Structural Eng. / Earthquake Eng. Vol. 3. No. 2, 215s-223s. October 1986
Japan Society of Civil Engineers (Proc. of JSCE No. 374/I-6)

MECHANICS OF UNSTABLE CRACK INITIATION : EFFECT OF RESIDUAL STRAIN

By Hideyuki HORII, Akio HASEGAWA** and Fumio NISHINO****

A model of unstable crack initiation from an original crack tip, at which a plastic zone has been formed, is presented. The model provides an answer to the question how and when an increasing load leads to unstable crack initiation instead of further growth of the plastic zone. It is shown that some energy must be supplied to initiate cracking because of the residual strain in the plastic zone, which implies that the crack initiation necessarily accompanies an instability. For brittle materials, the required energy is small and the Griffith criterion turns out to be valid even with the plastic zone at the original crack tip. With increasing ductility of the material, the energy required for the crack initiation increases dramatically, which may lead to the transition from brittle (unstable) crack growth to ductile (stable) one. The present study seems to provide a clue to the problem of the transition from stable, ductile crack growth to the following unstable crack growth. The emphasis is placed on the effect of the residual strain which seems to play an important role in all modes of crack growth. The results of this study suggest a way to estimate the increase in the material resistance to fracture induced by generating intensive plastic deformation at the crack tip prior to its extension.
Keyword : crack tip plasticity, nonlinear fracture mechanics, elastic-plastic fracture

1. INTRODUCTION

One of the great contributions which advanced the fracture mechanics forward is the Griffith criterion. It states that the crack growth can occur if the released energy of the system due to the crack extension equals the required energy to separate the material. For brittle materrials, the Griffith theory received a number of experimental supports and has been widely accepted as the fundamental concept of the linear fracture mechanics. The relation between the energy release rate and the stress intensity factor was proved, and the main concern of the linear fracture mechanics was led to the calculation of the stress intensity factor for different geometry and different loading conditions and to the measurement of the fracture toughness of materials[1].

Since the end of 1960's extensive efforts have been made to apply the fracture mechanics to ductile materials which involve large scale yielding at the crack tip. The J-integral[2] and CTOD (crack tip opening displacement)[3] have been used to characterize the onset of the crack extension. The concepts of the J-resistance curve and the tearing modulus[4] are introduced to study the stable crack growth possibly followed by the unstable crack growth.

Serious questions of the adequacy to use the J-integral for the ductile crack growth have been raised, whereas it was first considered to be the nonlinear counterpart of the linear energy release rate. Rice[5] pointed out that a Griffith-type energy balance for crack growth leads to paradoxical results for elastic-plastic materials, since such solids provide no energy surplus for the material separation in the

* Member of JSCE, Ph. D., Associate Professor, Department of Civil Engineering, University of Tokyo, (Bunkyo-ku, Tokyo)
** Member of JSCE, Dr. Eng., Associate Professor, Division of Structural Engineering and Construction, Asian Institute of Technology (Bangkok, Thailand), on leave from University of Tokyo.
*** Member of JSCE, Ph. D., Professor, Department of Civil Engineering, University of Tokyo, (Bunkyo-ku, Tokyo)

continuous crack advance. Hutchinson and Paris[6] discussed the limitation of the J-integral of the deformation theory which does not allow the unloading. Although they aimed to show that under certain conditions a J-controlled crack growth regime can exist, the use of J-integral turned out to be questionable for the ductile crack growth where the unloading and the residual strain play dominant roles. On the basis of those and related discussions, different models of crack growth and varions versions of modified J-integral and modified energy release rate have been proposed[7)~10)]. The emphasis has been cast on the necessity of constructing a model of the crack tip separation process, on which a suitable approach to the crack growth must be founded.

One way to set about constructing a unified model of crack growth is to answer the following questions ; how and when an increasing load leads to unstable crack initiation instead of further growth of the plastic zone, and how the unstable crack initiation is suppressed as the ductility of the material increases. Even for brittle materials the plastic zone exists at the original crack tip prior to the crack extension. The Griffith theory leaves the plastic zone out of considerations. As the ductility of materials increases, the size of the plastic zone increases. Hence the above questions seem to be fundamental for the mechanism of the crack growth. To answer them, we propose a model of the unstable crack initiation from the original crack tip at which a plastic zone has grown, with the final goal to extend it for the stable crack growth. The attention is paid to the effect of the residual strain in the plastic zone on the crack extension.

2. MODEL OF UNSTABLE CRACK INITIATION

One of the first models of the plastic yielding at a crack tip was proposed by Dugdale[11] and Bildy, Cottrell, and Swinden[12]. It considers a plane of plastic flow at the crack tip coplanar with the crack. Since the plastic zone at the crack tip in plane strain under tensile stress is of the shape shown in Fig. 1 (a), the Dugdale model was modified as is shown in Fig. 1 (b), where two planes of plastic flow inclined to the plane of the crack are considered ; see e. g. Bilby and Swinden[13], Rice[14], Vitek[15], and Riedel[16] where the growth of plactic zone at both ends of a finite crack is considered. Along the planes of plastic flow the yield condition that the shear stress equals the yield stress is assumed. Numerical results provide various features of the plastic zone at the crack tip ; size, orientation, CTOD, and so on. No information on the crack growth, however, is drawn fron this model.

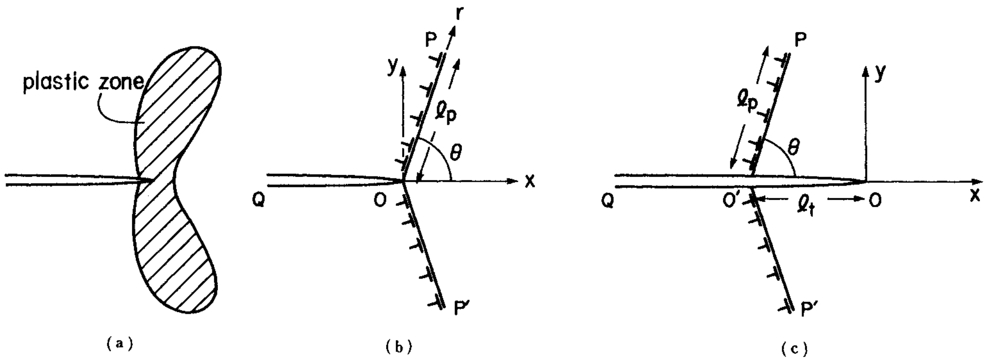

Fig. 1 (a) The plastic zone at the tip of a semi-infinite crack in plane strain, (b) planes OP, OP′ of the plastic flow at the tip of a semi-infinite crack OQ, and (c) the extended crack OQ with the residual strain along the plastic zones O′P, O′P′ at the original crack tip O′.

To study the possibility of the unstable crack growth from the original crack tip, we consider a model shown in Fig. 1 (c). The extended crack OQ is considered where the residual strain (the displacement gap) is distributed along the plastic zone O′P, O′P′ at the original crack tip O′. The distribution of the residual strain is obtained by solving the problem of Fig. 1 (b). Since we restrict our attention on the unstable crack growth where the strain rate is high enough, the plastic deformation at the tip of the extended crack is neglected. That is, we apply the linear fracture mechanics for the extended crack. Deformation in mode I under plane strain is considered.

We consider the growth of a semi-infinite crack in an infinite plane with plastic zones at its original tip. It corresponds to the assumption that the length of the plastic zone is small enough compared with that of the original crack and the specimen, but it is compatible with the size of the crack extension. [The interaction between the outer surface of the specimen and plastic zone often observed in ductile materials is out of focus in this paper.] It is not only to simplify the problem but also to exclude immaterial factors such as the variation of the stress intensity factor with the crack length. We introduce a loading parameter for this model, "the applied K-value", which is defined as the mode I stress intensity factor at the crack tip when the material is assumed to be elastic, in other words, when the plastic deformation and the residual strain do not exist. It is known that the stability of the crack growth highly depends on the geometry of the specimen and the way of loadings. In this paper we consider the crack growth under the constant applied K-value. Results may be easily applied for cases where the loading is the function of the crack length.

The problem consists of two parts : First we consider the plastic zone growth prior to the crack extension; see Fig. 1 (b). The problem is symmetric with respect to the x-axis. The length of the plastic zone, which makes an angle θ with the x-axis, is denoted by l_p. The boundary conditions are given by

$$\sigma_y=\tau_{xy}=0, \text{ on OQ} \quad (1)$$

$$u_\theta^+=u_\theta^-, \quad |\tau_{r\theta}|=\tau_Y, \text{ on OP and OP}' \quad (2)$$

where superscripts + and − indicate the value of the quantity on the upper and lower surfaces of plastic zone and τ_Y denotes the yielding shear stress. Since stresses are bounded at the end of the plastic zone, the condition

$$\lim_{r\to l_p^+} |\tau_{r\theta}|=\tau_Y \text{ at P and P}' \quad (3)$$

must be satisfied. With the applied K-value, one finds the solution which satisfies conditions (1)-(3). To solve this problem we use the Green's function technique with the solution of a dislocation near a semi-infinite crack. Distributed dislocations along plastic zones are introduced. The condition (1) is automatically satisfied and the condition (2) leads to the singular integral equation for the dislocation density. It is solved numerically with the condition (3).

Next, with the obtained distribution of the residual strain, we calculate the stress intensity factor at the tip of the extended crack, which is different from the applied K-value because of the effect of the residual strain. Mathematical formulations are shown in the following section. Those who are not interested in the mathematical details can skip the next section without loss of continuity. In section 4 numerical results are shown and their physical implications are discussed. Note that the mathematical formulation is easily modified for a finite crack since the stress functions for a semi-infinite crack are derived as the limiting case of those for the finite crack.

The interaction of plastic zones and cracks is also a fundamental factor in the micromechanism of the brittle-ductile transition under compression. Its analytical model is proposed by Horii and Nemat-Nasser[17]. The model indudes cracks and plastic zones emanating from an initial defect. Features of the brittle-ductile transition are explained in terms of numerical results.

3. MATHEMATICAL FORMULATION

For the mathematical formulation of the problem stated in the previous section, Muskhelishvili's complex stress functions Φ and Ψ are employed[18]. In terms of these potentials the stresses and displacements are given by

$$\sigma_x+\sigma_y=2(\Phi'+\overline{\Phi'}), \quad \sigma_y-\sigma_x+2i\tau_{xy}=2(\bar{z}\Phi''+\Psi'), \quad 2\mu(u_x+iu_y)=\kappa\Phi-z\overline{\Phi'}-\overline{\Psi} \quad (4)$$

where μ is the shear modulus; $\kappa=3-4\nu$ for plane strain, ν being Poison's ratio; $z=x+iy$ with $i=$

$\sqrt{-1}$; overbar denotes the complex conjugate ; and prime stands for differentiation with repect to the argument.

To solve the problem we consider a single dislocation at z_0 near a crack of length $2c$; see Fig. 2. We introduce stress functions $\Phi_D=\Phi_0+\Phi_R$ and $\Psi_D=\Psi_0+\Psi_R$ where Φ_0 and Ψ_0 are stress functions for a single dislocation in an infinite plane and Φ_R and Ψ_R are the complementary potentials to satisfy the stress free condition (1) along the crack surface. Φ_R and Ψ_R are obtained by the method of Muskhelishvili[18]. They are given by

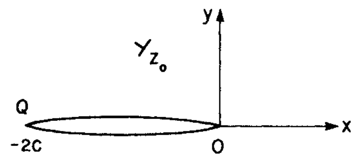

Fig. 2 A dislocation near a crack.

$$\Phi_0'=\frac{\alpha}{z-z_0},\quad \Psi_0'=\frac{\overline{\alpha}}{z-z_0}+\frac{\alpha\overline{z}_0}{(z-z_0)^2},\quad \Phi_R'=-\alpha[F(z,z_0)+F(z,\overline{z}_0)]-\overline{\alpha}(z_0-\overline{z}_0)G(z,\overline{z}_0),$$
$$\Psi_R'=\overline{\Phi}_R'-\Phi_R'-z\Phi_R'' \quad \cdots\cdots (5)$$

with

$$F(z,z_0)=\frac{1}{2}\left[1-\sqrt{\frac{z_0(z_0+2c)}{z(z+2c)}}\right]\frac{1}{z-z_0},\quad G(z,z_0)=\frac{\partial}{\partial z_0}F(z,z_0) \quad \cdots\cdots (6)$$

where $\alpha=\mu([u_r]+i[u_\theta])e^{i\theta}/\pi i(\kappa+1)$, $[u]=u^+-u^-$, and $\overline{\Phi}=\overline{\Phi(\overline{z})}$. For a semi-infinite crack, take the limit, $c\to\infty$, and Eqns. (6) become

$$F(z,z_0)=\frac{1}{2}\left[1-\sqrt{\frac{z_0}{z}}\right]\frac{1}{z-z_0},\quad G(z,z_0)=\frac{\partial}{\partial z_0}F(z,z_0) \quad \cdots\cdots (7)$$

Next we consider stress functions Φ_A and Ψ_A for the applied load. By the method of Muskhelishvili[18], stress functions for a crack of length $2c$ in an infinite plane under uniform tension σ_y^∞ at infinity are obtained as

$$\Phi_A'=\frac{1}{4}\sigma_y^\infty-\frac{1}{2}\sigma_y^\infty\left[1-\frac{z+c}{\sqrt{z(z+2c)}}\right],\quad \Psi_A'=\frac{1}{2}\sigma_y^\infty+\frac{1}{2}\sigma_y^\infty\frac{zc^2}{[z(z+2c)]^{3/2}} \quad \cdots\cdots (8)$$

For the semi-infinite crack we take the limit, $c\to\infty$, with $K_{IA}=\sigma_y^\infty\sqrt{\pi c}$ fixed constant to obtain from Eqns. (8)

$$\Phi_A'=\frac{1}{2}\frac{K_{IA}}{\sqrt{2\pi z}},\quad \Psi_A'=\frac{1}{4}\frac{K_{IA}}{\sqrt{2\pi z}} \quad \cdots\cdots (9)$$

K_{IA} is called, in this paper, "the applied K-value" which is the loading parameter for the semi-infinite crack. Obviously stress functions (9), through Eqns. (4), result in the well-known crack tip stress distribution.

Sterss functions Φ_D, Ψ_D and Φ_A, Ψ_A automatically satisfy the stress free condition (1) on the crack surface. We introduce distributed dislocations along the plastic zones OP and OP′ ; see Fig. 1 (b). From the first equation of (2) and the symmetry of the problem, the dislocation density is given by

$$\alpha(\xi)=-i\beta(\xi)e^{i\theta},\ \text{at}\ z_0=\xi e^{i\theta},\ \text{and}\ \alpha(\xi)=i\beta(\xi)e^{-i\theta},\ \text{at}\ z_0=\xi e^{-i\theta} \quad \cdots\cdots (10)$$

where $\beta(\xi)$, which is the derivative of the shear displacement gap across the plastic zone with respect to the distance ξ, is a real function to be determined. The remaining condition, that is the yield condition along the plastic zone [the second equation of (2)], leads to the singular integral equation for the dislocation density $\beta(\xi)$,

$$-2\int_0^{l_p}\frac{\beta(\xi)}{\xi-\eta}d\xi+\int_0^{l_p}\beta(\xi)K(\xi,\eta;\theta)d\xi+\frac{K_{IA}}{\sqrt{2\pi\eta}}\frac{1}{2}\sin\theta\cos\frac{\theta}{2}=\tau_Y \quad \cdots\cdots (11)$$

where

$$K(\xi,\eta;\theta)=Re\left\{e^{4i\theta}\left[\frac{2}{\xi-\eta e^{2i\theta}}+\frac{2i\eta\sin 2\theta}{(\xi-\eta e^{2i\theta})^2}\right]\right\}+4\eta\sin^2\theta Re\{e^{2i\theta}[F'(z,z_0)+F'(z,\overline{z}_0)$$
$$-\xi[e^{-i\theta}G'(z,\overline{z}_0)+e^{i\theta}G'(z,z_0)]]\} \quad \cdots\cdots (12)$$

with $Re\{\ \}$ for the real part of the argument ; $F'=\frac{\partial}{\partial z}F$. The first term of Eqn. (11) is the shear stress on the plastic zone OP due to the dislocation on OP, the second term due to the dislocation on OP′ and the complementary functions Φ_R, Ψ_R and the third term for the applied K-value.

The singular integral equation (11) for the dislocation density $\beta(\xi)$ [see Eqns. (10)] is solved numerically by the method of Gerasoulis and Srivastav[19] with the condltion (3) resulting in

$$\frac{K_{IA}}{\tau_Y\sqrt{\pi l_p}}=f(\theta) \quad (13)$$

which provides the relation between the applied K-value K_{IA} and the plastic zone length l_p where $f(\theta)$ is obtained numerically for a given value of θ. From the obtained distribution of the dislocation density, the crack tip opening displacement δ and the dissipated plastic work W_p are obtained as

$$\delta=2\sin\theta\frac{\pi(\kappa+1)}{\mu}\int_0^{l_p}\beta(\xi)d\xi,\quad W_p=2\tau_Y\frac{\pi(\kappa+1)}{\mu}\int_0^{l_p}\int_\eta^{l_p}\beta(\xi)d\xi d\eta \quad (14)$$

which are calculated in the forms

$$\frac{\delta}{\tau_Y l_p}\frac{\mu}{\pi(\kappa+1)}=d(\theta),\quad \frac{W_p}{\tau_Y^2 l_p^2}\frac{\mu}{\pi(\kappa+1)}=w(\theta) \quad (15)$$

The obtained distribution of the dislocation density is used as the residual strain when the extended crack is considered; see Fig. 1 (c). We calculate the stress intensity factor ΔK_I at the tip of the extended crack due to the residual strain along the plactic zones O′P and O′P′. The total stress intensity factor at the extended crack tip is given by the summation, $K_I=K_{IA}+\Delta K_I$. From Eqns. (4), (5), (7), and (10), ΔK_I is given by

$$\Delta K_I=-4\sqrt{2\pi}\sin\theta\int_0^{l_p}\beta(\xi)Re\left[\frac{1}{\sqrt{z_0}}+\frac{\xi e^{i\theta}}{2z_0\sqrt{z_0}}\right]d\xi,\quad \text{with } z_0=-l_t+\xi e^{i\theta} \quad (16)$$

which is calculated in the form

$$\frac{\Delta K_I}{\tau_Y\sqrt{\pi l_p}}=g(l_t/l_p;\theta) \quad (17)$$

In the following section, numerical results are shown and the growth of plastic zone followed by the unstable crack growth is discussed.

4. RESULTS AND DISCUSSIONS

In the previous section it is shown that by solving the singular integral equation numerlcally, one obatains

$$\frac{K_{IA}}{\tau_Y\sqrt{\pi l_p}}=f(\theta),\quad \frac{\delta}{\tau_Y l_p}\frac{\mu}{\pi(\kappa+1)}=d(\theta),\quad \frac{W_p}{\tau_Y^2 l_p^2}\frac{\mu}{\pi(\kappa+1)}=w(\theta),\quad \frac{\Delta K_I}{\tau_Y\sqrt{\pi l_p}}=g(l_t/l_p;\theta) \quad (18)$$

where δ and W_p are the crack tip opening displacement and the dissipated plastic work prior to the crack extension, respectively; ΔK_I is the stress intensity factor at the extended crack tip due to the residual strain. The total stress intensity factor at the extended crack tip is given by the summation, $K_I=K_{IA}+\Delta K_I$. The quantities $f(\theta)$, $d(\theta)$, $w(\theta)$, and $g(l_t/l_p;\theta)$ are the nondimensional values calculated for given θ and l_t/l_p.

Now we determine the orientation of the plastic zone such that the dissipated plastic work W_p is maximized for a constant applled K-value K_{IA}; see Fig. 1 (b). Eliminating l_p from $(18)_1$ [the first equation of (18)] and $(18)_3$, one obtains W_p normalized with K_{IA} which is plotted in Fig. 3 together with the length of the plastic zone. It is seen that the orientation which maximizes the length of the plastic zone is slightly less then that does the dissipated plastic work. From this result the orientation of the plastic zone θ is fixed at 76. 1^0 for which we have [see Eqns. (18)],

$$\frac{K_{IA}}{\tau_Y\sqrt{\pi l_p}}=2.35,\quad \frac{\delta E\sigma_Y}{K_{IA}^2(1-\nu^2)}=0.565,\quad \frac{W_p E\sigma_Y^2}{K_{IA}^4(1-\nu^2)}=0.0225 \quad (19)$$

where $\sigma_Y=2\tau_Y$ and E denotes the Young's modulus. Corresponding to this orientation and the associated

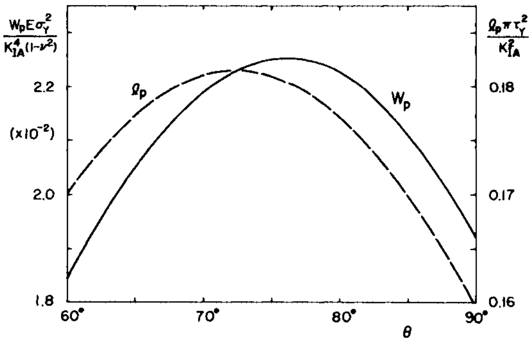

Fig. 3 The dissipated plastic work and the length of the plastic zone as a function of the orientation of the plastic zone.

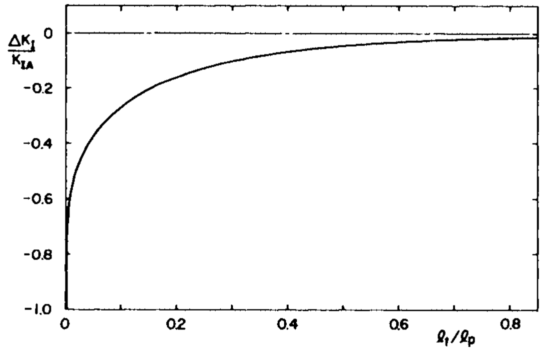

Fig. 4 The stress intensity factor at the tip of the extended crack due to the residual strain.

distribution of the residual strain, one calculates the stress intensity factor ΔK_I at the extended crack tip for different values of l_t/l_p; see Eqn. $(18)_4$ and Fig. 4.

So far the introduced material parameter is only the yielding shear stress τ_Y [except for E and ν]. To discuss the crack extension, we require the fracture toughness G_c or K_c which are related each other through the relation between the energy release rate G and the stress intensity factor K_I,

$$G=\frac{1-\nu^2}{E}K_I^2 \cdots\cdots (20)$$

Then the material property is represented by two parameters τ_Y and K_C. The characteristic length of the material r_p is defined by

$$r_p=\frac{K_C^2}{8\pi\tau_Y^2} \cdots\cdots (21)$$

which is the Irwin's plastic zone correction[1] with $K_I=K_C$. Each material has its own characteristic length. For example, the value of r_p is larger then 135 mm for low strength Carbon steel and about 0.1 mm for 4 340 steel. Quantities whose dimensions include the length are non-dimensionalized using r_p. With this characteristic length it follows from Eqn. $(19)_1$ that

$$\frac{K_{IA}}{K_C}=0.83\sqrt{\frac{l_p}{r_p}} \cdots\cdots (22)$$

which gives the relation between the length of the plastic zone and the applied K-value. From Eqns. $(18)_4$, (21), and (22), one obtains the stress intensity factor at extended crack tip as a function of l_t/r_p and l_p/r_p,

$$\frac{K_I}{K_C}=\frac{K_{IA}}{K_C}+\frac{\Delta K_I}{K_C}=\sqrt{\frac{l_p}{r_p}}\,[0.83+g(l_t/l_p)/2\sqrt{2}] \cdots\cdots (23)$$

which is shown in Fig. 5 where lines for constant values of K_I/K_C are plotted. Above the critical line for $K_I/K_C=1$, the stress intensity factor K_I at the crack tip is larger than the fracture toughness K_C. Once the crack is extended beyond the critical line, the crack grows in an unstable manner. This instability is possible if the applied K-value is greater than the fracture toughness as is seen in Fig. 5. However, the shaded area below the critical line where K_I is less then K_C obstructs the

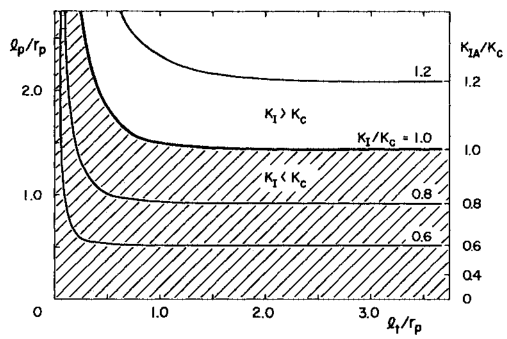

Fig. 5 Lines for constant values of the stress intensity factor at the tip of the extended crack.

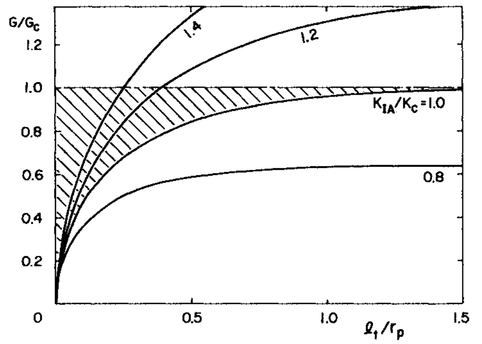

Fig. 6 The energy release rate vs. the extended crack length for the indicated applied K-value.

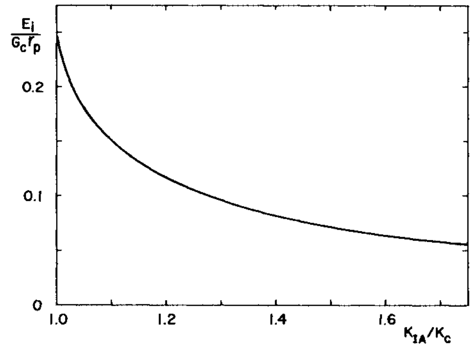

Fig. 7 The energy required for the crack extension vs. the applied K-value.

crack growth. To jump this obstacle some energy must be supplied since the released energy due to the crack extension is less than that required for the material separation. Once energy required to jump the obstacle is supplied, the unstable crack growth is materialized.

To calculate the energy required to initiate the crack extension, we plot the energy release rate as a function of the length of the crack extension for different values of the applied K-value in Fig. 6. [The stress intensity factor and the energy release rate are related through Eqn. (20).] It is seen that the energy release rate reaches the straight line, $G=G_c$, when K_{IA} is greater than K_c. The energy required for the crack extension is calculatec as the area surrounded by the corresponding curve, the straight line for $G=G_c$, and the vertical axis; the shaded area for $K_{IA}=K_c$. As the applied K-value increses, the associated area, that is the required energy, decreases. Calculated energy per unit thickness required for the crack extension, E_i, is shown in Fig. 7 as a function of the applied K-value.

As is seen from Fig. 7, the maximum energy required for the crack extension at $K_{IA}/K_c=1$ is given by $E_i=0.25\,G_c r_p$. It is equal to the energy for the material separation of length a quarter of r_p. It increases drastically as the ductility of the material increases. For example, E_i is 0.26 J/m for 4 340 steels and 7 300 J/m for Carbon steels. In Fig. 8 the characteristic length r_p and the maximum energy required for the crack initiation E_i are plotted as a function of τ_Y and K_c together with data for typical metals[1]; to calculate E_i, $E=20\times10^{10}\,\mathrm{Nm}^{-2}$ and $\nu=0.3$ are used.

For brittle materials such as Maraging steels, the energy required for the crack extension is so small that the crack extenslon is supposed to occur at the minimum value of the applied K-value which is the same as the fracture toughness. Therefore it is concluded that the Griffith criterion is valid for brittle materials even with the plastic zone at the crack tip prior to its extension.

As the ductility of the material increases, the required energy for the crack initiation increases drastically and it may not be possible to jump the obstacle at $K_{IA}=K_c$. Then two cases are possible. In the first case the required energy for the crack extension decreases with the increasing K_{IA} as shown in Fig. 7, and at a certain stage the unstable crack extension is materialized. In the second case, with the incresing K_{IA} a stable crack growth occurs where the applied K-value must be increased to advance the crack. Large plastic zones follow the crack tip as it advances. The prediction requires a criterion of the stable

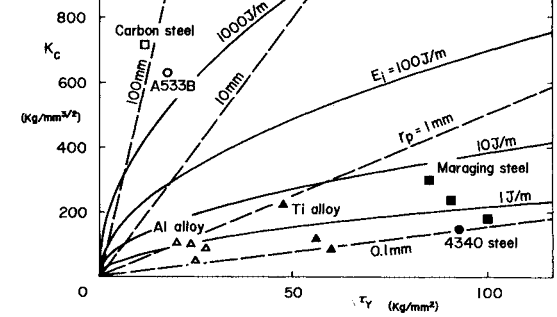

Fig. 8 The characteristic length and the maximum energy for the crack initiation together with data for typical metals.

(ductile) crack growth which must be based on the micro-events ahead of the crack tip such as the void growth and their coalescence.

As is shown above the stress intensity factor at the tip of the extended crack is less than the applied value. It is because of the residual strain in the plastic zone and it abstructs the crack initiation. In other words, the apparent toghness is increased. Our results suggest that the material resistance to fracture is increased if we generate artificially intensive plastic defomation at the crack tip prior to its extension. The same effect in fatigue crack growth is well-known as the retardation effect of overloads. This study shows a way to estimate the increase in the fracture toughness due to the residual strain.

Although the mechanism of the stable crack growth is out of focus in this paper, the residual strain is considered to play an important role in the stable crack growth and in the possibly following unstable crack growth. As the crack continues the stable growth, the plastic zone follows the advancing crack tip accumulating residual strains behind the crack tip. The resistance to fracture continues to rise due to the accumulated rasidual strain. With increasing crack length the applied K-value incrases, and the required energy for the unstable crack growth decreases similarly to the case shown in Figs. 6 and 7. [The accumulated residual strain might complicate the situation.]

In this paper the applied K-value is introduced as a loading parameter and is fixed constant for the crack extension to simplify the problem and to shed light on the most fundamental mechanism. In the actual situation, the applied K-value changes as the plastic deformation proceeds and as the crack extends. Their relation depends on the way of the loading in a very complex manner. Our results are easily modified for those cases if their relations are given. As is well-known, the stability of the crack growth depends on the way of the loading, the stiffness of the loading machine and other factors. However, it is seen from our results that the brittle crack initiation necessarily accompanies an instability because the crack initiation is accomplished by jumping the obstacle due to the residual strain.

5. SUMMARIES AND CONCLUSIONS

In this paper the plastic zone at the tip of a semi-infinite crack under the plane strain condition is modeled by symmetric planes inclined to the crack surface. Distributed dislocations are introduced along the plastic zones. The distribution of the dislocation density is obtained such that the yield condition along the plastic zone is satisfied. The orientation of the plastic zone is obtained to maximize the dissipated plastic work for the constant applied K-value which is introduced as the loading parameter. The configuration of the extended crack is considered and the stress intensity factor at the tip of the extended crack, which is different from the applied K-value due to the effect of the residual strain, is calculated. Numerical results are shown and their physical implications are discussed. The obtained conclusions are summarized as follows :

(1) The stress intensity factor at the tip of the extended crack is reduced because of the residual strain in the plastic zone at the original crack tip. As a consequence, some energy must be supplied to initiate the unstable crack growth which is possible if the applied K-value is greater than the fracture toughness.

(2) For brittle materials, the required energy for the crack extension is so small and the Griffith criterion turns out to be valid even with the plastic zone at the original crack tip.

(3) With increasing ductility of the material, the required energy for the crack initiation increases drastically, which may prevent the unstable crack extension and lead to the stable crack growth.

(4) The residual strain plays an important role in the unstable crack initiation as well as in the stable crack growth followed by the unstable crack growth.

(5) It seems to be the next step to extend the present model for the stable crack growth where the accumulated residual strain is one of the major factors which increase the resistance to fracture. The criterion for the stable crack growth, which must be based on the micro-events ahead of the crack tip such

as the void growth and their coalescence, seems to be necessary.

(6) Althongh perfect plasticity is assumed in the present study, the strain hardening can be introduced by assigning a relation between the amount of slip and the shear stress along the plastic zones.

ACKNOWLEDGEMENT

This dtudy is supported in part by the Grant-in Aid for Scientific Research from the Japanese Ministry of Education, Science and Culture.

REFERENCES

1) Broek, D. : Elementary Engineering Fracture Mechanice, Sijthoff & Noordhoff, 1978.
2) Begley, J. A. and Landes. J. D. : The J Integral as a Fracture Criterion, Fracture Toughness, Proceedings of the 1971 National Symposium on Fracture Mechanics Part II, ASTM, STP 514, pp. 1~23, 1971.
3) Robinson, J. N. and Tetelman, A. S. : Measurement of K_{Ic} on Small Specimens Using Critical Crack Tip Opening Displacement, Fracture Toughness and Slow-Stable Cracking, Proceedings of the 1973 National Symposium on Fracture Mechanics Part I, ASTM, STP 559, pp. 139~158, 1973.
4) Paris, P. C. Tada, H, Zahoor, A. and Ernst, H. : The Theory of Instability of the Tearing Mode of Elastic-Plastic Crack Growth, Elastic-Plastic Fracture, ASTM, STP 668, pp. 5~36, 1977.
5) Rice, J. R. : An Examination of the Fracture Mechanics Energy Balance from the Point of View of Continuum Mechanics, Proceedings of the First International Conference on Fracture, The Japanese Society for Strength and Fracture of Materials, Vol. 1, pp. 309~340, 1966.
6) Hutchinson, J. W. and Paris, P. C. : Stability Analysis of J-Controlled Crack Growth, Elastic-Plactic Fracture, ASTM, STP 668, pp. 37~64, 1977.
7) Kfouri, A. P. and Rice, J. R. : Elastic/Plactic Separation Enregy Rate for Crack Advance in Finite Growth Steps, Fracture 1977 Advances in Research on the Strength and Fracture of Materials, Vol. 1, pp. 43~59, 1977.
8) Wnuk, M. P. and Mura, T. : Extension of a Stable Crack at a Variable Growth Step, Fracture Mechanics : Fourteeth Symposium-Volume 1 : Theory and Analysis, ASTM, STP 791, pp. 96~127, 1981.
9) Miyamoto, H., Kageyama, K., Kikuchi, M. and Machida, K. : The J_{ext}-Integral Based on the Concept of Effective Energy Release Rate, Elastic-Plactic Fracture : Second Symposium, Volume 1-Inelastic Crack Analysis, ASTM, STP 803, pp. 116~129, 1981.
10) Saka, M., Shoji, T., Takahashi, H., and Abe, H. : A Criterion Based on Crack-Tip Energy Dissipation in Plane-Strain Crack Growth under Large-Scale Yielding, Elastic-Plastic Fracture : Second Symposium, Volume 1-Inelastic Crack Analysis, ASTM, STP 803, pp. 130~158, 1981.
11) Dugdale, D. S. : Yielding of Steel Sheets Containing Slits, Journal of the Mechanics and Physics of Solids, Vol. 8, pp. 100~108, 1960.
12) Bilby, B. A., Cottrell, A. H. and Swinden, K. H. : The Spread of Plastic Yield from a Notch, Proceedings of the Royal Society of London, Vol. A 272, pp. 304~310, 1963.
13) Bilby, B. A. and Swinden, K. H. : Representation of Plasticity at Notches by Linear Dislocation Arrays, Proceedings of the Royal Society of London, Vol. A 285, pp. 22~33, 1965.
14) Rice, J. R. : Limitations to the Small Scale Yielding Approximation for Crack Tip Plasticity, Journal of the Mechanics and Physics of Solids, Vol. 22, pp. 17~26, 1974.
15) Vitek, V. : Yielding on Inclined Planes at the Tip of a Crack London in Uniform Tension, Journal of the Mechanics and Physics of Solids, Vol. 24, pp. 263~275, 1976.
16) Riedel, H. : Plastic Yielding on Inclined Slip-Planes at a Crack Tip, Journal of the Mechanics and Physics of Solids, Vol. 24, pp. 277~289, 1976.
17) Horii, H. and Nemat-Nasser, S. : Brittle Failure in Compression : Splitting, Faulting, and Brittle-Ductile Transition, Philosophical Transactions of the Royal Society of London, 1986 (to appear).
18) Muskhelishvili, N. I. : Some Basic Problems in the Mathematical Theory of Elasticity, Noordhoff, 1963.
19) Gerasoulis A. and Srivastav, R. P. : A Method for the Numerical Solution of Singular Integral Equations with a Principal Value Integral, International Journal of Engineering Science, Vol. 19, pp. 1293~1298, 1981.

(Received September 26 1985)