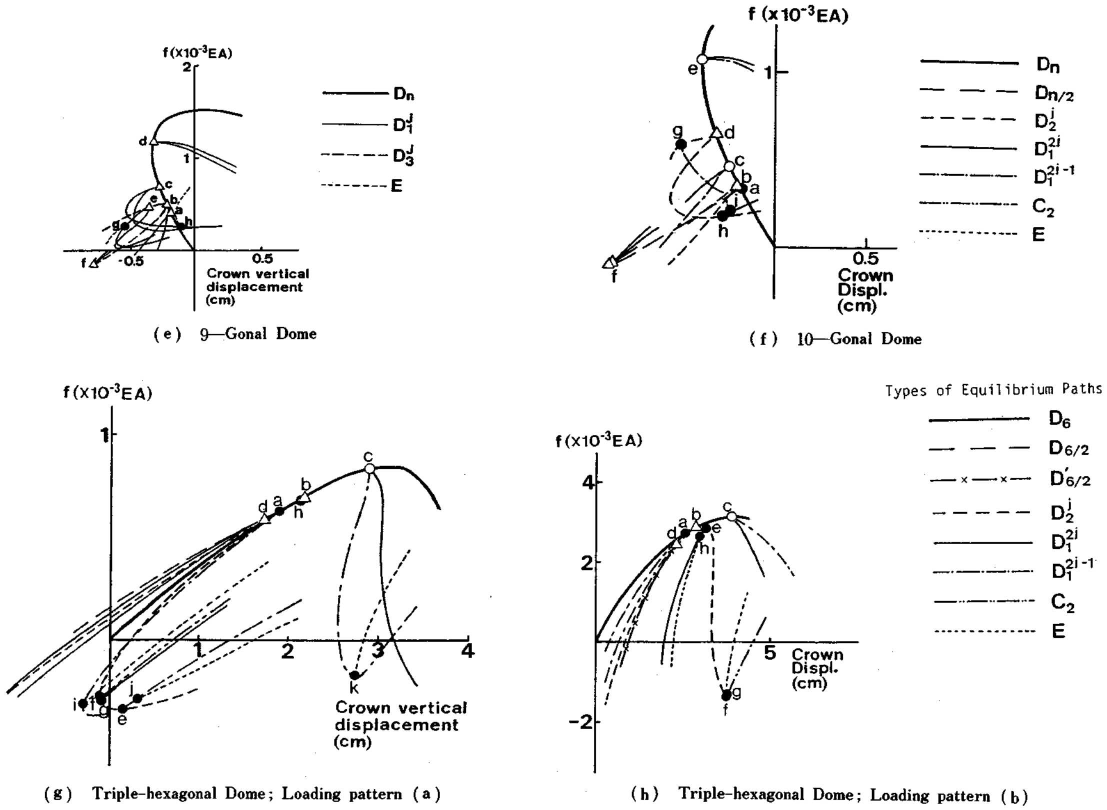


Fig. 5 Equilibrium Paths of Various Types of Regular Polygonal Domes.

1, and the triple-hexagonal dome (see Fig. 4) under the loading pattern (a) or (b) listed in Table 3. These domes, with various configurations, should offer us various types of bifurcation points. The single-layer domes should reflect the effects of degree of polygons and the triple dome does those of multiple layers. Figure 5(a) through (h) shows equilibrium paths of these domes obtained through the aforementioned analysis technique. The bifurcation paths were categorized by investigating deformation patterns of the domes at the course of the analyses based on the group theoretic categorization advanced in Ref. 10. Point symmetric modes represented by groups C_n and $C_{n/2}$ did not exist in these figures although their existence is theoretically feasible.

The authors inspected the bifurcation points of these domes to find that all the single roots were symmetric points of bifurcation and all the asymmetric points had double roots. This agrees with the general fact reported in Ref. 8) that single-rooted points on D_n are symmetric bifurcation points. The bifurcation points consequently belonged to one of the following three types : the symmetric bifurcation point with a single root, that with a double root, and the asymmetric point with a double root. The properties of these three types of points are studied below.

As reported in previous papers[11),12)] symmetry breaking bifurcation buckling behavior of dome structures occurs as a result of the loss in geometric symmetry. The nature of bifurcation points, therefore, is expected to be greatly influenced by the amount of reduction of symmtry encountered at a bifurcation point relative to the symmetry of a main path.

Such a reduction can be characterized by the index[13)], denoting the ratio of the orders of symmetry groups of main and of bifurcation paths, where the orders express the level of symmetry of the paths. Index, which enlarges in association with greater degradation of symmetry, will be employed in describing bifurcation points in the sequal, in addition to the other parameters.

Tables 4 and 5 tabulate a series of parameters characterizing the bifurcation points in the equilibrium paths shown in Fig. 5. These include : (1) the type of bifurcation point, (2) the symmetry groups representing main and bifurcation paths, (3) the number of branching paths, and (4) the index. As can be seen, the types of bifurcation points are uniquely determined by the types of main and of bifurcation paths, at the least for these bifurcation points. For example, bifurcation points a, e, g, j, and k of the hexagonal dome and the point a of the eight-gonal dome, defined as the intersection points of paths represented by D_n and by $D_{n/2}$ all belonged to the symmetric bifurcation point with a single root. Such dependency of the types of bifurcation points on the types of paths passing through the points suggests that the study of bifurcation points should be achieved with an emphasis on path types.

Table 4 Categorization of Bifuraction Points of Single-layer Domes.

Types of Domes	Name of Bif. Points	Types of Bif. Points	Symmetry Groups: Main Paths	Symmetry Groups: Bif. Paths	No. of Bif. Paths	Index Values
5-Gonal	a,b	Δ	D_n (10)	D_1^j (2)	10	5
	c	●	D_1^j (2)	E (1)	2	2
6-Gonal	a,e,g,j,k	●	D_n (12)	$D_{n/2}$ (6)	2	2
	b,d,h	Δ	D_n	D_2^j (4)	6	3
	c,f,i	○	D_n	D_1^{2j-1} & D_1^{2j} (2)	12	6
	m,n	Δ	$D_{n/2}$ (6)	D_1^{2j} (2)	6	3
	o,s,u	●	D_2^j (4)	D_1^{2j-1} (2)	2	2
	p,r	●	D_2^j	C_2 (2)	2	2
	q,t	●	D_2^j	D_1^{2j} (2)	2	2
	v,w	●	D_1^{2j} (2)	E (1)	2	2
7-Gonal	a,b,c	Δ	D_n (14)	D_1^j (2)	14	7
	d	●	D_1^j (2)	E (1)	2	2
8-Gonal	a	●	D_n (16)	$D_{n/2}$ (8)	2	2
	b,d	○	D_n	D_1^{2j-1} & D_1^{2j} (2)	16	8
	c	○	D_n	D_2^{2j-1} & D_2^{2j} (4)	8	4
	e,g	○	$D_{n/2}$ (8)	D_1^{2j} (2)	8	4
	f	●	$D_{n/2}$	D_2^{2j} (4)	2	2
	h	●	D_2^{2j-1} (4)	D_1^{2j-1} (2)	2	2
9-Gonal	a,c,d	Δ	D_n (18)	D_1^j (2)	18	9
	b	Δ	D_n	D_3^j (6)	6	3
	e,f	Δ	D_3^j (6)	D_1^j (2)	6	3
	g,h	●	D_1^j (2)	E (1)	2	2
10-Gonal	a	●	D_n (20)	$D_{n/2}$ (10)	2	2
	b,d	Δ	D_n	D_2^j (4)	10	5
	c,e	○	D_n	D_1^{2j-1} & D_1^{2j} (2)	20	10
	f	Δ	$D_{n/2}$ (10)	D_1^{2j} (2)	10	5
	g	●	D_2^j (4)	D_1^{2j-1} (2)	2	2
	h	●	D_2^j (4)	D_1^{2j} (2)	2	2
	i	●	D_1^{2j} (2)	E (1)	2	2

● : symmetric bifurcation point with a single root;
○ : symmetric bifurcation point with a double root;
Δ : asymmetric bifurcation point with a double root;
( ) : order of a symmetry group.

Table 5 Categorization of Bifuraction Points of The Triple-Hexagonal Domes (Obtained for Two Types of Loading Patterns).

Loading Pattern Type	Name of Points	Types of Points	Symmetry Groups: Main Paths	Symmetry Groups: Bifurcation Paths	No. of Bif. Paths	Index Values
(a)	a	●	D_n (12)	$D_{n/2}$ (6)	2	2
	b	Δ	D_n	D_2^j (4)	6	3
	c	○	D_n	D_1^{2j-1} & D_1^{2j} (2)	12	6
	d	Δ	$D_{n/2}$ (6)	D_1^{2j} (2)	6	3
	e,h	●	D_2^j (4)	D_1^{2j-1} (2)	2	2
	f	●	D_2^j	C_2 (2)	2	2
	g	●	D_2^j	D_1^{2j} (2)	2	2
	i,j,k	●	D_1^{2j-1} (2)	E (1)	2	2
(b)	a	●	D_n (12)	$D'_{n/2}$ (6)	2	2
	b	Δ	D_n	D_2^j (4)	6	3
	c	○	D_n	D_1^{2j-1} & D_1^{2j} (2)	12	6
	d	Δ	$D'_{n/2}$ (6)	D_1^{2j-1} (2)	6	3
	e	●	D_2^j (4)	D_1^{2j} (2)	2	2
	f	●	D_2^j	C_2 (2)	2	2
	g	●	C_2 (2)	E (1)	2	2
	h	●	D_1^{2j} (2)	E (1)	2	2

(2) Influence of index

As can be seen from Tables 4 and 5, the values of index greatly influenced the types of bifurcation points. Index values were equal to two for bifurcation points with a single root and greater than two for those with a double root. To be precise, their values equaled two for symmetric bifurcation points with a single root, even numbers other than two for symmetric points with a double root, and odd numbers for asymmetric points with a double root, respectively. These characteristics may serve as a convenient and promising alternative way to identify bifurcation point types. During the course of bifurcation tracing analyses, the types of bifurcation points can be determined easily by obtaining the orders of symmetry groups of main and bifurcation paths at the points and calculating the values of index.

As reported in Ref. 8, two bifurcation paths branch from bifurcation points with a single root. For double-rooted bifurcation points, we noted from Tables 4 and 5 that the numbers of branching paths were the twice of indexes, that is,

(Number of Branching Paths) $=2\times$ (Index) ·· (2)

although this formula may be hypothetical at this stage of research, it should be of great assistance in bifurcation tracing analyses if proved to be correct.

(3) Influence of degree of polygons

The influence of degree of polygons on bifurcation point properties can be monitored by referring to Lagrange's theorem, stating that the order of a group can be divided by the order of its subgroup[13]. Combining this theory with the fact that the order of group D_n equals $2n$, one can say that an index value is a factor of the number $2n$. Because this number always has 'two' as a factor, every polygonal dome can possess 'two' as an index, thereby having potential to hold a symmetric bifurcation point (with a single root). In particular, for a polygonal dome with a degree 2^m, the index is always an even number so that the dome should only possess symmetric bifurcation points. Domes with degrees other than 2^m possess an odd index(es), as well as an even index(es) so that they potentially hold both symmetric and asymmetric points.

We investigated the interrelationships between the degree of the domes and the presence of the three types of bifurcation points. Table 6 lists these interrelationships, where the symbol (E) denotes the existence of relevant bifurcation point type; (N) expresses the type which did not exist. The regular eight-gonal dome with a degree 2^3 only had symmetric points, whereas the other domes whose degrees had an odd factor(s) possessed both symmetric and asymmetric points. Domes with odd degrees did not hold symmetric points with a double root; domes with even degrees other than 2^m had all three types of points. As we have seen, the presence of bifurcation point types matched perfectly with what predicted above, thus validating the prediction.

Bifurcation points on fundamental equilibrium paths, determining buckling capacity of domes, possess much greater engineering importance. Let us recall the aforementioned rule that bifurcation points with a single-root occur only for the case where index equals two and that those with a double root do for an index greater than two. The index equals two only for the case where paths represented by $D_{n/2}$, $D'_{n/2}$ or C_n branch from the fundamental paths. Hence symmetric points with a single root on fundamental paths should

Table 6 Existence of Three Types of Bifuraction Points for the Domes with Various Degrees.

Degree of Polygons	Bif. Points (in general)			Bif. Points on Fundamental Paths		
	●	○	Δ	●	○	Δ
2^m (m is an integer)	E	E	N	E	E	N
Even No. other than 2^m	E	E	E	E	E	E
Odd Number	E	N	E	N	N	E

E : bifurcation point 'exists';
N : bifurcation point did 'not' exist;
● : symmetric bifurcation point with a single root;
○ : symmetric bifurcation point with a double root;
Δ : asymmetric bifurcation point with a double root.

have either $D_{n/2}$, $D'_{n/2}$, or C_n as a symmetry group of bifurcation paths. All the other types of paths, by contrast, branch from fundamental paths at bifurcation points with a double root. Only asymmetric bifurcation points existed in the fundamental paths of the domes with odd degrees.

(4) Influence of multiple layers

As can be seen from Tables 4 and 5, the bifurcation points of the triple-hexagonal (three-layer) dome had exactly the same properties as those for the single-layer ones. The rules regarding bifurcation points, therefore, appear to hold generality regarding the number of layers of the domes.

(5) Summary

The rules regarding bifurcation points advanced herein would permit one to develop fundamental understanding of bifurcation buckling behavior of dome structures. With the aid of such rules, bifurcation path tracing analyses can be performed in much more systematic manner (for example, in tracing bifurcation paths of a regular eight-gonal dome, one should expect only symmetric bifurcation points). The rules are very likely to hold for single-layer regular polygonal domes with relatively small degrees; furthermore, some of the rules may be valid for more general cases as well. It should be at the least appealing to academic interest that various bifurcational characteristics can be described well by means of a single variable, index.

4. SUMMARY AND CONCLUSIONS

The authors have advanced a group theoretic method for describing bifurcation paths of axisymmetric, polygonal truss dome structures in previous papers[11)-12)]. This method described well bifurcation behavior of the dome structures subjected to axisymmetric loadings in a methodical fashion and permitted us to arrive at several information concerning bifurcation phenomena simply through an observation of the domes' configuration. The method, however, somewhat disregarded the description of bifurcation points in favor of the study of bifurcation paths. The following studies were performed in this paper to make the method applicable to the description of bifurcation points.

Symmetry in bifurcation paths was inspected for a few symmetry breaking bifurcation points of a regular hexagonal truss dome. Two types of symmetries were seen in the bifurcation paths branching from these points. These symmetries, ascribed to the dome's geometric symmetry, include (1) the symmetry between two paths forming a pair and (2) the rotational symmetry among paths. Owing to the symmetries, as many as twenty paths were divided into only five independent paths. In post-bifurcation analyses of highly symmetric structures, the concept of symmetry can reduce the number of paths to be traced, thereby greatly simplifying such analyses.

We performed case studies on bifurcation points in the equilibrium paths of a series of regular polygonal, truss domes subjected to axisymmetric loadings. These bifurcation points belonged to three general types, including : the asymmetric point with a double root, and the symmetric points with a single and a double root. The index was advanced as a parameter to characterize bifurcation points. At the least for the equilibrium paths inspected herein, symmetric bifurcation points occurred for even-numbered indexes, while asymmetric ones did for odd ones. Single-rooted points had two as an index and double-rooted ones did three or greater number. These characteristics were suggested for use in distinguishing the types of points at the course of bifurcation tracing analyses. We combined the characteristics with Lagrange's theorem to conclude that every polygonal dome has potential to possess a symmetric bifurcation point (s). The polygonal domes with a degree 2^m, moreover, should possess only symmetric points, while do domes with other types of degrees both symmetric and asymmetric points. The number of paths branching from bifurcation points with double roots equaled the twice of index values. Index, influencing strongly various bifurcation properties, should greatly contribute to the description of bifurcation behavior.

In order to extend the generality of the rules of symmetry breaking bifurcation points advanced herein, we investigated bifurcation points of the triple-hexagonal (three-layer) dome to note that the rules described well its bifurcation points as well. The rules, therefore, appear to hold generality concerning the number of layers of domes.

The group theoretic method for the description of bifurcation points displayed a great promise in studying the bifurcation behavior of a series of truss domes. This method, combining completeness and simplicity, will be of great assistance in deriving general rules governing the nature of bifurcation points of domes. The rules advanced herein should form a basis in developing more general rules, or may eventually serve as general rules. Future studies should be directed toward developing such rules so as to enable one to perform bifurcation tracing analyses based on firm theoretical bases.

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