

EFFECT OF STRESS RATIO AND TENSILE RESIDUAL STRESS ON NEAR THRESHOLD FATIGUE CRACK GROWTH

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The effects of stress ratios on the fatigue crack growth in a high tensile residual stress field are examined. Specimens, made of 600 MPa class steel, are center-cracked type which contain longitudinal weld beads along the center of the plate. Fatigue crack growth rates and crack opening levels are measured at several stress ratios (−1 to 0.91). Under these test conditions, cracks fully open regardless of the value of stress ratio, and fatigue crack growth rates and threshold stress intensity factor ranges do not depend on the stress ratios.

1. INTRODUCTION

The greater part of the fatigue life of a welded joint is taken up by the growth process of fatigue cracks, and therefore, it is effective to apply the fracture mechanics analysis in predicting the fatigue strength of the joint¹⁾. The relationship of fatigue crack growth rate (da/dN) and stress intensity factor range (ΔK), which is a parameter of fracture mechanics analysis, becomes of greatest importance in such case. Particularly, the $da/dN - \Delta K$ relationship in the range of slow fatigue crack growth rate near the threshold value (ΔK_{th}) of fatigue crack growth greatly influences the estimates of fatigue strength.

Tensile residual stresses often exist in the vicinities of weld defects such as blowholes and slag inclusions or at toes of fillet welds which become causes of fatigue cracking at welded joints. Consequently, in order to estimate the life of a joint, it is necessary to investigate the influence of tensile residual stress on fatigue crack growth rate. The authors²⁾ showed in the previous report that the influence of tensile residual stress on fatigue crack growth rate is prominent especially in a range where the fatigue crack growth rate is low. Ohta et al.³⁾ and Horikawa et al.⁴⁾ have obtained similar results. The allowable sizes of defects remaining at truss corner welds and the allowable toe profile of fillet welds of Honshu-Shikoku bridges⁵⁾ have been calculated employing the constants (C, m)⁵⁾ in the Paris's law determined by rearranging existing data and the fatigue crack growth threshold values obtained in the study previously reported²⁾.

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Fatigue tests of joints are performed in many cases in zero to tension pulsating stress states (stress ratio $R=0$). However, stresses produced in various parts of a bridge are complex and the stresses repeated at welded joints are not always in zero to tension pulsating stress states. There are cases when repetitive stresses of very high stress ratios are produced depending on the member or location. It is therefore necessary to find how the stress ratio will influence the fatigue crack growth rate in a field where tensile residual stress exists. The allowable fatigue stress range in the design codes for railway bridges and the Honshu-Shikoku bridges is specified to be constant for the case of stress ratio higher than -1. Accordingly, in case the stress ratio has become especially high, it is important for ascertaining the design allowable stress range and applicability of the allowable size of the defect to investigate whether the fatigue crack growth threshold value will become lower than the value under zero to tension.

Ohta et al.⁷⁾ used transverse welded joint specimens and showed that fatigue crack growth characteristics did not change in a range of stress ratios of 0 to 0.9. In the present study, the influence of stress ratio in an especially low growth rate range was examined using longitudinal welded joint specimen in which the highest tensile residual stresses are introduced in a direction perpendicular to the direction in which fatigue cracks grow, in effect, specimens producing the severest conditions for crack growth.

2. TESTING METHOD

The steel used for the specimens was the 600 MPa class structural steel SM 58 having a plate thickness of 16 mm. The mechanical properties and chemical compositions of this steel are given in Table 1. Grooves of the shape and dimensions shown in Fig.1 were machined in the steel plate and MIG welding was performed under the conditions indicated in Table 2. The weldings were finished with single passes on both sides. Each longitudinal welded specimen was fabricated from this welded plate so that the weld bead would run through the middle of the plate width. The specimen configuration and dimensions are as shown in Fig. 2. Base metal specimens with no welding were also made. The base metal specimens were subjected to stress relief heat treatment (580°C, 1 hr) after fabrication to eliminate residual stresses due to rolling and machining.

Fatigue tests were carried out using an electro-hydraulic servo fatigue testing machine having a dynamic capacity of 500 kN. The load waveforms were sine with repetition rates from 8 to 25 Hz. In tests of gradual decrease in the stress intensity factor range to obtain the fatigue crack growth threshold value, the load range was reduced in steps of 5 percent each with crack growth of about every 0.25 mm. This condition meets the recommendations of ASTM on the rate of load reduction⁸⁾.

With the specimens used here fatigue cracks grew from both ends of artificial notches and the differences in growths of fatigue cracks on the two sides were extremely small. Therefore, the stress intensity factor

Table 1 Mechanical properties and chemical compositions of steel.

Mechanical Properties			Chemical Compositions				
yield strength (MPa)	tensile strength (MPa)	elongation (%)	C	Si	Mn	P	S
			x 100 (%)			x 1000 (%)	
640	690	34	8	24	166	20	4

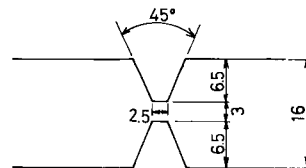


Fig. 1 Weld groove.

Table 2 Welding Conditions.

wire shield gas	YM-60A, ϕ 1.2 Ar90% + CO ₂ 10%
welding current	1st side : 250A 2nd side : 320A
arc voltage	1st side : 29V 2nd side : 31V
welding speed	1st side : 20cm/min 2nd side : 20cm/min

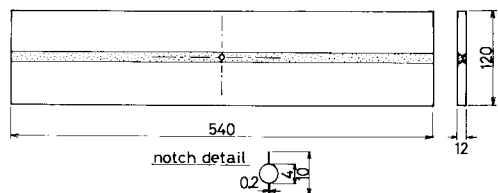


Fig. 2 Configuration and dimensions of specimen.

was calculated by the equation below⁹⁾.

$$K = \sigma_n \sqrt{\pi a} [1 - 0.025(2a/W)^2 + 0.06(2a/W)^4] \sqrt{\sec(\pi a/W)} \dots \dots \dots (1)$$

where,

σ_n : nominal stress, a : crack length, W : specimen width

The level of crack opening and closing was determined from the variation in compliance detected by strain gages mounted on the surface and sides of the specimen. In case closing of a crack could not be recognized within the range of testing loads, the closing point of the crack was obtained by lowering the load further. The effective stress intensity factor range (ΔK_{eff}) is defined as follows with crack opening point stress intensity factor as K_{op} ¹⁰⁾.

$$\Delta K_{eff} = U \cdot \Delta K \dots \dots \dots (2)$$

where,

$$U = (K_{max} - K_{op}) / \Delta K \quad (K_{op} > K_{min})$$

$$U = 1 \quad (K_{op} \leq K_{min})$$

In this case U is the effective stress intensity factor range ratio.

3. FATIGUE CRACK GROWTH RATE

Fig. 3 shows the relationships of fatigue crack growth rate (da/dN) and stress intensity factor range (ΔK) of base metal specimens with stress ratio (R) as 0 and 0.6 and longitudinal welded specimens with R as 0. The $da/dN - \Delta K$ relationship of base metal specimens and longitudinal welded specimens at $R=0$ is that of rough agreement in a range of ΔK larger than $12 \text{ MPa}\sqrt{\text{m}}$, but in a range of ΔK being smaller the difference between the two become larger. This is similar to the results given in the previous report²⁾. When stress ratio R is made 0.6 for the base metal specimen, the fatigue crack growth rate become fairly fast compared with the case of $R=0$, but is still slower than the case of $R=0$ with longitudinal welded specimens.

The $da/dN - \Delta K$ relationships of longitudinal welded specimens are shown in Fig. 4. The testing was done here under the conditions that stress ratio (R) was from -1 to 0.91 , but differences in the $da/dN - \Delta K$ relationship according to R could not be seen.

The $da/dN - \Delta K$ relationship with the longitudinal welded specimen declines linearly to the vicinity of 10^{-7} mm/cycle . In a range of da/dN lower than this the da/dN deviates from linear toward a lower value to approach the fatigue crack growth threshold value (ΔK_{th}). The fatigue crack growth threshold value (ΔK_{th}) with fatigue crack growth rate defined by 10^{-8} mm/cycle is shown in Fig. 5(a), (b). The value of

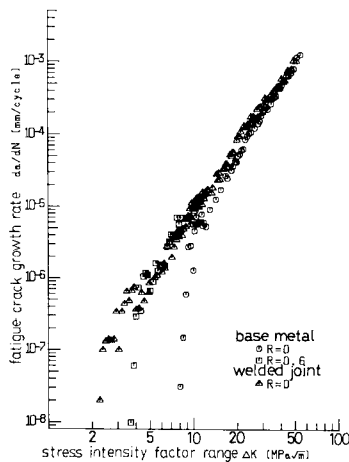


Fig. 3 $da/dN - \Delta K$ relationships of base metal and welded joint specimens.

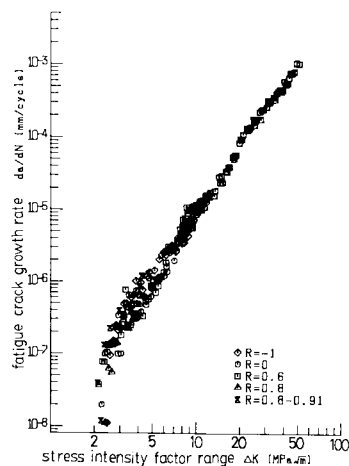


Fig. 4 Effect of stress ratios on $da/dN - \Delta K$ relationships of longitudinal welded specimens.

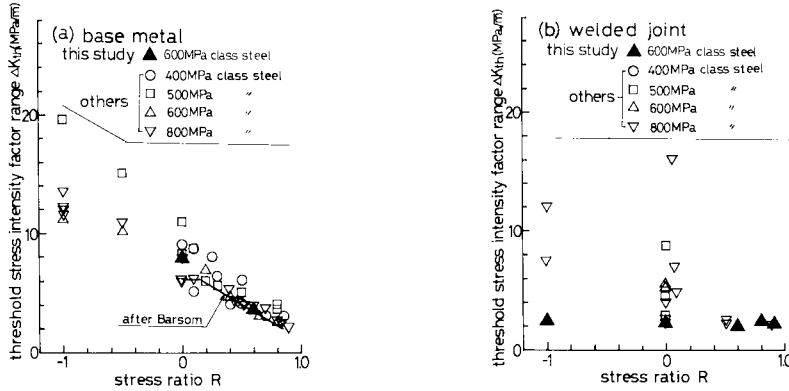


Fig.5 Relationship between threshold value and stress ratio.

$\Delta K_{th}^{2-4, 7, 11-16}$ obtained in Japan with various structural steels and welded joints are also shown in the figures. Barsom gave the relationship of ΔK_{th} and R as the conservative estimates for various steels from analysis of experimental results¹⁷⁾. The value of ΔK_{th} with the steel base metal decrease as R become higher, and the trend coincides with the relationship according to Barsom (Fig.5(a)). However, the values of ΔK_{th} obtained here with the longitudinal welded specimens are more or less constant regardless of R (−1 to 0.91). These values are from 1.9 to 2.4 $MPa\sqrt{m}$, roughly the minimum of the ΔK_{th} values obtained so far (Fig.5(b)). That is, tensile residual stresses of high intensity extinguish the influence of the stress ratio on da/dN and cause the lowering of ΔK_{th} .

In calculations of allowable defect sizes in corner welds of truss chord in Honshu-Shikoku bridges, $\Delta K_{th} = 8 \text{ kgf/mm}^{3/2}$ (2.5 $MPa\sqrt{m}$) was set referring to the previous test results²⁾ with $R=0$, and it may be said that the value was reasonable even for a case of high stress ratio. According to the results here it can be also said to be amply safe if $\Delta K_{th} = 2 \text{ MPa}\sqrt{m}$ were to be used regardless of the stress ratio for predicting the fatigue crack growth life of a welded joint.

4. FATIGUE CRACK CLOSURE

Strain gages were mounted on the surfaces and sides of specimens to examine the behaviors of crack closure, and the relationships between strains and loads were observed. Examples of the relationships between loads and strains at specimen surfaces are given in Fig. 6. As shown for $R=0.6$ with a base metal specimen and $R=0.6$ for a longitudinal welded specimen in the figure, when crack closure did not occur within the range of testing loads, the points of crack closure were sought by lowering loads further. The crack opening and closing points coincided approximately in all measurements. Hereafter, therefore, differentiations will not be made in particular between crack opening and closing points.

Fig.7 shows the ratios between crack opening stress intensity factors (K_{op}) and maximum stress intensity factors (K_{max}) in base metal specimens in relation to stress intensity factor range (ΔK). At stress ratio $R=0$, K_{op}/K_{max} is more or less constant, effective stress intensity factor range ratio (U) being constant, in a range of ΔK larger than 12 $MPa\sqrt{m}$ where the $da/dN - \Delta K$ relationship indicates linearity in terms of a logarithm, and when ΔK is approaches to the ΔK_{th} the value of K_{op}/K_{max} become high and U is decreased. Fig. 8 is a fractograph of a base metal specimen and adherence of oxides can be seen in the vicinities of points where ΔK_{th} value had been obtained. Consequently, it is considered that the rise in K_{op} in this region was due to closure of the crack that induced the oxide¹⁸⁾. Meanwhile, K_{op}/K_{max} with $R=0.6$, similar to the case of $R=0$, is approximately constant in a range of ΔK larger than 4 $MPa\sqrt{m}$ where the $da/dN - \Delta K$ relationship shows linearity in terms of logarithms, but K_{op} is lower than the minimum stress intensity factor (K_{min}), and crack closure does not occur within the range of testing loads. As ΔK

decreases and approaches ΔK_{th} , K_{op}/K_{max} increases, and opening and closing of cracks begin to occur. On observation of fracture surfaces the adherence of slight amounts of oxides was recognized at locations corresponding to ΔK_{th} . Since the fracture surfaces become rough in areas close to ΔK_{th} , the opening points of cracks become higher¹⁹⁾. Formation of oxide is also aided by the roughness of the fracture surfaces.

Closure of cracks did not occur in any of the tests with longitudinal welded specimens. As shown in Fig. 8, adherence of oxide cannot be seen. The relationships of crack opening load (P_{op}), maximum testing load (P_{max}), and minimum testing load (P_{min}) with crack length are shown in Fig. 9. The crack opening load P_{op} is affected by residual stresses and load inducing stresses distributed in the specimen. Accordingly, the value of P_{op} varies with crack length even though testing loads are the same. Also, even though crack lengths are the same the value of P_{op} differs according to testing load.

Using the solution of Chell²⁰⁾ in case of being subjected to an arbitrary pressure distribution ($\sigma(x)$) at the crack surface in the finite width plate indicated by the equation below, the stress intensity factor due to residual stress (K_{res}) was obtained by the method of superposition²¹⁾.

$$K_{res} = \frac{1}{2} \sqrt{\frac{\pi a}{1-a/W}} \left[2\sigma(0) + \int_0^a (1-x/W) \frac{2}{\pi} \cdot \arccos\left(\frac{x/a-x/W}{1-x/W}\right) \cdot \frac{\partial}{\partial x} \{(1+x/a)\sigma(x) + (1-x/a)\sigma(-x)\} dx \right] \dots \dots \dots (4)$$

where,

x : distance from the center of specimen

The residual stress distribution measured with a longitudinal welded specimen is shown in Fig. 10. The

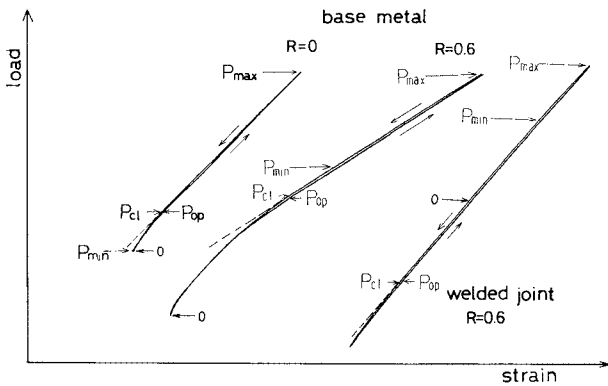


Fig. 6 Load-strain curve.

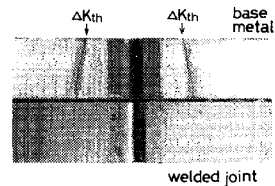


Fig. 8 Fracture surface.

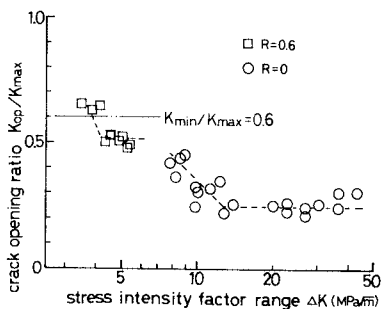


Fig. 7 Crack opening stress intensity factor of base metal specimen.

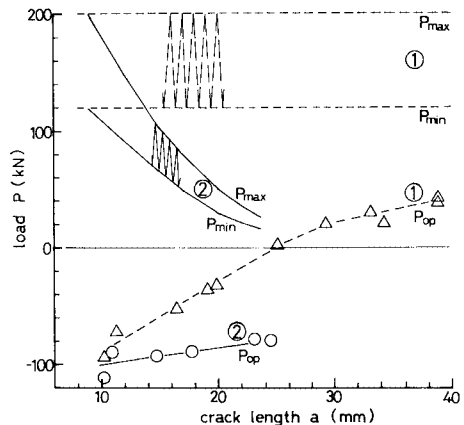


Fig. 9 Crack opening load of longitudinal welded specimen.

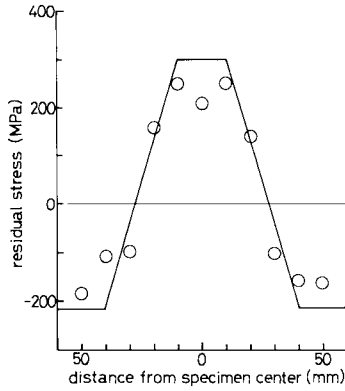


Fig. 10 Welding residual stress distribution.

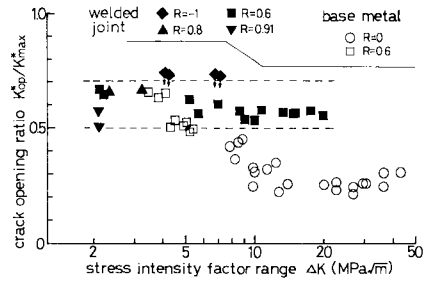


Fig. 11 Crack opening stress intensity factor of welded joint specimens.

value of K_{res} was calculated using the residual stress distribution indicated by the solid line in the figure. To obtain K_{res} by the method of superposition will involve problems²²⁾ in case of the crack becomes longer compared with plate width. But, in the case of crack length less than one-half of plate width, it is thought that K_{res} obtained by this method possesses ample precision for the study below. The K_{res} here agreed very well with the values obtained by Nihei et al.,²³⁾ using the boundary element method.

The stress intensity factor K^* for the crack in a longitudinal welded specimen is thought to be stress intensity factor due to applied stress (K_{min}, K_{max}) to which stress intensity factor due to residual stress (K_{res}) is added. Consequently, maximum and minimum stress intensity factors considering residual stress (K_{max}^*, K_{min}^*) and crack opening stress intensity factor (K_{op}^*) are given by the equations below.

$$K_{max}^* = K_{max} + K_{res} \quad K_{min}^* = K_{min} + K_{res} \quad (5)$$

$$K_{op}^* = K_{op} + K_{res}$$

Relationships between K_{op}^*/K_{max}^* and ΔK are shown in Fig. 11. Regardless of stress ratio, K_{op}^*/K_{max}^* is from 0.5 to 0.7 and roughly constant. The $K_{op}/K_{max} - \Delta K$ relationships with base metal specimens are also shown in the figure ($K_{op} = K_{op}^*, K_{max} = K_{max}^*$). At $R=0.6$ with base metal specimen, K_{op}/K_{max} roughly coincide with K_{op}^*/K_{max}^* in longitudinal welded specimens in the range of ΔK larger than $4 \text{ MPa}\sqrt{\text{m}}$ where K_{op}/K_{max} become approximately constant. This is thought to indicate that stress intensity factor of crack opening point depends on maximum stress intensity factor only in case crack closure does not occur within the range of applied loads. Consequently, it is possible to predict the crack opening point by computing the stress intensity factor due to residual stress.

At $R=0$ with base metal specimens, K_{op}/K_{max} is approximately 0.25 in a range of ΔK larger than $12 \text{ MPa}\sqrt{\text{m}}$ where there is no adherence of oxide, and is considerably lower compared with the case of $R=0.6$ with base metal and of longitudinal welded specimens. This is thought to be because the crack opening point was lowered by crack closure, and how to predict the crack opening point near threshold quantitatively will be a problem for future study.

5. EFFECTIVE STRESS INTENSITY FACTOR RANGE AND FATIGUE CRACK GROWTH RATE

Fatigue cracks are open at all times in the range of stress ratios in this study ($R=-1$ to 0.91) in case of longitudinal welded specimens, and therefore, effective stress intensity factor range (ΔK_{eff}) is equal to stress intensity factor range ($\Delta K = K_{max} - K_{min}$). The difference in the $da/dN - \Delta K_{eff}$ relationship according to the stress condition at crack tip was investigated using the effective stress ratio (R_{eff})¹⁸⁾ of the equation below for such cracks where $\Delta K_{eff} = \Delta K$.

$$R_{eff} = (K_{min}^* - K_{op}^*) / (K_{max}^* - K_{op}^*) \quad (6)$$

The relationship of fatigue crack growth rate (da/dN) and effective stress intensity factor range

(ΔK_{eff}) is shown in Fig. 12. The influence of residual stress on the value of R_{eff} is substantial, and R_{eff} is 0.5 or higher in all experiments. Particularly, in the vicinity of the fatigue crack growth threshold value ($\Delta K_{eff,th}$), the values of R_{eff} are 0.95 or higher in tests of all stress ratios, and this indicates that even when stress ratio R changes from -1 to 0.91 , there is hardly any difference produced by the influence of residual stress in the stress condition in the vicinity of the crack tip. This is thought to be the reason the value of ΔK_{th} was constant regardless of stress ratio (R). Therefore, it will suffice to use the results with longitudinal joint specimens at $R=0$ as employed here, while it may be said that the fatigue life of a welded joint will not be shorter than the life at $R=0$ even if stress ratio were to become high.

The $da/dN - \Delta K_{eff}$ relationships of base metal specimens with R at 0 and 0.6 roughly coincide. These relationships, except for da/dN in a low range, agree well with the $da/dN - \Delta K_{eff}(\Delta K)$ relationships with longitudinal welded specimens. However, the threshold values of fatigue crack growth when the effective stress intensity factor range is employed is approximately $3.0 \text{ MPa}\sqrt{\text{m}}$ for both R of 0 and 0.6, which is slightly higher than the 1.9 to $2.4 \text{ MPa}\sqrt{\text{m}}$ with longitudinal welded specimens. This is thought to be due to errors in measurements of opening points, but details are not clearly known at this time. There is also report that crack opening and closing points differ according to the measurement method²⁴⁾, and care will be required in using the $da/dN - \Delta K_{eff}$ relationship obtained in fatigue crack growth tests of base metal as the $da/dN - \Delta K$ relationship in a tensile residual stress field.

6. CLOSURE

Fatigue crack growth rates and crack opening points were measured using specimens with longitudinal welds and base metal specimens with the purpose of studying the influence of stress ratio on fatigue crack growth rates da/dN in a welding tensile residual stress field and on fatigue crack growth threshold value ΔK_{th} . The following results were obtained :

- (1) The influence of welding tensile residual stress on da/dN is especially prominent in the low range of fatigue crack growth rate.
- (2) The values of da/dN and ΔK_{th} in welding tensile residual stress field are not subject to the influence of stress ratio (-1 to 0.91). The values of ΔK_{th} determined here are from 1.9 to $2.4 \text{ MPa}\sqrt{\text{m}}$, and these are at the threshold of ΔK_{th} for structural steels and welded joints obtained up to this time.
- (3) The influence of residual stress is dominant with regard to the stress conditions in the vicinities of crack tips, contrasted to which the influence of stress ratio of applied stress is extremely small. It is considered that a difference in da/dN and ΔK_{th} due to stress ratio was not produced in the welding tensile residual stress field because of this.
- (4) The phenomenon of fatigue crack closure does not occur with a longitudinal welded specimen, and $\Delta K_{eff} = \Delta K$ was valid at all times. The $da/dN - \Delta K_{eff}$ relationships of base metal specimens coincided more and less with those of longitudinal welded specimens except near $\Delta K_{eff,th}$.
- (5) It is recommended to use the $da/dN - \Delta K$ relationship obtained from longitudinal welded specimens employed in these experiments for estimating the fatigue life of a welded joint regardless of stress ratios.

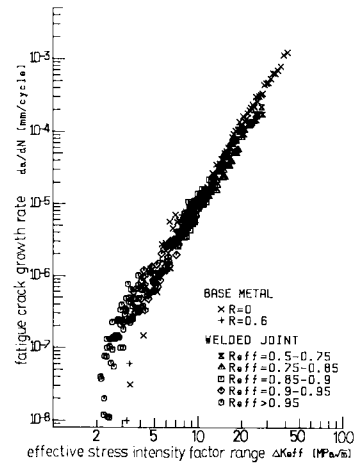


Fig. 12 Relationships between da/dN and ΔK_{eff} .

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