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A STOCHASTIC STUDY ON EFFECT OF MULTIPLE TRUCK PRESENCE ON FATIGUE DAMAGE OF HIGHWAY BRIDGES

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Very often there are two or more trucks present on the highway bridge, and the effect of their interaction on total fatigue damage to a bridge detail occurs in a non-linear fashion. This interaction-effect in a single-lane loading has been studied in terms of "multiple presence fatigue factor", γ_M. Explicit analytical expressions have been obtained for γ_M using renewal point process load with equal magnitude. Distribution of the headway distance between successive trucks has been found to have a significant effect on this factor. For normal traffic conditions, it has been found that the interaction effect in a single-lane loading is not so significant, and can be neglected for the practical design purposes.

Keyword : fatigue, highway bridges, live load, stochastic analysis, renewal point process

1. INTRODUCTION

In recent few years some steel highway bridges have experienced fatigue distress mainly due to the increase in truck traffic, overloading of trucks and the increased use of welded joints. The problem is rather recent and there are few highway bridge design codes which have fatigue provisions. As a result, of late, the fatigue aspect of bridge loading has been drawing increased attention.

In the case of bridges fatigue mostly occurs at bridge details. Very often, two or more trucks cross the bridge together and the net effect on total fatigue damage is not simply the sum of the individual effects. The interaction effect in a single-lane loading becomes important when the bridge-detail has a long influence line or the traffic is very dense.

Miki et. al. (1) carried out computer simulation and obtained average fatigue damage due to the passage of vehicles, in the actual traffic condition, for different lengths of influence line and traffic densities. Shilling (2) in his work did not consider quantitatively the interaction effect for a single-lane loading.

Since the traffic density on some of the Japanese highways is very high, we decided to study the effect of vehicle interaction through analytical means. Total fatigue damage to a bridge detail will be as the product of two factors :

Total fatigue damage= (interaction effect) × (fatigue damage due to the passages of individual vehicles)

The fatigue damage due to the passages of individual vehicles can be obtained easily, and the interaction

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effect for a single-lane loading will be quantified here in terms of *multiple presence fatigue factor*, γ_M.

In this paper the authors employ a renewal point process loading to study the effect of multiple presence of trucks on total fatigue damage to bridge details. Bending moment at the center of a simple span bridge is used as the reference for the fatigue causing stresses. An explicit analytical solution is obtained and this will be compared with the simulation results. We confine ourselves to the single lane problem, the free flowing traffic and the static response case.

2. MULTIPLE PRESENCE FATIGUE FACTOR (MPFF)

MPFF accounts for the interaction-effect on total fatigue damage due to simultaneous presence of two or more trucks on the bridge. Every truck that actually passes on the bridge is considered to be a part of an i-truck event, where i is the number of trucks simultaneously present on the span. Fig. 1 shows an example of a traffic situation that leads to two realizations of the 1-truck event, 3 realizations of the 2-truck event, and one realization of the 3-truck event. Since the traffic situation is clearly random, the occurrence of each i-truck event may only be classified in terms of its '*group probability*', p_i. This group probability is obtained through stochastic analysis of truck arrival process, taking into consideration the randomness of the headway distance between any two successive trucks. the MPFF, γ_M may be expressed as

$$\gamma_M=\sum_i i\times p_i\times\delta_i \qquad (1)$$

where, δ_i=fatigue factor for truck group of size i.

$$=\frac{\text{mean fatigue damage due to truck group of size } i}{i\times\text{mean fatigue damage due to single truck}}$$

p_i=probability of truck group of size i

In the following sections we discuss group probability p_i and group fatigue factor δ_i.

3. MODELS USED FOR TRUCK, TRUCK ARRIVAL AND FATIGUE ANALYSIS

Trucks are modeled as point load with equal weight, w. Though the above assumption means a gross simplification of the real problem, it makes the evaluation of MPFF mathematically tractable.

Truck arrival is modeled as a renewal point process with probability law which follows 1st-order Erlang (Exponential) headway distribution (1-EHD) or 3rd-order Erlang headway distribution (3-EHD).

For the fatigue analysis, constant amplitude stress-cycle relation, $NS^m=a$ with $m=3$, is used in addition to the Palmgren-Miner's linear fatigue damage accumulation rule and the rainflow counting method (4). Many fatigue test data suggest the value of m in the S-N relation of the weldments is about 3 and this value is used in this study. Effect of fatigue limit is neglected, because recent studies (3) show that even the stress range below the fatigue limit contributes to the fatigue damage once the crack is initiated.

4. GROUP PROBABILITY

Truck group should be counted such that (1) all the trucks in the traffic are accounted for, (2) each truck is counted only once and (3) a smaller truck group is not contained in a bigger truck group (see Fig. 1). Maximum size of the truck group, which needs to be considered, is determined by the length of the bridge as well as the value of group probability itself.

Fig. 2 shows an example string of trucks, where the intradistances are H_0, H_1, H_2, etc. The headway distance, H, between any two successive trucks is a random variable described by the probability density function $f_H(X)$. This same probability density function therefore applies to the varibles H_0, H_1, H_2, etc. The group probabilities may then be expressed in terms of $f_H(X)$; the corresponding complimentary cumulative distribution function, $G_H(X)=\text{Prob}[H>X]$; and the length of the influence line, L.

Without loss of generality, the expressions for group probabilities may be derived by considering truck 1 (Fig. 2) as the first member of the i-truck group. Values of i from 1 to 4 are considered below. Note that

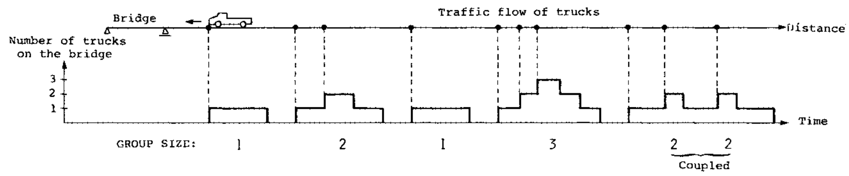

Fig. 1 An Example of Traffic Flow of Trucks, Time-history of Number of Trucks on the Bridge and Identification of Groups of Trucks| (indicates truck location on approach highwaw)

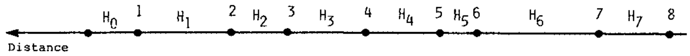

Fig. 2 A Sample string of Trucks Arrivals (H_i is headway distance)

the superscript, a, in every symbol of the form p_b^a does not denote exponentiation.

(a) Group of one truck.-

$$p_1 = P_r[H_0 > L \text{ and } H_1 > L] = G_H^2(L) \quad (2)$$

(b) Group of two trucks.-A simple definition would be,

$$p_2^0 = P_r[H_1 < L \text{ and } H_0 + H_1 > L \text{ and } H_1 + H_2 > L] = \int_0^L G_H^2(L-x) f_H(x)\,dx \quad (3)$$

Counting of group of 2 trucks in this way will have a problem of coupling or double counting. A string of 3 trucks may occur such that the consecutive trucks will form group of 2 trucks and hence only three trucks will be involved in the formation of two groups of 2 trucks as illustrated in Fig. 1. Fatigue damage due to this string of 3 trucks will be more than that of one group of 2 trucks, but certainly less than that of two groups of 2 trucks. We shall count the occurrence of this string of 3 trucks as 1.5 groups of 2 trucks. For this we have to deduct one half of the probability of occurence of this string of 3 trucks, p_2^1, from p_2^0 to obtain a reasonable p_2. This definition of truck groups is still an approximation, because yet longer string of trucks may be involved in the formation of 2-truck groups, but it should suffice. It will also ensure that the probabilities of different truck groups when multiplied by their sizes will add to approximately 1.0. Then p_2^1 is. given as

$$p_2^1 = P_r[H_1 < L \text{ and } H_2 < L \text{ and } H_0 + H_1 > L \text{ and } H_1 + H_2 > L \text{ and } H_2 + H_3 > L]$$
$$= \int_0^L f_H(x)\, G_H(L-x) \int_{L-x}^L G_H(L-y) f_H(y)\,dy\,dx \quad (4)$$

Finally, p_2 is given as,

$$p_2 = p_2^0 - \frac{1}{2} p_2^1 \quad (5)$$

(c) Group of three trucks.-In the case of group of three trucks, two trucks will be involved in the coupling and we shall have to deduct proper fractions of p_3^1 and p_3^2 from p_3^0 to obtain a reasonable p_3.

$$p_3^0 = P_r[H_1 + H_2 < L \text{ and } H_0 + H_1 + H_2 > L \text{ and } H_1 + H_2 + H_3 > L] = \int_0^L G_H^2(L-y) f_Y(y)\,dy \quad (6)$$

where $f_Y(y) = \int_0^L f_H(x) f_H(y-x)\,dx$

In the case of group of 3-trucks, we have the term p_3^1 when a string of 4 trucks is involved in coupling, and the term p_3^2 when a string of 5 trucks is involved in coupling. We don't consider a string longer than that of 5 trucks. We deduct two third of p_3^1 from p_3^0 to account for string of 4 trucks as 4/3 groups of 3-trucks, and similarly for others.

$$p_3^1 = P_r[H_1 + H_2 < L \text{ and } H_2 + H_3 < L \text{ and } H_0 + H_1 + H_2 > L \text{ and } H_1 + H_2 + H_3 > L \text{ and } H_2 + H_3 + H_4 > L]$$
$$= \int_0^L f_H(x) \int_0^{L-x} f_H(y)\, G_H(L-x-y) \int_{L-x-y}^{L-y} G_H(L-y-z) f_H(z)\,dz\,dy\,dx \quad (7)$$

$$p_3^2=\int_0^L f_H(x)\int_0^{L-x} f_H(y)\int_{L-x-y}^{L-y} f_H(z)\,G_H(L-x-y)\int_{L-y-z}^{L-z} G_H(L-z-w)\,f_H(w)\,dwdzdydx \quad (8)$$

Finally,

$$p_3=p_3^0-\frac{2}{3}p_3^1-\frac{4}{3}p_3^2 \quad (9)$$

(d) Group of four trucks. -

$$p_4^0=P_r[H_1+H_2+H_3<L \text{ and } H_0+H_1+H_2+H_3>L \text{ and } H_1+H_2+H_3+H_4>L]$$

$$=\int_0^L G_H^2(L-z)\,f_Z(z-x)\,dx \quad (10)$$

where $f_Z(z)=\int_0^z f_H(x)\,f_Y(z-x)\,dx$

$$p_4^1=P_r[H_1+H_2+H_3<L \text{ and } H_2+H_3+H_4<L \text{ and } H_0+H_1+H_2+H_3>L \text{ and } H_1+H_2+H_3+H_4>L \text{ and } H_2+H_3+H_4+H_5>L]$$

$$=\int_0^L f_M(x)\int_0^{L-x} f_H(y)\,G_H(L-x-y)\int_{L-x-y}^{L-x} G_H(L-x-z)\,f_H(z)\,dzdydx \quad (11)$$

where $f_M(m)=\int_0^m f_H(m-x)\,f_H(x)\,dx$

$$p_4^2=P_r[H_1+H_2+H_3<L \text{ and } H_2+H_3+H_4<L \text{ and } H_3+H_4+H_5<L \text{ and } H_0+H_1+H_2+H_3>L \text{ and } H_1+H_2+H_3+H_4>L \text{ and } H_2+H_3+H_4+H_5>L \text{ and } H_3+H_4+H_5+H_6>L]$$

$$=\int_0^L f_H(x)\int_0^{L-x} f_H(y)\int_0^{L-x-y} f_H(z)\int_{L-x-y-z}^{L-y-z} f_H(w)\,G_H(L-x-y-z)\int_{L-y-z-w}^{L-z-w} \cdot G_H(L-z-w-u)\,f_H(u)\,dudwdzdydx \quad (12)$$

$$p_4^3=P_r[H_1+H_2+H_3<L \text{ and } H_2+H_3+H_4<L \text{ and } H_3+H_4+H_5<L \text{ and } H_4+H_5+H_6<L \text{ and } H_0+H_1+H_2+H_3>L \text{ and } H_1+H_2+H_3+H_4>L \text{ and } H_2+H_3+H_4+H_5>L \text{ and } H_3+H_4+H_5+H_6>L \text{ and } H_4+H_5+H_6+H_7>L]$$

$$=\int_0^L f_H(x)\int_0^{L-x} f_H(y)\int_0^{L-x-y} f_H(z)\int_{L-x-y-z}^{L-y-z} f_H(w)\int_{L-y-z-w}^{L-z-w} f_H(u) \cdot G_H(L-x-y-z)\int_{L-z-w-u}^{L-w-u} G_H(L-w-u-v)\,f_H(v)\,dvdudwdzdydx \quad (13)$$

Finally,

$$p_4=p_4^0-\frac{3}{4}p_4^1-\frac{3}{2}p_4^2-\frac{9}{4}p_4^3 \quad (14)$$

Analytical expressions are developed for group probabilities with 1-EHD and 3-EHD using Eqs. (2)-(14) and are given in the appendix. Numerical results for three free-stream traffic conditions, as listed in Table 1, are shown in Table 2. The first traffic condition represents a very scarce traffic, the second represents a normal traffic and the last one represents an extremely dense traffic.

Simulation results shown in the bracket support our analytical results.

Table 1 Three traffic conditions.

Case	Volume (trucks/hr)	Velocity (km/hr)	L (meter)	λL	
				1-EHD	3-EHD
1	250	100	100	0.25	0.75
2	500	100	100	0.50	1.50
3	1000	50	100	2.00	6.00

Table 2 Group Probability. (in the bracket are shown the simuation results)

Group Pr.	1-EHD			3-EHD		
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
p_1	0.606 (0.604)	0.368 (0.366)	0.018 (0.018)	0.919 (0.919)	0.653 (0.653)	0.004 (0.004)
p_2	0.163 (0.163)	0.216 (0.216)	0.099 (0.098)	0.039 (0.040)	0.167 (0.167)	0.168 (0.167)
p_3	0.021 (0.021)	0.058 (0.053)	0.108 (0.103)	0.000 (0.000)	0.004 (0.004)	0.124 (0.162)
p_4	0.002 (0.002)	0.009 (0.009)	0.074 (0.064)	0.000 (0.000)	0.000 (0.000)	0.055 (0.038)

5. FATIGUE FACTOR FOR TRUCK GROUPS

To obtain the mean fatigue damage due to individual truck groups we need the joint distribution of the group-intradistances. This distribution will be conditioned upon the size of truck-groups and will be different from the parent headway distribution. We next obtain the conditional joint probability density function (CJPDF) of group-intradistances.

We denote the probability density function for intradistance X between first two trucks in a n-truck group, spread over a distance L, by $f_X(x|n, L)$, which is given as

$$f_X(x|n, L)\,dx = P_r[x<X<x+dx|\text{only } (n-1) \text{ trucks in distance } L] = \frac{f_H(x)\,P_r[n-2, L-x]}{P_r[n-1, L]}\,dx \quad (15)$$

where $P_r[n, L]$ is the probability of occurrence of exactly n trucks over distance L. It follows that for 2-truck groups with only one intradistance X,

$$f_X(|2, L)=\frac{f_H(x)\,P_r[0, L-x]}{P_r[1, L]} \quad (16)$$

Similarly for 3-truck groups with two intradistances X_1 and X_2

$$f_{X_1X_2}(x_1, x_2)=f_X(x_1|3, L)\cdot f_X(x_2|2, L-x_1) \quad (17)$$

For 4-truck groups with three intradistances X_1, X_2 and X_3

$$f_{X_1X_2X_3}(x_1, x_2, x_3)=f_X(x_1|4, L)\cdot f_X(x_2|3, L-x_1)\cdot f_X(x_3|2, L-x_1-x_2) \quad (18)$$

Results for CJPDF.-

Conditional Joint Probability Density Function for 1-EHD (Exponential)

$$f_H(x)=\lambda e^{-\lambda x} \quad (19)$$

where λ is the inverse of mean headway distance.

$$P_r[n, L]=\frac{(\lambda L)^n}{n!}e^{-\lambda L} \quad (20)$$

$$f_X(x|2, L)=\frac{1}{L} \quad (21)$$

$$f_{X_1X_2}(x_1, x_2)=\frac{2}{L^3} \quad (22)$$

$$f_{X_1X_2X_3}(x_1, x_2, x_3)=\frac{6}{L^6} \quad (23)$$

Conditional Joint Probability Density Function for 3-EHD

$$f_H(x)=\frac{1}{2}\lambda^3x^2e^{-\lambda x} \quad (24)$$

where λ is 3 times the inverse of mean headway distance.

$$P_r[n, L]=\frac{1}{(3n+1)!}\left[(3n+1)+\lambda L+\frac{(\lambda L)^2}{(3n+2)}\right](\lambda L)^{3n}e^{-\lambda L} \quad (25)$$

$$f_X(x|2, L)=\frac{12}{\left[4+\lambda L+\frac{1}{5}(\lambda L)^2\right]L^3}\left[1+\lambda(L-x)+\frac{1}{2}\lambda^2(L-x)^2\right]x^2 \quad (26)$$

$$f_{X_1X_2}(x_1, x_2)=\frac{1260}{\left[7+\lambda L+\frac{1}{8}(\lambda L)^2\right]L^6}\left[1+\lambda(L-x_1-x_2)+\frac{1}{2}\lambda^2(L-x_1-x_2)^2\right]x_1^2x_2^2 \quad (27)$$

$$f_{X_1X_2X_3}(x_1, x_2, x_3)=\frac{453600}{\left[10+\lambda L+\frac{1}{11}(\lambda L)^2\right]L^9}\left[1+\lambda(L-x_1-x_2-x_3)+\frac{1}{2}\lambda^2(L-x_1-x_2-x_3)^2\right]x_1^2x_2^2x_3^2 \quad (28)$$

Mean Fatigue Damage due to Truck Groups :

Our concern is the mean of the fatigue damage due to a truck group. When a truck group moves across the bridge, sometimes all the trucks and at other times only some of the trucks will have direct influence on the total stresses at a detail. As a consequence, the resulting moment history may have different patterns, depending upon the intradistances between trucks. For the bending moment at the center of a simple span, a moment history will have either one or two peaks. Only one peak results when the group is very compact i. e. when no two trucks are apart by more than half the span length. For the case of two peaks, the pattern of moment history may again differ, depending upon which and how many trucks give rise to which particular peak. Moment ranges for different patterns are obtained through the rainflow counting technique. If we write the equations for the moment ranges in different patterns, which will be in terms of the truck weights and the intradistances, and take its 3 rd statistical moment, for which we shall need CJPDF of the intradistances, we shall obtain a measure of the mean fatigue damage due to a truck group. We call it $E[M^3]_i$, where i denotes the size of the truck group. Extensive experimental studies indicate that fatigue of welding in bridge components can be evaluated by the 3 rd statistical moments of the stress range (1, 2).

(a) Single truck

$$E[M^3]_1=\left(\frac{wL}{4}\right)^3 \quad (29)$$

(b) Group of 2 Trucks

$$E[M^3]_2=\int_0^{L/2}\frac{w^3}{8}(L-x)^3 f_X(x)\,dx+\int_{L/2}^{L}\left(\frac{wL}{4}\right)^3 f_X(x)\,dx+\int_{L/2}^{L}\frac{w^3}{64}(2x-L)^3 f_X(x)\,dx \quad (30)$$

where $f_X(x)$ is given in Eq. 21 or Eq. 26.

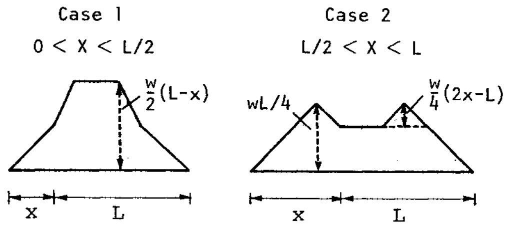

Fig. 3 Possible Patterns of Moment History for a 2-Truck group.

(c) Group of 3 trucks

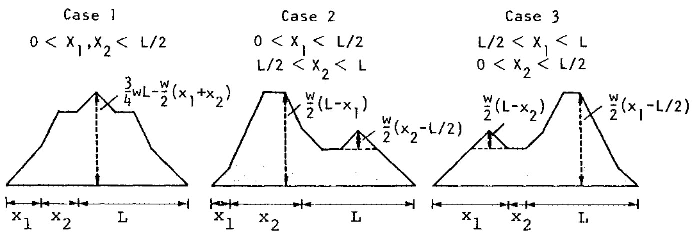

Fig. 4 Possible Patterns of Moment History for a 3-Truck Group.

$$E[M^3]_3=\int_0^{L/2}\int_0^{L/2}\left[\frac{3}{4}wL-\frac{w}{2}(x_1+x_2)\right]^3 f\,dx_2dx_1+\int_0^{L/2}\int_{L/2}^{L-x_1}\left[\frac{w}{2}(L-x_1)\right]^3 f\,dx_2dx_1$$
$$+\int_0^{L/2}\int_{L/2}^{L-x_1}\left[\frac{w}{2}\left(x_2-\frac{L}{2}\right)\right]^3 f\,dx_2dx_1+\int_{L/2}^{L}\int_0^{L-x_1}\left[\frac{w}{2}(L-x_2)\right]^3 f\,dx_2dx_1$$
$$+\int_{L/2}^{L}\int_0^{L-x_1}\left[\frac{w}{2}\left(x_1-\frac{L}{2}\right)\right]^3 f\,dx_2dx_1 \quad (31)$$

where $f=f_{X_1X_2}(x_1,x_2)$ is given in Eq. 22 or Eq. 28.

(d) Group of 4 Trucks

$$E[M^3]_4=\int_0^{L/2}\int_0^{L/2-x_1}\int_0^{L/2}\left[wL-\frac{w}{2}(x_1+2x_2+x_3)\right]^3 f\,dx_3dx_2dx_1$$
$$+\int_0^{L/2}\int_{L/2-x_1}^{L/2}\int_0^{L-x_1-x_2}\left[wL-\frac{w}{2}(x_1+2x_2+x_3)\right]^3 f\,dx_3dx_2dx_1$$

Table 3 Fatigue Factor δ_i for Truck Groups.

δ_i	1-EHD	3-EHD		
	All Cases	Case 1	Case 2	Case 3
δ_2	1.25	0.73	0.74	0.80
δ_3	2.33	1.15	1.19	1.40
δ_4	3.23	1.59	1.66	2.13

Table 4 MPFF, γ_M.

	Case 1	Case 2	Case 3
1-EHD	1.18	1.32	2.06
3-EHD	0.98	0.94	1.32

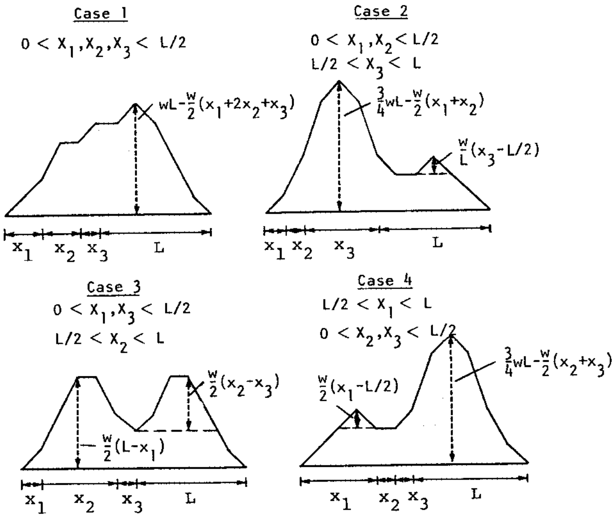

Fig. 5 Possible Patterns of Moment History for a 4-Truck Group.

$$+\int_{L/2}^{L}\int_{0}^{L-x_3}\int_{0}^{L-x_2-x_3}\left[\frac{3}{4}wL-\frac{w}{2}(x_1+x_2)\right]^3 f dx_1 dx_2 dx_3$$
$$+\int_{L/2}^{L}\int_{0}^{L-x_3}\int_{0}^{L-x_2-x_3}\left[\frac{w}{2}\left(x_3-\frac{L}{2}\right)\right]^3 f dx_1 dx_2 dx_3$$
$$+\int_{L/2}^{L}\int_{0}^{L-x_2}\int_{0}^{L-x_1-x_2}\left[\frac{w}{2}(L-x_1)\right]^3 f dx_3 dx_1 dx_2$$
$$+\int_{L/2}^{L}\int_{0}^{L-x_2}\int_{0}^{L-x_1-x_2}\left[\frac{w}{2}(x_2-x_3)\right]^3 f dx_3 dx_1 dx_2$$
$$+\int_{L/2}^{L}\int_{0}^{L-x_1}\int_{0}^{L-x_1-x_3}\left[\frac{3}{4}wL-\frac{w}{2}(x_2+x_3)\right]^3 f dx_2 dx_3 dx_1$$
$$+\int_{L/2}^{L}\int_{0}^{L-x_1}\int_{0}^{L-x_1-x_3}\left[\frac{w}{2}\left(x_1-\frac{L}{2}\right)\right]^3 f dx_2 dx_3 dx_1 \quad\cdots\cdots (32)$$

where $f=f_{X_1X_2X_3}(x_1, x_2, x_3)$ is defined in Eq. 23 or Eq. 30.

Fatigue factor of truck groups, δ_i is obtained as follows. Fatigue damage is assumed to be proportional to the 3rd statistical moment of the response.

Hence, for 1-EHD

$$\delta_i=\frac{E[M^3]_i}{i\times E[M^3]_1} \quad\cdots\cdots (33)$$

Note that δ_i is independent of the distribution parameters and hence is same for all traffic conditions. For 3-EHD, the expressions are given below. The numerical results are shown in Table 3.

$$\delta_2=\frac{6}{\left[4+\lambda L+\frac{1}{5}(\lambda L)^2\right]}[0.481+0.125(\lambda L)+0.029(\lambda L)^2] \quad\cdots\cdots (34)$$

$$\delta_3=\frac{3360}{\left[7+\lambda L+\frac{1}{8}(\lambda L)^2\right]}[0.0023+4.32\times10^{-4}(\lambda L)+6.59\times10^{-5}(\lambda L)^2] \quad\cdots\cdots (35)$$

$$\delta_4=\frac{907200}{\left[10+\lambda L+\frac{1}{11}(\lambda L)^2\right]}[1.695\times10^{-5}+2.354\times10^{-6}(\lambda L)+3.947\times10^{-7}(\lambda L)^2] \quad\cdots\cdots (36)$$

An interesting point to note in Table 3 is that δ_2 with 3-EHD is less than 1.0 for the traffic conditions given in Table 1. It means that, on the average, a truck crossing the bridge in a 2-truck group is less severe, from fatigue point of view, compared to a truck crossing the bridge alone. It is because the bending moment

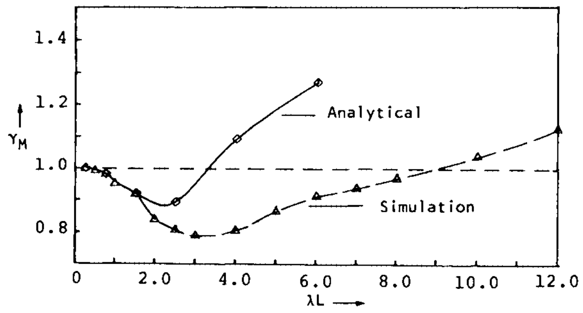

Fig. 6 Plot of Multiple Presence Fatigue Factor, γ_M vs. λL with 3-EHD.

due to a group of 2 trucks will exceed that due to a single truck only when the intradistance is less than half of the span length.

6. RESULTS AND DISCUSSION

The group probability, p_i, and the fatigue factor for truck groups, δ_i, both were found to depend upon only λL, which is a non-dimensional quantity. Therefore, the multiple presence fatigue factor, γ_M, will also depend upon only λL. This fact is very useful from design point of view, and implies that increasing the traffic density has the same effect on vehicle interaction as increasing the length of influence line.

Making use of Eq. (1) and the expressions for group probabilities and group fatigue factors, we obtain γ_M for the three cases listed in Table 1. The results are presented in Table 4.

The values of γ_M with 1-EHD, as shown in Table 4, are always greater than 1.0 and increases very fast as the increase of traffic density. This trend is because of the exponential distribution which assigns high probability density for small headway distances. As a consequence, the average fatigue damage due to truck groups becomes severe. In contrast, the values of γ_M with 3-EHD are small. It has also been shown in Fig. 6 as a function of λL. We observe that it first decreases with the increase of traffic density—in fact, for small traffic density it goes below 1.0—and only after certain traffic density does it start going up. Eventually, for dense traffic, it takes values greater than 1.0. The reason for this trend is that, for scarce traffic, most of the trucks cross the bridge either alone or in a group of two trucks. As shown in Table 3, δ_2 with 3-EHD is less than 1.0, as a result for small traffic density, γ_M goes below 1.0.

Table 4 also proves that the headway distribution has a great influence on the effect of truck interaction.

Annual average daily truck traffic (AADT) on most of the highways seldom exceeds 500 trucks per hour in one lane. This corresponds to the value of λL less than 1.5 even for a speed of 100 km/hr and a span less than 100 m. We observe in Fig. 6 that, the values of γ_M in this range of λL is very close to 1.0, which implies that the interaction effect in a single-lane loading is not so significant and can be neglected for practical design purposes.

7. APPLICABILITY OF THIS STUDY

Simulation results are also presented in Fig. 6 for comparison. Analytical solutions agree well with the simulation for the value of λL less than 2, but they overestimate γ_M for larger λL. This indicates that inclusion of more than 4-trucks group and more rational evaluation of fatigue damage due to consecutive truck groups are needed in the theoretical formulation. Nevertheless, our study shows that there is no real need to carry out the time-consuming simulation which takes into account the multiple presence of vehicles, to evaluate the fatigue damage under normal traffic conditions.

We have not studied the fatigue damage in the case of congestion traffic, but it seems plausible that, since in the congestion traffic condition seen in city highways, the stress level does not change much due to arrival or departure of an additional truck, the MPFF should not be more than 1.0.

Another limitation of our analytical model is that its extension to very long span bridges, where groups of more than 4-trucks will also be important, is rather difficult.

Finally it should be remembered that the present model replaced the actual vehicles by equal magnitude point loads. The actual configuration and the random nature of vehicle weights will certainly affect the multiple presence fatigue factor; however, the trend would not change much.

8. SUMMARY AND CONCLUSION

Effect of multiple presence of trucks on fatigue damage of highway bridges under a renewal point process loading has been studied analytically. Analysis is confined to the bending stress at the midspan of a simply-supported bridge and to single lane.

For normal traffic conditions, the analytical results show that the multiple presence fatigue factor, γ_M with 3rd-order Erlang headway distribution is somewhat smaller than 1.0 and hence the effect of the multiple presence of trucks on the fatigue damage to bridge-details is not significant. The total fatigue damage can be practically evaluated as the sum of the fatigue damage due to individual trucks. The mulitiple presence fatigue factor, γ_M would not change much for slightly different influence line shapes. Hence the above conclusion will also apply to other portions of bridges.

It should be mentioned that this analytical study gives a first-order evaluation of the multiple presence fatigue factor, γ_M and needs further improvement if very precise evaluation is need.

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Appendix: Analytical expressions for group probabilities

Analytical expressions for group probabilities with 1-EHD and 3-EHD are presented below. The group probabilities with 3-EHD are very difficult to compute through usual integration procedures, and hence a matrix-operation-based integration technique was developed for this purpose. Note that the superscript in p, $b2$, $b3$, $c2$, $c3$, and $d3$ does not denote exponentiation.

(a) 1 st-order erlang (exponential) headway distribution

$$f_H(x)=\lambda e^{-\lambda x} \quad \text{(A·1)}$$

where λ is the inverse of mean headway distance.

$$p_1=e^{-2\lambda L} \quad \text{(A·2)}$$

$$p_2^0=e^{-2\lambda L}(e^{\lambda L}-1) \quad \text{(A·3)}$$

$$p_2^1=\frac{1}{2}(\lambda L)^2 e^{-2\lambda L} \quad \text{(A·4)}$$

$$p_3^0=e^{-2\lambda L}[(\lambda L-1)e^{\lambda L}+1] \quad \text{(A·5)}$$

$$p_3^1=e^{-2\lambda L}\left[e^{\lambda L}-\frac{1}{2}(\lambda L)^2-\lambda L-1\right] \quad \text{(A·6)}$$

$$p_3^2=\frac{1}{24}(\lambda L)^4 e^{-2\lambda L} \quad \text{(A·7)}$$

$$p_4^0=e^{-2\lambda L}\left[\left(\frac{1}{2}(\lambda L)^2-\lambda L+1\right)e^{\lambda L}-1\right] \quad \text{(A·8)}$$

$$p_4^1=e^{-2\lambda L}\left[2(e^{\lambda L}-1)-2\lambda L-(\lambda L)^2-\frac{1}{3}(\lambda L)^3\right] \quad \text{(A·9)}$$

$$p_4^2=e^{-2\lambda L}\left[e^{\lambda L}-1-\lambda L-\frac{1}{2}(\lambda L)^2-\frac{1}{6}(\lambda L)^3-\frac{1}{24}(\lambda L)^4\right] \quad \text{(A·10)}$$

$$p_4^3=\frac{1}{720}(\lambda L)^6 e^{-2\lambda L} \qquad \text{(A·11)}$$

(b) 3rd-order erlang headway distribution

$$f_H(x)=\frac{1}{2}\lambda^3 x^2 e^{-\lambda x} \qquad \text{(A·12)}$$

where λ is 3 times the inverse of mean headway distance.

$$p_n^0=\frac{1}{(3n-4)!}e^{-2\lambda L}[Q^2 I_{3n-4}+2QTI_{3n-3}+(Q+T^2)I_{3n-2}+TI_{3n-1}+0.25I_{3n}] \qquad \text{(A·13)}$$

where $I_n=\lambda^{n+1}\int_0^L x^n e^{\lambda x}dx=n!\left[(-1)^{n+1}+\left\{\sum_{m=0}^{n}(-1)^{m+n}\frac{(\lambda L)^m}{m!}\right\}e^{\lambda L}\right]$

$Q=1+\lambda L+0.5(\lambda L)^2$

$T=-(1+\lambda L)$

$$p_2^1=\frac{1}{4}e^{-2\lambda L}\sum_{i=0}^{6}\frac{1}{i+4}cl_i(\lambda L)^{i+4} \qquad \text{(A·14)}$$

where $cl_i=\sum_{j=0}^{i} bl_j al_{i-j}$

$bl_i=\sum_{j=0}^{2} al_j rl_{ij}$; $i=0\sim1$

$bl_i=\sum_{j=i-2}^{2} al_j rl_{ij}$; $i=2\sim4$

$al_0=Q$; $al_1=T$; $al_2=0.5$

$rl_{ij}=\frac{1}{j+3}\left\{(-1)^i\frac{(j+3)!}{(i+1)!(j+2-i)!}\right\}$

$$p_3^1=\frac{1}{8}e^{-2\lambda L}\sum_{i=0}^{6}cl_i\frac{1}{i+4}\sum_{j=2}^{i+6}(-1)^{j-2}\frac{(i+4)!}{(j-2)!(i+6-j)!}I_j \qquad \text{(A·15)}$$

$$p_4^1=\frac{1}{480}e^{-2\lambda L}\sum_{i=0}^{6}cl_i\frac{1}{i+4}\sum_{j=5}^{i+9}(-1)^{j-5}\frac{(i+4)!}{(j-5)!(i+9-j)!}I_j \qquad \text{(A·16)}$$

$$p_3^2=\frac{1}{16}e^{-2\lambda L}\sum_{j=0}^{8}\frac{1}{j+4}\sum_{i=0}^{8}d2_{ij}\frac{(i+3)!(j+4)!}{(i+j+8)!}(\lambda L)^{i+j+8} \qquad \text{(A·17)}$$

where $d2_{ij}=\sum_{k=0}^{i}\sum_{l=0}^{j}a2_{kl}c2_{i-k-l}$

$c2_{ij}=c2_{ij}^1$; $j=0\sim6-i$; $i=0\sim6$

$c2_{ij}^1=\sum_{k=0}^{4}\sum_{l=0}^{i}b2_{1k}r2_{j,k,i-1}$; $j=0\sim1$

$c2_{ij}^1=\sum_{k=j-2}^{4}\sum_{1=0}^{i}b2_{1k}r2_{j,k,i-1}$; $j=2\sim6$

$b2_{ij}=b2_{ij}^1$; $j=0\sim4-i$; $i=0\sim4$

$b2_{ij}^1=\sum_{k=0}^{2}\sum_{l=0}^{i}a2_{1k}r2_{j,k,i-1}$; $j=0\sim1$

$b2_{ij}^1=\sum_{k=j-2}^{2}\sum_{1=0}^{i}a2_{1k}r2_{j,k,i-1}$; $j=2\sim4$

$a2_{00}=Q$; $a2_{01}=T$; $a2_{02}=0.5$; $a2_{10}=T$; $a2_{11}=1$; $a2_{20}=0.5$

$s2_{ni}=(-1)^i\frac{n!}{i!(n-i)!}(\lambda L)^{n-i}$; $i=0\sim n$

$r2_{mni}=s2_{2+n-m,i}$

$$p_4^2=\frac{1}{32}e^{-2\lambda L}\sum_{j=0}^{8}\frac{1}{j+4}\sum_{i=0}^{8}d2_{ij}\frac{(i+3)!(j+4)!}{(i+j+8)!}\sum_{k=2}^{i+j+10}(-1)^{k-2}\frac{(i+j+8)!}{(k-2)!(i+j+10-k)!}I_k \qquad \text{(A·18)}$$

$$p_4^3=\frac{1}{64}e^{-2\lambda L}\sum_{j=1}^{15}\frac{1}{j+3}\sum_{i=1}^{15}d2_{ij}\frac{(i+2)!(j+3)!}{(i+j+6)!}(\lambda L)^{i+j+6} \qquad \text{(A·19)}$$

where $g3_{ij}=\sum_{k=0}^{10} f3_{k,i-1,j-1}$

$$f3_{nij}=\frac{1}{n+4}\sum_{l=0}^{i}\sum_{m=0}^{j} e3_{nlm}s3_{n+4,i-1,j-m}$$

$$e3_{nij}=\sum_{p=0}^{n}\sum_{l=0}^{i}\sum_{m=0}^{j} d3_{plm}a3_{n-p,i-1,j-m}$$

$$d3_{nij}=d3^{1}_{nji}\,;\ d3^{1}_{nij}=d3^{2}_{jin}\,;\ j=0\sim 8-n-i\,;\ i=0\sim 8-n\,;\ n=0\sim 8$$

$$d3^{2}_{nij}=\sum_{k=0}^{6}\sum_{l=0}^{i}\sum_{m=0}^{j} c3_{klm}r3_{n,k,i-1,j-m}\,;\ n=0\sim 1$$

$$d3^{2}_{nij}=\sum_{k=n-2}^{6}\sum_{l=0}^{i}\sum_{m=0}^{j} c3_{klm}r3_{n,k,i-1,j-m}\,;\ n=2\sim 8$$

$$c3_{nij}=c3^{1}_{nji}\,;\ c3^{1}_{nij}=c3^{2}_{jin}\,;\ c3^{2}_{nij}=c3^{3}_{nji}\,;\ n=0\sim 6$$

$$c3^{3}_{nij}=\sum_{k=0}^{4}\sum_{l=0}^{i}\sum_{m=0}^{j} b3_{klm}r3_{n,k,i-1,j-m}\,;\ n=0\sim 1$$

$$c3^{3}_{nij}=\sum_{k=n-2}^{4}\sum_{l=0}^{i}\sum_{m=0}^{j} b3_{klm}r3_{n,k,i-1,j-m}\,;\ n=2\sim 6$$

$$b3_{nij}=b3^{1}_{jin}\,;\ n=0\sim 4$$

$$b3^{1}_{nij}=\sum_{k=0}^{2}\sum_{l=0}^{i}\sum_{m=0}^{j} a3_{klm}r3_{n,k,i-1,j-m}\,;\ n=0\sim 1$$

$$b3^{1}_{nij}=\sum_{k=n-2}^{2}\sum_{l=0}^{i}\sum_{m=0}^{j} a3_{klm}r3_{n,k,i-1,j-m}\,;\ n=2\sim 4$$

$$a3_{000}=Q\,;\ a3_{001}=T\,;\ a3_{002}=0.5\,;\ a3_{010}=T\,;\ a3_{011}=1\,;$$
$$a3_{020}=0.5\,;\ a3_{100}=T\,;\ a3_{101}=1\,;\ a3_{110}=1\,;\ a3_{200}=0.5$$

$$s3_{nij}=\sum_{k=0}^{n} u_{ik}v_{kj}$$

$$u_{ik}=(-1)^{i}\frac{(n-k)!}{i!(n-k-i)!}(\lambda L)^{n-k-i}\,;\ i=0,\ n\,;\ k=0\sim n-i$$

$$v_{jj}=(-1)^{j}\frac{n!}{j!(n-j)!}\,;\ j=0\sim n$$

$$r3_{mnij}=\frac{1}{n+3}\left\{(-1)^{m}\frac{(n+3)!}{(m+1)!(n+2-m)!}\right\}s3_{2+n-m,i,j}$$

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