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MODELLING CYCLIC PLASTICITY OF STRUCTURAL STEELS

By Masaru MINAGAWA, Takeo NISHIWAKI** and Nobutoshi MASUDA****

In this paper we propose a cyclic plasticity model with high accuracy to predict elasto-plastic behaviors of uniaxial steel members subjected to complicated repetitive loads. The model is based on the multi surface plasticity model and material property parameters introduced are a couple of fundamental surface size functions and weighting functions to describe sizes of state surfaces at arbitrary stress-strain phases. It is revealed that the hysteretic model proposed, by which quasi-static fluctuating stress-strain relations can be predicted, is sufficiently accurate from the engineering point of view.

Keywords : *cyclic plasticity, uniaxial loads, structural steel, multi surface model*

1. INTRODUCTION

To predict elasto-plastic hysteretic behaviors of structures or structural members made from structural metals is one of the important problems of the day in the structural engineering[1]. As an effective means to do this, numerical methods such as the finite element method are frequently used. In this case, assumptions introduced in calculation procedures and quality of modelling for analysis objects affect on predicted results. Especially, quality of a model for material properties affects considerably on the accuracy of elasto-plastic behavior prediction.

In the gross analyses of structures or structural components subjected to severe hysteretic loading, material properties are usually modeled to simplified forms such as the elastic perfectly plastic or the bilinear model because of the complexity associated with cyclic plasticity of structural metals. The results of prediction are not so hardly affected by modelling of material properties in these cases. On the contrary, for the local analyses such as local buckling analyses or crack propagation analyses, a more proper stress-strain model is needed in order to make predictions accurate[2].

The isotropic hardening model[3] and the kinematic hardening model[4,5] are primary models in the field of structural analyses. If the loading is confined to a limited region of the yield surface and to monotonous loading, the isotropic hardening model appears to be suitable[6]. For cyclic loading, the kinematic hardening model is more reasonable to represent Bauschinger effect, but it is not satisfactory because the change in strain hardening modulus and yield stress according to load reversals can not be evaluated sufficiently. In

* Member of JSCE, M. Eng., Research Associate, Musashi Institute of Technology
** Member of JSCE, Dr. Eng., Professor, Musashi Institute of Technology
*** Member of JSCE, Dr. Eng., Associate Professor, Musashi Institute of Technology
(Tamazutsumi 1-28-1, Setagaya-ku, Tokyo 158, Japan)

order to complement the shortcomings of these primary hardening models, a lot of constitutive models had been presented.

A model of a field of work hardening moduli, introduced by Mróz[7], corresponds to an extension of the sublayer model[8] to multi axial stress conditions. In this model, the change in the strain hardening modulus for cylic loading is represented by the concept of movements of multi surfaces defined in the stress field. This model has a feature that a piecewise linear stress-strain relation under proportional loading is assumed, and then the smooth transition from elastic to plastic state for reversed loading can not be represented[12]. Moreover, since a lot of surfaces should be dealt with in calculating hysteretic behaviors by using of the model, the calculation procedure appears to be complicated in general.

The two surface model was presented by Dafalias & Popov[9]~[11] and Krieg[12] individually in order to prevent these shortcomings. In this model, a bound surface, as a state surface corresponding to a extreme strain state, is introduced in addition to the yield surface. In Dafalias-Popov Model, the strain hardening modulus is defined by the configurations and locations of the bound surface and the yield surface. After the presentation of the two surface model, some amendments are presented[2),13]. But it was pointed out by one of the presenters that "if load reversals take place before any noticeable plastic flow took place in the opposite sense, the updating of the key parameter cannot be done correctly. In this case a progressively greater and greater overshooting of the probable stress path develops.... "[6]. In order to prevent this weak point, Petersson and Popov presented an improved model[6),14]. In this model, infinite intermediate surfaces were introduced between the yield surface and the bound surface and the complication due to treatment of many surfaces was prevented by using interpolation procedure. It was adopted to the predictions of tension-compression stress-strain relations of high strength steel as well as cyclic torsional stress-strain relations, its effectiveness was confirmed in some degree. It was, however, shown by the authors' investigations that the validity for material with yield plateau and conspicuous strain hardening characteristic may be doubtful. Moreover numerical trial-and-error must be done in order to estimate all of material property parameters introduced in the model and then a more rational method for evaluating them should be found. When these problems are solved, this cyclic plasticity model can become more practical.

In this paper we propose a modified cyclic plasticity model by which quasi-static fluctuating stress-strain relations of various structural steels can be predicted. In addition, presented is a reasonable measurement method in order to estimate material property parameters introduced in the proposed model. As an application of the model, tension-compression stress-strain relations of mild steel and high strength steel is predicted. Then the predicted results are compared with those obtained by the corresponding measurements and the appropriateness is verified. *

2. PETERSSON-POPOV MODEL[6),14]

Since a cyclic plasticity model proposed in this paper is based on Petersson-Popov Model, we refer to this model briefly.

In Petersson-Popov Model, stress-strain relations affected by loading histories are expressed by means of state surfaces defined in stress space. Fig. 1 (a) and (b) explains schematically the concept of this model by combinations of uniaxial stress-strain relations and behaviors of bi-axial multi surfaces. These figures show the cases of virgin material and pre-loaded material respectively.

In the former case, the elastic region pp′ in the left figure of Fig. 1 (a) is expressed by the inside of the yield surface f_0 in the right figure. Stress ranges a—a′, b—b′ and c—c′ corresponding to the equivalent plastic strain $\bar{\varepsilon}_{p1}$, $\bar{\varepsilon}_{p2}$ and $\bar{\varepsilon}_{p3}$ are expressed by the intermediate surfaces f_1, f_2 and f_3 respectively. In this case, since the stress-strain curve for the tension path is nearly equal to that for the compression path[15], the surfaces f_0, f_1, f_2 and f_3 constitute a set of ellipses with the same origin O if von-Mises yield criterion

* A part of this study was already presented in Proceedings of the 9th Symposium on Computational Methods in Structural Engineering and Relative Fields (1985).

is used. In the left figure of Fig. 1 (b), a difference between stress-strain relations of the tension path q–a–b–c from a reversed point q and those of the compression path q′–a′–b′–c′ is represented by movements and expansions and/or reductions of the surfaces f_0, f_1 and so on.

In the model, each surfaces are defined by a surface size function $\varkappa$, by which the size of each surfaces is expressed continuously, and a vector $\{\alpha\}$ indicating those central coordinates. The surface size function is evaluated as the weighted summation of two functions $\varkappa_a$ and $\varkappa_b$, which are surface size functions corresponding to two fundamental loading phases :

$$\varkappa = W\varkappa_a + (1-W)\varkappa_b \quad (1)$$

Where $\varkappa_a$ is the surface size function in the case where no hysteretic effect exists and $\varkappa_b$ is that in the case where the hysteretic effect becomes stationary. These functions are referred to as fundamental surface size functions in this paper. The weighting function W represents the change in the surface size function from $\varkappa_a$ to $\varkappa_b$ due to loading histories. The functions $\varkappa_a$ and $\varkappa_b$ are determined from experimental results obtained from a tension test and a tension-compression test and the function W is estimated by means of numerical trial and error.

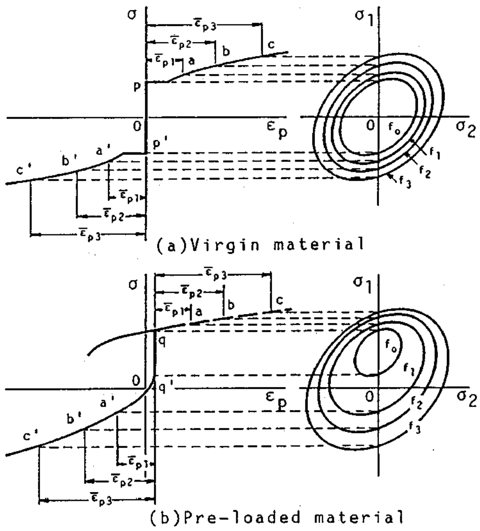


Fig. 1 Uniaxial stress-strain relations and corresponding biaxial multi surfaces in Petersson-Popov Model[6).

The following state variables describe the degree of hysteretic effect in this model :

$$\bar{\varepsilon}_p = \int_{t_0}^{t_c} d\,\bar{\varepsilon}_p, \qquad \bar{\varepsilon}_{pi} = \int_{t_c}^{t_i} d\,\bar{\varepsilon}_p, \qquad d\,\bar{\varepsilon}_p = \sqrt{\frac{2}{3}\,d\,\varepsilon_{ij}^p\,d\,\varepsilon_{ij}^p} \quad (2)$$

where $\bar{\varepsilon}_p$ is cumulative equivalent plastic strain from the start time (t_0) of loading to the time (t_c) of the last reversal on the stress-strain paths, and $\bar{\varepsilon}_{pi}$ is equivalent plastic strain increment from the time (t_c) to the time (t_i) when stress-strain relation is to be predicted. For instance, in the case where the stress-strain curve on the path q–q′–a′–b′–c′ in Fig. 1 should be predicted, cumulative equivalent plastic strain at the point q is $\bar{\varepsilon}_p$, and $\bar{\varepsilon}_{p1}$, $\bar{\varepsilon}_{p2}$ and $\bar{\varepsilon}_{p3}$ corresponds to $\bar{\varepsilon}_{pi}$ at the points a′, b′ and c′ respectively. The surface size function $\varkappa$ in a certain phase of loading can be determined as a function of $\bar{\varepsilon}_{pi}$, by means of $\varkappa_a$ and $\varkappa_b$, which are functions of $\bar{\varepsilon}_{pi}$, and the function W of $\bar{\varepsilon}_p$.

A vector $\{\alpha\}$ indicating the central coordinate of a state surface is updated in turn according to the progress of loading by the following equation :

$$\{\alpha_i\} = \{\tilde{\alpha}_j\} + \frac{\{\tilde{\sigma}\} - \{\tilde{\alpha}_0\}}{\tilde{\varkappa}_0}(\tilde{\varkappa}_j - \varkappa_i) \quad (3)$$

where subscript i and j indicate surfaces in a loading phase with equivalent plastic strain increment $\bar{\varepsilon}_{pi}$ and a phase with $\bar{\varepsilon}_{pi} + d\,\bar{\varepsilon}_p$ respectively. In Fig. 1, for example, when $\bar{\varepsilon}_{p2} = \bar{\varepsilon}_{pi}$ and $\bar{\varepsilon}_{p3} = \bar{\varepsilon}_{pi} + d\,\bar{\varepsilon}_p$ the surfaces f_2 and f_3 are the surface (i) and the surface (j) respectively. $\{\sigma\}$ is a stress vector, subscript o means that $\bar{\varepsilon}_{pi}=0$ and tilde represents the state of before the updating. In the states of before and after updating, cumulative equivalent plastic strain is $\bar{\varepsilon}_p$ and $\bar{\varepsilon}_p + d\bar{\varepsilon}_p$, respectively. Thus, $\{\alpha_i\} = \{\alpha(\bar{\varepsilon}_p + d\,\bar{\varepsilon}_p,\ \bar{\varepsilon}_{pi})\}$, $\{\tilde{\alpha}_j\} = \{\alpha(\bar{\varepsilon}_p,\ \bar{\varepsilon}_{pi} + d\,\bar{\varepsilon}_p)\}$, $\varkappa_i = \varkappa(\bar{\varepsilon}_p + d\,\bar{\varepsilon}_p,\ \bar{\varepsilon}_{pi})$, $\tilde{\varkappa}_j = \varkappa(\bar{\varepsilon}_p,\ \bar{\varepsilon}_{pi} + d\,\bar{\varepsilon}_p)$.

3. PROPOSED CYCLIC PLASTICITY MODEL

The model proposed in this paper is a stress-strain model constructed by refinement of Petersson-Popov Model to predict, with high accuracy, hysteretic stress-strain relations observed in measurements.

Important features of this model are referred to in this section.

(1) Evaluation of cumulative equivalent plastic strain

Accumulation of equivalent plastic strain in the process of repetitive loading is accomplished in a way consistent with the results of measurements. In **Fig. 2** a solid line shows an example of experimental results of repetitive loading tests and a broken line shows one of monotonous loading tests. In each test unloading took place at the point ① and the point ①′ respectively. If plastic strain is accumulated over all paths, the cumulative plastic strain at the point ① is fairly greater than that at the point ①′. In spite of this inference, the stress-strain curve on the path after the point ① is much the same as that on the path after the point ①′. From these experimental results we can understand the fact that the plastic strain produced in repetitive loading processes has to be separated in two components : one has an effect on following stress-strain relations and the other does not.

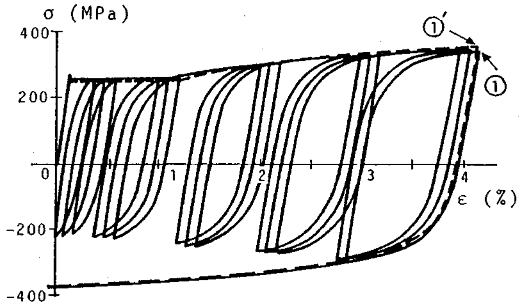


Fig. 2 Experimental stress-strain relations.

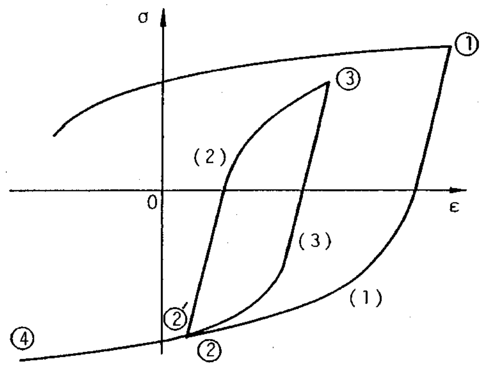


Fig. 3 Schematical figure showing ineffectiveness of a small strain amplitude on the following stress-strain relation.

Yokoo and Nakamura and et al. referred to the following phenomenon shown in **Fig. 3** as "Return Phenomenon"[16),17)] : "If a strain amplitude corresponding to a path (2) from the point ② to the point ③ is fairly small compared with its preceding strain amplitude, the stress-strain curve from the point ③ to the point ④, after passing through the point ②′ near the point ②, traces on such path that is the stress-strain curve in the case where a load reversal does not occur at the point ②. According to such a phenomenon, the stress-strain paths (2) and (3) do not affect the stress-strain curve following to these paths."

Basing on the above mentioned phenomena, the cumulative equivalent plastic strain is evaluated under the assumption that the plastic strain beyond the preceding plastic strain amplitude is effective. By the use of the estimation method, it appears to be not easy to concern cyclic hardening or cyclic softening phenomenon. However, the emphasis in this paper is to predict responses of structural steels under severe hysteretic loading and then cyclic number considered is not so large that cyclic hardening or cyclic softening occurs. As an example, thick lines in Fig. 4 show the paths on which the plastic strains are to be accumulated in the case of uniaxial loading. Since the estimation method of the cumulative equivalent plastic strain in the case of multi-axial loading is more complicated, we do not refer to in this paper.

(2) Choice of fundamental surface size functions and institution of weighting functions

Fig. 5 shows examples of tension-compression stress-strain relations of mild steel. In general, when a steel specimen is stretched and the unloading occurs on the yield plateau, Bauschinger effect part appears which is followed by an yield plateau in compression region. When it occurs at the start point of strain hardening in tension region, the yield plateau in the compression region does not appear any longer and the stress-strain curve becomes smooth. Although this phenomenon occurs similarly in the case where unloading occurs in strain hardening region, the degree of Bauschinger effect is not the same. Accordingly, the characteristic features of stress-strain relations of steels subjected to the repetitive loading are as follows ;

a) the appearance of the yield plateau in the hysteretic loading processes when the unloading occurs on the yield plateau and its gradual disappearance in the successive processes,

b) the change in the degree of Bauschinger effect when the unloading takes place in the strain hardening region.

It appears to be one of the necessary conditions for the stress-strain model compatible with experimental results that the model can represent these features. In the proposed model, to get better compatibility to experimental results, the surface size function κ_{ab} has been introduced as one of the fundamental surface size functions in addition to κ_a and κ_b. The function κ_{ab} is a surface size function of materials with loading history corresponding to a start point of the strain hardening.

Following to the introduction of κ_{ab}, W_1 and W_2 are defined as weighting functions. The function W_1 expresses a phenomenon that stress-strain curve changes continuously from the virgin stress-strain curve, which is characterized by the yield plateau and the strain hardening region, to the smooth curve, on which Bauschinger effect is characteristic. The function W_2 stands for the cyclic softening or cyclic hardening following to the progress of loading histories. As shown in Fig. 6 the surface size function at a certain hysteretic phase is defined by means of these weighting functions as follows;

$$\kappa=\begin{cases} W_1\,\kappa_a+(1-W_1)\,\kappa_{ab} & ;0\leqq\bar{\varepsilon}_p<\bar{\varepsilon}_{p,st} \\ W_2\,\kappa_{ab}+(1-W_2)\,\kappa_b & ;\bar{\varepsilon}_{p,st}<\bar{\varepsilon}_p\leqq\bar{\varepsilon}_{p,b} \\ \kappa_b & ;\bar{\varepsilon}_{p,b}\leqq\bar{\varepsilon}_p \end{cases} \cdots(4)$$

where $\bar{\varepsilon}_{p,st}$ is the plastic strain at the start point of strain hardening on virgin stress-surain curve. And $\bar{\varepsilon}_{p,b}$ is that at a loading phase in which the hysteretic effect becomes stationary. From the definitions of these weighting functions, $W_1=1$ at $\bar{\varepsilon}_p=0$, $W_1=0$ and $W_2=1$ at $\bar{\varepsilon}_p=\bar{\varepsilon}_{p,st}$ and $W_2=0$ at $\bar{\varepsilon}_p=\bar{\varepsilon}_{p,b}$. The weighting function W introduced in Petersson-Popov Model is defined as a function of $\bar{\varepsilon}_p$ and estimated by numerical trial and error as mentioned before. On the contrary, the function in the proposed model is determined clearly by means of results of several fundamental experiments. According to the authors' investiations, the weighting function has a tendency to change with $\bar{\varepsilon}_{pi}$. Thus the weighting function in our model is defined as a function of not just $\bar{\varepsilon}_p$ but $\bar{\varepsilon}_{pi}$.

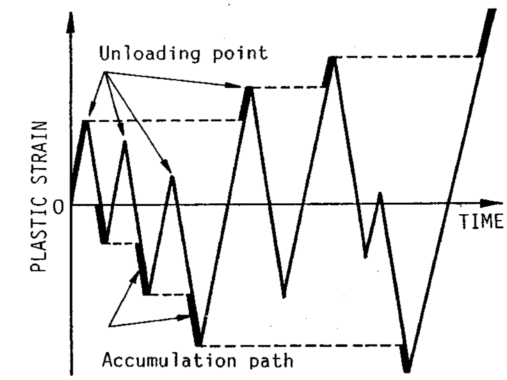


Fig. 4 Cumulative plastic strain.

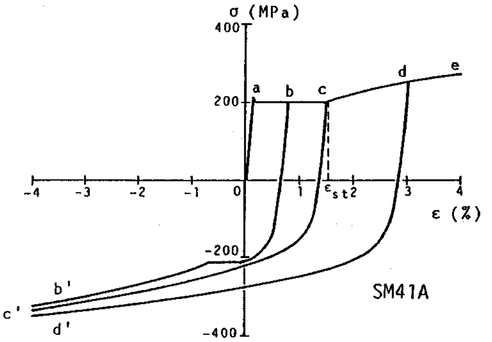


Fig. 5 Experimental stress-strain curves with unloading.

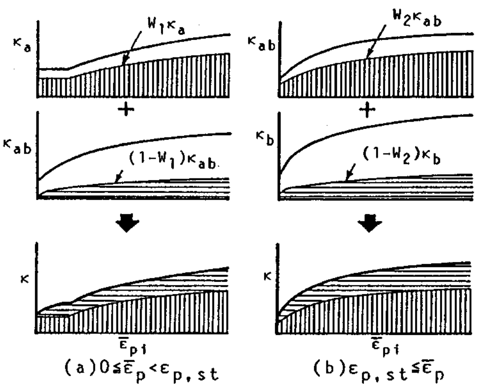


Fig. 6 Surface size functions.

(3) Estimation of material properties

In this section, presented is a method of experiments and its interpretation to evaluate meterial properties κ_a, κ_{ab}, κ_b, W_1 and W_2, which are necessary in order to predict hysteretic behaviors of steel by the proposed model. Since the model is based on the assumption that the stress-strain relations on the certain monotonous loading path or unloading path is determined by means of cumulative equivalent plastic strain $\bar{\varepsilon}_p$ at the start point of the loading or unloading, material properties can be estimated by a combination of a monotonous tension test and several tension-compression tests each including only one reversed point.

A procedure evaluating material properties is as follows.

a) Determination of κ_a.

Virgin stress-plastic strain curve represents κ_a.

b) Determination of surface size functions corresponding to some loading phases.

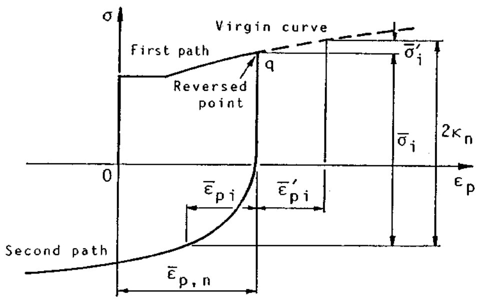


Fig. 7 Estimation of surface size function.

Surface sizes κ_n's corresponding to the cumulative plastic strains $\bar{\varepsilon}_{p,n}$'s have to be determined. Using the virgin stress-plastic strain curve and the stress-plastic strain curve obtained by unloading from the point where the cumulative plastic strain reaches $\bar{\varepsilon}_{p,n}$, κ_n is evaluated as the function of $\bar{\varepsilon}_{pi}$. Fig. 7 shows how to evaluate κ_n corresponding to the reversed plastic strain $\bar{\varepsilon}_{p,n}$.

c) Determination of κ_{ab} and κ_b

Since κ_{ab} corresponds to such κ_n that its parameter $\bar{\varepsilon}_{p,n}$ equals to $\bar{\varepsilon}_{p,st}$, it is gained by a combination of two curves. One of them is the virgin stress-plastic strain curve and the other is the stress-plastic strain curve, measured in a test including unloading at the start point of strain hardening. κ_n corresponding to the state in which the hysteretic effect becomes stationary is κ_b. The stationary state in hysteretic effect means that no difference is found among each κ_n's. Accordingly in the case where κ_n's do not converge within the experiments, κ_n for the measured maximum $\bar{\varepsilon}_{p,n}$ is adopted as κ_b. In this case it will be possible to correctly predict hysteretic behaviors within the limits where the cumulative equivalent plasic strain does not exceed the maximum $\bar{\varepsilon}_p$.

d) Determination of W_1 and W_2

By means of κ_a, κ_{ab} and κ_b, weighting values in order to evaluate κ_n corresponding to the values of $\bar{\varepsilon}_{p,n}$'s is determined by the next equation.

$$\left.\begin{array}{ll} W_1=(\kappa_n-\kappa_{ab})/(\kappa_a-\kappa_{ab}) & ;0\leqq\bar{\varepsilon}_p<\bar{\varepsilon}_{p,st} \\ W_2=(\kappa_n-\kappa_b)/(\kappa_{ab}-\kappa_b) & ;\bar{\varepsilon}_{p,st}\leqq\bar{\varepsilon}_p \end{array}\right\} \qquad (5)$$

Weighting function is determined by this formula which shows the relation of the weighting values and the corresponding $\bar{\varepsilon}_p$.

4. PREDICTION OF TENSION-COMPRESSION STRESS-STRAIN RELATIONS

(1) Specimens and testing apparatus

Structural steels of SM 41 A and HT 70 were used. Table 1 shows the mechanical properties of the steels presented by the steel makers. The configuration of the test specimens used is illustrated in Fig. 8. A testing machine with 30 tf capacity tension-compression actuator was employed and the oil pressure chucking system with 20 tf capacity was used for setting the specimens.

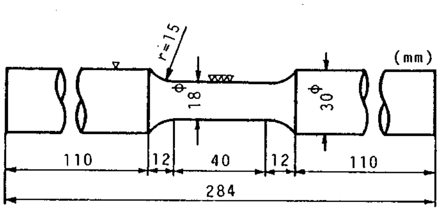


Fig. 8 Configuration of specimens.

The load was detected by a load-cell attached to the testing machine and the strain was detected by inelastic range strain gauges. The loading is controled by the strain at the central section of the test specimens. For all specimens, the strain rates were about 0.0001 mm/mm/sec.

Table 1 Mechanical properties of tested steels.

	Y.P. (MPa)	T.S. (MPa)	El. (%)
SM41A	284	421	37
HT70	622	661	–

(2) Experimental results and estimation of material

Properties

Fig. 9 shows stress-plastic strain relations obtained by the fundamental measurements in order to estimate the material properties. In these figures the stress value is normalized by the lower yield point. The fundamental surface size functions and the weighting functions estimated from these experimental results are shown in **Fig. 10** and **Fig. 11** respectivvely. The weighing function W_1 is assumed previsionally to decrease linearly according to the increase of the cumulative equivalent plastic strain.

(3) Comparison of experimental results and calculated results

Elasto-plastic finite element analyses were carried out for round-bar steel specimens, as shown by **Fig. 8**, subjected to tension-compression repetitive loading under strain control[18),19)]. Though specimens were subjected to uniaxial loads, actual stress conditions in the specimens were expected to be not uniaxial. So two-dimensional finite element analyses were carried out. Assumptions introduced in the analyses are as follows ;

a) constant strain triangular finite elements were used,

b) initial yielding complies with von Mises yield criterion,

c) yielding was judged by using the r-min method[20)],

d) incremental method was used as a nonlinear calculation procedure.

Fig. 12 shows stress-strain relations predicted by the proposed model and those gained by the corresponding experiments. These measured stress-strain relations are not referred to at all as the data for the purpose of estimating material properties used in analyses. **Fig. 12 (b)** for SM 41 A shows a tendency that Bauschinger effects are underestimated early in the load cycling. As a whole, however, in spite of the use of material properties determined from fundamental measurements only for some specimens, the stress-strain relations predicted by F. E. M. analyses coincide considerably with the measured stress-strain relations.

Secondly, how effective are the principal features of the proposed model for the purpose of

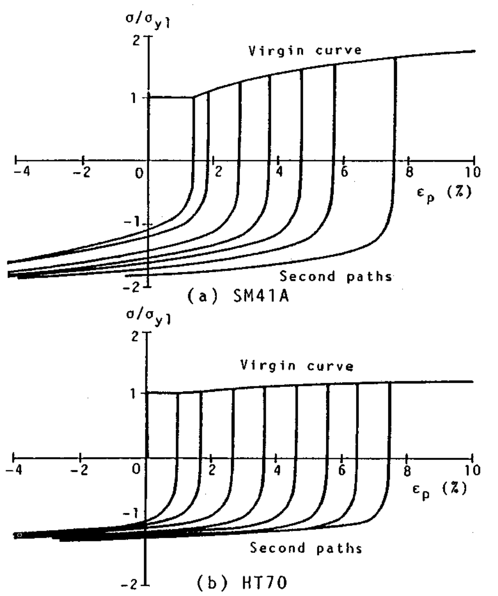


Fig. 9 Experimental stress-plastic strain relations for estimating material property parameters.

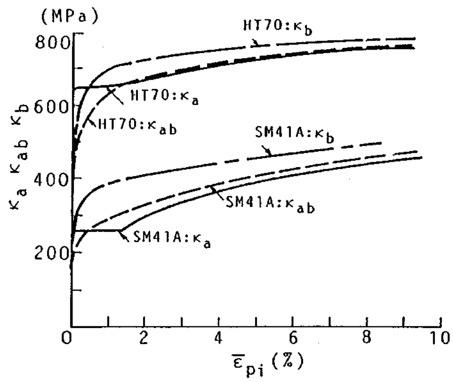


Fig. 10 Fundamental surface size functions.

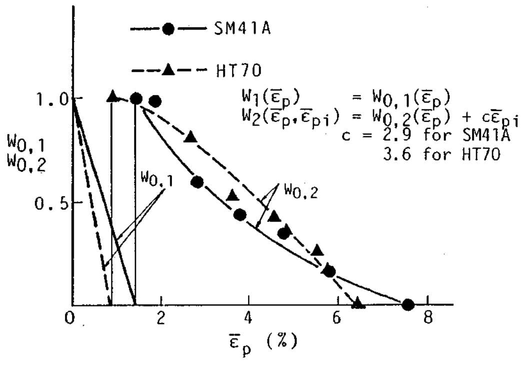
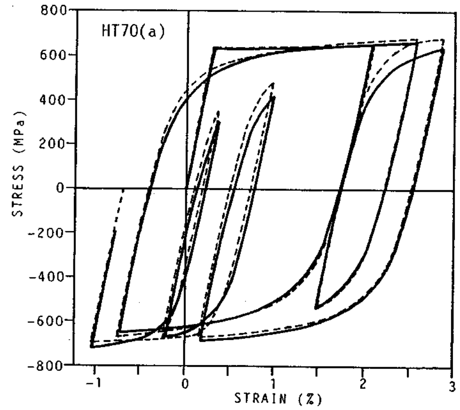


Fig. 11 Weighting functions.

improving the prediction accuracy is referred to. Fig. 13 shows stress-strain relations predicted by the model and those calculated by using the summations of the equivalent plastic strain over all strain paths as the cumulative equivalent plastic strain. In the latter case, according to the overestimations of the cumulative equivalent plastic strain, stress values are estimated larger by degrees. Fig. 14 shows stress-strain relations calculated by using the proposed model compared with those gained by the model of which fundamental surface size functions are only χ_a and χ_b in the same manner as Petersson-Popov Model. If the surface size function χ_{ab} is not introduced, the yield plateau represented by the function χ_a appears in the following processes of load repetitions. This appears to be caused by the use of the surface size function calculated as the weighted summations of χ_a and χ_b. In order to reproduce the disappearance of the yield plateau, the weighting value is to be zero at $\bar{\varepsilon}_p = \bar{\varepsilon}_{p,st}$. But the change in Bauschinger effect can not be represented any longer when $\bar{\varepsilon}_p$ becomes greater than $\bar{\varepsilon}_{p,st}$. Thus for the purpose of expressing the cyclic plasticity of the materials with the yield plateau and the notable strain hardening characteristic, it is necessary to use at least three fundamental surface size functions.

5. CONCLUSIONS

(1) Proposed was a new cyclic plasticity model by some refinements of the model presented by Petersson and Popov.



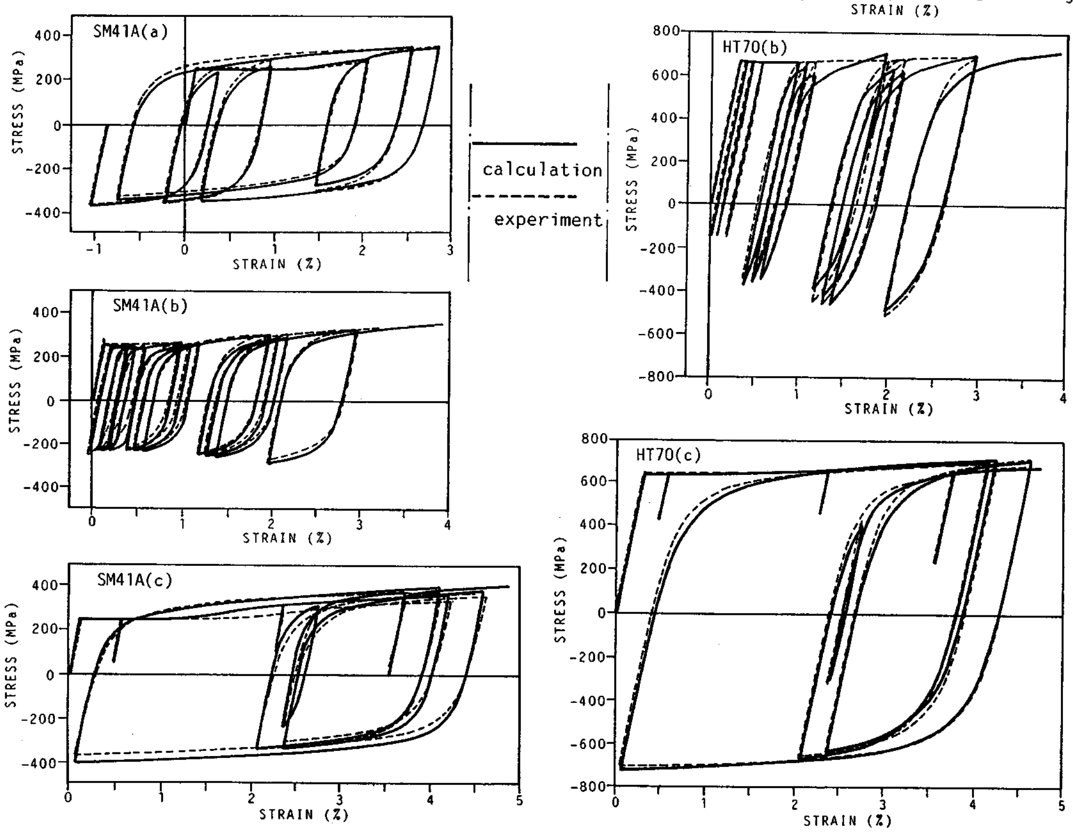


Fig. 12 Comparisons of stress-strain relations; experiments and calculations by the proposed model.

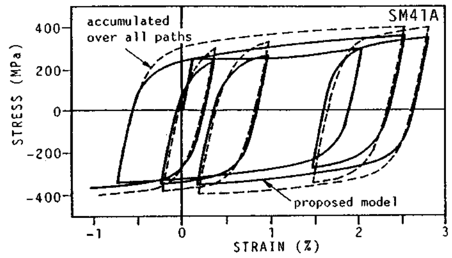


Fig. 13 Effect of estimation method of the cumulative equivalent plastic strain.

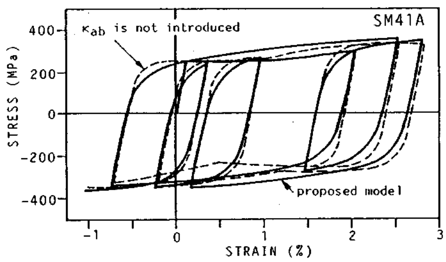


Fig. 14 Effect of the fundamental surface size function κ_{ab}.

(2) Material properties introduced in the proposed model can be easily estimated by a combination of the monotonous tension test and several tension-compression tests each including only one unloading.

(3) Tension-compression stress-strain relations of mild steel and high strength steel were predicted by means of the proposed model with high accuracy.

(4) The estimation method of the cumulative equivalent plastic strain as shown by Fig. 4 is effective for improving the accuracy of the predicted stress-strain relations.

(5) In the case of the materials with the yield plateau and the notable strain hardening such as the mild steel, the predictions of the actual stress-strain relations appears to be possible by the intoduction of the surface size function at the start point of strain hardening as a fundamental surface size function.

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