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A NON-ITERATIVE NONLINEAR ANALYSIS SCHEME OF FRAMES WITH THIN-WALLED ELASTIC MEMBERS

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The purpose of this study is to establish a non-iterative efficient computational scheme to trace the nonlinear finite displacement behaviour of space frames, using the tangent stiffness equation of linearized fininte displacement of a thin-walled elastic straight beam element. Direct solution of the tangent stiffness equation is used, imposing adequately small increments. Local coordinates are updated at each incremental step, utilizing a vector multiplication scheme. Numerical results for a wide variety of spatial structures are given, demonstrating the versatility of the present scheme.

Keywords : nonlinear analysis, non-interative procedures, FEM, space structures, thin-walled members

1. INTRODUCTION

The non-linear finite displacement behaviour of structures has attracted the attention of many researchers during the past few decades. Historically, Euler (see Ref. 1) happens to be among the earliest to investigate the post-buckling behaviour of elastic structutes, well-known as elastica, who pointed out that a slender column after buckling can increase its load carrying capacity significantly when the displacements become large. Following him, a considerable amount of works have been presented in the area of post-buckling behaviour of beams and frames. However, only a few of them have dealt with case of spatial structures. Zamost and Johnston[2] analysed the post lateral-buckling behaviour of a cantilever beam with uniform rectangular cross-section. The behaviour was found to be of the same nature as that of the planar problem. A similar case was also examined by Masur[3], who analysed a propped cantilever of narrow rectangular cross-section under a moment applid at the propped end.

Among those who treated the buckling and nonlinear behaviour of thin-walled members based on the finite element technique are Barsoum and Gallagher[4], Bazant and El Nimeiri[5], Ram and Osterrieder[6], Komatsu and Sakimoto et al.[7),8)], the last two of which have extended their analysis to the inelastic behaviour. Papadrakakis[9] analysed the finite displacement behaviour of spatial structures using vector iteration methods, demonstrating the application of the dynamic relaxation method and the first order conjugate gradient methods.

Even though a considerable amount of works have been reported in this area, it can be seen that most of

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the solution schemes available at present have certain drawbacks and many limitations in application. Under these circumstances, the aim of the present study is to establish a non-iterative efficient computational scheme to investigate the nonlinear displacement behaviour of thin-walled elastic beams and space frames with general loading and boundary conditions, applicable for structures with any original configuration, and which can be conveniently utilized for routine use in practical applications.

2. LOAD-DISPLACEMENT ANALYSIS

Common trend at present in the nonlinear finite displacement analysis is the adoption of iteration schemes such as Newton-Raphson technique. However, the necessity of calculating the internal force at each and every incremental step and of coninuous checking of convergence turns to be rather inconvenient for wide applications. On the contrary, the direct application of incremental relations with the consistent coordinate transformations and updating procedures and by proper selection of incremental sizes, as introduced in this study, seems to be more appropriate in general applications.

As is well-known, there exist two different methods of incremental procedures to analyse the finite displacement behaviour of structures undergoing large displacements and rotations. The first is the Lagrangian approach, in which all incremental quantities are referred to the original unstressed state, whereas in the second method, namely the updated Lagrangian approach, they are referred to the current equilibrium state. Owing to the fact that the tangent stiffness depends only on the current internal stress resultants, or in other words, is independant of the current displacements, the uqdated Lagrangian approach seems easier to handle and thus is employed in this study.

The nonlinear finite-displacement behaviour is traced in this study using the tangent stiffness equation of thin-walled straight beam element developed in Ref. 10, which can be written in the form

$$\tilde{\boldsymbol{F}}=\tilde{\boldsymbol{K}}(P^0)\tilde{\boldsymbol{d}} \quad (1)$$

for an arbitrary reference state in equlibrium where some internal stresses may exist, and $\tilde{\boldsymbol{F}}$, $\tilde{\boldsymbol{d}}$ and $\tilde{\boldsymbol{K}}$ are the incremental force and displacement vectors and the tangent stiffness matrix, respectively, and are the same as $\boldsymbol{F}$, $\boldsymbol{d}$ and $\boldsymbol{K}$ in Ref. 10, whereas P^0 stands for the internal stress resultants in the element at the current reference state. The stiffness equation is defined for a right hand cartesian coordinate system (x, y, z), with x along the beam axis, and y and z being the principal axes with origin at the centroid of the cross-section.

3. COORDINATE TRANSFORMATION AND ASSEMBLING

In the element stiffness equation (1), the components of the force and displacement vectors have been arranged separated to their groups of individual problems, namely, the axial, biaxial bending and torsional problems, in order to simplify the appearance of the stiffness matrix. Nevertheless, before the assembling process and transformation, the end displacements need to be given in vector representation, and also they should be rearranged separated to those corresponding to the end nodes. This can be achieved by making use of the following transformations.

$$\tilde{\boldsymbol{F}}=\boldsymbol{P}\boldsymbol{F}^{cs} \quad (2\cdot a)$$

$$\tilde{\boldsymbol{d}}=\boldsymbol{P}\boldsymbol{d}^{cs} \quad (2\cdot b)$$

where

$$\boldsymbol{P}=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\0&0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&1\end{bmatrix} \quad\cdots\cdots(3)$$

and

$$\boldsymbol{F}^{cs}=\langle \boldsymbol{F}_i^{csT}, \boldsymbol{F}_j^{csT}\rangle^T, \quad \boldsymbol{d}^{cs}=\langle \boldsymbol{d}_i^{csT}, \boldsymbol{d}_j^{csT}\rangle^T \quad\cdots\cdots(4\cdot a, b)$$

in which

$$\boldsymbol{F}_i^{cs}=\langle F_{xi}, F_{yi}, F_{zi}, C_{xi}, C_{yi}, C_{zi}, C_{\omega i}\rangle^T \quad\cdots\cdots(5\cdot a)$$

$$\boldsymbol{d}_i^{cs}=\langle u_{ci}, v_{si}, w_{si}, \phi_i, -w'_{si}, v'_{si}, -\phi'_i\rangle^T \quad\cdots\cdots(5\cdot b)$$

In Eq. (5), u, v and w are the displacements in the directions x, y and z respectively, and ϕ is the rotation of the cross-section. Subscripts c and s refer to the centroid and the shear center of the cross-setion, respectively, whereas i and j correspond to the element ends at the two ends at $x=0$ and $x=L$, respectively. Prime $(\ )'$ denotes differentiation with respect to x. The components at the j-node of Eq. (4) is defined in the same way as Eq. (5).

As is clear from Eq. (5), some quantities in the force and displacement vectors ($\boldsymbol{F}^{cs}$ and $\boldsymbol{d}^{cs}$) are referred to the shear center, whereas the remaining ones are referred to the centroid of the cross-sention. However, before transforming into the global coordinates, it is necessary that all quantities be referred to a single point on the cross-section. The cross-sectional centroid is used herein as the reference point. The quantities defined in the single point reference system (i. e. all the quantities are referred to the centroid) will be assigned by the superscript 'c'.

The relations between quantities defined in two different systems can be written in the form

$$\boldsymbol{d}^{cs}=\boldsymbol{Q}\boldsymbol{d}^{c}, \quad \boldsymbol{F}^{c}=\boldsymbol{Q}^T\boldsymbol{F}^{cs} \quad\cdots\cdots(6\cdot a, b)$$

with

$$\boldsymbol{Q}=\left[\begin{array}{ccccccc|ccccccc}1&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&-z_s&0&0&0&0&0&0&0&0&0&0\\0&0&1&y_s&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&y_s&0&0&0&0&0&0&0\\0&0&0&0&0&1&z_s&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0&0\\\hline 0&0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&-z_s&0&0&0\\0&0&0&0&0&0&0&0&0&1&y_s&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0&y_s\\0&0&0&0&0&0&0&0&0&0&0&0&1&z_s\\0&0&0&0&0&0&0&0&0&0&0&0&0&1\end{array}\right] \quad\cdots\cdots(7)$$

in which y_s and z_s are the coordinates of the shear center. By performing the transformations (2) and (6), the element stiffness equation (1) can be expressed in terms of the local coordinates as

$$\boldsymbol{K}^c \boldsymbol{d}^c = \boldsymbol{F}^c \quad \text{in which} \quad \boldsymbol{K}^c = (\boldsymbol{PQ})^T \tilde{\boldsymbol{K}} (\boldsymbol{PQ}) \cdots (8, 9)$$

Global cartesian coordinate system (X, Y, Z) and moving local cartesian coordinate system (x, y, z) defined at element level are being used in this study, as shown in Fig. 1 (a). The unit vectors along the axes X, Y and Z are $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ whereas those along the axes x, y and z in terms of global coordinates are denoted by $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, respectively. The direction cosines of x, y and z with respect to the global coordinates are $(a_X, a_Y, a_Z)(b_X, b_Y, b_Z)$ and (c_X, c_Y, c_Z), respectively. Hence, the relations between quantities defined in local and global coordinates can be expressed in the form

$$\boldsymbol{d}^c = \boldsymbol{T}\boldsymbol{d}, \quad \boldsymbol{F}^c = \boldsymbol{T}\boldsymbol{F} \cdots\cdots (10\cdot a, b)$$

where $\boldsymbol{d}$ and $\boldsymbol{F}$ are the displacement and load vectors in global coordinates, and the transformation matrix $\boldsymbol{T}$ is given by

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{t} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{t} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & 1 & \boldsymbol{0} & \boldsymbol{0} & 0 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 1 \end{bmatrix} \cdots\cdots (11)$$

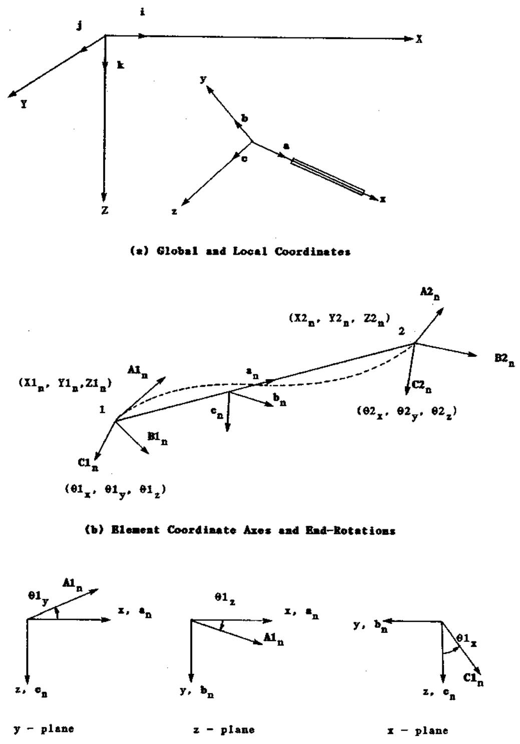

Fig. 1 Coordinate Axes and Reference Base Vectors.

in which

$$\boldsymbol{t} = \begin{bmatrix} a_X & a_Y & a_Z \\ b_X & b_Y & b_Z \\ c_X & c_Y & c_Z \end{bmatrix} \cdots\cdots (12)$$

Hence, by performing the coordinate transformation (10), the element stiffness equation (8) can be expressed in terms of the global coordinates in the form

$$\boldsymbol{K}\boldsymbol{d} = \boldsymbol{F} \cdots\cdots (13)$$

in which

$$\boldsymbol{K} = (\boldsymbol{PQT})^T \tilde{\boldsymbol{K}} (\boldsymbol{PQT}) \cdots\cdots (14)$$

Global stiffness equation is assembled for the whole structure and solved for the next incremental step. In order to overcome the singularity of the stiffness matrix at limit and bifurcation points, and also to achieve higher accuracy for the same computational effort, the path length control technique[11] is adopted in this study with some modifications of simplified linear form of absolute values, i. e. in addition to the stiffness equation, the following condition is imposed

$$\sum_{i=1}^{n} |\alpha_i \ d_i| + |\alpha_{n+1} \ f| = c \cdots\cdots (15)$$

in which n is the dimension of the stiffness matrix, d_i is displacement component nondimensionalized by its norm, and f is the load factor defined by

$$\boldsymbol{F} = f \cdot \boldsymbol{f} \cdots\cdots (16)$$

where $\boldsymbol{f}$ is the proportional load vector. Magnitudes of the nondimensional constants α_1 to α_{n+1} are so selected that the order of all terms in the left hand side side of Eq. (15) remains of similar magnitude, and c is another constant defining the magnitude of the incremental step. In numerical calculations, the absolute values appeared in Eq. (15) are achieved by defining the sign of α_i to be the same as that of the corresponding displacement or load step in the previous increment.

4. UPDATING PROCEDURE

(1) Updating of Coordinates

Consider a beam element in equilibrium with external loads at an arbitrary state, of which the configuration is already known. The unit vectors along the element axes (x, y, z) are denoted by $\boldsymbol{a}_n^m$, $\boldsymbol{b}_n^m$, $\boldsymbol{c}_n^m$ respectively as shown in Fig. 1 (b), in which the subscript 'n' refers to the number of the current reference state, whereas the superscript 'm' refers to the element number. Due to deformation of the element, the systems of beam axes at the two ends may not lie along the same directions as the system of local coordinate axes. The unit vectors along the beam axes at the two ends are denoted by $\mathbf{A1}_n^m$, $\mathbf{B1}_n^m$ and $\mathbf{C1}_n^m$ at $x=0$, and $\mathbf{A2}_n^m$, $\mathbf{B2}_n^m$ and $\mathbf{C2}_n^m$ at $x=L$, following the same notation as for the element local axes. The projected rotations at the two ends about the local axes (all of which are already known) are denoted by $(\theta 1_x)_n^m$, $(\theta 1_y)_n^m$, and $(\theta 1_z)_n^m$ at $x=0$, and $(\theta 2_x)_n^m$, $(\theta 2_y)_n^m$, and $(\theta 2_z)_n^m$ at $x=L$. Consider the rotations at $x=0$ through Fig. 1 (c). The angles $\theta 1_y$ and $\theta 1_z$ are those between the element local x-axis along the longitudinal direction and the projections of the element end longitudinal direction $\mathbf{A1}$ to the respective local y- and z-planes, whereas $\theta 1_x$ is the angle between local z-axis and the projection of $\mathbf{C1}$ to the x-plane. The rotations at $x=L$ can be defined in the same way as above. All the angles are measured starting from the respective local axis, with positive sign given when measured in positive direction of the axis of the plane, using the right hand screw rule. The superscript 'm' is omitted in further explanations, for the purpose of reducing complexity in appearance of the expressions. However, it should be understood that all expressions correspond to the m-th element.

For the ease of clear understanding, the tangent stiffness equation, whichever appropriate, Eq. (8) for local coordinates or Eq. (13) for global coordinates, is simply re-written as

$$\boldsymbol{K}_T \Delta \boldsymbol{d} = \Delta \boldsymbol{F} \quad \cdots\cdots (17)$$

where

$$\Delta \boldsymbol{F} = \langle \Delta \boldsymbol{F}_i^T, \Delta \boldsymbol{F}_j^T \rangle^T, \quad \Delta \boldsymbol{d} = \langle \Delta \boldsymbol{d}_i^T, \Delta \boldsymbol{d}_j^T \rangle^T \quad \cdots\cdots (18)$$

with

$$\Delta \boldsymbol{F}_i = \langle F_{xi}, F_{yi}, F_{zi}, C_{xi}, C_{yi}, C_{zi}, C_{\omega i} \rangle^T = \langle \Delta F1_x, \Delta F1_y, \Delta F1_z, \Delta C1_x, \Delta C1_y, \Delta C1_z, \Delta C1_\omega \rangle^T \quad \cdots\cdots (19)$$

$$\Delta \boldsymbol{d}_i = \langle u_{ci}, v_{ci}, w_{ci}, \phi_i, -w'_{ci}, v'_{ci}, -\phi'_i \rangle^T = \langle \Delta u1, \Delta v1, \Delta w1, \Delta\theta 1_x, \Delta\theta 1_y, \Delta\theta 1_z, \Delta\theta 1_\omega \rangle^T \quad \cdots\cdots (20)$$

Note that $\Delta F1_x$ and $\Delta u1$, for example, are replaced to $\Delta F2_x$ and $\Delta u2$ for the element end j. After solving the incremental equation for the assemblage of Eq. (17), the new position coordinates at the ends of the element are obtained by the relations

$$X1_{n+1} = X1_n + \Delta u1_n, \quad Y1_{n+1} = Y1_n + \Delta v1_n, \quad Z1_{n+1} = Z1_n + \Delta w1_n \quad \cdots\cdots (21\cdot\text{a, b, c})$$

$$X2_{n+1} = X2_n + \Delta u2_n, \quad Y2_{n+1} = Y2_n + \Delta v2_n, \quad Z2_{n+1} = Z2_n + \Delta w2_n \quad \cdots\cdots (21\cdot\text{d, e. f})$$

in which all the increments are defined in global coordinates. However, regarding incremental rotations it is necessary that they are transformed into local coordinates, before adding to the previous rotations. Hence, the new rotations at the two ends of the beam element can be obtained by the relations

$$(\theta 1_x)_{n+1} = (\theta 1_x)_n + (\Delta\theta 1_x)_n, \quad (\theta 1_y)_{n+1} = (\theta 1_y)_n + (\Delta\theta 1_y)_n, \quad (\theta 1_z)_{n+1} = (\theta 1_z)_n + (\Delta\theta 1_z)_n \quad \cdots\cdots (22\cdot\text{a, b, c})$$

$$(\theta 2_x)_{n+1} = (\theta 2_x)_n + (\Delta\theta 2_x)_n, \quad (\theta 2_y)_{n+1} = (\theta 2_y)_n + (\Delta\theta 2_y)_n, \quad (\theta 2_z)_{n+1} = (\theta 2_z)_n + (\Delta\theta 2_z)_n \quad \cdots\cdots (22\cdot\text{d, e, f})$$

in which all the incremental rotations are defined with respect to the current local coordinates.

The unit vectors along the new directions of the beam axes at the two ends can then be determined by the relations

$$\boldsymbol{A1}_{n+1} = \frac{\boldsymbol{a}_n + \tan[(\theta 1_z)_{n+1}]\boldsymbol{b}_n - \tan[(\theta 1_y)_{n+1}]\boldsymbol{c}_n}{|\boldsymbol{a}_n + \tan[(\theta 1_z)_{n+1}]\boldsymbol{b}_n - \tan[(\theta 1_y)_{n+1}]\boldsymbol{c}_n|} \quad \cdots\cdots (23\cdot\text{a})$$

$$\boldsymbol{C1}_{n+1} = \frac{\boldsymbol{c}_n + \tan[(\theta 1_y)_{n+1}]\boldsymbol{a}_n - \tan[(\theta 1_x)_{n+1}]\boldsymbol{b}_n}{|\boldsymbol{c}_n + \tan[(\theta 1_y)_{n+1}]\boldsymbol{a}_n - \tan[(\theta 1_x)_{n+1}]\boldsymbol{b}_n|}, \quad \boldsymbol{B1}_{n+1} = \boldsymbol{C1}_{n+1} \times \boldsymbol{A1}_{n+1} \quad \cdots\cdots (23\cdot\text{b, c})$$

$$A2_{n+1}=\frac{a_n+\tan[(\theta 2_z)_{n+1}]b_n-\tan[(\theta 2_y)_{n+1}]c_n}{|a_n+\tan[(\theta 2_z)_{n+1}]b_n-\tan[(\theta 2_y)_{n+1}]c_n|} \quad (23\cdot d)$$

$$C2_{n+1}=\frac{c_n+\tan[(\theta 2_y)_{n+1}]a_n-\tan[(\theta 2_x)_{n+1}]b_n}{|c_n+\tan[(\theta 2_y)_{n+1}]a_n-\tan[(\theta 2_x)_{n+1}]b_n|},\quad B2_{n+1}=C2_{n+1}\times A2_{n+1} \quad (23\cdot e,\ f)$$

Next, the procedure to find the updated system of local coordinates, which are to be used as the local coordinates for the next incremental step, is explained. The local axis in the longitudinal direction of the beam element is selected along the line joining the two ends of the element, and hence the corresponding unit vector can be expressed in the form

$$a_{n+1}=\frac{(X2_{n+1}-X1_{n+1})i+(Y2_{n+1}-Y1_{n+1})j+(Z2_{n+1}-Z1_{n+1})k}{|(X2_{n+1}-X1_{n+1})i+(Y2_{n+1}-Y1_{n+1})j+(Z2_{n+1}-Z1_{n+1})k|} \quad (24)$$

The local z-axis is selected to be along the average of the corresponding directions at the two ends of the beam element. Hence,

$$c_{n+1}=\frac{C1_{n+1}+C2_{n+1}}{|C1_{n+1}+C2_{n+1}|} \quad (25)$$

and the local y-axis is obtained by using the orthogonal property with x and z axes. Therefore, the unit vector along y-axis can be obtained using the condition

$$b_{n+1}=\frac{c_{n+1}\times a_{n+1}}{|c_{n+1}\times a_{n+1}|} \quad (26)$$

As the present c_{n+1} may not be perpendicular to a_{n+1}, a modified c_{n+1} is obtained by making use of the condition

$$c_{n+1}=a_{n+1}\times b_{n+1} \quad (27)$$

Next, the end rotations with respect to the new set of local coordinates are re-computed using the following relations.

$$(\theta 1_x)_{n+1}=\tan^{-1}\left(-\frac{C1_{n+1}\cdot b_{n+1}}{C1_{n+1}\cdot c_{n+1}}\right),\quad (\theta 1_y)_{n+1}=\tan^{-1}\left(-\frac{A1_{n+1}\cdot c_{n+1}}{A1_{n+1}\cdot a_{n+1}}\right) \quad (28\cdot a,\ b)$$

$$(\theta 1_z)_{n+1}=\tan^{-1}\left(-\frac{A1_{n+1}\cdot b_{n+1}}{A1_{n+1}\cdot a_{n+1}}\right),\quad (\theta 2_x)_{n+1}=\tan^{-1}\left(-\frac{C2_{n+1}\cdot b_{n+1}}{C2_{n+1}\cdot c_{n+1}}\right) \quad (28\cdot c,\ d)$$

$$(\theta 2_y)_{n+1}=\tan^{-1}\left(-\frac{A2_{n+1}\cdot c_{n+1}}{A2_{n+1}\cdot a_{n+1}}\right),\quad (\theta 2_z)_{n+1}=\tan^{-1}\left(-\frac{A2_{n+1}\cdot b_{n+1}}{A2_{n+1}\cdot a_{n+1}}\right) \quad (28\cdot e,\ f)$$

The calculations can then be proceeded to the next incremental step.

It is assumed in the numerical examples explained later that the inital element length L does not change, simply for the ease of computation, although the change can be evaluated from the new positions (21) of the element ends, if required. It is also worthwhile to mention that, as far as the in-plane behaviour is concerned, the evaluation only of a_{n+1} which is obtainable from the new positions (21) is required to proceed the computations.

(2) Updating of Stress Resultants

The procedure for updating the stress resultants is explained in this section. Consider the structure in equilibrium at the reference state 'n' and suppose that the configuration and the end forces of the individual elements are already known. The stiffness equation for the whole structure is solved for the next incremental step, thus leading to the incremental displcements. The coordinates can then be updated following the procedure explained in the preceeding section.

Next, the increments in the element end forces are found by the re-substitution of the incremental displacements to the individual element stiffness equations as given by

$$\Delta F^e=K_T^e\Delta d^e \quad (29)$$

Hence, the element end forces can be updated by adding the increments of the element end forces to the previous values in terms of global coordinates, thus leading to the next reference state, of which the configuration and the element end forces are known. The individual element end forces are next transformed to the respective new element local coordinates and the stress resultants at the new reference state can then be found. The stress resultants are computed by taking the average of the stress resultants at the two ends of the element, except for the axial stress resultant which is constant throughout the element length.

5. NUMERICAL EXAMPLES

The proposed method is applied to investigate the nonlinear finite displacement behaviour of a number of important spatial structures. In some cases, the structure itself is spatial, whereas, regarding the others the original structure is plane while the displacements occur in the out-of-plane direction as well. All the computations are performed using HITACHI HITAC M-280H computer. The following cases are considered.

1) Cantilever beam : Investigated first is the out-of-plane load-displacement behaviour of a uniform cantilever beam with doubly symmetric I-section under vertical load at the free end. The lateral buckling mode is initiated by a torsional moment at the free end, applied at the beginning. The displacements at the free end are plotted against the load, as shown in Fig. 2. Also shown in Fig. 3 are the deformed configurations of the structure at various load levels.

2) Symmetric circular fixed arch : Considered next is the out-of-plane load-displacement behaviour of a uniform circular arch with doubly symmetric I-section fixed at both ends under a concentrated vertical load at the crown with initially applied disturbing torque at the same point of application. The lateral and vertical displacements at the crown of the arch are plotted against the load, as shown in Fig. 4. The corresponding deformed configurations are shown in Fig. 5.

3) Twelve member hexagonal frame : The twelve member hexagonal frame with hinged supports shown in Fig. 6 under vertical load applied at the crest is considered next. All members are of uniform solid square cross-section without warping. The tangent stiffness equation for non-warping members is found in Ref. 12. The crest of the structure is allowed for the displacements in the vertical direction only. The displacements at the loading point are plotted against the load, as shown in Fig. 6. The deformed configurations are also shown in Fig. 7. It should be noted that the vertical dimensions in the deformed configurations have been exaggerated by three times, to distinguish the deformations.

4) Space frame : Investigated in the fourth is the finite displacement behaviour of the space frame shown in Fig. 8. The structure is loaded with four vertical concentrated loads of equal magnitude P at the joints, in addition to two disturbing

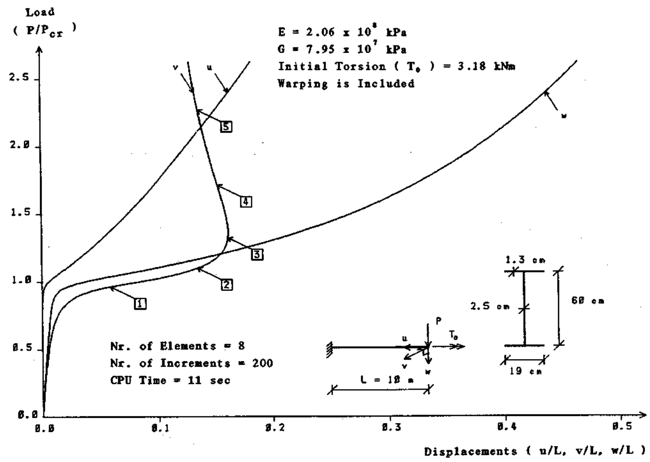

Fig. 2 Spatial Load-Displacement Behaviour of a Cantilever Beam.

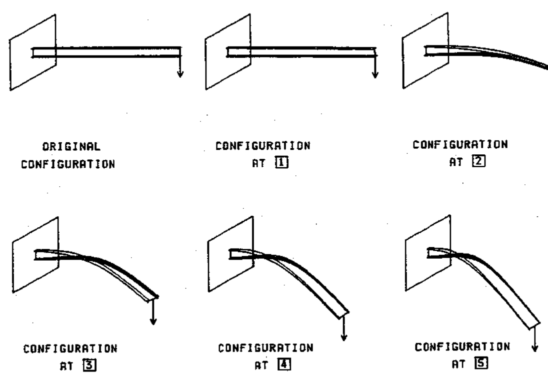

Fig. 3 Deformed Configurations of a Cantilever Beam.

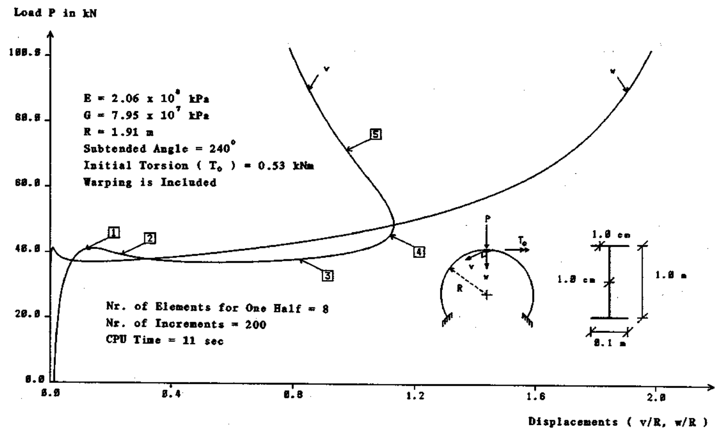

Fig. 4 Spatial Load-Displacement Behaviour of a Fixed Circular Arch.

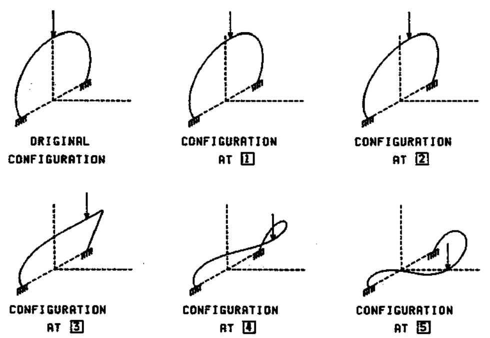

Fig. 5 Deformed Configurations of a Fixed Circular Arch.

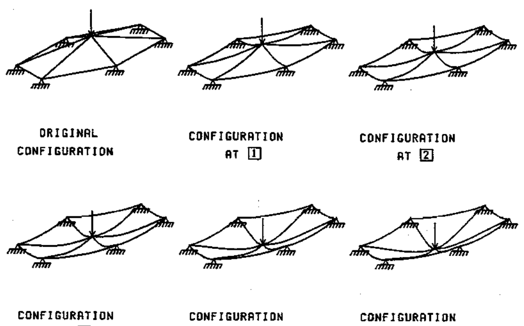

Fig. 7 Deformed Configurations of a Hexagonal Frame with Hinged Supports (3 times enlarged in Vertical Scale).

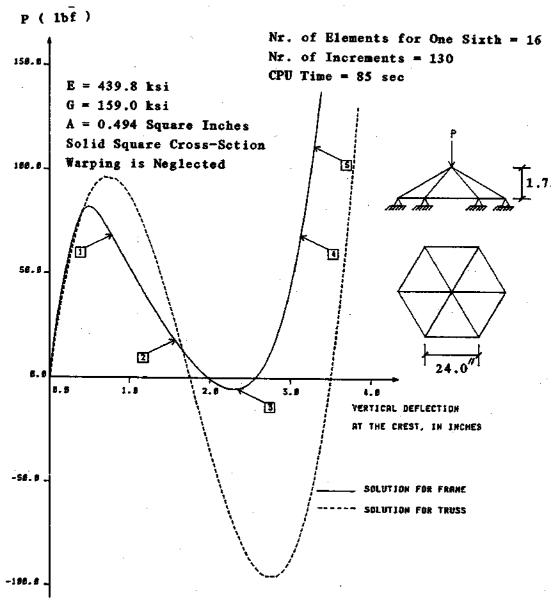

Fig. 6 Load-Displacement Behaviour of A Hexagonal Frame with Hinged Supports.

horizontal lateral loads of maguitude $P/1\,000$. All members of the structure are of uniform doubly symmetric I-section. The lateral and vertical displacements v and w at one joint are plotted against the load, as shown in Fig. 8. The deformed configurations are also shown in Fig. 9.

5) Truss Structures : In addition to the frame structures, the proposed method is applied for some truss assemblages also, for the confirmation of applicability. The stiffness equation for trusses is found in Ref. 12. The load-displacement behaviour for the truss idealization (all joints are assumed to be hinged, instead of the rigid joint condition assumed in frame analysis) of the twelve member hexagonal frames considered previously is obtained, as shown in Fig. 6. Also analysed is a reticulated space truss shown in Fig. 10, under a system of vertical loads at the joints. The result is presented in Fig. 10, showing the

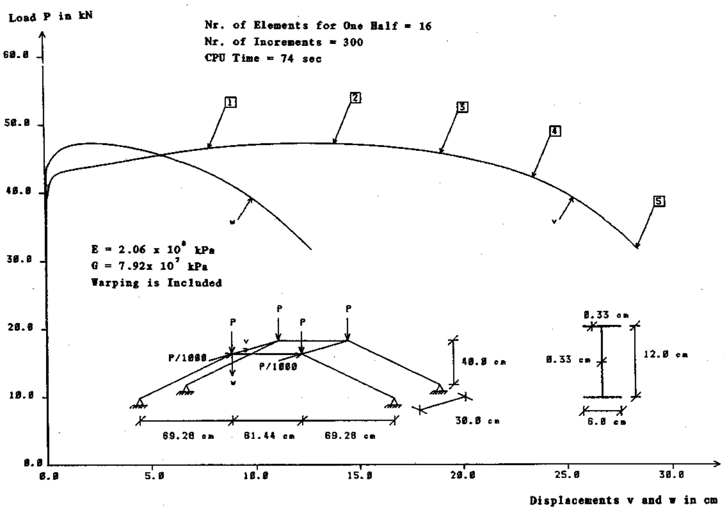

Fig. 8 Load-Displacement Behaviour of a Space Frame.

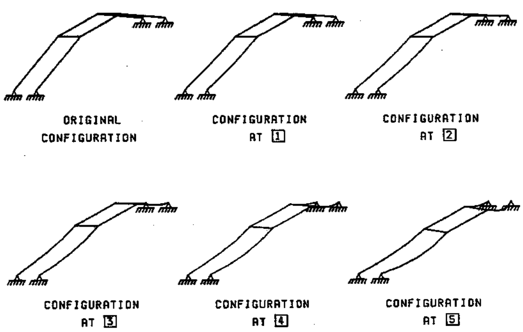

Fig. 9 Derormed Configurations of a Space Frame.

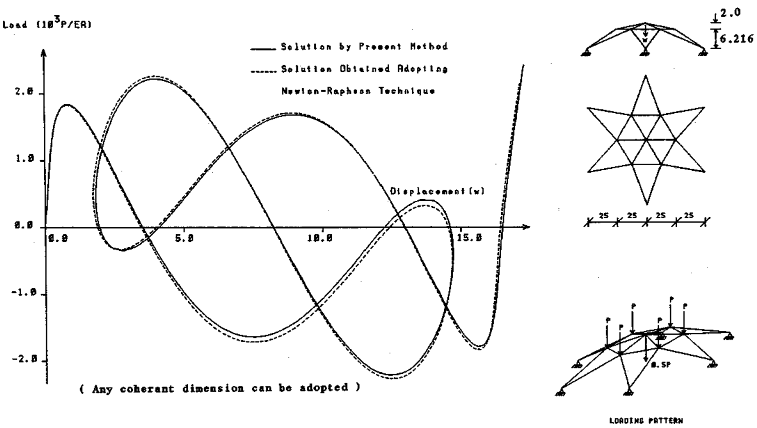

Fig. 10 Load-Displacement Behaviour of a Reticulated Space Truss.

relation of the load versus the displacement at the center. For comparison purposes, a more precise solution is also obtained by using the so-called Newton-Raphson technique, and it is seen that the results predicted by the present scheme closely agree with it.

A number of numerical examples have been limited in this paper because of page limitation. More extensive examples are seen in Ref. 13, including the rather familiar cases of the plane behaviour of structures.

6. CONVERGENCE STUDY

As is explained in the previous sections, the load-displacement behaviour is obtained in the present scheme by the direct solution of the incremental equation, without performing iterations. The important factors affecting the accuracy and the efficiency are the size of the incremental step and the selection of the coefficients in the path length control equation.

The solution is expected to converge to the exact solution, when the number of incremental steps approches infinity in a similar way as the convergence with respect to the number of elements. Numerical results for convergence analysis are obtained for the out-of-plane behaviour of cantilever beam considered in the first example, as shown in Fig. 11. As mentioned before, the rate of convergence depends also on the coefficients in the path length control equation (15). However, it seems difficult to present a general criterion applicable for all the structures, to select the most appropriate values for them. Nevertheless, in general, good estimates for the coefficients can be obtained by selecting them such that all the terms in the left hand side of the equation are of approximately similar magnitude.

From the results obtained in this study, a question may be raised regarding the rather wide-spread understanding that reliable solutions for finite displacements are obrainable only through the use of some iterative procedures with the inevitable consideration for large rotations. Although one may argue that the present results do not have any guarantee for 'true' solutions, a serious question is also raised what is a 'true' solution without any exact analytical solution. Having confirmed the numerically stable outputs which satisfy the equilibrium conditions for a wide variety of structures, the results can be identified as solutions under 'some' assumptions. Knowing the fact that the beam mechanics itself is not exact in the sense of, for example, the theory of elasticity, exactness may not be so imperative as in the case of explicit mathemmatical solutions, particularly when the result can only be found numerically. The most important and the most seriously considered for structural mechanics problems seems equilibrium condition to be satisfed. The solution procedure presented in this paper may be understood as the solutions with some sacrifice of the accumulated stress resultant-displacement relations which have not been examined carefully in the present scheme. However, this sacrifice may be accepted, when one may realize the fact that some level of approximation inevitably be involved in this sort of nonlinear numerical computations even with the considerations of iterations and large rotations particularly for space structures.

7. SUMMARY AND CONCLUSIONS

A computationsl scheme to investigate the nonlinear finite displacement behaviour of thin-walled beams and their assemblages, making use of the tangent stiffness matrix for thin-walled straight beam element, has been presented. Direct solution of the tangent stiffness matrix helped by a proposed updating procedure and a modified form of the path length control technique was utilized to trace the

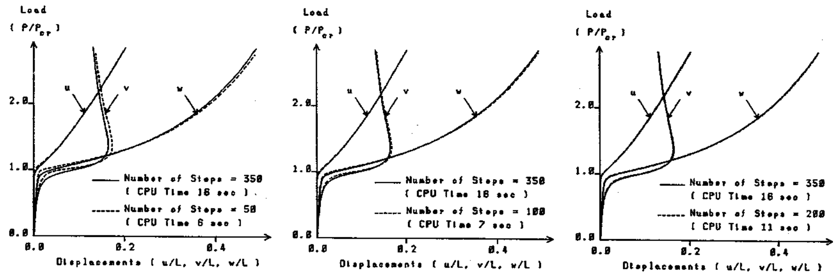

Fig. 11 Convergence Analysis.

load-displacement behaviour. Imposing adequately small increments, the accumulation of error has been reduced to an acceptable level.

Numerical results are presented for a variety of spatial structures having various initial geometries, boundary and loading conditions, including structures with snap-through type load-displacement behaviour. Buckling of bifurcation type is avoided by applying appropriate inital disturbing loads of relatively small magnitude. Noting that neither iteration nor checking of convergence is involved, it can be concluded that the present scheme is quite appropriate for routine use in practical applications.

8. ACKNOWLEDGEMENT

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