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A UNIFIED APPROACH TO THE ELASTO-PLASTIC STRENGTH OF COMPRESSED CYLINDRICAL SHELLS

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Proposed herein are two procedures for the strength of compressed cylindrical shells on the basis of so-called nonlinear bifurcation theory : one being a direct approach making use of the pathological curve representing the stability limit for the elasto-plastic behaviors near the elasto-plastic buckling point, and another being a unified approach making use of the pathological curve associated with the plastic failure mechanism but near the equivalent bifurcation point.

Several numerical demonstrations are provided to give the imperfection sensitivity curves, and the ultimate strength curves as the reasonable lower limit of several experimental data, yet showing good correlation with the DASt and ECCS design curves.
Keywords : cylindrical shells, elasto-plastic buckling, nonlinear bifurcation theory

1. INTRODUCTION

Nowadays, cylindrical shells are being widely used in many civil engineering structures such as offshore constructions, nuclear power plants, pipe lines and storage oil tanks. These cylindrical shells are often subjected to uniaxial compression either independently or in combination with hydraulic pressure or torsion. The compressed shells are generally believed to be very sensitive to even small initial imperfections, and thus may undergo global collapse easily. This is why the problems of the stability and the strength of cylindrical shells are of great significance.

Studies on the strength of cylindrical shells were initiated in the late nineteenth century[1]. Lorenz, von Kármán and Flüge obtained the buckling loads of cylinders, subjected to axial compression, uniform external pressure and their combinations, respectively[2]. Then, Donnell derived the fundamental equations of equilibrium for shells under torsion[3]. However, the results through the analytical investigations had been considerably discrepant from the corresponding experimental results by Flüge, Donnell and et al[4]. This discrepancy was attempted to be interpreted clearly in terms of the so-called imperfection sensitivity by such people as von Kármán and Tsien[5]. Moreover, Donnell and Wan clarified the post-bifurcation and the general equilibrium paths of compressed cylindrical shells[6].

Koiter discussed the stability and the imperfection sensitivity of cylinders on the basis of the potential energy function using his general theory of elastic stability[7]. He also analyzed the imperfection sensitivity curves of compressed cylindrical shells with axi-symmetric modes of initial deflections[8]. Arbocz and Babcock reported on relationships between the initial deflection modes and the buckling configuration

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modes from many test data[9]. Similarly, Hutchinson, Hansen and Croll focused on the imperfection sensitivity of axially loaded cylindrical shells[10)~12)]. On the other hand, advanced studies on the inelastic strength of cylinders have been vigorously performed by Batterman, Hutchinson and Croll[13)~15)]. Also, Vandepitte, Rathe and Bornscheuer reviewed many test results supporting the ECCS strength curves in the elastic and elasto-plastic ranges[16),17)].

Some types of shells are being designed so that they may fail in the elasto-plastic range. For the evaluation of their inelastic strength, the incremental materially and geometrically nonlinear numerical procedures seem to be being preferably adopted in recent years. These are time-consuming, nevertheless in general and besides, the strength can only be determined in an isolated form for a specified set of material and geometrical parameters.

Presented herein is a unified approach to the elasto-plastic strength of compressed cylindrical shells considering residual stresses where the effects of the initial deflections are explicitly designated by a unified strength formula[18)~22)].

2. BASIC CONCEPTS

(1) Elasto-plastic buckling strength

A cylindrical shell model of length L under uniaxial compression as shown in Figs. 1 is analyzed herein. It is assumed to be welded along a longitudinal line with an appropriate distribution of longitudinal residual stress uniformly in that direction. The distribution of the residual stress is assumed to be such that the maximum compressive stress is σ_r and to be either parabolic, triangular or trapezoidal in the circumferential direction as shown in Figs. 2(a)-(c), respectively. Herein, only a half of the distribution of each type residual stress is shown for the developed surface of the cylinder. Then, the relationships among the tangent modulus E_t, the secant modulus E_s, the average axial stress σ and the average axial strain ε can be obtained as[18)~22)]:

$$\sigma=\sigma(k),\quad \varepsilon=\varepsilon(k),\quad E_t=\frac{d\sigma}{d\varepsilon}=kE,\quad \text{and}\quad E_s=\frac{\sigma}{\varepsilon} \qquad (1)$$

where, both the stress and the strain are functions of the factor k, for given elastic Young's modulus E and yielding stress σ_Y, indicating the ratio of the elastic cross-section to the total section of the cylinder, that is, the factor k denotes the non-dimensionalized tangent modulus E_t by the Young's modulus, E.

The out-of-plane deflection, W, is assumed to be represented by the combination of an asymmetric and an axi-symmetric mode both in the elastic and the elasto-plastic ranges in the coordinate system of Fig. 1 as follows[8),9)]:

$$W=w_1\sin\frac{m\pi x}{L}\cos\frac{ny}{r}+w_2\cos\frac{l\pi x}{L} \qquad (2)$$

Herein, r, w_1 and w_2 refer to the radius of the cylinder considered, the magnitude of the asymmetric and the axi-symmetric buckling mode, respectively. Moreover, the magnitudes of the initial out-of-plane deflections corresponding to w, w_1 and w_2 are denoted as w_0, w_{01} and w_{02}, respectively.

The equilibrium of the cylinder can be determined from the modified Donnell's fundamental equations using the Airy's stress function, F, and the secant modulus E_S for the in-plane deformations and the Bleich's factor[23)] for the out-of-plane deformations, similar to the cases of plate members[18),20),22)]:

$$\left.\begin{aligned}&\nabla^4F+E_St\left(\frac{1}{r}W_{,xx}+W_{,xx}W_{,yy}-W_{,xy}^2+W_{,xx}W_{0,yy}-2W_{,xy}W_{0,xy}+W_{,yy}W_{0,xx}\right)=0\\&D\nabla_p^4W-\left[\frac{1}{r}F_{,xx}+F_{,xx}(W+W_0)_{,yy}-2F_{,xy}(W+W_0)_{,xy}+F_{,yy}(W+W_0)_{,xx}\right]=0\end{aligned}\right\} \qquad (3)$$

where

$$\nabla^4=(\nabla^2)^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2,\quad \nabla_p^4=(\nabla_p^2)^2=\left(\sqrt{\tau}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2,\quad D=\frac{Et^3}{12(1-\nu^2)}$$

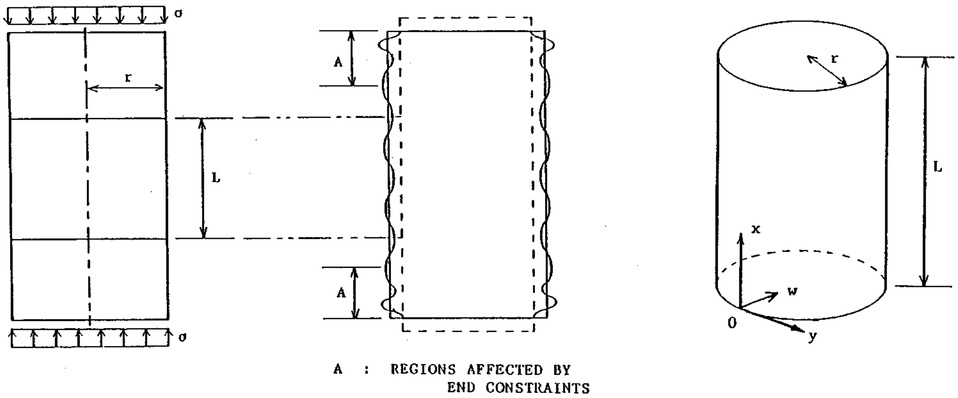


Fig. 1 Cylindrical Shell under Uniaxial Compression.

$\tau=\frac{E_t}{E}$, $W_{,xx}=\frac{\partial^2 W}{\partial x^2}$, $W_{,xy}=\frac{\partial^2 W}{\partial x \partial y}$, $W_{,yy}=\frac{\partial^2 W}{\partial y^2}$, t : shell-thickness.

Upon substitution of W in Eq. (2) into Eqs. (3) letting W_0 be zero and through the Galerkin's method, the elasto-plastic buckling stress σ_{cr} can be obtained as follows : The elasto-plastic buckling stresses σ_{cr1} and σ_{cr2} for the asymmetric mode w_1 and the axi-symmetric mode w_2 can be independently obtained in the similar form. Then, the interactive elasto-plastic buckling can occur at the near-coincident critical stress and the buckling mode.

$$\tilde{\sigma}_{cr1}=f_1^c\tilde{\sigma}_E \text{ and } \tilde{\sigma}_{cr2}=f_2^c\tilde{\sigma}_E \quad \cdots\cdots (4)$$

where

$$\tilde{\sigma}_{cr1}=\frac{\sigma_{cr1}}{\sigma_Y},\quad \tilde{\sigma}_{cr2}=\frac{\sigma_{cr2}}{\sigma_Y},\quad \tilde{\sigma}_E=\frac{\sigma_E}{\sigma_Y}=\frac{1}{R^2},\quad \sigma_E=\frac{E}{\sqrt{3(1-\nu^2)}}\frac{t}{r},\quad R=\sqrt{\frac{r}{t}\frac{\sigma_Y}{E}\sqrt{3(1-\nu^2)}}$$

$$f_1^c=\beta f_2^c,\quad f_2^c=\sqrt{\frac{k^c E_s}{E}},\quad \text{and}\quad \beta=\frac{\sigma_{cr1}}{\sigma_{cr2}}=\frac{1}{2}+\frac{(\sqrt{k^c}+1)^2}{8\,k^c}$$

in which, σ_E and R refer to the Euler stress and the generalized radius-thickness ratio for the compressed cylinder, respectively. In order to consider the effect of residual stress on the elasto-plastic buckling stress σ_{cr}, the Bleich's factor τ is taken conveniently herein to be equal to the tangent modulus k in Eq. (1). Then, k^c and f^c refer to the critical value of the factor k, evaluated at the elasto-plastic buckling point, and that of the ratio of the elasto-plastic buckling stress to the Euler stress, respectively[18]. Taking into account the interaction between asymmetric and axi-symmetric modes, the buckling mode can be determined so that

$$\left(\frac{l\pi}{L}\right)^2=\frac{2\sqrt{3(1-\nu^2)}}{k^c t r}\sqrt{\frac{E_s}{E}} \text{ and } \left(\frac{l\pi}{L}\right)=2\left(\frac{m\pi}{L}\right)=2\left(\frac{n}{r}\right) \quad \cdots\cdots (5)$$

Then, since $0<k^c\leq 1$, $\beta\geq 1$. In this paper, the least elasto-plastic buckling stress σ_{cr} is taken to be σ_{cr2} for the axi-symmetric mode w_2 of buckling, considering the interaction with the asymmetric mode w_1. It is needless to say that in Eq. (4), σ_{cr2} is equal to σ_{cr1} when $\beta=1$, i. e., $k^c=1$ in the purely elastic range, showing the complete compound bifurcation buckling.

(2) Elastic pseudo-potential energy

The elasto-plastic equilibrium paths for the imperfect cylinder and the postbuckling path for the perfect cylinder can also be obtained substituting the elasto-plastic buckling modes of Eq. (2) into equations of equilibrium, Eqs. (3) :

$$\frac{1}{8}(\beta-\lambda)x-\frac{1}{8}\lambda\varepsilon_1+2\,\alpha_s xy=0 \quad \cdots\cdots (6\cdot a)$$

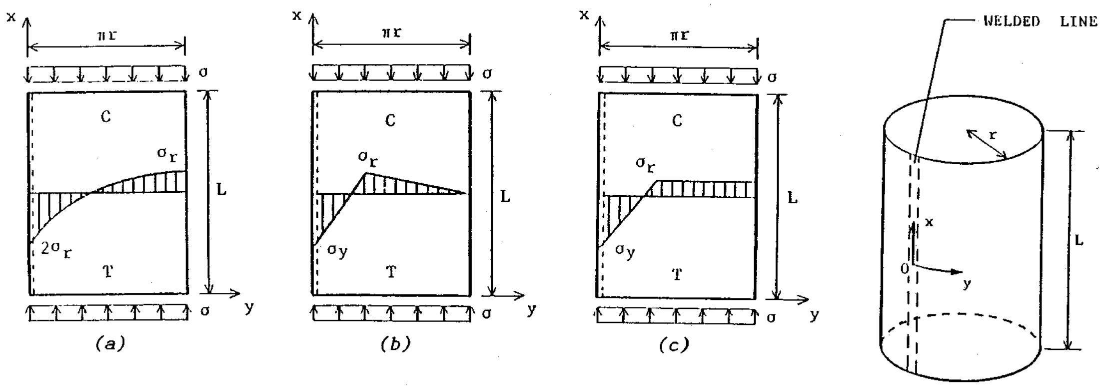


Fig. 2 Several Circumferential Distributions of Longitudinal Residual Stress.
(a) Parabola (b) Triangle (c) Trapezoid

$$(1-\lambda)y-\lambda\varepsilon_2+\alpha_s x^2=0 \qquad (6\cdot b)$$

where

$$\lambda=\frac{\sigma}{\sigma_{cr}},\quad \sigma_{cr}=\sigma_{cr2},\quad \alpha_s=\frac{3\sqrt{3(1-\nu^2)}}{32\sqrt{k^c}}\sqrt{\frac{E_s}{E}},\quad x=\frac{w_1}{t},\quad \varepsilon_1=\frac{w_{01}}{t},\quad y=\frac{w_2}{t},\quad \text{and}\quad \varepsilon_2=\frac{w_{02}}{t}$$

in which, the corresponding buckling mode is given by Eq. (5). Therefore, an elastic pseudo-potential energy A can be defined so that the equilibrium equations can be derived by the first differentiation of A near the elasto-plastic buckling point :

$$A=\frac{1}{16}(\beta-\lambda)x^2-\frac{1}{8}\lambda\varepsilon_1 x+\frac{1}{2}(1-\lambda)y^2-\lambda\varepsilon_2 y+\alpha_s x^2 y \qquad (7)$$

The potential may be truncated up to the 3rd-order terms of the modes, leading to the incomplete parabolic umbilic catastrophe. The elasto-plastic and the elastic bucklings correspond to the near-coincident and completely simultaneous bucklings, respectively[24].

3. DIRECT STRENGTH PREDICTION (Method I)

(1) Ultimate strength

Described herein first is called Method I based on a direct evaluation of the stability limit curve using an elastic pesudo-potential energy near the elasto-plastic buckling point. The elasto-plastic stability limit of compressed cylindrical shells is obtained directly from the singularity of the "Hessian" matrix of the elastic pseudo-potential energy, Eq. (7).

Solving simultaneously for the nonlinear process of Eqs. (6) and vanishing the Hessian, the elasto-plastic ultimate strength formula λ_m can be obtained as the imperfection sensitivity surfaces at the elasto-plastic buckling point, their typical configuration being illustrated by Fig. 3 :

$$\frac{16}{3\sqrt{6}\,\alpha_s}\left[2\,\alpha_s\lambda_m\varepsilon_2+\frac{1}{8}(1-\lambda_m)(\beta-\lambda_m)\right]^{\frac{3}{2}}=\lambda_m(1-\lambda_m)\varepsilon_1 \qquad (8)$$

Apparently, in the purely elastic range and in case $\varepsilon_1=0$, this strength is identically equal to the Koiter's imperfection sensitivity formula with only finite axi-symmetric mode of initial deflections for compressed cylinders[8].

(2) Modification of imperfection

The concept of the equivalent initial imperfection is adopted herein following the cases of columns, beams, plate panels and stiffened plates[18)~22)] in the following form :

$$\varepsilon_i^*=\mu(R)\varepsilon_i \qquad (i=1,2) \qquad (9)$$

where

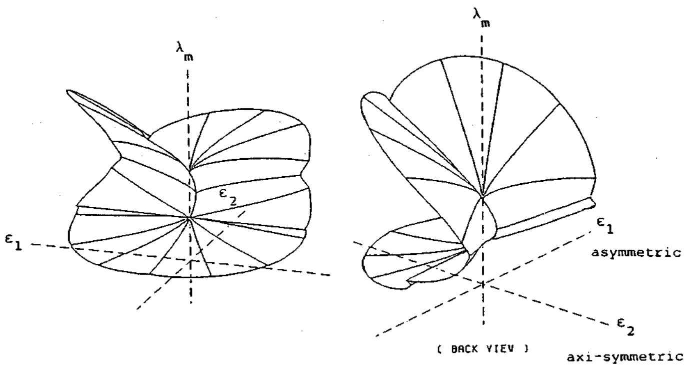


Fig. 3 Typical Imperfection Sensitivity Surfaces by Method Ⅰ.

$$\mu(R)=\mu_c\left(\frac{R}{R_p}\right)^{\beta^*},\quad \mu_c=1,\quad \text{and}\quad \beta^*=2\left(1-\frac{R}{R_p}\right)$$

In which, ε_i^*, ε_i and R_p denote the equivalent and the initial deflections, non-dimensionalized by the shell-thickness, and the transient value of R from the elasto-plastic to purely elastic, respectively, and the form of $\mu(R)$ is determined taking into account many test results and the ECCS strength design curves. Finally, the imperfection sensitivity or the load-carrying capacity can be determined by Eq. (8) with the equivalent imperfections of Eq. (9)[18)~22)].

4. UNIFIED STRENGTH PREDICTION (Method Ⅱ)

(1) Elasto-plastic postbuckling paths and failure mechanisms

Since Method Ⅰ fails to take into account the effects of the failure mechanisms upon the ultimate strength, another method, referred to as Method Ⅱ will be proposed herein to consider the effects. In Method Ⅱ, the strength is obtained from the imperfection sensitivity near the equivalent bifurcation point. The point is defined as one of the intersections of the elasto-plastic postbuckling paths as shown in **Fig. 4**, with the plastic failure mechanism surfaces as shown in Figs. 5. The original concept underlying this method has been proposed by Watanabe and Isami and has been applied to columns, beams, unstiffened plates and stiffened plates[18)~22)].

Two types of failure mechanisms of compressed cylindrical shells are considered herein : one is a ring type corresponding to the axi-symmetric mode, and another being a diamond type corresponding to the asymmetric mode. Then, the relationship between the deflection and the average axial stress of the cylinder for each failure mechanism can be obtained respectively as follows[18)] :

$$\tilde{w}=A_p\frac{1-\tilde{\sigma}^2}{\tilde{\sigma}} \qquad (10)$$

where

$$\tilde{w}=\frac{w}{t},\quad \tilde{\sigma}=\frac{\sigma}{\sigma_Y}\quad \text{and}\quad A_p=\frac{3}{4}\ \text{for diamond-type or}\ A_p=\frac{1}{4}\ \text{for ring-type.}$$

(2) Ultimate strength

Rigorously speaking, the ultimate strength should be obtained like in Method Ⅰ, in terms of the imperfection sensitivity surfaces against two independent modes of imperfections such as those illustrated in Fig. 3. One of the most convenient and simplified concepts to obtain the imperfection sensitivity surfaces would be to make use of the so-called one-dimensional imperfection sensitivity curve for each of two independent initial modes of axi-symmetric and asymmetric deflection corresponding to each failure mechanism while considering two independent modes of axi-symmetric and asymmetric deflections in the postbuckling behavior simultaneously.

Consequently, the ultimate strength can be formulated simply in the following form :

$$\frac{\tilde{\sigma}}{\tilde{\sigma}^*}=1+\alpha^*\tilde{w}_{0i}-\sqrt{2\,\alpha^*\tilde{w}_{0i}\left(1+\frac{1}{2}\,\alpha^*\tilde{w}_{0i}\right)} \qquad (i=1,2) \cdots\cdots \quad (11)$$

near the equivalent bifurcation point $c(\tilde{w}_1^*,\ \tilde{w}_2^*,\ \tilde{\sigma}^*)$. Besides, $\tilde{\sigma}^*$ can be obtained from the intersection of the postbuckling curves and the surface of the failure mechanism as a real root of the following polynomial equation :

$$(9\,c_1^2-1)\tilde{\sigma}^{*4}+(\tilde{\sigma}_{cr1}+\tilde{\sigma}_{cr2})\tilde{\sigma}^{*3}-(18\,c_1^2+\tilde{\sigma}_{cr1}\tilde{\sigma}_{cr2})\tilde{\sigma}^{*2}+9\,c_1^2=0 \cdots\cdots \quad (12\cdot a)$$

for the diamond-type mechanism, $A_p=3/4$ in Eq. (10), or

$$\tilde{\sigma}^*=\tilde{\sigma}_{cr} \cdots\cdots \quad (12\cdot b)$$

for the ring-type mechanism, $A_p=1/4$ in Eq. (10). In which,

$$c_1=\tilde{\sigma}_E\frac{3\sqrt{3(1-\nu^2)}}{32}\frac{E_s}{E}$$

Moreover, the factor α^* is determined approximately from the slope of the failure mechanism curve at the equivalent bifurcation point C ; the slope in the $\tilde{w}_1$-direction ($\tilde{w}_2=\tilde{w}_2^*$) for the diamond-type mechanism or in the $\tilde{w}_2$-direction ($\tilde{w}_1=\tilde{w}_1^*$) for the ring-type mechanism, respectively :

$$\alpha^*=-\frac{1}{\tilde{\sigma}^*}\frac{d\tilde{\sigma}}{d\tilde{w}}\bigg|_{(\tilde{w}_1^*,\,\tilde{w}_2^*,\,\tilde{\sigma}^*)}=\frac{\tilde{\sigma}^*}{A_p(1+\tilde{\sigma}^{*2})} \cdots\cdots \quad (13)$$

5. NUMERICAL ILLUSTRATIONS

Several numerical demonstrations are provided on the elasto-plastic strength of the compressed cylindrical shells. The type of residual stress distribution is mainly assumed to be parabolic as shown in Fig. 2 (a) with the maximum compressive residual stress $\sigma_r=0.4\,\sigma_Y$, since the difference of the distribution does not essentially affect the ultimate strength[18]. Moreover, the magnitude of initial out-of-plane deflection is also assumed to be $l_r/100$ for both asymmetric and axi-symmetric modes. The value is prescribed on the basis of the maximum limitation as specified by the ECCS-Recommendations, where l_r refers to the gauge length specified as $l_r=4\sqrt{rt}$ [25]. Therefore, the magnitudes of the non-dimensionalized initial deflections can be obtained as follows :

$$\varepsilon_i=\frac{w_{0i}}{t}=\frac{w_{0i}}{l_r}\frac{l_r}{t}=0.04\sqrt{\frac{r}{t}} \qquad (i=1,2) \cdots\cdots \quad (14)$$

Fig. 6 shows the predicted ultimate strength curves, Eq. (11), for the axi-symmetric initial deflection of ε_2 of Eq. (14), with only the ring-type failure mechanism being taken into account by Method II, together with several experimental results[16),17)], results by Method I and two design formulas of DASt and ECCS[25]

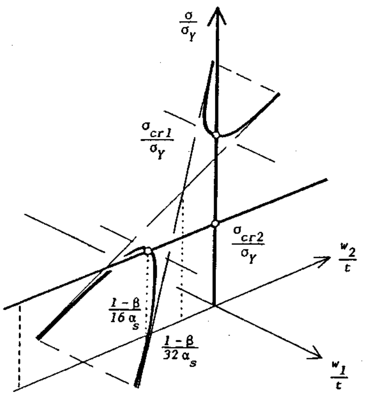
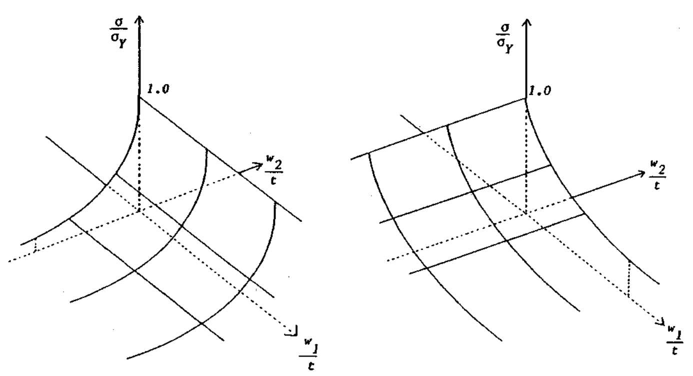


Fig. 4 Postbuckling Paths considering both Asymmetric and Axi-Symmetric Modes.



(a) Ring-Type (b) Diamond-Type

Fig. 5 Simplified Failure Mechanisms.

plotted at the same time.

Also illustrated in Fig. 7 are the comparison of the imperfection sensitivity curves by Method I, and Method II in which the ring-type failure mechanism is assumed, for generalized radius-thickness ratio of R=0.516, 0.807, 1.291, and 1.504, respectively. The abscissa and the ordinate designate the initial deflection of $w_0/(Rt)$ multiplied by a constant, and the non-dimensionalized strength, respectively. The specific value of the abscissa, 0.04, corresponds to the one as specified by the ECCS recommendations.

From Figs. 6 and 7, it is found that, regardless of the values of R, the results by Method II are generally more conservative than those by Method I and are in good correlation with the DASt and the ECCS curves. The imperfection sensitivity in the range of small imperfections is noted to be quite significant when R is close to the transient value, R_p. The difference between the results by two methods seems to diminish for larger generalized-thickness ratios.

On the other hand, Fig. 8 shows the ultimate strength curves for cylinders, Eq. (11) for asymmetric initial deflection of ε_1 of Eq. (14), with only the diamond-type failure mechanism being taken into account by Method II, together with several experimental results[16),17)] and two design formulas of DASt and ECCS[25)] plotted simultaneously. In the range of $R>0.58$, the results by Method II assuming the diamond-type failure mechanism cease to exist since the elasto-plastic postbuckling path as shown in Fig. 4 does not intersect with the failure mechanism surface of Fig. 5 any more. However, should a compound failure mechanism be considered between the ring- and diamond-type failure mechanisms, the continuous strength curves could be obtained for $R>0.58$ as well. Therefore, In the range of $R>0.58$, the ultimate strength may have to be predicted somehow by using some transient curve connecting the point of the strength curve II at $R=0.58$ with certain point of the strength curve I at the intermediate value of R near $R=1$.

Illustrated in Fig. 9 are the comparison of the imperfection sensitivity curves by Methods I and II for the generalized radius-thickness ratio of R=0.516, 0.807, 1.291, and 1.504, similarly to Fig. 7. It is also shown that the results by Method II assuming the diamond-type failure mechanism is more conservative than those by Method I for $R<0.58$, similarly to the case of the ring-type failure mechanism as shown in Fig. 7.

Finally, from the comparison of Figs. 7 and 9, the strength prediction of the compressed cylindrical shells by Method II assuming the ring-type failure mechanism will be the most conservative for wide range of generalized radius-thickness ratios, if the magnitude of the initial deflection is within the values prescribed by the ECCS-Recommendations. Thus, this prediction may serve as a simple and convenient basic strength formula of compressed cylindrical shells.

6. CONCLUSIONS AND ACKNOWLEDGEMENT

Two approaches to the ultimate strength of compressed cylindrical shells are proposed. The main conclusions are summarized as follows:

(1) Two different procedures for the strength of compressed cylinerical shells are presented: Method I being a direct approach making use of the slope of the pathological curve representing the stability limit for the elasto-plastic behaviors near the elasto-plastic buckling point; and Method II being a unified approach making use of the slope of the plastic unloading curve associated with the failure mechanism near the equivalent bifurcation point.

(2) The initial deflection is conveniently modified and replaced by the equivalent imperfection proposed herein to be in good correlation with several test data and design curves in DASt and ECCS.

(3) The elasto-plastic buckling may occur simultaneously in the axi-symmetric buckling and in the square asymmetric mode. In the elastic range, the present formula through Method I identically gives the complete simultaneous bucklings, and the imperfection sensitivity for the axi-symmetric mode is just equal to the Koiter's formula.

(4) From the comparison of several demonstrated results, the strength prediction of the compressed cylindrical shells by Method II assuming the ring-type failure mechanism is the most conservative for wide range of generalized radius-thickness ratios, if the magnitude of the initial deflection is within the values as limited by the ECCS-Recommendations. Thus, this prediction may serve as a simple and convenient basic strength formula for compressed cylindrical shells.

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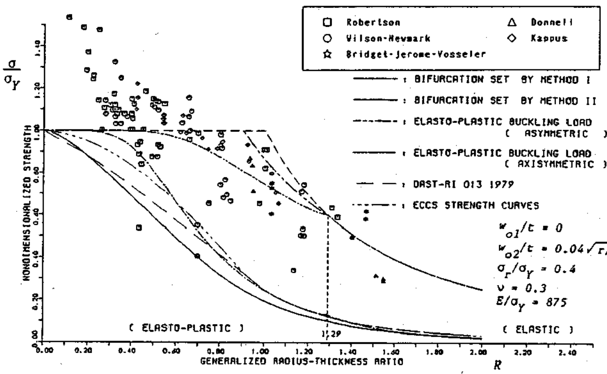

Fig. 6 Strength of Compressed Cylindrical Shells. Axi-Symmetric Imperfections.

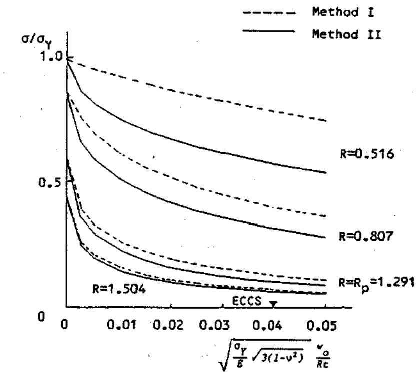

Fig. 7 Comparison of Imperfection Sensitivity Curves. Axi-Symmetric Imperfections.

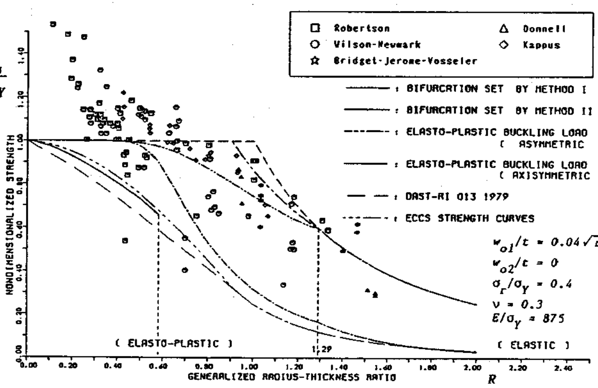

Fig. 8 Strength of Compressed Cylindrical Shells. Asymmetric Imperfections.

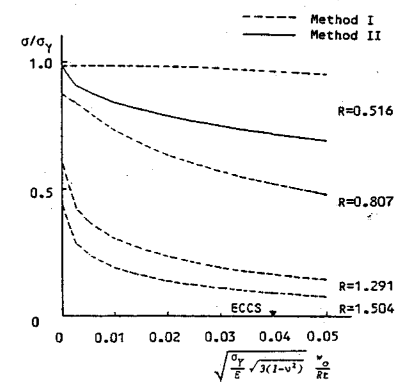

Fig. 9 Comparison of Imperfection Sensitivity Curves. Asymmetric Imperfections.

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